ISSN 2042-2695

CENTRE *for* ECONOMIC PERFORMANCE

CEP Discussion Paper No 1158

July 2012

Steady-State Equilibrium in a Model of Short-Term Wage-Posting

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THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

Abstract
This paper takes the canonical Burdett-Mortensen model of wage- posting and relaxes the assumption that wages are set once-for-all, instead assuming they can only be committed one period at a time. It derives a closed-form solution for a steady-state Markov Rank-Preserving Equilibrium and shows how this relates to the canonical model and performs some comparative statics on it. By means of example it is shown that a Rank-Preserving Equilibrium may fail to exist and that this non-existence can be a problem for plausible parameter values. The paper discusses how the model can be modified to ensure existence of a Rank-Preserving equilibrium. It is also shown, by means of example, how the opposite, a Rank-Inverting Equilibrium may exist.

Keywords: Wage-posting, search
JEL Classifications: J31, J42

This paper was produced as part of the Centre's Labour Markets Programme. The Centre for Economic Performance is financed by the Economic and Social Research Council.

Acknowledgements
I would like to thank Philipp Kircher, Giuseppe Moscarini and Fabien Postel-Vinay for very helpful comments for very helpful comments. This paper was produced under ESRC Research Grant RES-062-23-1434 'Wage-Setting in Britain' and I am grateful for their financial support.

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Published by
Centre for Economic Performance
London School of Economics and Political Science
Houghton Street
London WC2A 2AE

Introduction

The model proposed by Burdett and Mortensen (1998) (henceforth BM) has rightly become regarded as the canonical way in which to analyze labour markets with frictions in which employers post wages. BM showed how in a world where all workers and employers were ex ante identical, frictions could lead to ex post differences in wages across workers and firms with larger firms paying higher wages. The economic mechanism underlying it - that employers who pay higher wages find it easier to recruit and retain workers - is intuitively very appealing. Given this it is not surprising that labour economists have wanted to use the model to address a wider range of phenomena than were the subject of the original paper. One particularly appealing direction in which to take the paper is to try to use the wage-posting model to address the age-old question of how employment and wages vary over the business cycle. But, in order to do this some modifications to the canonical model need to be made. The canonical model follows the sensible principle of only making the assumptions you need for the phenomenom you want to explain, but, if one wants to use the model to analyze business cycles, the assumptions that wages are set once-for-all and that firms want to maximize steady-state profits are obviously problematic.

This paper analyzes a version of the canonical model in which firms and workers are identical, but (in contrast to the canonical model) employers can only commit to wages for the current period and forward-looking workers base their decisions not just on the current wage offered but on what they rationally expect to happen in the future. Those future expectations are tied down by the assumption that firms follow a stationary Markov strategy. In essence this is a finite-period version of the canonical model - that sounds very simple but some of the results are perhaps surprsing. First, the paper seeks the steady-state equilibrium in which the wage paid this period by each firm is also paid next period - this is a rank-preserving equilibrium (RPE) to use the terminology of Moscarini and Postel-Vinay (2010) as a firm that is high-wage one period must be high-wage the next. Using a 'first-order' approach based on necessary conditions the paper derives a unique closed form solution for a RPE, shows, as one would expect, this corresponds to the equilibrium in the canonical Burdett-Mortensen model when employers do not discount the future, but that when there is discounting of the future, wages are lower than in the canonical model. If workers and firms have different discount factors equilibrium wages are higher the more myopic are workers and the more forward-looking are firms.

However, all of these results are derived under the assumption that an RPE exists. The paper then shows that an RPE may not exist i.e. if we start from the proposed steady-state rank-preserving equilibrium some firms can increase profits by deviating frm the proposed equilibrium strategy. The paper shows how a sufficient condition for the non-existence of a RPE is that the probability of workers losing their job per period is below some critical value. If workers and firms are myopic this critical value is 0.5. As we get arbitrarily close to the canonical model (i.e. as the discount factor of both employer and workers approaches one) the critical value becomes $1-\sqrt{\frac{1}{2}} \approx 0.293$. As actual flows are lower than this for a reasonable definition of a 'period' (which is perhaps best thought of as the length of time for which employers can commit to wages), this suggests that non-existence could be more than a hypothetical concern.

This result might be thought surprising but the intuition is the following. Think of a model in which a myopic firm just wants to maximize current profits but it has inherited an employment level of N_0. If the quit rate is $q(w)$ and the flow of recruits $R(w)$ (both depending only on the current wage because of the myopia assumption) then profits will be given by $[p-w]\{[1-q(w)]N_0+R(w)\}$. Like any good monopsonist the employer will choose a higher wage the higher the elasticity of the labour supply curve facing it. That labour supply elasticity will be a weighted average of the elasticity of $[1-q(w)]$ with respect to the wage and the elasticity of $R(w)$ with respect to the wage with the weight on the elasticity of $[1-q(w)]$ being higher the higher the inherited level of employment. So if the elasticity of $[1-q(w)]$ with respect to the wage is lower than the elasticity of $R(w)$ with respect to the wage (and the paper shows how that can easily be the case) then a higher inherited level of employment leads to a lower elasticity in the labour supply curve facing a firm and this induces it to choose a lower wage. This has been for a myopic firm and the non-myopic case is more complicated but the point remains.

This non-existence result is a problem because the world seems to resemble a RPE - there is a very high correlation in wages over time within firms. The paper then discusses ways to ensure existence of a RPE - by introducing productivity differentials across firms, by allowing non-Markov strategies (as in Coles-Mortensen, 2011, henceforth, CM), by allowing different form of contracts (one version of which is the contract assumed by Moscarini and Postel-Vinay, 2010, henceforth MP-V). As CM and MP-V both establish existence of a RPE one might think that either or both of their solutions are the right way to go, especially as both of these papers are much more ambitious than this one - analyzing equilibrium in a model with heterogeneity in employers, potentially transitional dynamics and in an economy subject to aggregate productivity shocks. But both of their solutions to the problem do raise other issues - in the case of CM they assume that any firm that deviates from equilibrium is assumed to pay the reservation wage thereafter, an assumption that is somewhat arbitrary and quite extreme (and does not naturally carry over to a model set in finite time). And in MP-V firms are assumed to be able to commit to offering future wages not just a current wage so makes much the same commitment assumption as the canonical model and considerably more than one might think reasonable. In addition the MP-V model differs in assumptions about the timing of decisions within a period, with the current wage not being allocational.

Finally, the paper investigates the nature of the equilibrium if an RPE does not exist. The paper does not provide a complete characterization of equilibrium but shows by construction how a Rank-Inverting equilbirium may exist in which a high-wage firm one period is a low-wage firm the next. Although wages in individual firms cycle, the aggregate wage distribution is constant through time.

The plan of the paper is as follows. The second section lays out the discrete-time version of the Burdett-Mortensen model analyzed in this paper. The third section then shows how one can derive a unique steady-state RPE under the assumption that this equilibrium exists. The fourth section then discusses the existence of the RPE, the fifth ways in which the model might be modified to ensure existence of an RPE, and the sixth section describes the possibility of a rank-inverting equilibrium.

2 The Model

This is a discrete-time version of the Burdett-Mortensen model in which wages are only set for a single period. The firms have constant returns to scale with marginal revenue of labour equal to p. There are L identical workers and M identical firms - we will, without loss of generality assume that (L/M)=1 - the constant returns assumption means this has no consequence for the equilibrium wage distribution. Workers get a flow utility of b when unemployed.

The timing of each period is the following

1. at the start of each period, workers are either attached to a firm or unemployed. The state variable for each firm is the number of workers inherited from the previous period.
2. employers then set wages for that period

3. employed workers lose their job with probability δ and become unemployed.

4. those workers who did not suffer job loss, whether previously employed or unemployed receive a job offer with probability λ, drawn at random from the firms. Job loss and a job offer are assumed to be mutually exclusive probabilities so we must have $(\delta + \lambda) < 1$. This assumption implies that workers who become unemployed do not have a chance to be re-employed within the period.

5. workers with job offers accept or reject job offers, there is production and wage payments.

With this set-up the state variable for each employer in each period will be the inherited level of employment. We restrict attention to stationary Markov strategies in which the wage offered by an employer is only a function of the state variable and this function remains the same over time. It is sometimes convenient to use as the state variable not the actuall level of employment but the position in the steady-state distribution of employer sizes - for the cases we consider there is a one-to-one mapping between the two.

3 Constructing a Rank-Preserving Equilibrium

We start by looking for a Rank-Preserving Equilibrium (RPE) in which not only is the wage distribution constant through time but the wages paid by individual employers are as well. Denote the steady-state equilibrium wage distribution by $w(f)$ where f is the position of the firm in the wage offer distribution which, in an RPE, must be constant over time. An argument familiar from the canonical Burdett and Mortensen model shows that this cannot contain any mass point. We start by deriving some facts about employment flows in a RPE.

3.1 Employment Flows in a RPE

One of the convenient characteristics of the Burdett-Mortensen model with a rank-preserving equilibrium is that the distribution of employment across firms can be solved for separately from the wage offer distribution. The reason for this is that workers will always move for a wage gain, however small, so the size of wage differentials between firms is irrelevant for the worker mobility decision. This is a convenient proprty although one that is not particularly realistic e.g. because the incentive for on-the-job search will be related to the size of wage differentials between firms. Everything here is standard but reproduced for reference.

In steady-state, the unemployment rate will be given by:

$$u = \frac{\delta}{\delta + \lambda} \tag{1}$$

The fraction of employed workers employed in firms at position f in the distribution or lower, $G(f)$, will, equating employment flows into and out of the firm solve:

$$[\delta + \lambda(1-f)]\, G(f)(1-u) = \lambda f u \tag{2}$$

that can be solved to yield:

$$G(f) = \frac{\delta f}{\delta + \lambda(1-f)} \tag{3}$$

A firm at position f in the wage offer distribution will, in equilibrium, have quits equal to recruits so will have the following level of employment:

$$[\delta + \lambda(1-f)]\, N(f) = \lambda[u + (1-u)\, G(f)] \tag{4}$$

that can be solved to yield:

$$N(f) = \frac{\delta\lambda}{[\delta + \lambda(1-f)]^2} = \frac{\delta\lambda}{q(f)^2} \tag{5}$$

where the quit rate of the firm is given by:

$$q(f) = [\delta + \lambda(1-f)] \tag{6}$$

and the flow of recruits is:

$$R(f) = \frac{\delta\lambda}{[\delta + \lambda(1-f)]} = \frac{\delta\lambda}{q(f)} \tag{7}$$

What follows below will make extensive use of these equations. In addition we will use:

$$R'(f) = \frac{\delta\lambda^2}{q(f)^2} = \lambda N(f) \tag{8}$$

One other feature of the Burdett-Mortensen model that will be used later and turns out to be important is that:

$$\frac{R'(f)}{R(f)} = \frac{\lambda}{q(f)} = \frac{-q'(f)}{q(f)} \tag{9}$$

which implies that the elasticity of recruits with respect to the position in the wage distribution is equal to minus the elasticity of quits with respect to the same variable (this is true in a wider class of search models and has been proposed by Manning, 1993, to estimate the elasticity of recruits to the firm). This result does not depend on the equilibrium wage distribution. These equations are identical to those obtained in the canonical Burdett-Mortensen model which is set in continuous time. This makes comparison easy.

3.2 The Value Functions of Workers

Denote the value function for an unemployed worker by V^u, and for a worker employed at position f by $V(f)$. Define the value functions to be the value of that state before any job offers or unemployment shocks arrive i.e. at the beginning of a period. Assume the discount factor for workers is β_w. Give this, we will have:

$$V^u = (1-\lambda)\left[b + \beta_w V^u\right] + \lambda \int \left[w(f) + \beta_w V(f)\right] df \tag{10}$$

and:

$$\begin{aligned} V(f) &= \delta\left[b + \beta_w V^u\right] \\ &\quad + \left[1-\delta-\lambda(1-f)\right]\left\{w(f) + \beta_w V(f)\right\} + \lambda \int_f \left[w(f') + \beta_w V(f')\right] df' \end{aligned} \tag{11}$$

Differentiating (11) we have that:

$$V'(f) = \left[1 - q(f)\right]\left\{w'(f) + \beta_w V'(f)\right\} \tag{12}$$

that solves to:

$$V'(f) = \frac{\left[1-q(f)\right] w'(f)}{1 - \beta_w\left[1-q(f)\right]} \tag{13}$$

This is useful in what follows. Note that the reservation wage of workers will be equal to b because of the assumption that job offers arrive at the same rate whether employed or unemployed.

3.3 The Employer's Decision

One can think of the state variable of a firm as being its initial position in the steady-state employment distribution, f_0 - as we are interested in the steady-state and not transitional dynamics there is no loss of generality in this and it makes the derivation of the equilibrium a lot easier. Denote by $\Pi(f_0)$ the value function of a firm at position f_0. Each period the firm will choose the wage it pays, w. Workers will then quit from and be recruited to this firm according to a rule that will be derived shortly. But the bottom line is that, next period, the firm will end up with a level of employment that must be in the range $[N(0), N(1)]$ - so that one can think of the firm as ending up at some point in the steady-state employment distribution if we are considering a steady-state in which (5) is satisfied. The reason is that $N(0)$ is the level of employment in a firm that was at that level last year, only ever attains recruits from unemployment and always loses workers to other firms when they have alternative job offers - as long as $w \geq b$ (and all firms will do this) one can never do worse than this. Similarly one can never do better than $N(1)$.

The position in the employment distribution next period will be a function of the initial position and the wage offered this period - denote this function by $\psi(f_0, w)$ - we derive this function below. So under the assumption that firms follow a stationary Markov strategy so that the wage offer in future periods will only be a function of the state variable, the value function for the employer can be written as:

$$\Pi(f_0) = \max_w \; [p - w] N(\psi(f_0, w)) + \beta \Pi(\psi(f_0, w)) \tag{14}$$

where β is the discount factor of the employer (we retain the possibility that this differ from the workers' discount factor). In a steady-state rank-preserving equilibrium we will have:

$$\Pi(f) = \frac{[p - w(f)] N(f)}{1 - \beta} \tag{15}$$

Substituting 15 into 14 we have, after some re-arrangement, that:

$$(1 - \beta)\Pi(f_0) = \max_w \; [p - (1 - \beta)w - \beta w(\psi(f_0, w))] N(\psi(f_0, w)) \tag{16}$$

so one can think of the firm as maximizing a level of current profits where the employment it will have is the current employment level and the 'wage' is a weighted average of the current wage and the future wage with the weight on the future wage being the discount factor.

To make any progress we need to derive an expression for the function $\psi(f_0, w)$ - this comes from the mobility rule followed by workers.

3.4 Workers' Mobility Decision

Now let us consider how we can derive $\psi(f_0, w)$. At this point we need to specify the expectations held by workers about what wage will be offered next period by a firm that, this period, is deviating from equilibrium behaviour. Suppose all workers think that a firm currently at position f_0 and offering w will be at position $\psi(f_0, w)$ next period. Suppose that workers can observe the current employment level in the firm and the current wage offered - we return to this below. If workers are assumed to believe that employers are following a Markov strategy and they expect reversion to equilibrium behaviour, the state variable of the firm next period will be $\psi(f_0, w)$ and the value of a job at such a firm is $V(\psi(f_0, w))$. Now define f in the following way:

$$w(f) + \beta_w V(f) = w + \beta_w V(\psi(f_0, w)) \tag{17}$$

The right-hand side is the value one obtains from a job in the deviating firm - the current offered wage and then the future value of a job at the position the firm ends up with. The left-hand side is the value from accepting a job at a firm that is following the equilibrium strategy and is at position f. The implication of (17) is that a worker in the deviating firm will accept a wage offer from a firm at a position above f and, symmetrically, a worker in a firm at position below f will accept a wage offer from the deviating firm.

We can use (17) to eliminate w from (16) and change, without loss of generality, the variable of choice for the employer from the current wage to f - with some change of notation let us write this as $\psi(f_0, f)$. Now we have the mobility decision for workers one can derive what the relationship between f, ψ, and f_0 will be. From (17) we know that the offer from the deviating firm will be regarded as equivalent to a firm at position f so that quit and recruitment decisions will be based on this and hence employment will satisfy:

$$N\left(\psi\left(f_0,f\right)\right)=\left[1-q\left(f\right)\right]N\left(f_0\right)+R\left(f\right) \tag{18}$$

And with the change in the decision variable to f , and using (17), (16) becomes:

$$\begin{aligned}(1-\beta)\Pi\left(f_0\right) &= \max_f\ \pi\left(f,f_0\right)\equiv \\ &\{p-(1-\beta)w(f)-\beta w(\psi)-(1-\beta)\beta_w[V\left(f\right)-V(\psi)]\}N\left(\psi\right)\end{aligned} \tag{19}$$

where $\psi=\psi\left(f_0,f\right)$. If employers do not discount the future so that $\beta=1$, the maximand in (19) reduces to $\{p-w(\psi)\}N\left(\psi\right)$ i.e. it is only the steady-state level of profits that is relevant as one would expect. But if there is some discounting of the future this is not the case. And if workers discount the future then if a firm is currently at a position f_0 but wants to be at position ψ next period it has to make sure workers evaluate its current job offer at a position $f>\psi>f_0$ so that the term $[V(f)-V\left(\psi\right)]$ is positive and this adds to the employer costs. Intuitively, the labour supply curve facing an employer is less elastic in the wage when there is discounting and short-term wage contracts - this intuition will be important later.

It is also useful to think about how the labour supply curve facing an employer is affected by the assumption of no-commitment in wage-setting. Figure 1 sets out what happens. It first shows the equilibrium relationship between wages and employer size - this can be thought of as the long-run labour supply curve facing the firm as if it paid a wage w for ever it would end up with a level of employment on this curve. Now consider a firm starting on this long-run labour supply curve and considering paying a wage that deviates from it. There are two reasons for why the short-run labour supply curve will be less elastic than the long-run labour supply curve. First, because it takes time for employment to adjust. Suppose that workers expect the new wage to be paid for ever (or that workers are myopic) so that the quit rate and recruitment flow faced by a firm doing this deviation is the same as that faced by a firm offering the new wage on a permanent basis. But employment in the deviating firm this period will by $\left[1-q\right]N_0+R$ and this will be lower than the employment level in a firm that has always paid the new wage because the initial level of employment is lower - call this the naive short-run labour supply curve. But there is a second effect at work if workers are forward-looking and not myopic. Because employment is lower at the end of the period in a firm that has only paid w this period compared to one that has always paid it, workers do not expect the new higher wage to be sustained - they expect the wage to fall back in the subsequent period. This means that the quit rate will be higher and flow of recruits lower giving a second reason why employment will be less sensitive to current wages than the long-run labour supply curve would suggest. The size of this second effect depends on how forward-looking are workers - if they do not care about the future at all they will only pay attention to the current wage in making mobility decisions. This is represented by the 'sophisticated' labour supply curve in Figure 1 This way of understanding the model will be of use in giving an intuition for some comparative statics results later.

In deriving the above, we assumed that workers can observe both the current level of the offered wage and the current level of employment in the firm. They can then deduce whether the firm is deviating from the steady-state and then use this information to derive what they expect future wages to be. But, suppose that workers only observe the current wage offer and not the current level of employment. Workers would then be unable to detect deviation from the steady-state as long as the offered wage is an equilibrium offer for some firm. As, no firm will deviate from the equilibrium, workers would simply assume that the offered wage is the equilibrium offer for the firm. Decisions would then be based on the current wage alone as workers would assume this wage will be maintained in the future. From the employer perspective, this case is then isomorphic to the case $\beta_w = 0$ where workers are myopic. This makes the myopic worker case of more interest than one might have thought.

3.5 Solving for the Rank-Preserving Equilibrium

We are now in a position to derive the RPE, assuming that it exists (an issue we return to later). We will use the first-order conditions that are necessary for profit maximization and show that these have a unique solution.

Proposition 1 *An RPE, if it exists must be the unique solution to the differential equation:*

$$\frac{\partial \ln [p - w(f)]}{\partial f} = -2\lambda \frac{1 - \beta_w[1 - q(f)]}{1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]} \tag{20}$$

with the initial condition $w(0) = b$

Proof. Taking the derivative of the log of (19) with respect to f, leads to the following first-order condition:

$$\frac{\partial \ln \pi\left(f, f_0\right)}{\partial f}=\frac{N'\left(\psi\right)}{N\left(\psi\right)}\frac{\partial \psi}{\partial f}-\frac{(1-\beta)w'(f)+\beta w'(\psi)\frac{\partial \psi}{\partial f}+(1-\beta)\beta_w[V'\left(\psi\right)\frac{\partial \psi}{\partial f}-V'(f)]}{\{p-(1-\beta)w(f)-\beta w(\psi)-(1-\beta)\beta_w[V\left(\psi\right)-V(f)]\}} \tag{21}$$

Using (13) this can be simplified to:

$$\frac{\partial \ln \pi\left(f, f_0\right)}{\partial f}=\frac{N'\left(\psi\right)}{N\left(\psi\right)}\frac{\partial \psi}{\partial f}-\frac{\frac{(1-\beta)w'(f)}{1-\beta_w[1-q(f)]}+\frac{[\beta-\beta_W+\beta_W q(\psi)]w'(\psi)}{1-\beta_w[1-q(\psi)]}\frac{\partial \psi}{\partial f}}{\{p-(1-\beta)w(f)-\beta w(\psi)-(1-\beta)\beta_w[V\left(\psi\right)-V(f)]\}} \tag{22}$$

By differentiating (18) we have that:

$$N'\left(\psi\right)\frac{\partial \psi}{\partial f}=-q'\left(f\right)N\left(f_0\right)+R'\left(f\right)=\lambda[N\left(f_0\right)+N\left(f\right)] \tag{23}$$

A necessary condition that an RPE must satisfy is that the first-order condition for the maximization of (19) must be satisfied at $f=\psi=f_0$. Putting this into (22) leads to the following:

$$2\lambda=\frac{(1-\beta)+q(f)[\beta-\beta_W+\beta_W q\left(f\right)]}{1-\beta_w\left[1-q\left(f\right)\right]}\frac{w'(f)}{[p-w(f)]} \tag{24}$$

and re-arrangement leads to (20). The lowest wage offered must be equal to the reservation wage b. Given this, the RPE must be unique. ■

First, let us use this differential equation to obtain some comparative static results.

Proposition 2 *Assuming that an RPE exists, the equilibrium wage distribution is:*

(a) increasing (in the sense of first-order stochastic dominance) in β *holding* β_w *fixed.*

(b) decreasing (in the sense of first-order stochastic dominance) in β_w *holding* β *fixed.*

(c) increasing (in the sense of first-order stochastic dominance) in β *if* $\beta=\beta_w$.

Proof. The general pattern of proof is the following. Because all equilibrium wage distributions start from $w(0) = b$, one can rank them in terms of stochastic dominance if the right-hand side of (20) is higher or lower for all f. A higher value of the right-hand side is associated with profit per worker that declines faster which implies wages rise faster so are higher.

(a) By inspection one can see that the right-hand side of (20) is decreasing in β holding β_w fixed proving the result.

(b) Differentiating the right-hand side of (20) with respect to $\beta_w$we obtain:

$$\frac{\partial^2 \ln[p - w(f)]}{\partial f \partial \beta_w} = 2\lambda \frac{[1 - q(f)]^2 (1-\beta)}{[1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]]^2} > 0 \tag{25}$$

so this proves the result.

(c) If $\beta = \beta_w$ then (20) can be written as:

$$\frac{\partial \ln[p - w(f)]}{\partial f} = -2\lambda \frac{1 - \beta[1 - q(f)]}{1 - \beta[1 - q(f)^2]} \tag{26}$$

and differentiating leads to:

$$\frac{\partial^2 \ln[p - w(f)]}{\partial f \partial \beta} = -2\lambda \frac{q(f)[1 - q(f)]}{[1 - \beta[1 - q(f)^2]]^2} < 0 \tag{27}$$

■

Let us provide some intuition for these results. First, consider part(a). If employers have a higher discount factor, they put greater weight on future relative to current profits. This reduces the temptation to exploit the short-run immobility of workers by cutting current wages so tends to make employers choose higher wages. The higher wages of one employer then spills over to raise the wages of other employers. There is a certain paradox here - one can interpret (somewhat loosely) a higher β as a longer period of committed wages. It would seem in the interest of an individual employer to commit to wages for a longer period but this is to the collective disadvantage of employers.

Now, consider part (b). If workers have a higher discount factor, they put greater weight on future wages relative to current wages. This makes their mobility decision less responsive to current wage offers from employers as was explained earlier. This has the effect of making the labour supply curves to employers less elastic and this leads to lower wages. Again there is a certain paradox here - workers want to make employers think they are more responsive to current wages than they really are and being myopic is one way to do this.

Part (c) shows that when one puts the two effects together it is that in part(b) that dominates. Wages are higher the lower is the discount factor.

It is perhaps useful to compare the RPE with the equilibrium in the canonical B-M model in which employers make a one-off decision about wages to maximize steady-state profits. The equilibrium in this case is well-known - all firms must make the same level of profits that is given by:

$$[p - w(f)]\, N(f) = [p - b]\, N(0) \tag{28}$$

Using (5) and (6) this can be written as:

$$[p - w(f)] = [p - b]\, \frac{q\,(0)^2}{q\,(f)^2} \tag{29}$$

which, differentiating, can be written as:

$$\frac{\partial \ln\left[p - w\,(f)\right]}{\partial f} = -\frac{2\lambda}{q\,(f)} \tag{30}$$

Comparison of (20) and (30) shows that the equilibria are identical when $\beta = 1$ (though note that no assumption is needed on β_w). This is what we would expect - if firms do not discount future profits they only care about steady-state profits and this is essentially the canonical model. More generally, if we compare the two equilibria we have the following result:

Proposition 3 *Wages are lower (in the sense of first-order stochastic dominance) in the RPE than in the canonical equilibrium.*

Proof. This is the case if profit per worker declines slower in the canonical case than the short-run case as long as $\beta < 1$ as we have:

$$-2\lambda \frac{1 - \beta_w[1 - q\,(f)]}{1 - \beta[1 - q\,(f)] - \beta_w q(f)[1 - q\,(f)]} > -2\lambda \frac{1}{q\,(f)} \tag{31}$$

Re-arranging this can be written as:

$$(1 - \beta)[1 - q\,(f)] > 0 \tag{32}$$

which is true if $\beta < 1$. ■

One implication of these results is that the degree of monopsony power possessed by employers is more than the canonical model would suggest.

These results have been derived assuming that job offer arrival rates are the same whether employed or unemployed - what would we find if they were different? As is well-known the lowest wage would be the reservation wage and that this is itself a function of the wage offer distribution. If (as most estimates suggest) the offer arrival rate is higher for the unemployed than the employed then a result like Proposition 2 would probably be true as the reservation wage is higher the higher are wages generally.

For what it is worth[1], the closed-form solution for the equilibrium wage distribution is given by:

$$[p - w(f)] = [p - b] \frac{1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]}{1 - \beta[1 - q(0)] - \beta_w q(0)[1 - q(0)]} . \exp[Z(f) - Z(0)] \tag{33}$$

where:

$$Z(f) \equiv \sqrt{\frac{2 - \beta - \beta_w}{(1 - \beta)\beta_w}} \arctan\left(\sqrt{\frac{\beta_w}{1 - \beta}} q(f) + \frac{\beta - \beta_w}{2\sqrt{(1 - \beta)\beta_w}}\right) \tag{34}$$

All of this has assumed that an RPE exists, and used necessary conditions to show that there is a unique RPE if one exists. But as the next section shows this is problematic.

4 The Existence of an RPE

A sufficient condition for non-existence of an RPE is that at the point $f = f_0$ profits are convex in f for some f in the unit interval. The following result provides some useful information on this.

Proposition 4 *At the constructed RPE if* $\beta < 1$*:*

(a) A necessary (sufficient) condition for the existence (non-existence) of an RPE is that:

[1] Probably not much except that it involves the differentiation of an inverse trigonometrical function, knowledge not used since secondary school.

$$\delta \geq (<) q^{*}(\beta, \beta_w) \tag{35}$$

where $q^{*}(\beta, \beta_w)$ is the unique solution in the unit interval to the equation:

$$[2q-1] - \beta[2q-1](1-q)^2 + 2\beta_w q (1-q)^3 = 0 \tag{36}$$

Proposition 5 *(b)* $q^{*}(\beta, \beta_w)$ *is decreasing in* β *and* β_w*, reaching a minimum value of* $1 - \sqrt{\frac{1}{2}} \approx 0.293$ *when* $\beta = \beta_w = 1$ *a maximum value of* 0.5 *when* $\beta_w = 0$*.*

Proof. See Appendix. ■

It is natural to ask whether this non-existence result is likely to be relevant in practice. To answer that question needs a definition of a 'period'. The most natural interpretation of a period is that it represents the time for which employers can commit to a certain level of wages. That would suggest a period of a year at most. Even over the time horizon of a year, turnover rates are below the critical value for non-existence suggesting this may be more than a hypothetical concern. Howver, this definition of a period is somewhat loose as the model assumes that workers can move at most once within a period, while in reality they are moving in contniuous time even if wages are set in discrete time.

The general intuition for why a RPE may fail to exist is that given in the introduction - that the elasticity of the labour supply curve facing a firm may be lower if it has more current workers inducing it to want to pay a lower wage. But it is perhaps useful to see a particular case worked out - the myopic case is easiest for which we can also provide a necessary condition for non-existence of an RPE.

Proposition 6 *If* $\beta = \beta_w = 0$*,* $\delta < 0.5$ *is a necessary and sufficient condition for non-existence of an RPE.*

Proof. See Appendix. ■

Let us try to give some intuition for the result in the myopic case which is the simplest to analyze. (20) then becomes:

$$\frac{\partial \ln [p - w(f)]}{\partial f} = -2\lambda \tag{37}$$

with solution

$$[p - w(f)] = [p - b]\, e^{-2\lambda f} \tag{38}$$

In the myopic case workers will base their mobility decisions only on the current wage offered so that $\psi(f_0, f) = f$. So we can think of a firm as choosing its current position in the wage offer distribution. If it inherits employment of position f_0 and chooses f then profits will be given by:

$$\begin{aligned} \pi(f, f_0) &= [p - w(f)]N(f, f_0) = [p - w(f)][(1 - q(f))N(f_0) + R(f)] \quad (39) \\ &= [p - b]e^{-2\lambda f}[(1 - q(f))N(f_0) + R(f)] \end{aligned}$$

One way to write the first-order condition for the maximization of the profit function in (39) is:

$$\frac{\partial \ln[p - w(f)]}{\partial \ln f} = \frac{\partial \ln N(f, f_0)}{\partial \ln f} \tag{40}$$

This is a version of the standard first-order condition for the optimal wage of a monopsonist - the right-hand side of (40) is the elasticity of the labour supply curve facing the employer. The left-hand side of (40) does not vary with the initial position , f_0, so the question of how the optimal choice of position relates to the initial position depends on how the elasticity of the labour supply curve varies with the initial level of employment. Using (39) the labour supply elasticity can be written as:

$$\frac{\partial \ln N(f, f_0)}{\partial \ln f} = f\frac{-q'(f)N(f_0) + R'(f)}{N(f, f_0)} = \frac{-\varepsilon_{qf} q(f)N(f_0) + \varepsilon_{Rf} R(f)}{N(f, f_0)} \tag{41}$$

where ε_{qf} is the elasticity of the quit rate with respect to f, and where ε_{Rf} is the elasticity of recruitment with respect to f. Neither of these elasticities depend on f_0. Now define $\rho(f, f_0)$ to be the share of total current employment that is from retained workers $\rho(f, f_0) = [1-q(f)]N(f_0)/N(f, f_0)$. Then, using these bits of notation we can re-write (41) as:

$$\frac{\partial \ln N(f, f_0)}{\partial \ln f} = f\frac{-q'(f)N(f_0) + R'(f)}{N(f, f_0)} = [-\varepsilon_{qf}\frac{q(f)}{1 - q(f)} - \varepsilon_{Rf}]\rho(f, f_0) + \varepsilon_{Rf} \tag{42a}$$

When one increases the initial level of employment the only part of the right-hand side of (42a) that changes is that there is a rise in $\rho(f, f_0)$. (42a) then tells us that the effect of a rise in the initial level of employment on the elasticity of the labour supply curve facing the employer depends on the sign of $[-\varepsilon_{qf}\frac{q(f)}{1-q(f)} - \varepsilon_{Rf}]$. But from (9) we have that $-\varepsilon_{qf} = \varepsilon_{Rf}$ so that (42a) can be written as:

$$\frac{\partial \ln N(f, f_0)}{\partial \ln f} = \varepsilon_{Rf}\Big[\frac{q(f)}{1-q(f)} - 1\Big]]\rho(f, f_0) + \varepsilon_{Rf} = \varepsilon_{Rf}\frac{2q(f)-1}{1-q(f)}\rho(f, f_0) + \varepsilon_{Rf} \tag{43}$$

so that a rise in the initial level of employment raises the elasticity of the labour supply curve facing the employer if $q(f) > 0.5$, and reduces it otherwise. This is exactly the condition that came out of (36).

Given this non-existence result one might wonder whether an RPE ever exists. In the myopic case note from (43) that the sign of $\frac{\partial^2 \ln N(f,f_0)}{\partial \ln f \partial f_0}$ depends only on whether $q(f)$ is above or below half. So if this is above 0.5 for all f then it is always the case that initially large employers will prefer to choose higher wages. And the condition that $q(f) \geq 0.5$ for all f corresponds to the condition that $\delta \geq 0.5$ so this is a necessary and sufficient condition for the RPE to exist in the myopic case.

Proposition 4 has as a condition that $\beta < 1$ i.e. it applies arbitrarily close to but not at the canonical equilibrium with $\beta = 1$ - one might wonder what happens in the limit. When $\beta = 1$, the objective function of the firm does not depend on its initial condition and in the RPE every derivative of the profit function with respect to f is equal to zero. This can be thought of as the canonical equilibrium being on a knife-edge between existence and non-existence. And if we are arbitrarily close to $\beta = \beta_w = 1$, then whether the RPE exists or not depends on whether $\delta > 0.293$ or not. If, on the other hand we consider the myopic worker case $\beta_w = 0$ (or equivalently, where workers can only observe the wage paid by a firm and not its level of employment), then a sufficient condition for non-existence is $\delta > 0.5$.

5 Altering the Model to Ensure Existence of a RPE

An RPE is appealing not just because it simplifies the mathematics (though it does) but because it is empirically appealing - firm wages seem very highly correlated over time. If the model presented here does not guarantee existence for realistic parameter values then one might wonder about making changes to the model to guarantee the existence of an RPE. This section discusses a number of possibilities. This section also discusses Cole-Mortensen (CM) and Moscarini-Postel-Vinay (MP-V) who both always have an RPE.

5.1 Heterogeneity in Productivity

This may help to make the RPE exist as high productivity firms have an ever present incentive to be the larger firms. Let us consider the consequences of this in the myopic model. Denote by $p(f)$ the marginal product of labour in a firm at position f in the productivity distribution. In any RPE it must be the case that the most productive firms have more workers so that:

$$\pi(f, f_0) = [p(f_0) - w(f)][(1 - q(f))N(f_0) + R(f)] \tag{44}$$

Each employer will maximize this with respect to f - this leads to the first-order condition:

$$-w'(f)[(1-q(f))N(f_0)+R(f)]+[p(f_0)-w(f)][R'(f)-q'(f)N(f_0)] = 0 \tag{45}$$

In an RPE this must be satisfied for $f = f_0$. Using (7), (6) and (5) this can be written as:

$$2\lambda w(f) + w'(f) = 2\lambda p(f) \tag{46}$$

a differential equation with solution:

$$e^{2\lambda f} w(f) = w(0) + \int_o^f 2\lambda e^{2\lambda g} p(g) dg \tag{47}$$

Integrating by parts and using the fact that $w(0) = b$, this can be written as:

$$p(f) - w(f) = e^{-2\lambda f}[p(0) - b] + \int_o^f e^{2\lambda(g-f)} p'(g) dg \tag{48}$$

For the heterogeneous productivity case $p'(f) = 0$, this reduces to the previous solution. Substituting into (44) leads to the following expression:

$$\begin{aligned} \pi(f, f_0) = \quad & [p(f_0) - p(f) - e^{-2\lambda f}[p(0) - b] \\ & - \int_o^f e^{2\lambda(g-f)} p'(g) dg][(1 - q(f))N(f_0) + R(f)] \end{aligned} \tag{49}$$

and, for the RPE to exist we need to check that $f = f_0$ is a global maximum of this function. One can do this through simulation. In Figure 2 we compare the regions of the parameter space where an RPE exists for the case when there is no heterogeneity in productivity (this is the case $\delta > 0.5$) and the case when there is productivity heterogeneity and $p(0) - b = 1$ and $p'(f) = 1$. As can be seen the region of existence of an RPE is much larger when there is heterogeneity in productivity. Both CM and MP-V have productivity heterogeneity but do not put restrictions on the extent of productivity heterogeneity so that their results on the existence of an RPE should apply in the limit as productivity heterogeneity vanishes. So it would seem that some other aspects of their models are also important - we now turn to a discussion of what that it.

5.2 Non-Markov Strategies

Coles-Mortensen do not assume that agents pursue Markov strategies and this gives them an RPE. In particular they assume that an employer who deviates from the steady-state equilibrium is assumed to pay the reservation wage thereafter. Given these expectations there is no point in an employer doing anything different so this is an equilibrium. However, they do not claim the equilibrium is unique and it probably is not though Coles (2001) does show that the proposed equilibrium tends to the canonical equilibrium as the interest rate goes to zero so one could argue that this gives the choice of their non-Markov strategy some plausibility.

The motivation for this assumption about out-of-equilibrium behaviour is not primarily to ensure existence of an RPE but to avoid a version of the Diamond paradox in a model which is set in continuous time. Continuous time implies that firms are unable to commit to paying wages higher than b for more than an instant, so workers' mobility decisions do not respond to wage changes and the labour supply to employers becomes completely inelastic inducing employers to pay the minimum possible wage. Coles and Mortensen express well the intuition about what will happen if there is Markov behaviour "if employees anticipate the firm will, after a small delay of length dt, return to the equilibrium wage strategy, then the wage deviation will have (almost) no effect on the expected value of the worker's future earnings and thus there will be no turnover response. But if turnover does not respond to the wage deviation, then announcing w=0 is a profitable deviation". Coles and Mortensen want to avoid this prediction but one could argue that it is sensible. Somewhat loosely (because transitions are certainly happening in continuous time so more than one transition might be made per 'period') one can interpret the length of a period as the length of time for which employers are able to commit to paying a certain level of wages. Then the limiting result is what we might expect to happen if firms cannot commit to wages for more than an instant (a second, a millisecond?) - one goes to work in the morning and the employer changes the wage in the course of the working day. If this equilibrium does not correspond to what we observe this is because in the real world firms are able to commit to wages for a certain period of time so that the finite time period model is the correct one to use and the Diamond paradox problem does not arise in that model. This is a case where the continuous time model is not simply a convenient way to represent a finite time model - there is a substantive difference in the nature of the equilibrium. We should not be surprised by this - essentially a conclusion that the extent of possible commitment affects the real equilibrium.

In the model here, if we make the length of a period smaller and smaller the discount factors will tend to one and (δ, λ) to zero. Our earlier result tells us that the RPE will eventually fail to exist. This leaves open what is the equilibrium but as the period becomes very small all wages will collapse to b, the reservation wage, for the reasons given by Coles and Mortensen.

5.3 Different Labour Contracts

In Moscarini-Postel-Vinay different assumptions are made about the contracts that can be offered by employers and the timing of decisions within a period. On the timing they assume that wages each period are paid before mobility decisions are made so are not allocational. They assume that employers can commit to offering not just a current wage but to all future wages that, potentially, depend on all future pay-off relevant states. So, they relax the assumption in the canonical model that wages have to be the same in the future though they do retain the assumption that all workers in every state must be paid the same wage. They do not allow the firm to offer contracts in which wages increase with job tenure as would be known to be optimal (Burdett and Coles, 2003). They show the assumed contract is equivalent to assuming that firms commit to offering current workers a value of the job. An analysis in the Appendix takes a very stripped-down version of their model and shows how this works. It also shows how the steady-state equilibrium in the MP-V model corresponds to the equilibrium in the canonical BM model - perhaps not surprising given that offering current workers a lifetime utility is essentially the same as offering them a constant wage for ever. The question is how plausible is their assumption - do employers really have this level of commitment to future wages?

5.4 Different Model

The intuition behind the failure of the RPE to exist is that the elasticity of $[1 - q]$ with respect to the wage is less than the elasticity of recruits with respect to the wage for plausible value of the quit rate. If one could manage to alter this then an RPE would exist. This section sketches two possible approaches though only in an informal manner. First, one could endogenize the job search activity of the employed. Workers in high-wage firms would search less for alternative jobs and this would make the quit rate more sensitive to the wage than the standard model.

Alternatively one might think about modifying the model so that quits to and/or recruits from non-employment are sensitive to the wage. This could be done by introducing heterogeneity into the reservation wage (this makes recruits from non-employment sensitive to the wage being offered) and variation in the reservation wage (this will make quits to non-employment sensitive to the wage as most empirical estimates suggest that they are).

6 The Possibility of a Rank-Inverting Equilibrium

The discussion above has shown that a RPE may not exist. This raises the obvious question of what is the equilibrium in this case. I do not have a full characterization of equilibrium. But what this section shows, by means of example, is that the exact opposite type of equilibrium to an RPE may exist - what we call a Rank-Inverting Equilibrium (RIE). In a rank-inverting equilibrium those firms that inherit a large stock of employment from last period choose the lowest wages this period. Note that in a RIE we cannot have a steady-state in which the employment levels in a particular firm remain constant (except for the median firm). To understand why, suppose the contrary, that firms' employment levels are constant through time which also means their position in the employment distribution must also be constant through time. Consider for clarity the largest firm. This must every period be setting the lowest wage in a RIE so must have the lowest recruitment rate and highest quit rate. But this means that it cannot remain the largest firm for ever, contradicting the steady-state assumption.

The particular example constructed in this section is the myopic model introduced above because that is the simplest case. Because of the argument above, a RIE must have non-constant employment levels for firms and, for the myopic case, it must have a cycle lasting two periods. However this is consistent with the overall distribution of firm sizes and wages being constant through time as firms simply swap places from one period to the next (with the exception of the median firm). A firm that starts period 0 as the largest firm sets the lowest wage and becomes the smallest firm entering period 1. It then sets the highest wage in period 1 and becomes the largest firm entering period 2, reproducing the situation in period 0. More generally a firm starting at position f in the distribution will be at position $(1-f)$ next period before returning once more to position f. The following result derives the distribution of workers across firms.

Proposition 7 *If $G(f)$ is the fraction of workers at position f or below in last period's wage distribution, then in a RIE $G(f)$ must satisfy:*

$$G(f) = \frac{f\{[1-q(f)](\delta+\lambda)+\delta\}}{1-[1-q(f)][1-q(1-f)]} \tag{50}$$

where, to keep notation simple, we have used (6).

Proof. See Appendix ■

From this one can derive the employment level in a firm at position f as $N(f) = (1-u)G'(f)$. Now consider a firm that inherits a level of employment $N(f_0)$ and chooses a current position in the wage offer distribution of f. In the myopic model its log profits must be given by:

$$\ln \pi(f, f_0) = \ln[p - w(f)] + \ln[R(f) + (1-q(f))N(f_0)] \tag{51}$$

Taking first-order conditions leads to:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = \frac{\partial \ln[p-w(f)]}{\partial f} + \frac{R'(f) + q'(f)N(f_0)}{[R(f) + (1-q(f))N(f_0)]} \tag{52}$$

Now in a RIE we have that $R'(f) = \lambda N(1-f)$ so (52) can be written as:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = \frac{\partial \ln[p-w(f)]}{\partial f} + \frac{\lambda[N(1-f) + N(f_0)]}{[R(f) + (1-q(f))N(f_0)]} \tag{53}$$

In a RIE this first-order condition must be equal to zero at $f = 1 - f_0$ or, equivalently, $f_0 = 1 - f$. Putting this in (53) leads to:

$$\frac{\partial \ln[p-w(f)]}{\partial f} = -\frac{2\lambda N(1-f)}{N(f)} \tag{54}$$

that has, as a solution:

$$[p - w(f)] = [p-b]e^{-2\lambda \int_0^f \frac{N(1-x)}{N(x)}dx} \tag{55}$$

where we have used the fact that the lowest offered wage must be equal to b. [2] This RIE has been derived using the first-order conditions. But one has to check that at this proposed solution the choice of $f = 1 - f_0$ is a global maximum. Substituting (55) into (51) leads to:

$$\ln \pi(f, f_0) = \ln[p - b] - 2\lambda \int_0^f \frac{N(1-x)}{N(x)} dx + \ln[R(f) + (1 - q(f))N(f_0)] \quad (56)$$

A sufficient condition for the RIE to exist is for this function to be concave in f. There are parameter values for which this is satisfied. One can also prove a version of the limit result as the length of the period goes to zero for the RIE - wages must collapse to the reservation level - one can see this from (55) as $\lambda \to 0$.

7 Conclusion

This paper has taken the classic canonical Burdett-Mortensen (1998) model of wage-posting and changed the assumption that firms set wages once-for-all. Instead it has been assumed that firms set wages one period at a time without commitment. Workers take account of this fact in making their mobility decisions. The focus has been on the steady-state equilibrium of this model. It was shown how one can derive the unique Rank-Preserving Equilibrium but that the RPE may fail to exist. Ways to alter the model to ensure existence of an RPE were discussed - perhaps persistent productivity differentials across firms is the most plausible explanation. The model presented here does have its limitations. Because this note is just about the steady-state it can not be directly applied to questions like the cyclical variability of wages and employment. Nor does it produce more realistic predictions about things like the equilibrium distribution of wages than the canonical model. But until the steady-state is properly understood, those applications should perhaps wait.

[2] This is a feature of the myopic case but if workers are not myopic, the lowest wage will be lower than b because workers expect the lowest-wage firm this period to be a higher-wage firm next period

8 Appendix

8.1 Proof of Proposition 4

First, let us change the variable of choice for the firm from f to ψ. There will be a mapping from ψ and f_0 to f that will be given by (18) - denote this by $f(f_0, \psi)$. From (19) the log of profits can be written as:

$$\begin{aligned} & \ln \pi(f_0, \psi) \\ = \; & \ln N(\psi) + \ln[p - (1-\beta) w(f) - \beta w(\psi) - (1-\beta)\beta_w\{V(f) - V(\psi)\}] \\ \equiv \; & \ln N(\psi) + \ln Z(f_0, \psi) \end{aligned} \tag{57}$$

The first-order condition for the choice of ψ can be written as:

$$\frac{\partial \ln \pi(f_0, \psi)}{\partial \psi} = \frac{1}{Z(f_0, \psi)} \frac{\partial Z(f_0, \psi)}{\partial \psi} + \frac{N'(\psi)}{N'(\psi)} \tag{58}$$

and that, in the RPE this must be zero at $\psi = f_0$ for all f_0 in the unit interval. This means that we must have:

$$\frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} + \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi^2} = 0 \tag{59}$$

when evaluated at the RPE and $\psi = f_0$. One implication of (59) is that:

$$sgn \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} = -sgn \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi^2} \tag{60}$$

when evaluated at the RPE and $\psi = f_0$. One implication of this is that the log profit function will be convex in ψ if and only if the cross-partial is positive. The RPE will fail to exist if, for a firm at any position, the log profit function is convex at the proposed RPE so this result tells us to look for the sign of the cross-partial. This is Proposition 3a.

Differentiating (58) we have that:

$$\frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} = -\frac{1}{Z(f_0, \psi)^2} \frac{\partial Z(f_0, \psi)}{\partial \psi} \frac{\partial Z(f_0, \psi)}{\partial f_0} + \frac{1}{Z(f_0, \psi)} \frac{\partial^2 Z(f_0, \psi)}{\partial \psi^2} \tag{61}$$

from which, after some rearrangement one can derive that:

$$sgn \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi^2} = -sgn \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} = sgn \frac{\partial \ln \frac{\partial Z(f_0, \psi)}{\partial f_0}}{\partial \psi} - \frac{\partial \ln Z(f_0, \psi)}{\partial \psi} \tag{62}$$

Now consider the last two terms in (62). First, $\frac{\partial \ln \frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial \psi}$ - from (57) we have that:

$$\frac{\partial Z\left(f_0,\psi\right)}{\partial f_0} = -\left(1-\beta\right)\left[w'\left(f\right)+\beta_w V'\left(f\right)\right]\frac{\partial f}{\partial f_0} \tag{63}$$

Using (13) this can be written as:

$$\frac{\partial Z\left(f_0,\psi\right)}{\partial f_0} = \frac{-\left(1-\beta\right)w'\left(f\right)}{1-\beta_w+\beta_w q(f)}\frac{\partial f}{\partial f_0} \tag{64}$$

and, at the proposed RPE (20) this can be written as:

$$\frac{\partial Z\left(f_0,\psi\right)}{\partial f_0} = \frac{-2\lambda\left(1-\beta\right)\left[p-w\left(f\right)\right]}{1-\beta[1-q(f)]-\beta_w q(f)[1-q(f)]}\frac{\partial f}{\partial f_0} \tag{65}$$

Taking logs we have that:

$$\begin{aligned} & \ln\frac{\partial Z\left(f_0,\psi\right)}{\partial f_0} \qquad (66)\\ = \; & \ln 2\lambda\left(1-\beta\right)+\ln[p-w\left(f\right)]-\ln\left[1-\beta[1-q(f)]-\beta_w q(f)[1-q(f)]\right]+\ln\left(-\frac{\partial f}{\partial f_0}\right)\end{aligned}$$

Now, differentiating this with respect to ψ as (62) says we need to we have that:

$$\begin{aligned} & \frac{\partial \ln \frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial \psi} \qquad (67)\\ = \; & \frac{\partial \ln[p-w\left(f\right)]}{\partial f}\frac{\partial f}{\partial \psi}+\frac{\lambda\left[(\beta-\beta_w)+2\beta_w q(f)\right]}{1-\beta+(\beta-\beta_w)q(f)+\beta_w q(f)^2}\frac{\partial f}{\partial \psi}+\frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi}\end{aligned}$$

Using (20) this can be written as:

$$\begin{aligned} & \frac{\partial \ln \frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial \psi} \qquad (68)\\ = \; & \frac{-2\lambda[1-\beta_w+\beta_w q(f)]\frac{\partial f}{\partial \psi}+\lambda\left[(\beta-\beta_w)+2\beta_w q(f)\right]\frac{\partial f}{\partial \psi}}{1-\beta+(\beta-\beta_w)q(f)+\beta_w q(f)^2}+\frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi}\end{aligned}$$

Now consider the values of $\frac{\partial f}{\partial \psi}$ and $\frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi}$ - these can be derived from (18). From (18) we have that:

$$N'(\psi) = \lambda\left[N(f_0)+N(f)\right]\frac{\partial f}{\partial \psi} \tag{69}$$

We are interested in evaluating this expression at $\psi = f = f_0$ (the steady-state) when (69) becomes:

$$\frac{\partial f}{\partial \psi} = \frac{N'(f)}{2\lambda N(f)} = \frac{1}{q(f)} \tag{70}$$

where the final equality comes from differentiation of (5).

Now differentiate (18) with respect to f_0 to give:

$$0 = \lambda \left[N(f_0) + N(f)\right] \frac{\partial f}{\partial f_0} + [1 - q(f)]N'(f_0) \tag{71}$$

from which we can derive:

$$\ln\left(-\frac{\partial f}{\partial f_0}\right) = \ln[1 - q(f)] + \ln N'(f_0) - \ln\lambda - \ln\left[N(f_0) + N(f)\right] \tag{72}$$

Differentiating (72) with respect to ψ (as required for (68)) leads to:

$$\frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi} = \frac{\lambda}{[1 - q(f)]}\frac{\partial f}{\partial \psi} - \frac{N'(f)}{[N(f_0) + N(f)]}\frac{\partial f}{\partial \psi} \tag{73}$$

We are interested in evaluating this expression at $\psi = f = f_0$ (the steady-state) when (73) becomes, after using (5) and (70):

$$\frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi} = \frac{\lambda\left[2q(f) - 1\right]}{q(f)^2[1 - q(f)]} \tag{74}$$

Now putting (74) into (68) we have that at $\psi = f = f_0$

$$\frac{\partial \ln \frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial \psi} = \frac{-2\lambda[1 - \beta_w + \beta_w q(f)] + \lambda\left[(\beta - \beta_w) + 2\beta_w q(f)\right]}{q(f)\left[1 - \beta + (\beta - \beta_w)q(f) + \beta_w q(f)^2\right]} + \frac{\lambda\left[2q(f) - 1\right]}{q(f)^2[1 - q(f)]} \tag{75}$$

Now consider the other term in (62), $\frac{\partial \ln Z(f_0,\psi)}{\partial \psi}$. This is simple to evaluate at the proposed steady-state because, from the first-order condition for profit maximization we must have:

$$\frac{N'(\psi)}{N(\psi)} + \frac{\partial \ln Z\left(f_0, \psi\right)}{\partial \psi} = 0 \tag{76}$$

so that at the steady-state:

$$\frac{\partial \ln Z\left(f_0, \psi\right)}{\partial \psi} = -\frac{N'(\psi)}{N(\psi)} = -\frac{2\lambda}{q(f)} \tag{77}$$

Combining (75) and (77) with (62) we have that the second-order conditions for a maximum at the profit functioning the proposed steady state is that:

$$\frac{-2\lambda[1-\beta_w+\beta_w q(f)]+\lambda\left[(\beta-\beta_w)+2\beta_w q(f)\right]}{q(f)\left[1-\beta+(\beta-\beta_w)q(f)+\beta_w q(f)^2\right]}+\frac{\lambda\left[2q(f)-1\right]}{q(f)^2[1-q(f)]}\geq-\frac{2\lambda}{q(f)} \tag{78}$$

A necessary condition for the existence of an RPE is that this is true for all f. Hence a sufficient condition for the non-existence of an RPE is that the inequality in (78) is violated for any f. After some rearrangement the sufficient condition for non-existence can be written as:

$$A\left(q,\beta,\beta_w\right)=[2q-1]-\beta[2q-1]\left(1-q\right)^2+2\beta_w q\left(1-q\right)^3<0 \tag{79}$$

for some $q(f)$. Inspection reveals that $A\left(0,\beta,\beta_w\right)<0$ and $A\left(1,\beta,\beta_w\right)>0$. We will show that that there is only one value of q at which $A\left(0,\beta,\beta_w\right)=0$. We will show that at the point where $A\left(0,\beta,\beta_w\right)=0$ we have $\frac{\partial A(q,\beta,\beta_w)}{\partial q}>0$. Note that when $A\left(0,\beta,\beta_w\right)=0$ we must have, from (79) $q\leq 0.5$. Differentiate (79) to give:

$$\frac{\partial A\left(q,\beta,\beta_w\right)}{\partial q}=2\left[1-\beta\left(1-q\right)^2\right]+\beta[2q-1]\left(1-q\right)+2\beta_w\left(1-q\right)^2\left[1-4q\right] \tag{80}$$

Using (79) to eliminate the term in β_w we can write (80) as:

$$\begin{aligned}\frac{\partial A\left(q,\beta,\beta_w\right)}{\partial q} &= 2\left[1-\beta\left(1-q\right)^2\right]+\beta[2q-1]\left(1-q\right) \\ &\quad+\frac{A\left(q,\beta,\beta_w\right)-[2q-1]\left[1-\beta\left(1-q\right)^2\right]}{q\left(1-q\right)}\end{aligned} \tag{81}$$

At the point where $A\left(q,\beta,\beta_w\right)=0$, we have, from (81) that, after some rearrangement:

$$q\left(1-q\right)\frac{\partial A\left(q,\beta,\beta_w\right)}{\partial q}=\left[1-\beta\left(1-q\right)^2\right]\left(1-2q^2\right)+\beta[2q-1]q\left(1-q\right)^2 \tag{82}$$

(82) is decreasing in β when $q<0.5$. Hence:

$$q\left(1-q\right)\frac{\partial A\left(q,\beta,\beta_w\right)}{\partial q}\geq[2-q]\,q\left(1-2q^2\right)-[1-2q]q\left(1-q\right)^2>0 \tag{83}$$

where the final inequality follows from the fact that $[2-q]>(1-q)^2$ and $(1-2q^2)>[1-2q]$

This shows that there is a unique value $q^*(\beta, \beta_w)$ such that $A(q, \beta, \beta_w) < 0$ when $q < q^*(\beta, \beta_w)$ and $A(q, \beta, \beta_w) > 0$ when $q > q^*(\beta, \beta_w)$. A necessary condition for existence (or sufficient condition for non-existence) of an RPE is that $A(q(f), \beta, \beta_w) > 0$ for all f - this is hardest to satisfy when the quit rate is lowest which is in the highest wage firm when the quit rate is δ. Hence the condition in (36) and this proves Proposition 3b.

To prove the first part of Proposition 3c note that from $A(q^*(\beta, \beta_w), \beta, \beta_w) = 0$ we must have:

$$\frac{\partial A(q^*, \beta, \beta_w)}{\partial q} \frac{\partial q^*}{\partial \beta} + \frac{\partial A(q^*, \beta, \beta_w)}{\partial \beta} = 0 \tag{84}$$

As $\frac{\partial A(q^*, \beta, \beta_w)}{\partial q} > 0$, this implies that:

$$sgn \frac{\partial q^*}{\partial \beta} = -sgn \frac{\partial A(q^*, \beta, \beta_w)}{\partial \beta} = -sgn[1 - 2q](1 - q)^2 < 0 \tag{85}$$

as, from (79) we must have $2q^*(\beta, \beta_w) - 1 < 0$.

To prove the second part of Proposition 3c a similar argument shows that:

$$sgn \frac{\partial q^*}{\partial \beta_w} = -sgn \frac{\partial A(q^*, \beta, \beta_w)}{\partial \beta_w} = -sgn 2q(1 - q)^3 < 0 \tag{86}$$

8.2 Proof of Proposition 5

In the myopic case, $\beta = \beta_w = 0$, (20) then becomes:

$$\frac{\partial \ln [p - w(f)]}{\partial f} = -2\lambda \tag{87}$$

with solution

$$[p - w(f)] = [p - b] e^{-2\lambda f} \tag{88}$$

This has been derived using the first-order conditions for profit maximization i.e. necessary conditions that must be satisfied in an RPE. But they are not sufficient conditions - for that we need to ensure global profit maximization. So let us now consider whether those conditions are satisfied. In the myopic case workers will base their mobility decisions only on the current wage offered so that $\psi(f_0, f) = f$. So we can think of a firm as choosing its current position in the wage offer distribution. If it inherits employment of position f_0 and chooses f then profits will be given by:

$$\begin{aligned}\pi(f,f_0) &= [p-w(f)]N(f,f_0) = [p-w(f)][(1-q(f))N(f_0)+R(f)] \quad (89)\\ &= [p-b]e^{-2\lambda f}[(1-q(f))N(f_0)+R(f)]\end{aligned}$$

Using (88). It is easiest to analyze this by taking logs to give us:

$$\ln\pi(f,f_0) = \ln[p-b] - 2\lambda f + \ln[(1-q(f))N(f_0)+R(f)] \qquad (90)$$

Differentiating with respect to f leads to:

$$\begin{aligned}\frac{\partial \ln\pi(f,f_0)}{\partial f} &= -2\lambda + \frac{[-q'(f))N(f_0)+R'(f)]}{[(1-q(f))N(f_0)+R(f)]} \qquad (91)\\ &= -2\lambda + \lambda\frac{[N(f_0)+N(f)]}{[(1-q(f))N(f_0)+R(f)]}\end{aligned}$$

This is equal to zero when evaluated at $f = f_0$. But another necessary condition is that the second-order conditions are satisfied. Differentiating (91) again with respect to f, we have that:

$$\begin{aligned}&\frac{\partial^2 \ln\pi(f,f_0)}{\partial f^2} \qquad (92)\\ &= -\frac{\lambda^2[N(f_0)+N(f)]^2]}{[(1-q(f))N(f_0)+R(f)]^2} + \frac{\lambda N'(f)}{[(1-q(f))N(f_0)+R(f)]}\\ &= -[2\lambda + \frac{\partial \ln\pi(f,f_0)}{\partial f}]^2 + \frac{2\lambda^2 N(f)/q(f)}{[(1-q(f))N(f_0)+R(f)]}\end{aligned}$$

The sign of (92) depends on the sign of the numerator. At $f = f_0$, we have that the first derivative is zero so that:

$$sgn\frac{\partial^2 \ln\pi(f,f_0)}{\partial f^2} = sgn\{-4\lambda^2 + \frac{2\lambda^2}{q(f)}\} = sgn[1-2q(f)] \qquad (93)$$

If $q(f) < 0.5$ for any f, which corresponds to $\delta < 0.5$ then (93) shows that the profit function is convex in f, so that, far from the proposed RPE picking out a maximum of the profit function it actually picks out a minimum. In this case the RPE fails to exist.

8.3 Proof of Proposition 6

Denote, as usual, by $G(f)$ the fraction of workers at position f or below in last period's wage distribution. Now consider the fraction of workers who, in a rank-inverting equilibrium will end up at position f or below in this period's wage distribution. In the steady-state this must also be equal to $G(f)$. This will, as usual, be the fraction of workers who are in this group who do not get a shock that causes them to leave it plus the recruits into this group from unemployment. Because of the rank-inverting nature of the proposed equilibrium the proportion of workers who are in this group is not $G(f)$ as in a RPE but $[1 - G(1 - f)]$. Putting this together we have that, in a steady state:

$$(1-u)G(f) = [1-(\delta+\lambda(1-f)](1-u)[1-G(1-f)] + \lambda f u \tag{94}$$

Using the expression for the unemployment rate 1 this can be written as:

$$G(f) = [1-(\delta+\lambda(1-f)][1-G(1-f)] + \delta f \tag{95}$$

Now invert the roles of f and $(1-f)$ and write this as:

$$G(1-f) = [1-(\delta+\lambda f)][1-G(f)] + \delta(1-f) \tag{96}$$

(95) and (96) can be solved to give the following expression for $G(f)$:

$$G(f) = \frac{f\{[1-q(f)](\delta+\lambda)+\delta\}}{1-[1-q(f)][1-q(1-f)]} \tag{97}$$

where, to keep notation simple, we have used (6). This expression is what had to be proved.

8.4 Proof of the Possibility of a RIE

We have that:

$$\ln \pi(f, f_0) = \ln[p-b] - 2\lambda \int_0^f \frac{N(1-x)}{N(x)} dx + \ln[R(f) + (1-q(f))N(f_0)] \tag{98}$$

Differentiating this with respect to f we have that:

$$\begin{aligned}
\frac{\partial \ln \pi(f, f_0)}{\partial f} &= -2\lambda \frac{N(1-f)}{N(f)} + \frac{[R'(f) - q'(f)N(f_0)]}{[R(f) + (1-q(f))N(f_0)]} \\
&= -2\lambda \frac{N(1-f)}{N(f)} + \frac{\lambda[N(1-f) + N(f_0)]}{[R(f) + (1-q(f))N(f_0)]}
\end{aligned} \tag{99}$$

Differentiating this again with respect to f we have that:

$$\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} = 2\lambda \frac{N'(1-f)N(f) + N'(f)N(1-f)}{N(f)^2} - \frac{\lambda N'(1-f)}{[R(f) + (1-q(f))N(f_0)]} - \frac{\lambda^2 [N(1-f) + N(f_0)]^2}{[R(f) + (1-q(f))N(f_0)]^2} \tag{100}$$

Now, in a RIE, we must have that:

$$N(f) = R(f) + [1 - q(f)]N(1-f) \tag{101}$$

and, differentiating this with respect to f we have that:

$$N'(f) = R(f) - q'(f)N(1-f) - [1-q(f)]N'(1-f) = 2\lambda N(1-f) - [1-q(f)]N'(1-f) \tag{102}$$

Using (102) to eliminate $N'(f)$ from (100) we have that:

$$\begin{aligned} &\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} \\ = \; & 2\lambda \frac{N'(1-f)[N(f) - (1-q(f))N(1-f)] + 2\lambda N(1-f)^2}{N(f)^2} \\ & - \frac{\lambda N'(1-f)}{[R(f) + (1-q(f))N(f_0)]} - \frac{\lambda^2 [N(1-f) + N(f_0)]^2}{[R(f) + (1-q(f))N(f_0)]^2} \\ = \; & 2\lambda \frac{N'(1-f)R(f) + 2\lambda N(1-f)^2}{N(f)^2} \\ & - \frac{\lambda N'(1-f)}{[R(f) + (1-q(f))N(f_0)]} - \frac{\lambda^2 [N(1-f) + N(f_0)]^2}{[R(f) + (1-q(f))N(f_0)]^2} \end{aligned} \tag{103}$$

Using (99), and after some rearrangement (103) can be written as:

$$\begin{aligned} \frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} = \; & \\ & \frac{\lambda N'(1-f)[2R(f)[R(f) + (1-q(f))N(f_0)] - N(f)^2]}{N(f)^2 [R(f) + (1-q(f))N(f_0)]} \\ & + \frac{4\lambda^2 N(1-f)^2}{N(f)^2} - [\frac{\partial \ln \pi(f, f_0)}{\partial f} + \frac{2\lambda N(1-f)}{N(f)}]^2 \end{aligned} \tag{104}$$

Using (101) this can be -rearranged to give:

$$\frac{\partial^2 \ln \pi(f,f_0)}{\partial f^2} = \tag{105}$$
$$\frac{\lambda N'(1-f)[N(f)[2R(f)-N(f)]+2R(f)(1-q(f))[N(f_0)-N(1-f)]]}{N(f)^2[R(f)+(1-q(f))N(f_0)]}$$
$$-\frac{\partial \ln \pi(f,f_0)}{\partial f}[\frac{\partial \ln \pi(f,f_0)}{\partial f}+\frac{4\lambda N(1-f)}{N(f)}]$$

We can use 105 to provide necessary and sufficient conditions for an RIE to exist. A necessary condition is that at $f_0 = 1-f$, the second derivative of the profit function is negative. Substituting in $f_0 = 1-f$ and using the fact that $\frac{\partial \ln \pi(f,f_0)}{\partial f} = 0$ at this point we have that:

$$\frac{\partial^2 \ln \pi(f,f_0)}{\partial f^2} = \frac{\lambda N'(1-f)[N(f)[2R(f)-N(f)]}{N(f)^2[R(f)+(1-q(f))N(f_0)]} \tag{106}$$

and this will be negative if $2R(f) < N(f)$ for all f. This is a condition that can be satisfied if λ and δ are small enough.

This is only a necessary condition for the RIE to exist - now consider how we can provide a sufficient condition. If $2R(f) < N(f)$ for all f the above argument shows that the profit function must have at least a local maximum at $f = 1-f_0$. If this is not a global maximum then there must also be a local interior minimum of the profit function. At a local minimum we have that $\frac{\partial \ln \pi(f,f_0)}{\partial f} = 0$ and that $\frac{\partial^2 \ln \pi(f,f_0)}{\partial f^2} > 0$. But this will not be possible if the first term in (105) is everywhere negative. So the first term in (105) being everywhere negative is a sufficient condition for the existence of an RIE. This can be written as the condition:

$$N(f)[2R(f)-N(f)]+2R(f)(1-q(f))[N(f')-N(1-f)] \leq 0 \tag{107}$$

This condition is harder to satisfy the larger is $f_0$so a sufficient condition is:

$$N(f)[2R(f)-N(f)]+2R(f)(1-q(f))[N(1)-N(1-f)] \leq 0 \tag{108}$$

After some re-arrangement, 108 can be written as:

$$N(f) \geq 2R(f)[1+\frac{(1-q(f))[N(1)-N(1-f)]}{N(f)}] \tag{109}$$

One can choose the parameters of the model so that this is satisfied e.g. if λ and δ are small enough.

8.5 Moscarini and Postel-Vinay (2010)

Moscarini and Postel-Vinay (2010) consider a contract-posting model that is similar in many ways to the one considered here but have existence of an RPE. This section tries to explain the difference between their results and the one considered here. Moscarini and Postel-Vinay (2010) is much more ambitious in scope than this note allowing for heterogeneity in productivity across firms, aggregate productivity shocks and transitional dynamics. But I think the fundamental driver of the difference in results is that they make different assumptions about the contract structure and the timing of decisions. MP-V assume that employers can commit to a state-contingent path of future wages but at each point in time must pay all existing workers the same contract. They show this is equivalent to offering workers a value of the job and this is the formulation adopted here.

MP-V assume that production and payments take place at the beginning of each period using the level of employment attained at the end of the previous period. In that previous period employers had committed to offering workers a level of expected discounted future utility V in the firm and workers had accepted and rejected job offers on that basis. So, at the start of any period a firm has two state variables, the inherited level of employment N and the level of utility promised to existing workers V. We will be looking for a steady-state equilibrium that can be represented by the value offer function $V(f)$. It is convenient to work not with V as the second state variable but f' the position in the equilibrium value offer distribution. As we are only interested in the steady-state no firm will ever want to make a job offer outside the distribution so this is not restrictive. Accordingly, we will model the firm's decision this period as the position in the value offer distribution utility to be be offered to workers from next period f and the current wage w. These two are linked because the employer must keep its promise to the existing workers to deliver $V(f')$ - this is what MP-V call the 'promise-keeping' constraint.

So this section considers a stripped-down version of their model in order to get to the essence of the difference in which we consider a static environment with no productivity differences and no aggregate productivity shocks. Define by $\Pi(N, f)$ the value function for an employer as a function of the state variables. We will have:

$$\Pi(N, f') = \max_{w,f}[p - w]N + \beta\Pi(N', f) \tag{110}$$

subject to the constraint on the dynamics of employment that we will write as:

$$N' = [1 - q(f)]N + R(f) \tag{111}$$

Note that quit rates and recruits depend only on f and are exactly the same dynamics as in the main body of this note. Note that in this set-up, in contrast to the model considered in this note, the current wage has no allocational consequences. Given that, one might think that employers will lower the current wage as much as possible but they are assumed not to be able to do this because they must stick to offering the current workers a value of the job equal to the pre-determined V. This promise-keeping constraint can be written as:

$$V(f') = w + \beta[V(f) + \lambda \int \max[V(x) - V(f), 0]dx + \delta(V^u - V(f))] = w + \beta Z(f) \tag{112}$$

Note that $Z'(f) = [1 - q(f)]V'(f)$. Using (112) to eliminate w from (110) we can write the value function as:

$$\Pi(N, f') = \max_f [p - V(f')]N + \beta[\Pi(N', V(f)) + Z(f)N] \tag{113}$$

Note that a solution to this must have the form $\Pi(N, f') = \phi(N) - V(f')N$ in which case (113) can be written as:

$$\phi(N) = \max_f pN + \beta[\phi(N') - N'V(f)) + Z(f)N] \tag{114}$$

Taking the first-order condition with respect to f we have that:

$$[\phi'(N') - V(f)]\frac{\partial N'}{\partial f} - N'V'(f) + NZ'(f) = 0 \tag{115}$$

This can be written as:

$$[\phi'(N') - V(f)]\lambda[N + N'] - V'(f)R(f) = 0 \tag{116}$$

The envelope condition from (114) can be written as:

$$\phi'(N) = p + \beta[[\phi'(N') - V(f)]\frac{\partial N'}{\partial N} + Z(f)] \tag{117}$$

Re-arranging (117) we have that:

$$\phi'(N) - V(f) = \frac{p - V(f) + \beta Z(f)}{1 - \beta[1 - q(f)]} \tag{118}$$

Substituting (118) into (116) and evaluating at the RPE in which $N(f) = N$,

$$\left[\frac{p - V(f) + \beta Z(f)}{1 - \beta[1 - q(f)]}\right] 2\lambda N(f) = V'(f) R(f) \tag{119}$$

In a steady-state with constant wages and constant V we will, using (112) in the steady-state have:

$$V'(f) = \frac{w'(f)}{1 - \beta[1 - q(f)]} \tag{120}$$

Note that this is different to the value function used earlier because of the different timing assumed.

Converting this to wages we can write this as:

$$\left[\frac{p - w(f)}{1 - \beta[1 - q(f)]}\right] \frac{2\lambda}{q(f)} = \left[\frac{w'(f)}{1 - \beta[1 - q(f)]}\right] \tag{121}$$

which can be reduced to (30) i.e. the RPE in MP-V is the same as in the canonical model. Perhaps this is not surprising as, while there is no commitment to a time profile of future wages there is a commitment to a given value of the job and this perhaps amounts to the same thing.

So far, we have not proved that the RPE exists i.e. that it is profit maximizing each period for a firm with inherited employment level $N(f)$ to choose a position in the value distribution equal to f. A necessary condition for this is that employers with larger inherited employment levels will choose a higher position in the value distribution. From (116) this requires that:

$$\lambda[N + N']\phi''(N')\frac{\partial N'}{\partial N} + \lambda[\phi'(N') - V(f)][1 + \frac{\partial N'}{\partial N}] > 0 \tag{122}$$

A sufficient condition for this is that $\phi(N)$ is convex in N. To see that this must be satisfied in the steady-state RPE consider the following. Consider two initial levels of employment N_0 and N_1. Denote by f_0 the position chosen by the firm with N_0 and assume we are in steady-state. Obviously the firm with N_1 could choose this so must, in equilibrium do at least as well as this. So using (114) we must have that:

$$\phi(N_1) - \phi(N_0) \geq [p + \beta Z(f_0) - \beta V(f_0)[1 - q(f_0)]] (N_1 - N_0) + \beta[\phi(N_1') - \phi(N_0)] \tag{123}$$

where N_1' is the employment level the N_1 firm will end up with next period and we have used the fact that the firm with N_0 choosing f_0 is in steady state so will end up in the same position next period. In deriving this we have used the fact that recruits and the quit rate must be the same for the two firms if they both choose f_0 . Now lets do the same again, comparing a firm with $N_1'$and one with N_0 if they both choose f_0. Keep running this forward until we end up with:

$$\phi(N_1)-\phi(N_0) \geq \frac{p+\beta Z(f_0)-\beta V(f_0)[1-q(f_0)]}{1-\beta[1-q(f_0)]}(N_1-N_0) = \phi'(N_0)(N_1-N_0) \tag{124}$$

where the equality follows from (118). Hence we have convexity.

References

[1] Burdett, K and M.G.Coles (2003) "Equilibrium Wage-Tenure Contracts", Econometrica, 71, 1377-1404.

[2] Burdett, K. and D.Mortensen (1998) "Wage Differentials, Employer Size and Unemployment", International Economic Review, 39, 257-273.

[3] Coles, M.G. (2001) "Equilibrium Wage Dispersion, Firm Size and Growth", Review of Economic Dynamics, 4, 159-187.

[4] Coles, M. G. and D. Mortensen (2011) "Dynamic Monopsonisitic Competition and Labor Market Equilibrium, unpublished.

[5] Manning, A. (2003) "Monopsony in Motion: Imperfect Competition in Labor Markets", Princeton: Princeton University Press.

[6] Moscarini. G. and F. Postel-Vinay (2010) "Sotchastic Search Equilibrium", Yale University

Figure 1

Long- and Short-Run Labour Supply Curves

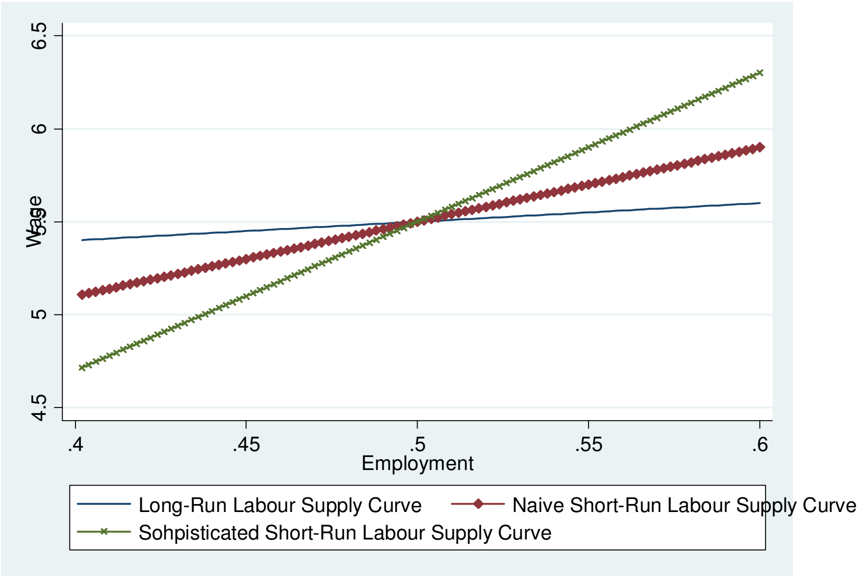
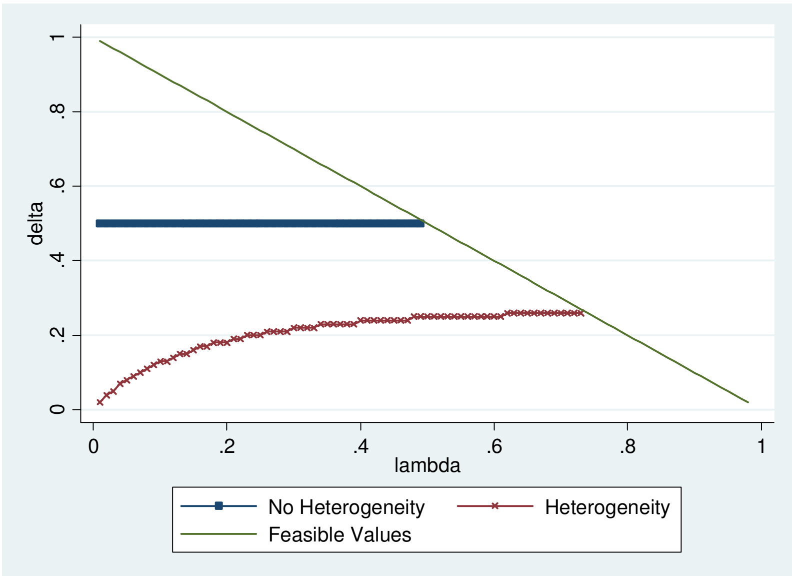


Figure 2

Existence of a Rank-Preserving Equilibrium with and without Heterogeneity in Productivity



Notes: The areas where an RPE exists are above the relevant lines.

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