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Fiscal Multipliers over the Business Cycle

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Abstract
This paper illustrates why fiscal policy becomes more effective as unemployment rises in recessions. The theory is based on the equilibrium unemployment model of Michaillat (forthcoming), in which jobs are rationed in recessions. Fiscal policy takes the form of government spending on public-sector jobs. Recessions are periods of acute job shortage without much competition for workers among recruiting firms; hiring in the public sector does not crowd out hiring in the private sector much; therefore fiscal policy reduces unemployment effectively. Formally the fiscal multiplier—the reduction in unemployment rate achieved by spending one dollar on public-sector jobs—is countercyclical. An implication is that available estimates of the fiscal multiplier, which measure the average effect of fiscal policy over the business cycle, do not apply in recessions because the multiplier is much higher in recessions than on average.

JEL Classifications: : E24, E32, E62, J64
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1 Introduction

Many economists and policymakers believe that fiscal policy may be especially effective to stimulate the economy in recessions. Recent research substantiates this belief by showing that fiscal policy may especially increase output during extraordinary recessions when the zero lower bound on nominal interest rates binds [for example, Christiano et al., 2011; Eggertsson, 2011; Eggertsson and Krugman, 2011]. This result surely applies to the US during the Great Depression and the years following the 2008 financial crisis, when the nominal interest rate reached a near-zero value regarded as effective lower bound. But we have no reason to believe that fiscal policy is particularly effective otherwise, when the economy is in recession away from the zero lower bound.

This paper argues that the result from the zero-lower-bound research is in fact more general. It shows that fiscal policy may be especially effective to stimulate the economy not just at the zero lower bound, but at any time when the labor market is depressed. A fiscal policy that achieves this property is public employment. I measure the effectiveness of public employment by a fiscal multiplier—defined as the reduction in unemployment rate achieved by spending one dollar on public employment. I prove that the fiscal multiplier is countercyclical: the higher the unemployment rate, the more effective public employment, irrespective of the zero lower bound.

By providing a model in which the fiscal multiplier is much higher in recessions than on average, this paper reinforces the argument in Parker [2011] that most available estimates of the fiscal multiplier, obtained by averaging the effects of fiscal policy over the business cycle, cannot apply in recessions. Parker [2011] points out that numerical and empirical methods used to estimate the multiplier average the effects of fiscal policy over all states of the economy. He concludes that these methods are inappropriate to describe the effectiveness of fiscal policy in recessions if fiscal policy is more effective in recessions than on average. That is the case in my calibrated model: the fiscal multiplier increases more than threefold when the unemployment rate rises from 4% to 10%.

The core of the analysis characterizes the fiscal multiplier in presence of job rationing—the property that the labor market does not converge to full employment when matching frictions are arbitrarily small, or equivalently when unemployed workers exert an arbitrarily large effort to search for jobs. The paper augments the equilibrium unemployment model of Michaillat [forthcoming] with public employment. The model combines (a) equilibrium unemployment arising from frictions in matching workers to firms as in Pissarides [2000]; (b) unemployment fluctuations arising from technology shocks and real wage rigidity as in Hall [2005]; (c) job rationing in recessions resulting from the combination of wage rigidity and diminishing marginal returns to labor. Public employment is financed by a labor tax such that the government budget remains balanced each period.

While public employment has been relatively neglected by a literature in which fiscal policy typically consists of government purchase of consumption goods, it has been widely used in the US (and elsewhere) to tackle unemployment in recessions.[1] Statistically, public employment is countercyclical: the correlation between public employment and the unemployment rate is 0.66 in the US for the 1970–2007 period [Gomes, 2010]. During the Great Depression the Roosevelt administration hired millions of unemployed workers to build dams, bridges, and roads [Fishback and Kachanovskaya, 2010; Fleck, 1999; Neumann et al., 2010]. More recently the American Jobs Act presented by the Obama administration to Congress in 2011 proposed to spend $130 billion to tackle high unemployment by hiring teachers, construction workers allocated to infrastructure projects, and other public-sector workers.[2] Indeed, public employment seems particularly adapted to reduce unemployment in the short run.[3]

I present the basic framework in Section 2. Section 3 demonstrates that the sheer presence of unemployment is not sufficient to explain why fiscal policy is more effective in recessions. The section consider two limit cases of the model in which there is no job rationing: a model with constant marginal returns to labor and Nash bargaining, and a model with constant marginal returns to labor and rigid wages. In these models, public employment crowds out private employment one-

[1] Previous literature on public employment includes Finn [1998] and Cavallo [2005], who study government purchases of hours in the public sector in a standard real business cycle model. But these models do not have unemployment. Holmlund and Linden [1993], Hörner et al. [2007], Quadrini and Trigari [2007], Gomes [2010], and Burdett [in press] to study the effects of public employment in an equilibrium unemployment model. But these models do not isolate the effects of public employment in recessions.

[2] Details are provided at http://www.whitehouse.gov/the-press-office/2011/09/08/fact-sheet-american-jobs-act.

[3] See Pappa [2010] for empirical evidence that government spending on public employment is more effective than government spending on investment or consumption goods.

for-one and fiscal policy has no effect on aggregate employment.

In Section 4, I prove that the public-employment multiplier is positive in a model with job rationing. Next, I write the fiscal multiplier as a continuous function of the underlying state of the economy and prove that the multiplier is countercyclical. The result hinges on the nature of unemployment in times of trouble. While public employment mechanically increases aggregate labor demand, it also crowds out private employment by making it more difficult for firms to hire workers. The crowding out of private employment partially offsets the increase in public employment. The effectiveness of fiscal policy depends on the extent of crowding out. In recessions, crowding out is weak: when jobs are rationed and unemployment is high, there is not much competition for job applicants between firms and the government, so hiring in the public sector does not displace hiring in the private sector much. Thus, fiscal policy is particularly effective.

Section 5 proposes three natural extensions of the basic model, and proves that the fiscal multiplier remains positive and countercyclical in these cases. The multiplier is actually larger in all three extensions than in the basic model. In the first extension, unemployed workers control their future employment rate (their labor supply) by choosing how much to search for a job. Public employment increases the return to job search by increasing the probability to find a job. Thus, public employment also stimulates labor supply. In the second extension, the output of the public sector contributes to a stock of public capital, which in turn contributes to the productivity of firms. Since public employment increases the marginal productivity of firms, it also stimulates the demand for labor of firms. In the third extension, wages differ in the public and private sectors. Unemployed workers direct their search to one of the two sectors, depending on the wage and job-finding probability in each sector. If wages in the public sector are below those in the private sector, the probability to find a job is higher in the public sector, and creating public-sector jobs is more effective because it attracts workers toward the sector with the highest job-finding rate.

Section 6 calibrates the model with US data to illustrate that the fluctuations of the fiscal multiplier over the business cycle are sizable. The model is too simple to be the basis for quantitative estimates of the effects of actual fiscal policy interventions. Nonetheless, the calibration illustrates how much the effectiveness of fiscal policy varies over the business cycle, and how the effectiveness varies with some features of the policy, such as the level of wages in the public sector. In the basic model and its extensions the fiscal multiplier is in the 0.6–1.4 range for the average US unemployment rate of 6%, which means that spending 1% of GDP on fiscal policy reduces unemployment by 0.6–1.4 percentage points on average. The multiplier fluctuates significantly over the business cycle. It increases nearly fourfold when the unemployment rate increases from 4% to 10%: for instance in the basic model, the multiplier increases from 0.27 to 1.15.

By theoretically characterizing the effects of fiscal policy over the business cycle, the model illustrates why fiscal policy may be more desirable in times of trouble. The fiscal multiplier in a stochastic dynamic model is usually calculated by log-linearizing the system of equilibrium relationships around the deterministic steady state, as in Woodford [2011]. By construction the fiscal multiplier is valid only around the steady state. The multiplier cannot be a function of the state of the economy and cannot be valid in recessions, which are some distance away from the steady state. To circumvent these difficulties, I use a deterministic dynamic model, calculate fiscal multipliers in the nonlinear model by comparative statics, and compare fiscal multipliers across steady states with different unemployment rates. I am also able to represent a steady-state equilibrium diagrammatically to depict the mechanisms at play. The model, however, falls short of providing estimates of the fiscal multiplier in a stochastic environment. Section 7 concludes by discussing such shortcomings of the theory and possible resolutions. Proofs are collected in the Appendix.

2 The Model

A technology parameter a captures the position of the economy in the business cycle. To strip the model to its essence, I abstract from workers' choice of labor supply, from public capital, and from the existence of two separate labor markets for public jobs and private jobs. These extensions will be introduced in Section 5.

Labor market. There is a unit mass of workers in the labor market. Two sectors compose the labor market: a private sector with l_t workers, and a public sector with g_t workers. Aggregate employment is $n_t = l_t + g_t$. At the end of period $t-1$, a fraction s of the n_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job apply for a new job immediately. At the beginning of period t, $u_t = 1-(1-s)\cdot n_{t-1}$ unemployed workers search for a job. Jobseekers apply to jobs randomly: they do not direct their search toward private or public jobs.Those who find a job participate in production in period t with the $(1-s)\cdot n_{t-1}$ incumbent workers.

By posting vacancies, firms hire workers in the private sector and the government hires workers in the public sector. The number h_t of matches made in a period is given by a Cobb-Douglas matching function of unemployment u_t and vacancies o_t: $h_t = \omega_h \cdot u_t^{\eta} \cdot o_t^{1-\eta}$, with the restriction that $h_t \leq \min\{u_t, o_t\}$. $\omega_h > 0$ and $\eta \in (0,1)$ are parameters. Labor market conditions are summarized by labor market tightness $\theta_t \equiv o_t/u_t$. The matching technology prevents all unemployed workers from finding a job and all vacancies from being filled. Jobseekers find a job with probability $f(\theta_t) = \omega_h \cdot \theta_t^{1-\eta}$. Vacancies in the public and private sectors are filled with the same probability $q(\theta_t) = \omega_h \cdot \theta_t^{-\eta}$. In a tight market it is easy to find jobs—the job-finding probability $f(\theta_t)$ is high—but difficult to find workers—the vacancy-filling probability $q(\theta_t)$ is low.

Keeping a vacancy open has a per-period cost $r\cdot a$ in units of private good, where $r>0$ captures resources spent recruiting workers.[4] A worker is hired with certainty by opening $1/q(\theta_t)$ vacancies and spending $r\cdot a/q(\theta_t)$.

Firm. A representative firm produces a private good sold to workers taking the price (normalized to 1) as given. The firm's production function is

$$y_t = a \cdot x(l_t), \tag{1}$$

where y_t is output of private good, a is technology level, l_t is employment in the firm, $x(l) = l^{\alpha}$, and $\alpha > 0$ is a parameter. A risk-neutral entrepreneur, with the same discount factor $\delta < 1$ as workers, owns the firm. Given wage and tightness $\{w_t, \theta_t\}_{t=0}^{+\infty}$, the *firm's problem* is to choose employment

[4] As in Pissarides [2000] the cost of opening a vacancy is proportional to technology a. This is equivalent to assuming that the recruiting technology itself is independent of technology, but that it uses labor as unique input [Shimer, 2010]. This assumption is appealing since recruiting is a time-intensive activity.

$\{l_t\}_{t=0}^{+\infty}$ to maximize present discounted profits $\sum_{t=0}^{+\infty} \delta^t \cdot \Pi_t$, where the profit in period t is given by

$$\Pi_t = a \cdot x(l_t) - w \cdot l_t - \frac{r \cdot a}{q(\theta_t)} \cdot [l_t - (1-s) \cdot l_{t-1}] .$$

$l_t - (1-s) \cdot l_{t-1} \geq 0$ is the number of hires in period t. In steady state the first-order condition with respect to l_t is

$$x'(l) = \frac{w}{a} + [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)}. \tag{2}$$

Firms hire labor until until marginal product of labor $a \cdot x'(l)$ equals marginal cost of labor, which is the sum of the wage w plus the amortized hiring cost $[1 - \delta \cdot (1-s)] \cdot r \cdot a/q(\theta)$.

Workers. Workers have separable utility over consumption c_t of private good (produced in the private sector) and consumption p_t of public good (produced in the public sector) of the form $\omega_v \cdot v(p_t) + v(c_t)$. ω_v is a parameter, $v(x) = x^{1-\nu}/(1-\nu)$ is a utility function with constant relative risk aversion ν.

Employed workers earn a wage w_t, taxed at rate τ_t. Unemployed workers receive unemployment benefits $b_t \cdot w_t$. Workers neither borrow nor save, so consumption is $c_t^e = w_t \cdot (1-\tau)$ when employed and $c_t^u = b_t \cdot w_t$ when unemployed. The replacement rate $\rho \equiv c_t^u / c_t^e = b_t/(1-\tau_t)$, which measures the generosity of the unemployment insurance (UI) system, is assumed to remain constant.

Unlike the standard macroeconomic literature, but in line with the equilibrium unemployment literature, I abstract from changes in the number of hours worked by employed workers. This choice is consistent with the empirical evidence that most cyclical variations in total hours worked are due to variations in the number of employed workers and not to variations in hours per worker [Shimer, 2010]. Instead, I focus on changes in workers' employment rate. In steady state the inflow to unemployment $s \cdot n$ must equal the outflow from unemployment $[1 - (1-s) \cdot n] \cdot f(\theta)$. Therefore employment rate n is related to labor market tightness θ by a Beveridge curve:

$$n = \frac{f(\theta)}{s + (1-s) \cdot f(\theta)}. \tag{3}$$

If firms post more vacancies, tightness θ and job-finding probability $f(\theta)$ increase, which raises employment rate n.

Government. The government's production function is:

$$p_t = \omega_p \cdot a \cdot x(g_t). \tag{4}$$

g_t is public employment, p_t is the output of public good, which is consumed by workers, and ω_p is a parameter that scales the productivity of the public sector relative to that of the private sector.

The government has two sources of expenditure: (a) it employs g_t workers in the public sector at the prevailing private-sector wage w_t; and (b) it provides unemployment benefits $b_t \cdot w_t$ to all unemployed workers. Government spending is financed by a labor tax τ_t paid by all employees, and by the firm's profit Π_t, which is entirely taxed by the government. In the absence of saving and borrowing the resource constraint imposes that the government's budget be balanced each period:

$$n_t \cdot \tau_t \cdot w_t + \Pi_t = b_t \cdot w_t \cdot (1 - n_t) + g_t \cdot w_t + \frac{r \cdot a}{q(\theta_t)} \cdot [g_t - (1 - s) \cdot g_{t-1}] \,.$$

The government's budget constraint is equivalent to the resource constraint in the economy:[5]

$$a \cdot x(l_t) = (1 - n_t) \cdot c_t^u + n_t \cdot c_t^e + \frac{r \cdot a}{q(\theta_t)} \cdot h_t, \tag{5}$$

which says that all private output must be either consumed or used as a resource for recruiting.

Since the government's budget remains balanced each period, the fiscal multiplier analyzed in the paper is a balanced-budget multiplier. The balanced-budget multiplier is particularly relevant for two reasons. First, exploding public debt in most developed countries seems to call for fiscal restraint, making the use of deficit financing less desirable. Second, from an historical perspective, governments have not always resorted to deficit financing in recessions. In the 1930s the Hoover and Roosevelt administrations financed most of the increases in federal spending with taxes and

[5]The government's budget constraint is written as the resource constraint using the definition of the firm's profit.

ran relatively small deficits [Fishback and Kachanovskaya, 2010].

Wage. The wage in the private sector is set once worker and firm have matched. Since search costs are sunk at the time of matching, there are always mutual gains from trade. There is no compelling theory of wage determination in such an environment [Hall, 2005]. Hence I assume that the private-sector wage follows the simple wage schedule from Blanchard and Galí [2010]:

$$w_t = \omega \cdot a^{\gamma}. \tag{6}$$

$\omega > 0$ is a parameter. The parameter γ captures the flexibility of wages over the business cycle. If $\gamma = 0$, wages are completely fixed over the cycle. If $\gamma = 1$, wages are proportional to technology and fully flexible over the cycle. To simplify I assume that the wage is constant over time. This assumption is innocuous since the analysis focuses on the steady state of the economy.

It becomes clear in Section 4 that any wage (6) is privately efficient in a deterministic environment as long as the initial values of employments l_0 and g_0 are close enough to their steady-state values. Private efficiency guarantees that worker-firm pairs exploit all opportunities for mutual improvement, such that the wage never causes the destruction of a match generating a positive bilateral surplus. Private efficiency is a reasonable equilibrium requirement when rational workers and firms engage in long-term interactions [Barro, 1977].[6]

Steady-state equilibrium. In a steady state parameterized by technology a, given public employment g and replacement rate ρ, the equilibrium is characterized by eleven variables: wage w, private output y, public output p, aggregate employment n, unemployment u, private employment l, labor market tightness θ, consumption of employed workers c^e, consumption of unemployed workers c^u, labor tax rate τ, and unemployment benefit rate b. The eleven variables are determined by eleven relationships: wage schedule (6); production functions (1) and (4); accounting identities $n = l + g$ and $u = 1 - (1 - s) \cdot n$; Beveridge curve (3); firm's profit-maximization condition (2);

[6] In a stochastic environment, private efficiency may not be respected after large negative technology shock. But under a condition on the standard deviation of technology shocks and the wage flexibility γ, private efficiency holds with high enough probability [Michaillat, forthcoming].

resource constraint (5) and replacement rate definition $\rho = c^u/c^e$; budget constraints of employed and unemployed workers: $c^e = (1-\tau)\cdot w$ and $c^u = b\cdot w$. The government's budget constraint is redundant once workers' budget constraints and the resource constraint are accounted for.

Efficient allocation. The efficient allocation solves the problem of a benevolent planner who faces the technological constraints and labor market frictions present in the economy. Proposition A1 in the Appendix proves that in the efficient allocation, labor market variables (labor market tightness, public and private employment) are independent of technology a. When technology falls, productivities in the private and public sectors as well as recruiting costs all fall in concert. Thus the trade-offs between productions in the private and public sector and between production and job search are unaffected by technology fluctuations. As a result it is socially optimal to stabilize labor market variables completely, irrespective of the technology level.[7] In other words, unemployment spikes during recessions are socially inefficient. In response, the government uses fiscal policy in the form public employment to reduce unemployment.

3 Zero Fiscal Multiplier in the Absence of Job Rationing

The next sections characterize the fiscal multiplier in the presence of job rationing—the property that the labor market does not converge to full employment when matching frictions are arbitrarily small. Before presenting the core of the analysis, this section argues that in the absence of job rationing, the sheer presence of unemployment does not explain why fiscal policy is particularly effective in recessions. To illustrate this point, I confine my analysis to a relatively special case of the equilibrium unemployment model that shares the main features of the Pissarides [2000] model.[8] In this model public employment necessarily crowds out private employment one-for-one, whether wages are perfectly flexible or somewhat rigid. Fiscal policy cannot stimulate employment and the

[7]This result is not new to the literature. Blanchard and Galí [2010] shows that in a new-Keynesian model augmented with matching frictions in the labor market, the efficient allocation implies a constant unemployment rate over the business cycle. Shimer [2010] finds the same result in a neoclassical model augmented with matching frictions.

[8]Michaillat [forthcoming] proves that in the canonical equilibrium unemployment model of Pissarides [2000] and in the variant with rigid wages of Hall [2005], there is no job rationing.

fiscal multiplier is zero.

ASSUMPTION 1. The production function has constant marginal returns to labor: $\alpha = 1$.

ASSUMPTION 2. The wage w_t is determined by generalized Nash bargaining in any period t. Worker's bargaining power is $\chi \in (0,1)$.

These two assumptions capture the main features of the equilibrium unemployment model in Pissarides [2000]. The generalized Nash bargaining solution allocates a fraction χ of the match surplus to the worker and the rest to the firm. Lemma 1 characterizes the outcome of bargaining:

LEMMA 1. *Under Assumptions 1 and 2, the steady-state wage w satisfies (6) with $\gamma = 1$ and*

$$\omega = \frac{\chi}{1-\chi} \cdot \frac{1-\nu}{1-\rho^{1-\nu}} \cdot \left\{ [1-\delta \cdot (1-s)] \cdot \frac{r}{q(\theta)} + \delta \cdot (1-s) \cdot r \cdot \theta \right\},$$

where equilibrium labor market tightness θ is an implicit function of the parameters defined by

$$1 = \left[1 + \frac{\chi}{1-\chi} \cdot \frac{1-\nu}{1-\rho^{1-\nu}}\right] \cdot [1-\delta \cdot (1-s)] \cdot \frac{r}{q(\theta)} + \frac{\chi}{1-\chi} \cdot \frac{1-\nu}{1-\rho^{1-\nu}} \cdot \delta \cdot (1-s) \cdot r \cdot \theta. \quad (7)$$

In steady state the Nash-bargained wage is proportional to technology ($\gamma = 1$). Therefore the equilibrium system of two equations $\{(3), (7)\}$ and two variables $\{n, \theta\}$ is independent of technology. Fluctuations in technology do not lead to any fluctuations in labor market variables because the equilibrium wage is fully flexible.[9] Since the unemployment rate remains constant when technology fluctuates, there are no proper recessions in this model.

Even though there are no recessions, the rate of unemployment is not necessarily low: if workers have strong bargaining power, the unemployment rate could be high. In that case the government may want to reduce unemployment through public employment. Is that feasible? No it is not. The

[9] When technology falls, marginal product of labor, wage, and recruiting cost fall in proportion. Marginal product of labor and marginal cost of labor move in proportion with technology. Therefore the profit-maximizing level of private employment is invariant to technology: technology a drops out of equation (2). The wage moves in proportion to technology because the recruiting cost (which determines the outside option of the firm in bargaining) is proportional to technology and unemployment benefits (which determine the outside option of the worker in bargaining) are a constant fraction ρ of the post-tax wage. In slightly different models, Blanchard and Galí [2010] and Shimer [2010] also show that the unemployment rate is invariant to technology when wages are determined by Nash bargaining.

equilibrium system of two equations $\{(3), (7)\}$ and two variables $\{n, \theta\}$ is also independent of public employment g. Therefore public employment has no effect on aggregate employment. When public employment increases, it reduces private employment by the same amount and aggregate employment is unchanged: the fiscal multiplier is zero. Public jobs replace private jobs one-for-one with public jobs because aggregate employment is solely a function of tightness through the Beveridge curve (3), and tightness is solely a function of model parameters through (7), unaffected by public employment.

In fact the fiscal multiplier remains zero even if I replace the Nash-bargained wage of Assumption 2 by a rigid wage that satisfies the schedule (6) with $\gamma < 1$. The model with a rigid wage generates employment fluctuations when technology fluctuates.[10] But the equilibrium system remains independent of public employment. Therefore public employment cannot stimulate aggregate employment: the fiscal multiplier remains zero.

The result that public employment cannot stimulate aggregate employment is obviously a special one, arising in presence of a number of simplifying assumptions. For instance the result would no longer hold if public-sector workers produced public capital contributing to firm's productivity. Nevertheless, this section illustrates that fiscal policy is not necessarily effective in a model of equilibrium unemployment, even when unemployment is inefficiently high.

4 Fiscal Multiplier in the Presence of Job Rationing

In the models studied in the previous section, fiscal policy is ineffective. These models, however, do not provide a good description of recessionary unemployment. As argued by [Michaillat, forthcoming], these models converge to full employment when matching frictions are arbitrarily small. This property is at odds with the long queues of jobseekers observed in front of factory gates during the Great Depression. This section proposes a better model of recessions and shows that in this better model, fiscal policy does reduce unemployment. Following Michaillat [forthcoming], I

[10] The equilibrium system depends on technology a because $w/a = \omega \cdot a^{\gamma-1}$ in equation (2). When technology falls the normalized wage w/a increases and private employment falls.

make two assumptions on the production function and wage schedule faced by firms:

ASSUMPTION 3. The production function has diminishing marginal returns to labor: $\alpha < 1$.

ASSUMPTION 4. The wage schedule is somewhat rigid: $\gamma < 1$.

Under these assumptions, the model captures two key properties of recessions: (a) unemployment is higher in recessions; and (b) jobs are rationed in recessions, as some unemployment remains even if matching frictions vanish. Assumption 3 is motivated by the observation that at business cycle frequency, some production inputs adjust slowly. Assumption 4 is motivated by historical, ethnographic, and empirical studies that document and explain the sources of wage rigidity [for example, Bewley, 1999; Jacoby, 1984; Kahn, 1997; O'Brien, 1989].

I measure the effectiveness of fiscal policy with a fiscal multiplier, defined as the reduction in unemployment achieved by spending one unit of private good on public employment. I study the fiscal multiplier in two steps. In the first part of the section, I represent the steady-state equilibrium with a labor demand-labor supply diagram in a price θ-quantity n plan. This representation provides graphical intuition for the behavior of the model over the business cycle and the impact of fiscal policy. It also simplifies the theoretical characterization of the sign and cyclicality of the fiscal multiplier, in the second part of the section.

To represent the equilibrium in a labor demand-labor supply diagram, I need to modify the conventional definition of labor supply. In standard macroeconomic models, labor supply designates the number of hours of work that a worker supplies given economic conditions. But in this model the number of hours worked per worker is fixed. Instead, I use labor supply to designate the employment rate of a worker given labor market conditions. I define labor supply $n^s(\theta)$ as the employment rate that satisfies the Beveridge curve (3) for a given tightness θ. Labor supply $n^s(\theta)$ increases with θ, because the job-finding probability $f(\theta)$ increases with θ. I define the private labor demand $l^d(\theta, a)$ as the private-sector employment rate that satisfies the firm's profit-maximization condition (2). Assumption 3 implies that $l^d(\theta, a)$ is a well-defined, decreasing function of θ because the marginal product of labor $x'(l)$ decreases with l and the vacancy-filling probability $q(\theta)$ decreases with θ. When the labor market is slack, it is cheap for firms to recruit, stimulating hiring. Assumption 4 implies that $l^d(\theta,a)$ is a decreasing function of technology a because the normalized wage $w/a = \omega \cdot a^{\gamma-1}$ decreases with a. When technology is low, wages are relatively high, depressing hiring. Aggregate labor demand is the sum of public and private labor demands:

$$n^d(\theta,a,g) = g + l^d(\theta,a). \tag{8}$$

In presence of matching frictions, the wage itself is exogenous and cannot equalize labor supply and labor demand. Instead, tightness θ acts as a price equilibrating labor supply and labor demand:

$$n^s(\theta) = n^d(\theta,a,g). \tag{9}$$

Equation (9) implicitly defines equilibrium tightness θ. As technology a decreases, θ decreases because labor demand $n^d(\theta,a,g)$ is lower for all θ. Equilibrium employment n can be directly read off the labor demand curve:

$$n = n^d(\theta,a,g), \tag{10}$$

where equilibrium labor market tightness θ satisfies (9). As technology a decreases, n decreases.

Figure 1 depicts the equilibrium for high (left panel) and low (right panel) technology in a price θ-quantity n diagram. Equilibrium employment n and equilibrium tightness θ are given by the intersection of the downward-sloping labor demand curve with the upward-sloping labor supply curve. In a recession technology decreases and labor demand shifts inwards. The new equilibrium has lower aggregate and private employment, lower tightness, and higher unemployment. Labor market tightness equalizes labor supply and labor demand. While tightness acts as a price, it is not a price and the usual adjustment to reach equilibrium through bidding does not apply. Instead the adjustment to reach the labor market equilibrium is achieved through vacancy posting. For instance if labor demand is above labor supply at the current tightness, the number of vacancies posted by firms is not sufficient to hire the desired number of workers. Consequently firms post more vacancies to hire more workers. But, while more vacancies generate from matches and allow firms to hire more workers, they also increase labor market tightness. This increase reduces the probability to fill each vacancy and augments hiring costs. Higher hiring costs imply a higher marginal cost of labor, reducing firms' demand for labor. To summarize, firms increase labor market tightness and reduce the gap between labor supply and labor demand by posting more vacancies. This adjustment mechanism operates until the labor market equilibrium is reached. At this point tightness equalizes labor demand and labor supply. As I discuss the tightness adjustments necessary to reach equilibrium in the text, the reader must be aware that vacancy posting is the actual mechanism equalizing demand and supply in the labor market.

Figure 1 also illustrates job rationing. After a negative technology shock the marginal product of labor falls but the rigid wage adjusts downward only partially, so private labor demand shifts inward (left to right panel). If the adverse shock is large enough the marginal product of the last workers in the labor force, who are least productive due to diminishing marginal returns to labor, falls below the wage. It becomes unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$ because the labor demand curve cuts the x-axis at $n^R < 1$. Jobs are therefore rationed: the labor market fails to clear and some unemployment remains even when matching frictions vanish.[11] The private efficiency of existing worker-firm matches, however, is always respected. This is because the wage is always below the marginal product of employed workers; otherwise the firm would not hire them. On the other hand, the wage is above the marginal product of the least productive workers. But the firm never offers a job to these unproductive workers, who are never matched with the firm.

The fiscal multiplier λ is the marginal increase in aggregate employment achieved by spending one unit of private good on public employment:

$$\lambda = \frac{1}{mc} \cdot \frac{\partial n}{\partial g},$$

where $mc = w + [1 - \delta \cdot (1-s)] \cdot r \cdot a/q(\theta)$ is the per-period marginal cost of public employment. What is the size of the fiscal multiplier when technology varies over the business cycle ? The answer does not solely depend on the structure of the model. I must also specify the level of public employment associated with each technology level. One should aim to characterize the effects of a marginal increase in public employment over the business cycle, leaving public employment policy

[11] Formally when frictions vanish, recruiting costs $r \to 0$ and labor demand $n^d(\theta, a, g) \to n^R$.

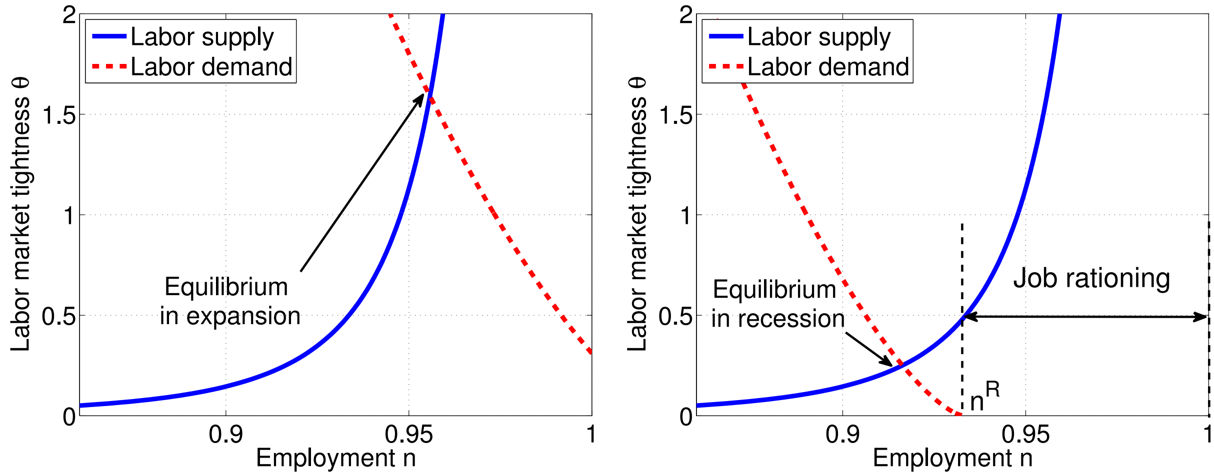

Figure 1: Steady-state equilibrium in a price θ-quantity n diagram

unchanged. But one must specify what exactly is assumed to be unchanged. Below I consider a policy experiment in which the share of public employment in total employment is unchanged over the business cycle.

ASSUMPTION 5. For any technology level a, the government sets $g = \zeta \cdot n$, where n is equilibrium employment and $\zeta \in (0,1)$.

Under Assumption 5, fiscal multiplier and comparative statics with respect to a are simple to calculate. Proposition 1 establishes a few properties of the fiscal multiplier λ:

PROPOSITION 1 (Multiplier in the basic model)**.**

(a) $\partial n/\partial g < 1$. *Under Assumption 3,* $\partial n/\partial g > 0$ *such that* $\lambda > 0$.

(b) Under Assumptions 3, 4, and 5, $d\left[\partial n/\partial g\right]/da < 0$ *such that* $d\lambda/da < 0$.

Part (a) shows that the change in aggregate employment dn following a marginal increase in public employment $dg > 0$ is necessarily less than dg, but is positive under Assumption 3. This result is illustrated in Figure 2. The government decides to recruit $dg > 0$ additional workers. In the figure, the labor demand curve shifts right by $dn^d = dg > 0$. At the current tightness, the level of labor supply is below the new level of labor demand. To reach the new equilibrium, tightness must increase by $d\theta > 0$. Effectively firms face more competition in recruiting because the government recruits from the same pool of unemployed workers as them. At this higher tightness, firms face a lower vacancy-filling probability, a higher hiring cost, and a higher marginal cost of labor, which forces them to reduce employment by $dl = \left(\partial l^d / \partial\theta\right) \cdot d\theta < 0$. This reduction is illustrated by a movement to the left along the demand curve. That is, public employment necessarily crowds out private employment because $dn = dl + dg < dg$. Under Assumption 3, however, public jobs crowd out private jobs strictly less than one-for-one and $dn = dg + dl > 0$. This is because if public jobs crowded out private jobs one-for-one, the new equilibrium would have the same labor market tightness θ but lower private employment l. The marginal cost of labor $w + [1 - \delta \cdot (1 - s)] \cdot (r \cdot a)/q(\theta)$ would remain constant, but the marginal product of labor $a \cdot x'(l)$ would be higher by diminishing marginal returns to labor. Hence the marginal product of labor would be above the marginal cost of labor, violating the firm's profit-maximizing condition (2).

While Part (a) shows that the fiscal multiplier λ is always positive, Part (b) shows that the multiplier is high in recessions when technology a is low and unemployment is high, and that it is low in expansions when technology is high and unemployment is low. That is, fiscal policy is especially effective to reduce unemployment when the labor market is depressed. This property arises mostly from the crowding-out of private employment by public employment being weaker in recessions.[12] This result is illustrated by comparing left (an expansion) and right (a recession) panels in Figure 2. Under Assumption 4, private and aggregate labor demands fall when technology falls because wages are somewhat rigid. The labor demand curve shifts left. Equilibrium labor market tightness and employment are lower in recession. What is the effect of public hiring in recession? Since the matching process is congested by the high volume of unemployment, each vacancy posted is filled with high probability. The government only needs to open a few additional vacancies to hire dg additional workers, barely rising tightness. The equilibrium increase $d\theta > 0$ in tightness is small, such that the reduction in private employment $dl = \left(\partial l^d / \partial\theta\right) \cdot d\theta < 0$ imposed by the increase in public employment is small. That is, hiring in the public sector does not displace hiring in the private sector much in recessions.

[12] The small decrease of the marginal cost of labor mc in recessions—because wage and hiring cost both fall—also contributes to the increase of the multiplier.

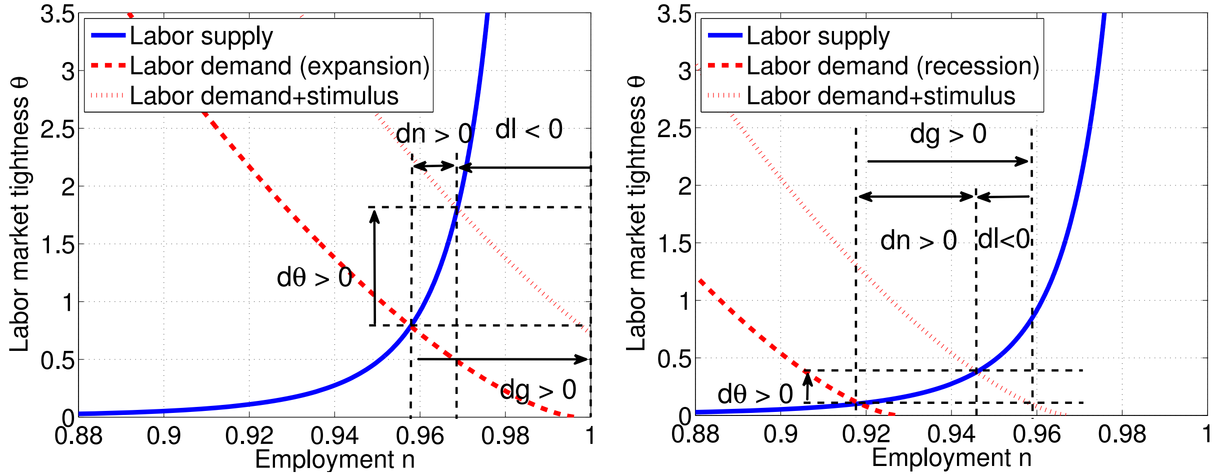

Figure 2: Effect of fiscal stimulus in the form of a marginal increase in public employment

The proof of the proposition is relegated to the Appendix, but I provide a sketch here. The core of the proof characterizes the marginal effect $\partial n/\partial g$ of public employment g on aggregate employment n. I define the tightness-elasticities $\varepsilon^s \equiv (\theta/n^s)\cdot(\partial n^s/\partial\theta) > 0$ and $\varepsilon^d \equiv -\left(\theta/n^d\right)\cdot\left(\partial n^d/\partial\theta\right) > 0$ of labor supply and labor demand. ε^d is normalized to be positive. First, I express $\partial n/\partial g$ as a function of ε^s and ε^d by differentiating (9) and (10):

$$\frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g}\cdot\left[1 - \frac{1}{1+\left(\varepsilon^s/\varepsilon^d\right)}\right]. \tag{11}$$

The formula says that the increase dn in aggregate employment following an increase dg in public employment equals the mechanical shift dn^d of labor demand attenuated by a factor $1/\left[1+\left(\varepsilon^s/\varepsilon^d\right)\right] < 1$. This factor captures the reduction dl in private employment caused by the equilibrium increase $d\theta$ in labor market tightness. The magnitude of the reduction depends on the elasticities ε^s and ε^d because they characterize the slopes of the labor demand and labor supply curves in Figure 2. As $\varepsilon^s > 0$ and $\varepsilon^d > 0$, $\partial n/\partial g \in (0,1)$. Second, I determine how $\partial n^d/\partial g$, ε^s, and ε^d vary over the cycle. From (8), $\partial n^d/\partial g = 1$, so the shift of labor demand is constant over the cycle. The proof shows that (a) ε^s is countercyclical because it moves in proportion with unemployment u; and (b) ε^d is procyclical because it moves in proportion with the share of the hiring cost in the marginal cost of labor (in recessions, this share is small because it is easy to hire workers). Hence $\varepsilon^s/\varepsilon^d$ as well as $\partial n/\partial g$ are countercylical.[13]

5 Extensions

In this section, I extend the model in three directions and examine how these extensions affect the fiscal multiplier. First, I allow workers to adjust their labor supply to labor market conditions. Second, I assume that the output of the public sector contributes to a stock of public capital that enters the production function of firms. Third, I allow the government to set a public-sector wage different from the private-sector wage.

5.1 Endogenous labor supply

In the basic model, workers have no control over their employment rate, which is determined by the separation rate s and the job-finding rate $f(\theta)$—both exogenous to the worker—through the Beveridge curve (3). But in practice people do have some control over their employment rate. For instance when unemployment benefits become less generous, unemployed workers search more intensively to find a job more rapidly.[14] It is possible that fiscal policy, by creating new jobs in the public sector and improving the prospects of jobseekers, stimulates labor supply.

To account for a possible response of labor supply to fiscal policy, I give workers control over their future employment rate through the choice of a job-search effort e_t.[15] At the beginning of period t, unemployed workers search for a job with effort e_t and incur a utility cost $z(e_t) = \omega_z \cdot e_t^{1+\kappa}/(1+\kappa)$, where $\omega_z > 0$, and $\kappa > 0$ are parameters. The number of matches is a Cobb-Douglas matching function of aggregate search effort $e_t \cdot u_t$ and vacancies o_t: $h_t = \omega_h \cdot (e_t \cdot u_t)^{\eta} \cdot o_t^{1-\eta}$. I

[13] The results from Section 3 also derive from (11). Under Assumption 1, private and aggregate labor demands l^d and n^d are perfectly elastic: $\varepsilon^d = +\infty$. Formula (11) implies that $\partial n/\partial g = 0$. In Figure 2, labor demand is horizontal.

[14] See Krueger and Meyer [2002] for an overview of the response of labor supply to unemployment insurance.

[15] The model with endogenous job-search effort closely follows Landais et al. [2010]. Alternatively, other papers give households control over their future employment rate through the allocation of non-employed workers between unemployment, which allows workers to find jobs, and inactivity out of the labor force, which provides leisure [for example, Bruckner and Pappa, forthcoming; Krusell et al., 2011; Shimer, 2011]. Both approaches are broadly equivalent for the task at hand.

redefine labor market tightness as $\theta_t \equiv o_t/(e_t \cdot u_t)$. $f(\theta_t)$ becomes the job-finding probability per unit of effort. That is, a jobseeker searching with effort e_t finds a job with probability $e_t \cdot f(\theta_t)$. $q(\theta_t)$ remains the vacancy-filling probability.

Given labor tax rate, unemployment benefit rate, wage, and tightness $\{\tau_t, b_t, w_t, \theta_t\}_{t=0}^{+\infty}$, the *worker's problem* is to choose search effort $\{e_t\}_{t=0}^{+\infty}$ to maximize utility

$$\sum_{t=0}^{+\infty} \delta^t \cdot \left\{(1-n_t^s) \cdot v(b_t \cdot w_t) + n_t^s \cdot v((1-\tau_t) \cdot w_t) - \left[1-(1-s) \cdot n_{t-1}^s\right] \cdot z(e_t)\right\},$$

subject to the law of motion of the probability n_t^s to be employed in period t

$$n_t^s = (1-s) \cdot n_{t-1}^s + \left[1-(1-s) \cdot n_{t-1}^s\right] \cdot e_t \cdot f(\theta_t).$$

I simplify derivations by assuming that the coefficient of relative risk aversion $\nu = 1$. In steady state, the worker's first-order condition with respect to effort e_t is

$$[1-\delta \cdot (1-s)] \cdot \frac{z'(e)}{f(\theta)} + \kappa \cdot \delta \cdot (1-s) \cdot z(e) = \ln\left(\frac{1}{\rho}\right). \tag{12}$$

$\rho = b/(1-\tau)$ is the replacement rate of the UI system.[16] This equation implicitly defines the supply of search effort $e^s(\theta)$ as an increasing function of labor market tightness θ.

The mechanism through which public employment reduces unemployment is almost identical to that in the basic model. The recruitment of $dg > 0$ additional workers by the government leads to an increase $d\theta > 0$ of labor market tightness. The only difference is that this increase raises not only the per-unit job-finding probability $f(\theta)$, but also unemployed workers' search effort $e^s(\theta)$. For a given $d\theta > 0$, the search effort increase $de > 0$ also contributes to the increase in labor supply. This effect appears transparently when I compute labor supply. Using the equality of flows into

[16]Unemployment benefit rate b and labor tax rate τ do not matter: the utility gap between employment and unemployment is solely determined by replacement rate ρ. Therefore the government can levy any amount of revenue without affecting search effort by reducing benefits, increasing taxes, while keeping the ratio $\rho = b/(1-\tau)$ constant. That is why I do not worry about the government's budget constraint when I study different policy interventions.

and out of unemployment in steady state:

$$n^s(\theta) = \frac{e^s(\theta) \cdot f(\theta)}{s + (1-s) \cdot e^s(\theta) \cdot f(\theta)}. \tag{13}$$

Labor supply $n^s(\theta)$ is the employment rate chosen by jobseekers through their choice of search effort, for a given tightness θ. $n^s(\theta)$ increases with θ, because both the supply of effort $e^s(\theta)$ and the per-unit job-finding probability $f(\theta)$ increase with θ. In figure 2, the labor supply would be steeper. Equivalently, the elasticity ε^s of labor supply with respect to tightness would be larger. The only consequence is that a smaller increase $d\theta > 0$ is required to re-establish equilibrium after the increase $dg > 0$ in public employment. Hence, the crowding out $dl = \left(\partial l^d / \partial \theta\right) \cdot d\theta < 0$ of private employment is bound to be smaller. The adjustment in search effort notwithstanding, Proposition 2 establishes that the properties derived in the basic model obtain:

PROPOSITION 2 (Multiplier with endogenous labor supply)**.** *Assume that* $\nu = 1$.

(a) $\partial n / \partial g < 1$. *Under Assumption 3,* $\partial n / \partial g > 0$ *such that* $\lambda > 0$.

(b) Under Assumptions 3, 4, and 5, $d\left[\partial n / \partial g\right] / da < 0$ *such that* $d\lambda / da < 0$.

The proof of the proposition is almost identical to that of Proposition 1. The only difference is that the elasticity ε^s, which also accounts for the response of search effort to a change in tightness, is no longer proportional to unemployment. But it remains procyclical and the proof carries through.

5.2 Public capital

In the basic model, the public good produced by public employment is consumed by workers. But in practice public employment may improve institutions or infrastructure, which in turn improve the productivity of firms. In fact, fiscal policy in the form of public employment often aims to improve infrastructure. That was the case with the New Deal under the Roosevelt administration. This is the case with the American Recovery and Reinvestment Act of 2009, which includes $44 billion for infrastructure expenditures on water quality, transportation, or housing, and $88 billion in federal spending on energy, innovative technology, or federal buildings [Leeper et al., 2010].

Public capital represents institutions or infrastructure. I follow Baxter and King [1993] to model public capital. I assume that in period t, public-sector workers produce a public good p_t that is invested in public capital k_{t+1}. The production function of public capital is

$$k_{t+1} = (1-\beta)\cdot k_t + p_t.$$

β is the capital depreciation rate. The firm's production function takes as argument private employment l_t and public capital k_t:

$$y_t = a\cdot k_t^{\xi}\cdot x(l_t)$$

a is technology, y_t is output of private good, $x(l) = l^{\alpha}$, and $\alpha \in (0,1)$ is a parameter. $\xi > 0$ is the elasticity of output with respect to public capital, which indicates the productiveness of public capital. To simplify the analysis I assume that public capital does not enter the production function of public good, which remains (4).

The mechanism through which public employment reduces unemployment is almost identical to that in the basic model. The only difference is that the recruitment of $dg > 0$ additional workers by the government leads not only to a mechanical increase in aggregate labor demand but also to an increase in private labor demand because firm's productivity is enhanced by the higher level of public capital. In steady state public capital k remains constant over time so $\beta\cdot k = p$. Therefore the relationship between public capital and public employment is

$$k = \frac{\omega_p}{\beta}\cdot a\cdot x(g), \tag{14}$$

The firm's profit-maximization condition (2) is modified because its production function depends on public capital. In steady state the firm's labor demand $l^d(\theta,a,g)$ solves

$$\left[\frac{\omega_p}{\beta}\cdot a\cdot x(g)\right]^{\xi}\cdot x'(l) = \frac{w}{a} + [1-\delta\cdot(1-s)]\,\frac{r}{q(\theta)}. \tag{15}$$

Private labor demand $l^d(\theta,a,g)$ increases with public employment g because the production function $x(g)$ increases with g whereas the marginal productivity $x'(l)$ decreases with l. In figure 2, the shift of the labor demand curve following an increase in public employment (a fiscal stimulus) would be larger. The response of private labor demand notwithstanding, Proposition 3 establishes that the properties of the fiscal multiplier derived in the basic model obtain:

PROPOSITION 3 (Multiplier with public capital)**.**

(a) $\partial n/\partial g < 1+\xi\cdot[\alpha/(1-\alpha)]\cdot[(1-\zeta)/\zeta]$. *Under Assumption 3,* $\partial n/\partial g > 0$ *such that* $\lambda > 0$.

(b) Under Assumptions 3, 4, and 5, $d\left[\partial n/\partial g\right]/da < 0$ *such that* $d\lambda/da < 0$.

The proof of the proposition is almost identical to that of Proposition 1. The only difference is that the direct effect of public employment on labor demand $\partial n^d/\partial g \neq 1$. Increasing public employment increases steady-state public capital, firm's marginal productivity, and firm's labor demand. Therefore public employment has a stronger desirable effect on aggregate labor demand:

$$\frac{\partial n^d}{\partial g} = 1+\xi\cdot\frac{\alpha}{1-\alpha}\cdot\frac{1-\zeta}{\zeta}. \tag{16}$$

But the direct effect $\partial n^d/\partial g$ remains a constant, and the proof carries through.

5.3 Public-sector wages

In the basic model, public-sector wages equal private-sector wages. But in practice, these wages may differ. On average public-sector jobs enjoy a wage premium compared to private-sector jobs.[17] Public-sector jobs created in recessions as part of a countercyclical fiscal policy could have wages below those of private-sector jobs. For example during the New Deal, relief work programs—administered first by the Federal Emergency Relief Administration (FERA) from 1933–1935, then by the Works Progress Administration (WPA) established in 1935—paid substantially lower hourly wage than private-sector jobs [Margo, 1991; Neumann et al., 2010].

I allow the public-sector wage w_t^g to differ from the private-sector wage w_t: $w_t^g = \pi_t\cdot w_t$, where the public-wage premium π_t may not be one. If $\pi_t < 1$, the public-sector wage is below the private-

[17] See Gregory and Borland [1999] for an overview of empirical evidence on the public-sector wage premium.

sector wage, as in the work relief programs of the New Deal. To simplify, all unemployed workers receive the same unemployment benefits $b_t \cdot w_t$.

Since wages differ across sectors, unemployed workers direct their search either to the private sector or to the public sector, as in Quadrini and Trigari [2007] and Gomes [2010]. Their choice depends on the job-finding probability and wage in each sector. Let σ_t be the share of unemployed workers searching in the public sector. Let θ_t^g be the labor market tightness in the public sector, and θ_t the labor market tightness in the private sector. The government attracts jobseekers to the public sector by choosing the public-wage premium π_t. To simplify derivations, I assume that π_t is chosen such that the ratio of job-finding probabilities $\psi \equiv f(\theta_t^g)/f(\theta_t)$ be constant at any time and for any technology level. In a steady state equilibrium the optimal search behavior of unemployed workers across labor markets creates a relationship between ψ and π, and the Appendix shows how to recover π from a choice of ψ. In particular, $\pi < 1$ if and only if $\psi > 1$: it takes longer to find jobs in the private sector when private-sector jobs are more attractive.

When labor markets are in steady state:

$$\sigma \cdot u \cdot f(\theta^g) = s \cdot g \tag{17}$$

$$(1-\sigma) \cdot u \cdot f(\theta) = s \cdot l. \tag{18}$$

$\sigma \cdot u$ is the number of unemployed workers searching in the public sector, and $f(\theta^g)$ is their probability to find a job, so $\sigma \cdot u \cot f(\theta^g)$ is the number of unemployed workers who find a job in the public sector each period. $s \cdot g$ is the number of public-sector jobs destroyed each period. The first equation says that in steady state, flows into and out of public employment balance each other. The second equation is the same condition for private employment. Taking the ratio of (17) and (18) gives an expression for σ:

$$\sigma = \frac{1}{1+\psi \cdot (l/g)}. \tag{19}$$

Summing equations (17) and (18) and recombining them, I can express labor supply $n^s(\theta,\sigma)$ as a function of the tightness θ in the private sector and the share σ of unemployed workers in the public sector:

$$n^s(\theta, \sigma) = \frac{[1 + \sigma \cdot (\psi - 1)] \cdot f(\theta)}{s + (1 - s) \cdot [1 + \sigma \cdot (\psi - 1)] \cdot f(\theta)}. \tag{20}$$

$n^s(\theta, \sigma)$ gives workers' employment rate when unemployed workers search optimally across the public and private sectors, for a given tightness θ in the private sector. Labor supply $n^s(\theta, \sigma)$ increases with θ and, when the ratio $\psi > 1$, with σ. When $\psi > 1$, the job-finding rate is higher in the public sector than in the private sector. When the share σ of unemployed workers searching the public sector increases, the number of unemployed workers finding a job each period mechanically increases because it is faster to find a job in the public sector. Thus the employment rate increases.

In spite of the presence of two distinct labor markets, the structure of the equilibrium is quite similar to that in the basic model. The main difference is that that there is one additional equilibrium variable—the share σ of unemployed workers searching the public sector—accompanied by one additional equilibrium relationship. Given technology a, public employment g, and ratio ψ, the equilibrium is described by three variables: aggregate employment n, labor market tightness in the private sector θ, and σ. These variables are determined by three relationships: (19) (replace l by $n - g$), (10), and the equilibrium condition $n^s(\theta, \sigma) = n^d(\theta, a, g)$. The mechanism through which public employment reduces unemployment can be represented with a diagram similar to those in Figure 2. The government aims to recruit $dg > 0$ additional workers, which shifts the labor demand curve to the right by $dn^d = dg > 0$. There is an additional effect: newly created public-sector jobs attract jobseekers towards the public sector, where they find jobs more rapidly. This reallocation of jobseekers from the private to the public sector increases the share σ (as appears in (19)), and shifts the labor supply curve $n^s(\theta, \sigma)$ to the right. The crowding-out of private-sector jobs by public-sector jobs is mitigated by the labor supply shift.

Proposition 4 establishes that the fiscal multiplier has the same properties as in the basic model as long as it is easier to find jobs in the public sector than in the private sector (which implies that wages are lower in the public sector than in the private sector):

PROPOSITION 4 (Multiplier with public-sector wages)**.** *Assume that* $\psi \geq 1$.

(a) $\partial n / \partial g < 1$. *Under Assumption 3,* $\partial n / \partial g > 0$ *such that* $\lambda > 0$.

(b) Under Assumptions 3, 4, and 5, $d[\partial n/\partial g]/da < 0$ *such that* $d\lambda/da < 0$*.*

The proof of the proposition is somewhat different to that of Proposition 1. The marginal effect of public employment g on total employment n is

$$\frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g} \cdot \left[1 - \frac{1}{1 + \left(\varepsilon^s/\varepsilon^d\right)}\right] + \frac{1}{1 + \left(\varepsilon^s/\varepsilon^d\right)} \cdot \frac{\partial n^s}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial g}$$

instead of (11). This expression takes into account the shift in labor supply $n^s(\theta, \sigma)$ when the share σ increases after an increase $dg > 0$ in public employment. The proof characterizes $\partial n^s/\partial \sigma$ and $\partial \sigma/\partial g$ before characterizing the multiplier λ using the expression above for $\partial n/\partial g$.

6 Calibration

This section calibrates the model with US data to illustrate numerically the cyclical fluctuations of the fiscal multiplier. While these simulations are too simple to estimate quantitatively the effects of actual policy interventions, they illustrate how much the effectiveness of fiscal policy varies over the business cycle, and how the effectiveness is influenced by various features of the model. I also compare the outcome of the simulations with some empirical estimates of the fiscal multiplier.

I calibrate all parameters at a weekly frequency as summarized in Table 1. I calibrate as many parameters as possible directly from micro-evidence and macro-data for the December 2000–June 2010 period. Following Michaillat [forthcoming], I set $\delta = 0.999$, $s = 0.0094$, $r = 0.32 \cdot \omega$, and $\gamma = 0.7$. Following Landais et al. [2010], I set $\eta = 0.7$, $\nu = 1$, and $\rho = 65\%$. Following common practice, I set $\alpha = 0.66$. I calibrate ζ to the average share of public employment in total employment in the seasonally-adjusted, monthly data from the Current Employment Survey (CES) collected by the Bureau of Labor Statistics (BLS). Public employment is the employment level in the government super sector. Total employment is the employment level in the total nonfarm super sector. I find $\zeta = 0.16$. I calibrate the remaining parameters by matching the steady-state values of variables in the model to the average of their empirical counterpart. I normalize steady-state technology $\bar{a} = 1$, steady-state effort $\bar{e} = 1$, steady-state public capital $\bar{k} = 1$. Following Landais et

Table 1: Parameter values in simulations (weekly frequency)

	Interpretation	Value	Source
δ	Discount factor	0.999	Corresponds to 5% annually
s	Separation rate	0.94%	JOLTS, 2000–2010
η	Unemp.-elasticity of matching	0.5	Petrongolo and Pissarides [2001]
r	Recruiting cost	0.23	Barron et al. [1997], Silva and Toledo [2009]
ζ	Share of public employment	0.16	CES, 2000–2010
γ	Real wage flexibility	0.7	Haefke et al. [2008]
ν	Relative risk aversion	1	Chetty [2006]
ρ	UI replacement rate	65%	Pavoni and Violante [2007]
α	Marginal returns to labor	0.66	Convention
ω_h	Effectiveness of matching	0.19	Matches JOLTS, 2000–2010
ω	Steady-state real wage	0.70	Matches JOLTS, 2000–2010
	With endogenous labor supply		
κ	Elasticity of disutility of effort	2.1	Matches micro-elasticity of 0.9 [Meyer, 1990]
ω_z	Disutility of effort	0.58	Matches effort of 1 for $\rho = 65\%$
	With public capital		
ξ	Productivity of public capital	0.05	Baxter and King [1993], Leeper et al. [2010]
β	Depreciation rate of public capital	0.21%	Baxter and King [1993], Leeper et al. [2010]
ω_p	Relative public-sector productivity	0.71%	Matches public capital of 1
	With public-sector wages		
ψ	Job-finding rate ratio $f(\theta^g)/f(\theta)$	2	Arbitrary policy choice

al. [2010], I set $\overline{\theta} = \overline{v}/\overline{u} = 0.47$ and $\overline{u} = 5.9\%$, which implies $\overline{n} = 0.950$, $\overline{g} = 0.157$, $\overline{l} = 0.793$, and which allows me to recover $\omega_h = 0.19$, $\omega = 0.70$, and $r = 0.32 \cdot \omega = 0.23$.

Figure 3 plots the fiscal multiplier λ times the gross domestic product (GDP) for a series of technology levels, in a variety of models.[18] A graph represents the reduction of unemployment rate (measured in percentage points) achieved by spending 1% of GDP on public employment as a function of the corresponding equilibrium unemployment rate. In the basic model (black solid line), the fiscal multiplier is strongly countercyclical: it increases fourfold from 0.27 to 1.15 when

[18] GDP is defined as $a \cdot x(l) + w^g \cdot g - (r \cdot a)/q(\theta) \cdot (s \cdot n)$.

the unemployment rate increases from 4% to 10%.

The other graphs describe fiscal multipliers in the three extensions of the basic model. All the parameters calibrated in the basic model remain valid in these extensions. In the extension with endogenous labor supply, I also set $\kappa = 2.1$ and $\omega_z = 0.58$, following Landais et al. [2010]. The multiplier (red line with diamonds) is almost identical to that in the basic model. The reason is that search effort e is quite inelastic with respect to θ, such that the adjustment in search effort when the government creates new jobs in the public sector is small. In the extension with public capital, I calibrate the depreciation rate of public capital at 2.5% at quarterly frequency as Leeper et al. [2010] and Baxter and King [1993], implying $\beta = 0.21\%$ at weekly frequency. I use (14) to set the relative productivity of the public sector at $\omega_p = \beta \cdot \bar{k}/\bar{g}^{\alpha} = 0.71\%$. The productivity of public capital ξ is critical to determine the size of the fiscal multiplier. A large literature studies the impact of public capital on economic outcomes and social welfare, but no consensus emerges.[19] Given the lack of reliable estimates for ξ, I follow Baxter and King [1993] and Leeper et al. [2010] and plot the multiplier for $\xi = 0.05$. The multiplier (blue dashed line) is always about 50% larger than in the basic model because $(1-\alpha)+\xi \cdot \alpha \cdot (1-\zeta)/\zeta = 0.50$: the multiplier increases about fourfold from 0.41 to 1.68 when unemployment increases from 4% to 10%. As a robustness check, I plot the multiplier for $\xi = 0.1$ (blue dashed line with dots): the multiplier is larger but it retains its cyclical properties. Finally in the extension with public-sector wages, I plot the multiplier for an arbitrary choices of $\psi = f(\theta^g)/f(\theta)$. I choose $\psi = 2$, which entails a share $\sigma = 9.0\%$ of jobseekers in the public sector. The value of ψ is greater than one so wages are lower (by about 2%) in the public sector to make workers indifferent between searching in the two sectors. Compared to the basic model, the multiplier (green dot-dashed line with circles) is higher but its cyclical fluctuations are milder: when unemployment increases from 4% to 10%, the multiplier increases only threefold from 0.35 to 1.19. As a robustness check, I plot the multiplier for $\psi = 5$ (green dot-dashed line): the multiplier is slightly larger but it retains its cyclical properties.

Quantitatively my calibration suggests that for the average US unemployment rate of 6%, the multiplier could be in the 0.6–1.4 range, depending on the specificities of the model. How do these

[19] See for instance Aschauer [1989], Garcia-Milà and McGuire [1992], Holtz-Eakin [1993], Evans and Karras [1994], and Nadiri and Mamuneas [1994].

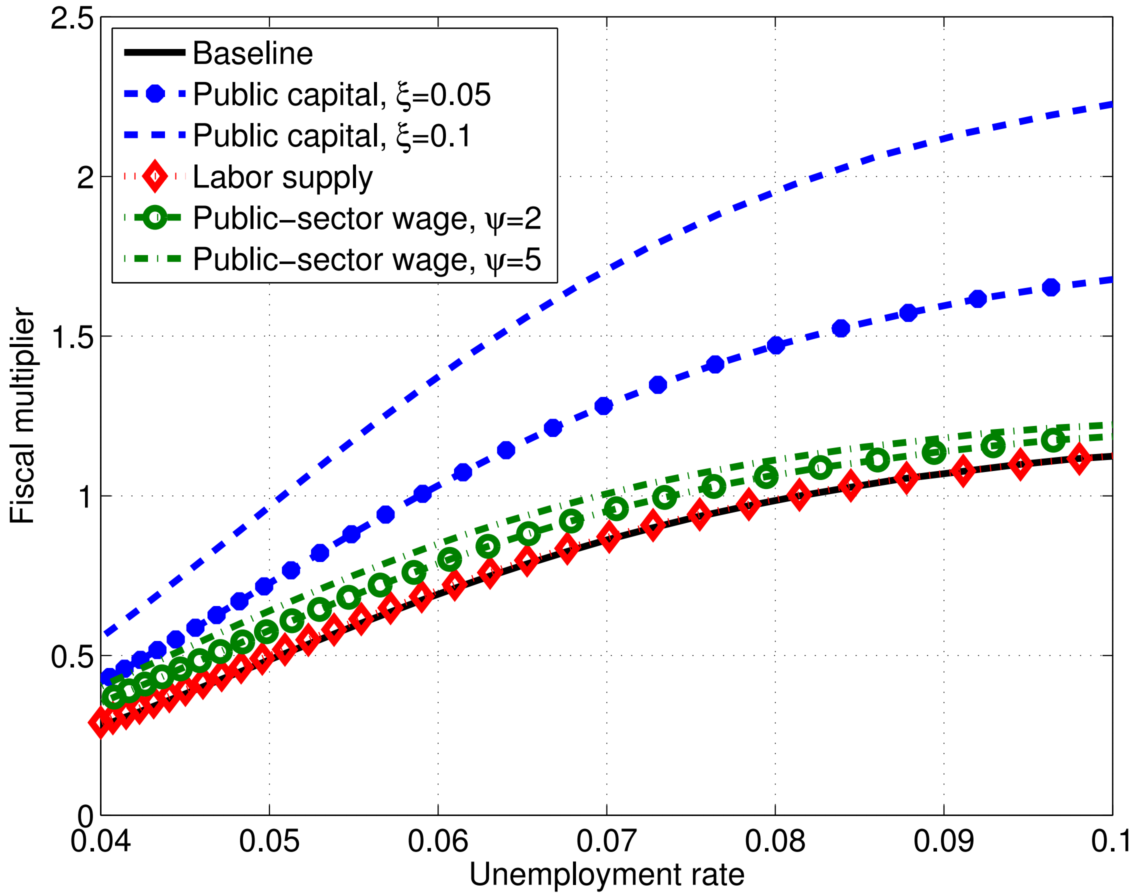

Figure 3: Fluctuations of the fiscal multiplier over the business cycle

Notes: The graphs represent a measure of the fiscal multiplier: the reduction of unemployment rate (measured in percentage points) achieved by spending 1% of GDP on public employment. Calibration is in Table 1. Multipliers are computed using expressions collected in Section C in the Appendix.

values match empirical estimates? Monacelli et al. [2010] use a structural vector autoregression (SVAR) framework to estimate the fiscal multiplier in the US for the 1954–2006 period. They find that an increase in government spending on goods and services by 1% of GDP increases the employment rate by 0.6 percentage point at the peak, which implies a multiplier of 0.6. Using a SVAR with alternative identification restrictions in US data for the 1970–2008 period, Pappa [2010] finds that an increase in government spending on public employment by 1% of GDP increases the employment rate by 1.4 percentage point at the peak, which implies a multiplier of

1.4.[20] These empirical estimates fall in the range suggested by my simulations.[21]

My calibration also suggests that when the unemployment rate is 10% the fiscal multiplier is three to four times higher than when the unemployment rate is 4%. Do we find empirical evidence that the fiscal multiplier is strongly countercyclical? Unfortunately empirical evidence about the behavior of fiscal multipliers over the business cycle is still scarce, because empirical studies do not usually account for the state of the economy when fiscal measures are enacted [Parker, 2011]. While more empirical work is required to reach a consensus about the effects of fiscal policies over the business cycle, recent studies suggest that fiscal multipliers may indeed be countercyclical. For instance using a regime-switching SVAR model in US data for the 1947–2009 period, Auerbach and Gorodnichenko [2010] find that in recessions fiscal multipliers are at least twice as large than in expansions. Auerbach and Gorodnichenko [2011] confirm these results using data for a large set of OECD countries and allowing the multiplier to vary smoothly with the state of the economy. Holden and Sparrman [2011] also investigate empirically the effect of government spending on unemployment in a large set OECD countries for the 1960–2007 period. They confirm that higher government spending leads to lower unemployment, and that the effect is greater in recessions. At the state level in the US, Nakamura and Steinsson [2011] investigate the increase in employment following government spending during military build-ups. They find that in high-unemployment periods the increase is roughly twice larger than in low-unemployment periods. The estimated increases of the fiscal multiplier between low-unemployment and high-unemployment regimes are consistent with my simulations: in the basic model, the multiplier increases twofold from 0.5 to 1 when the unemployment rate increases from 5% (low-unemployment regime) to 8% (high-unemployment regime); in the extensions, the multiplier increases are commensurable.

[20] Pappa [2010] finds that a 1% increase in government spending on public employment induces a 0.15% increase in employment at the peak. I translated her results using the facts that on average in the US over the 1970–2008 period, the employment rate is 94% and the spending on public employment is 10% of GDP.

[21] The SVAR methodology estimates the average fiscal multiplier over all possible economic states. Accordingly, I compare the SVAR estimates with the fiscal multiplier in my model at the average US unemployment rate of 6%.

7 Discussion

During the Great Depression, the Roosevelt administration was concerned that public jobs newly created as part the New Deal might make it more difficult for private firms to hire workers by taking away job applicants [Neumann et al., 2010]. The paper directly addresses this practical concern by proving that in recessions the crowding out of private jobs by public jobs is much weaker than in expansions. The paper also suggests that the Roosevelt administration responded appropriately by hiring workers as part of large infrastructure projects, and by paying relief-job wages typically well below private-sector wages.

Besides its practical implications, the paper provides a theoretical foundation for the critique in Parker [2011] that most available estimates of the fiscal multiplier do not apply in recessions. In the calibrated model, recessionary multipliers are much higher than average multipliers: in the basic model the multiplier is 1.15 when unemployment is 10%, although it is only 0.70 when unemployment is at its average level of 6%. As pointed out by Parker [2011], most empirical studies estimating fiscal multipliers do not control for the economic conditions when fiscal policies are implemented; and most macroeconomic studies estimate fiscal multipliers by computing impulse response functions in models linearized around their steady state. The resulting estimates are only valid in average economic conditions. For example, these methods would broadly yield a multiplier of 0.70 in my model. This value does not convey much useful information to policymakers who ponder whether enacting a fiscal stimulus in a recession, because the recessionary multiplier is 1.15 and not 0.70.[22]

[22] The countercyclicality of the fiscal multiplier is quite general. It does not only hold when fiscal policy takes the form of public employment: for instance, Proposition A2 in the Appendix shows that the fiscal multiplier is also countercyclical when fiscal policy takes the form of a wage subsidy. For two specific reasons, however, I am more hesitant about the effectiveness of a wage subsidy than about the effectiveness of public employment. First, a wage subsidy may not be effective if demand shocks drive fluctuations. Assume that firms face a downward-sloping goods demand, that prices are completely sticky in the short run, and that demand falls. In that case firm's labor demand is fully determined by goods demand and production function so that a wage subsidy is ineffective. Second, the impact of a wage subsidy depends on the bargaining mechanism between firm and workers. If workers capture all wage subsidy transferred by the government to firms, then the post-tax wage faced by firms is unaffected by the subsidy, and the subsidy is completely ineffective. On the contrary, the effectiveness of public employment is to the source of job rationing, the source of aggregate fluctuations, and the wage-setting mechanism (as long as the creation of public-sector jobs does not lead to drastic increases in private-sector wages).

Several restrictions, however, limit the degree to which the theory moves us toward a full understanding of the role for fiscal policy in recessions. A weakness that the theory shares with other macroeconomic models relying on rigid wages is that wages adhere to an exogenous wage schedule. As discussed in Michaillat [forthcoming], this rigid schedule is theoretically valid because any privately efficient wage is a possible Nash equilibrium; and it is empirically valid because it is calibrated with the estimate of wage rigidity obtained by Haefke et al. [2008] using micro-data on wages for new hires. But this wage does not resolve the indeterminacy of the outcome of wage setting, and as such it cannot capture the influence of fiscal policy on private-sector wages. If private-sector wages, and thus private labor demand, respond markedly to conditions on public-sector jobs as in Quadrini and Trigari [2007] and Gomes [2010], the fiscal multiplier should be amended to capture this influence. Future research should design a richer yet tractable wage-setting mechanism explaining the wage rigidity observed in the data to improve our understanding of the role for fiscal policy. Some papers offer a promising start on this research agenda: Hall and Milgrom [2008] and Menzio [2005] propose theories of wage rigidity based on bargaining; Rudanko [2009] describes wage rigidity as the result from implicit contracts between firm and workers; an Elsby [2009] models wage rigidity as the result of workers' loss-aversion.

Another limitation that the model shares with most equilibrium unemployment models is that the only source of recessions is a technology shock. In reality, this is not plausible. The contraction experienced after the 2008 financial crisis is not apparently caused by a collapse of productivity: in the US, the BLS reports that while multifactor productivity in the private non-farm business sector fell in 2008, it was constant in 2009 and it rose sharply during 2010. Future work should explore how other shocks influence the effectiveness of fiscal policy. Demand shocks and financial shocks are promising candidates [Barnichon, 2010; Christiano et al., 2007]. Existing papers already examine the influence of non-technology shocks in macroeconomic models of equilibrium unemployment: a monetary policy shock in Walsh [2003] and Trigari [2009]; shocks to monetary policy, preferences, investment, government spending, price markup, and workers' bargaining power in Gertler et al. [2008]. I do not anticipate that alternative shocks would change the results as long as jobs are rationed in recessions, because the behavior of the multiplier does not rely on the source of the shock but on labor market conditions in recessions.

To conclude, although my analysis builds on a dynamic model, it focuses on its steady state. It would be instructive to study the role for fiscal policy in a stochastic environment. A difficulty arising in a stochastic environment is that there is no standard method to study, analytically or numerically, fiscal multipliers over the business cycle. Numerical estimates of the fiscal multiplier are usually obtained by computing impulse response functions after a government spending shock in a macroeconomic model log-linearized around its deterministic steady state. The resulting multiplier is only valid around the steady state and not in recessions. Studying fiscal multipliers over the business cycle would require new numerical methods. In presence of stochastic fluctuations, it would be interesting to explore various avenues: the financing of government spending using debt in recessions; investment and accumulation of private capital; and borrowing and saving by workers such that workers are able to substitute consumption intertemporally.[23]

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[23] In the current model all workers behave as the "rule-of-thumb" consumers of Campbell and Mankiw [1990] who consume their entire income each period. This assumption simplifies the analysis, but it is also supported by a large empirical literature highlighting substantial deviations from the permanent income hypothesis by providing evidence of "excessive" dependence of consumption on current income [for example, Card et al., 2007; Parker, 1999; Zeldes, 1989]. Future research could move beyond a model with rule-of-thumb consumers only and introduce a mix of rule-of-thumb and permanent-income workers as in Galí et al. [2007].

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Appendix

A Some Derivations

A.1 Extension: endogenous labor supply

The Lagrangian of the worker's problem described in Section 5.1 is

$$\sum_{t=0}^{+\infty} \delta^t \Big\{ (1-u_t)\cdot v((1-\tau_t)\cdot w) + u_t \cdot [z(e_t) + v(b_t \cdot w) + e_t \cdot f(\theta_t)\cdot [v((1-\tau_t)\cdot w) - v(b_t\cdot w)]]$$

$$+ A_t \cdot \left[u_t \cdot e_t \cdot f(\theta_t) + (1-s)\cdot n^s_{t-1} - n^s_t\right] \Big\},$$

where n^s_t is the probability to be employed in period t, $u_t \equiv 1-(1-s)\cdot n^s_{t-1}$ is the probability of being unemployed at the beginning of period t, and $\{A_t\}$ are Lagrange multipliers. The first-order condition with respect to effort e_t gives (after dividing by $u_t \cdot f(\theta_t)$):

$$\frac{z'(e_t)}{f(\theta_t)} = [v((1-\tau_t)\cdot w) - v(b_t \cdot w)] + A_t.$$

The first-order condition with respect to the probability of being employed n^s_t yields:

$$A_t = \delta \cdot (1-s)\cdot \{z(e_{t+1}) + [A_{t+1} + [v((1-\tau_{t+1})\cdot w) - v(b_{t+1}\cdot w)]]\cdot [1-e_{t+1}\cdot f(\theta_{t+1})]\}$$

Merging the two first-order conditions together yields:

$$\frac{z'(e_t)}{f(\theta_t)} = [v((1-\tau_t)\cdot w) - v(b_t\cdot w)] + \delta\cdot(1-s)\cdot\left[z(e_{t+1}) + \frac{z'(e_{t+1})}{f(\theta_{t+1})}\cdot[1-e_{t+1}f(\theta_{t+1})]\right]$$

When $\nu = 1$, l'Hospital's rule implies that $v((1-\tau_t)\cdot w) - v(b_t \cdot w) = \ln((1-\tau_t)/b_t) = \ln(1/\rho)$. With the isoelastic disutility of effort, $e_{t+1}\cdot z'(e_{t+1}) = (1+\kappa)\cdot z(e_{t+1})$. Thus the optimal effort function therefore satisfies equation (12).

A.2 Extension: public-sector wages

LEMMA A1. *In steady state, $\psi \equiv f(\theta^g)/f(\theta) \geq 1$ if and only if $\pi \equiv w^g/w \leq 1$.*

Proof. Let $\mathcal{U}_t$ denote the present discounted value of unemployment (whether the unemployed worker searches for a job in the public or private sector). Let $\mathcal{W}^g_t$ denote the present discounted value of employment in the public sector. Let $\mathcal{W}_t$ denote the present discounted value of employment in the private sector. These values are related by:

$$\begin{aligned}
\mathcal{U}_t &= f(\theta_t^g)\cdot \mathcal{W}_t^g + \left[1-f(\theta_t^g)\right]\cdot[v(b_t\cdot w)+\delta\cdot \mathcal{U}_{t+1}] \\
\mathcal{U}_t &= f(\theta_t)\cdot \mathcal{W}_t + [1-f(\theta_t)]\cdot[v(b_t\cdot w)+\delta\cdot \mathcal{U}_{t+1}] \\
\mathcal{W}_t &= v((1-\tau)\cdot w)+\delta\cdot[(1-s)\cdot \mathcal{W}_{t+1}+s\cdot \mathcal{U}_{t+1}] \\
\mathcal{W}_t^g &= v((1-\tau)\cdot\pi\cdot w)+\delta\cdot\left[(1-s)\cdot \mathcal{W}_{t+1}^g+s\cdot \mathcal{U}_{t+1}\right]
\end{aligned}$$

In steady state, these values are constant over time: $\mathcal{U}_t=\mathcal{U}$, $\mathcal{W}_t^g=\mathcal{W}^g$, $\mathcal{W}_t=\mathcal{W}$. Hence:

$$\psi\cdot[\mathcal{W}^g-[v(b\cdot w)+\delta\cdot \mathcal{U}]] = \mathcal{W}-[v(b\cdot w)+\delta\cdot \mathcal{U}] \tag{A1}$$

$$\mathcal{W}-\mathcal{W}^g = \frac{v((1-\tau)\cdot w)-v((1-\tau)\cdot\pi\cdot w)}{1-\delta\cdot(1-s)}. \tag{A2}$$

Therefore $\pi\geq 1 \Leftrightarrow \mathcal{W}^g\geq \mathcal{W}$ (using (A2), since utility $v(\cdot)$ is increasing) $\Leftrightarrow \mathcal{W}^g-[v(b\cdot w)+\delta\cdot \mathcal{U}]\geq \mathcal{W}-[v(b\cdot w)+\delta\cdot \mathcal{U}]>0$ (since $1>\rho$, $\mathcal{W}>\mathcal{U}$ which imposes $\mathcal{W}>[v(b\cdot w)+\delta\cdot \mathcal{U}]$ because $\mathcal{U}$ is a convex combination of $\mathcal{W}$ and $[v(b\cdot w)+\delta\cdot \mathcal{U}]$) $\Leftrightarrow \psi\leq 1$ (using (A1)). □

To recover the wage premium π from ψ and θ, notice that (after some algebra) $\hat{\mathcal{U}}\equiv \mathcal{U}/v((1-\tau)\cdot w)$, $\hat{\mathcal{W}}\equiv \mathcal{W}/v((1-\tau)\cdot w)$, $\hat{\mathcal{W}}^g\equiv \mathcal{W}^g/v((1-\tau)\cdot w)$, and $\pi^{1-\nu}$ are related by a linear system:

$$\begin{bmatrix}
[1-\delta\cdot(1-\psi\cdot f(\theta))] & 0 & -\psi\cdot f(\theta) & 0 \\
[1-\delta\cdot(1-f(\theta))] & -f(\theta) & 0 & 0 \\
-\delta\cdot s & [1-\delta\cdot(1-s)] & 0 & 0 \\
-\delta\cdot s & 0 & [1-\delta\cdot(1-s)] & -1
\end{bmatrix}\cdot\begin{bmatrix}\hat{\mathcal{U}}\\ \hat{\mathcal{W}}\\ \hat{\mathcal{W}}^g\\ \pi^{1-\nu}\end{bmatrix} = \begin{bmatrix}[1-\psi\cdot f(\theta)]\cdot\rho^{1-\nu}\\ [1-f(\theta)]\cdot\rho^{1-\nu}\\ 1\\ 0\end{bmatrix}.$$

The matrix on the left-hand side is invertible. Given $f(\theta)$ and ψ, this linear system uniquely pins down $\pi^{1-\nu}$ and the wage premium π. To keep the ratio $\psi=f(\theta^g)/f(\theta)$ constant, the wage premium needs to be adjusted over the business cycle.

B Proofs

B.1 Proof of Lemma 1

Under Assumption 2, wages $\{w_t\}_{t=0}^{+\infty}$ are determined using the generalized Nash solution to the bargaining problem faced by a firm-worker pair. $\mathcal{E}_t$ denotes the value of being employed, and $\mathcal{U}_t$ the value of being unemployed (both are evaluated after the matching process):

$$\begin{aligned}
\mathcal{E}_t &= v\left((1-\tau_t)\cdot w_t\right)+\delta\cdot[\{1-s\cdot(1-f(\theta_{t+1}))\}\,\mathcal{E}_{t+1}+s\cdot(1-f(\theta_{t+1}))\cdot \mathcal{U}_{t+1}] \\
\mathcal{U}_t &= v\left(b_t\cdot w_t\right)+\delta\cdot[(1-f(\theta_{t+1}))\cdot \mathcal{U}_{t+1}+f(\theta_{t+1})\cdot \mathcal{E}_{t+1}].
\end{aligned}$$

These continuation values are the sum of current payoffs, plus discounted continuation values. Combining both conditions yields the worker's surplus $\mathcal{W}_t$ from a relationship with a firm:

$$\mathcal{W}_t = \mathcal{E}_t - \mathcal{U}_t = [v((1-\tau_t)\cdot w_t) - v(b_t \cdot w_t)] + \delta \cdot (1-s) \cdot [1 - f(\theta_{t+1})] \cdot \mathcal{W}_{t+1}. \tag{A3}$$

Since workers consume exactly the post-tax wage $(1-\tau_t)\cdot w_t$ in the current period, a wage brings utility $v((1-\tau_t)\cdot w_t)$ to the employed worker in the current period and

$$\frac{d\mathcal{W}_t}{dw_t} = (1-\tau_t)\cdot v'((1-\tau_t)\cdot w_t) = (1-\tau_t)^{1-\nu}\cdot v'(w_t).$$

In equilibrium the firm's surplus from an established relationship is simply given by the hiring cost since a firm can immediately replace a worker at that cost during the matching period: $\mathcal{F}_t = r\cdot a/q(\theta_t)$. Assume that wages are continually renegotiated. Then the wage bargained in the current period only influences payoffs in the current period. Accordingly, since the firm's utility is simply its profits, a wage w_t brings a utility $-w_t$ to the firm (or its owners) and $d\mathcal{F}_t/dw_t = -1$.

The generalized Nash solution to the bargaining problem faced by a firm-worker pair is the wage w_t that maximizes

$$\mathcal{W}_t(w_t)^{\chi}\cdot \mathcal{F}_t(w_t)^{1-\chi},$$

where χ is the worker's bargaining power. The first-order condition of the maximization problem implies that the worker's surplus each period is related to the firm's surplus by

$$\mathcal{W}_t = \frac{\chi}{1-\chi}\cdot\frac{d\mathcal{W}_t}{dw_t}\cdot\mathcal{F}_t.$$

Plugging this relationship into the recursive equation (A3) for the worker's surplus $\mathcal{W}_t$, and using the above expressions for $\mathcal{F}_t$ and $d\mathcal{W}_t/dw_t$, I obtain the relationship between equilibrium variables imposed by Nash bargaining over wages:

$$\begin{aligned}
\frac{r\cdot a}{q(\theta_t)}\cdot(1-\tau_t)^{1-\nu}\cdot v'(w_t) = &= \frac{1-\chi}{\chi}\cdot[v((1-\tau_t)\cdot w_t) - v(b_t\cdot w_t)] \\
&\quad + \delta\cdot(1-s)\cdot[1-f(\theta_{t+1})]\cdot\frac{r\cdot a}{q(\theta_{t+1})}\cdot(1-\tau_t)^{1-\nu}\cdot v'(w_{t+1}) \\
\frac{r\cdot a}{q(\theta_t)}\cdot v'(w_t) = &= \frac{1-\chi}{\chi}\cdot\left[1-\rho^{1-\nu}\right]\cdot v(w_t) + \delta\cdot(1-s)\cdot v'(w_{t+1})\cdot\left[\frac{1}{q(\theta_{t+1})} - \theta_{t+1}\right].
\end{aligned}$$

In steady state:

$$\begin{aligned}
\left[1-\rho^{1-\nu}\right]\cdot\frac{v(w)}{v'(w)} &= \frac{\chi}{1-\chi}\cdot a\cdot\left\{[1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)} + \delta\cdot(1-s)\cdot r\cdot\theta\right\} \\
\frac{w}{a} &= \frac{\chi}{1-\chi}\cdot\frac{1-\nu}{1-\rho^{1-\nu}}\cdot\left\{[1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)} + \delta\cdot(1-s)\cdot r\cdot\theta\right\}.
\end{aligned}$$

Combining this expression for w/a with the firm's profit-maximization condition (2) yields an expression for tightness θ as a function of the parameters of the model only (in particular, θ does not depend on technology a).

B.2 Proof of Proposition 1.

Part 1. In a steady state parameterized by technology a and public employment g, the key equilibrium condition is (9). Differentiating the condition with respect to g yields

$$\frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} = \frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} + \frac{\partial n^d}{\partial g}.$$

This equation allows me to express $\partial\theta/\partial g$ as a function of $\partial n^d/\partial\theta$, $\partial n^s/\partial\theta$, and $\partial n^d/\partial g$. In fact,

$$\frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} = -\frac{\partial n^d}{\partial g} \cdot \frac{1}{1+\left(\varepsilon^s/\varepsilon^d\right)}. \tag{A4}$$

where I define the tightness-elasticities ε^s and ε^d of labor supply and labor demand:

$$\varepsilon^s \equiv \frac{\theta}{n^s} \cdot \frac{\partial n^s}{\partial \theta} > 0, \tag{A5}$$

$$\varepsilon^d \equiv -\frac{\theta}{n^d} \cdot \frac{\partial n^d}{\partial \theta} > 0. \tag{A6}$$

ε^d is normalized to be positive. The effect of public employment g on aggregate employment n appears by differentiating (10), and using the result from (A4):

$$\frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g} \cdot \left[1 - \frac{1}{1+\left(\varepsilon^s/\varepsilon^d\right)}\right]. \tag{A7}$$

Part 2. To determine how the multiplier λ fluctuates over the business cycle, I examine the tightness-elasticities ε^s and ε^d. Labor supply (3) implies an equilibrium relationship between elasticity ε^s and unemployment u:

$$\varepsilon^s = (1-\eta) \cdot u. \tag{A8}$$

The tightness-elasticity of labor supply is countercyclical because unemployment is countercyclical. Next, I calculate the tightness-elasticity ε^d. Equation (2) implies that the elasticity ε^l_θ of private labor demand $l^d(\theta, a)$ with respect to tightness θ is

$$\varepsilon^l_\theta \equiv \frac{\theta}{l} \cdot \frac{\partial l^d}{\partial \theta} = -\frac{\eta}{1-\alpha} \cdot \Omega,$$

where I define

$$\Omega \equiv \frac{[1-\delta \cdot (1-s)] \cdot r/q(\theta)}{[1-\delta \cdot (1-s)] \cdot r/q(\theta) + w/a}. \tag{A9}$$

Ω measure the share of the marginal recruiting costs in the marginal cost of labor. Aggregate labor demand (8) relates ε^d to Ω in equilibrium:

$$\varepsilon^d = -(1-\zeta) \cdot \varepsilon^l_\theta = \eta \cdot \frac{1-\zeta}{1-\alpha} \cdot \Omega. \tag{A10}$$

LEMMA A2. *Under Assumptions 3, 4, and 5, equilibrium variables satisfy:* $d\theta/da > 0$, $dn/da > 0$, $dl/da > 0$, $du/da < 0$.

Proof. Labor supply $n^s(\theta)$ satisfies (3). That is,

$$n^s(\theta) = \frac{f(\theta)}{s + (1-s) \cdot f(\theta)}.$$

Therefore $\partial n^s / \partial \theta > 0$. Private labor demand $l^d(\theta, a)$ solves (2). That is,

$$l^d(\theta, a) = \left[\frac{1}{\alpha} \cdot \left(\omega \cdot a^{\gamma-1} + \frac{r}{q(\theta)}\right)\right]^{-1/(1-\alpha)}.$$

Under Assumptions 3 and 4, $\partial l^d/\partial \theta < 0$, $\partial l^d / \partial a > 0$. Under Assumption 5, equilibrium condition (9) becomes $l^d(\theta, a) = (1-\zeta) \cdot n^s(\theta)$. Differentiating this equilibrium condition with respect to a yields:

$$(1-\zeta) \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{d\theta}{da} = \frac{\partial l^d}{\partial a} + \frac{\partial l^d}{\partial \theta} \cdot \frac{d\theta}{da}$$

$$\frac{d\theta}{da} = \underbrace{\frac{\partial l^d}{\partial a}}_{+} \cdot (1-\zeta) \cdot \left[\underbrace{\frac{\partial n^s}{\partial \theta}}_{+} - \underbrace{\frac{\partial l^d}{\partial \theta}}_{-}\right]^{-1}.$$

Thus $d\theta/da > 0$. I conclude by using $\partial n^s / \partial \theta > 0$ and noting that in equilibrium $n = n^s(\theta)$, $u = 1 - (1-s) \cdot n$, and $l = (1-\zeta) \cdot n$. □

Using Lemma A2, the facts that $q'(\theta) < 0$ and $d[w/a]/da < 0$, and definition (A9): $d\Omega/da > 0$. Using Lemma A2 and relations (A8) and (A10): $d\varepsilon^d/da > 0$ and $d\varepsilon^s/da < 0$. In addition, $\partial n^d / \partial g = 1$. Hence (A7) implies that $d[\partial n/\partial g]/da < 0$. The marginal cost of public employment is $mc = w + [1-\delta \cdot (1-s)] \cdot r \cdot a/q(\theta)$. Using Lemma A2, and the facts that $q'(\theta) < 0$ and $w'(a) > 0$: $dmc/da > 0$. Since $\lambda = (\partial n/\partial g)/mc$: $d\lambda/da < 0$.

B.3 Proof of Proposition 2.

The proof of Proposition 1 remains mostly valid. The proof of Part 1 is unaltered. There are slight differences in the proof of Part 2. A first difference is that there is an additional equilibrium variable: search effort e.

LEMMA A3. *Under Assumptions 3, 4, and 5, equilibrium variables satisfy:* $d\theta/da > 0$, $de/da > 0$, $dn/da > 0$, $dl/da > 0$, $du/da < 0$.

Proof. Effort supply $e^s(\theta)$, given by (12), and labor supply, given by (13), satisfy: $\partial e^s/\partial\theta > 0$, $\partial n^s/\partial\theta > 0$, $\partial n^s/\partial e > 0$. Under Assumptions 3 and 4, private labor demand $l^d(\theta, a)$, given by (2), satisfies $\partial l^d/\partial\theta < 0$, $\partial l^d/\partial a > 0$. Proceeding as in the proof of Lemma A2, I obtain:

$$\frac{d\theta}{da} = \underbrace{\frac{\partial l^d}{\partial a}}_{+} \cdot \left[(1-\zeta)\cdot\left(\underbrace{\frac{\partial n^s}{\partial e}}_{+}\cdot\underbrace{\frac{\partial e^s}{\partial\theta}}_{+}+\underbrace{\frac{\partial n^s}{\partial\theta}}_{+}\right)-\underbrace{\frac{\partial l^d}{\partial\theta}}_{-}\right]^{-1}.$$

Thus $d\theta/da > 0$. I conclude by using $\partial e^s/\partial\theta > 0$, $\partial n^s/\partial\theta > 0$, $\partial n^s/\partial e > 0$ and noting that in equilibrium $e = e^s(\theta)$, $n = n^s(e,\theta)$, $u = 1-(1-s)\cdot n$, and $l = (1-\zeta)\cdot n$. □

Equilibrium variable behave as in the basic model. In addition, effort e is procyclical.

A second difference is that the elasticity ε^s of labor supply is affected by the presence of endogenous labor supply: search effort e responds to a change in tightness θ. I log-linearize the worker's optimality condition (12) to obtain the effect of a marginal change in dg in public employment. Using the iso-elasticity of the disutility $z(e)$ I obtain:

$$\begin{aligned}
\ln(1/\rho) &= [1-\delta\cdot(1-s)]\cdot\frac{z'(e)}{f(\theta)}+\kappa\cdot\delta\cdot(1-s)\cdot z(e)\\
0 &= (1-\delta\cdot(1-s))\cdot\left[\frac{z'(e)}{f(\theta)}\right]\left[\kappa\check{e}-(1-\eta)\cdot\check{\theta}\right]+\kappa\cdot\delta\cdot(1-s)\cdot z(e)\cdot[1+\kappa]\check{e}\\
0 &= \left[\kappa\check{e}-(1-\eta)\cdot\check{\theta}\right]+\check{e}\cdot\kappa\cdot\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\frac{(1+\kappa)z(e)}{z'(e)}\\
(1-\eta)\cdot\check{\theta} &= \kappa\cdot\check{e}\cdot\left[1+\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\cdot e\right]\\
\varepsilon^e_\theta &= \frac{\check{e}}{\check{\theta}} = (1-\eta)\cdot\frac{1}{\Lambda},
\end{aligned}$$

where

$$\Lambda = \kappa\cdot\left[1+s\cdot\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\cdot e\right]. \tag{A11}$$

Labor supply (13) implies an equilibrium relationship between ε^s, u, and Λ:

$$\varepsilon^s = u \cdot [\varepsilon^e_\theta + (1-\eta)] = (1-\eta) \cdot \left[1 + \frac{1}{\Lambda}\right] \cdot u.$$

This elasticity is similar to that in the basic model: there is only an additional $1/\Lambda$ term. Using definition (A11) and Lemma A3: $d\Lambda/da > 0$. Therefore it remains that $d\varepsilon^s/da < 0$. The rest of the proof proceeds as the proof of Proposition 1.

B.4 Proof of Proposition 3.

The proof of Part 1 is unaltered. There are slight differences in the proof of Part 2. A first difference is that equilibrium labor demand is affected by the introduction of public capital. Under Assumption 5, I rewrite the firm's optimality condition (15) to obtain equilibrium labor demand:

$$l^d(\theta, a) = a^{\frac{\xi}{(1-\alpha)\cdot\alpha\cdot\xi}} \cdot \left\{ \left[\frac{\beta}{\omega_p}\right]^{\xi} \cdot \left[\frac{1-\zeta}{\zeta}\right]^{\alpha\cdot\xi} \cdot \frac{1}{\alpha} \cdot \left[\frac{w}{a} + \frac{r}{q(\theta)}\right] \right\}^{-\frac{1}{(1-\alpha)\cdot\alpha\cdot\xi}}.$$

While the expression for equilibrium private labor demand l^d is different here, its properties remain the same as in the basic model: $l^d(\theta, a)$ is increasing in a, decreasing in θ. Thus the results from Lemma A2 remain valid: the qualitative behavior of equilibrium variables when technology varies is unchanged.

A second difference is that public employment g now influences private labor demand l^d; thus I need to re-calculate $\partial n^d/\partial g = 1 + \partial l^d/\partial g \neq 1$. Using the implicit definition (15) of private labor demand l^d, consider the effect dl^d of a marginal change dg in public employment, keeping tightness θ constant:

$$0 = \xi \cdot \alpha \cdot \check{g} + (\alpha - 1) \cdot \check{l}^d$$
$$\frac{\partial n^d}{\partial g} = 1 + \frac{\partial l^d}{\partial g} = 1 + \frac{l}{g} \cdot \frac{\check{l}^d}{\check{g}} = 1 + \xi \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1-\zeta}{\zeta}.$$

Since $\partial n^d/\partial g$ remains a function of parameters only, the proof from Proposition 1 carries through.

B.5 Proof of Proposition 4.

Part 1. In steady state parameterized by technology a and public employment g, the key equilibrium condition is the labor market clearing condition:

$$n^s(\theta, \sigma) = n^d(\theta, a, g).$$

Differentiating this equilibrium condition with respect to g yields

$$\frac{\partial \theta}{\partial g} = \left[\frac{\partial n^s}{\partial \theta} - \frac{\partial n^d}{\partial \theta}\right]^{-1} \cdot \left[\frac{\partial n^d}{\partial g} - \frac{\partial n^s}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial g}\right].$$

Proceeding as in the proof of Proposition 1 then yields

$$\frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g} \cdot \left[1 - \frac{1}{1 + \left(\varepsilon^s / \varepsilon^d\right)}\right] + \frac{1}{1 + \left(\varepsilon^s / \varepsilon^d\right)} \cdot \frac{\partial n^s}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial g}, \tag{A12}$$

where the elasticities ε^s and ε^d of labor supply and labor demand are defined by (A5) and (A6). It is easier to study $\partial l / \partial g = \partial n / \partial g - 1$. Since $\partial n^d / \partial g = 1$, (A12) simplifies to

$$\frac{\partial l}{\partial g} = -\frac{1}{1 + \left(\varepsilon^s / \varepsilon^d\right)} \cdot \left[1 - \frac{\partial n^s}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial g}\right] \tag{A13}$$

I start from the share of unemployed workers in the public sector given by (19), and I consider a small deviation dg in public employment:

$$\check{\sigma} = -\frac{\psi}{\psi + \frac{\zeta}{1-\zeta}} \cdot \left[\check{l} - \check{g}\right].$$

$$\frac{\partial \sigma}{\partial g} = \frac{\sigma}{g} \cdot \frac{\psi}{\psi + \frac{\zeta}{1-\zeta}} \cdot \left[1 - \varepsilon^l_g\right].$$

The labor supply is given by (20).

$$\frac{\partial n^s}{\partial \sigma} = \frac{u}{s} \cdot f(\theta) \cdot (\psi - 1) - n \cdot \frac{u}{s} \cdot (1-s) \cdot (\psi - 1) \cdot f(\theta) = \frac{g}{\sigma} \cdot u \cdot \frac{\psi - 1}{\psi},$$

where I used the balance of flows in and out of public employment (17), and the definitions $f(\theta) = f(\theta^g)/\psi$ and $u = 1 - (1-s) \cdot n$. These two results allow me to rewrite (A13) as

$$\frac{\partial l}{\partial g} = -\frac{\varepsilon^d}{\varepsilon^s + \varepsilon^d} \cdot \left[1 - u \cdot \frac{\psi - 1}{\psi + \frac{\zeta}{1-\zeta}} \cdot \left[1 - \frac{\zeta}{1-\zeta} \frac{\partial l}{\partial g}\right]\right].$$

Notice that using (19) that $1/\left[\psi + \zeta/(1-\zeta)\right] = \sigma \cdot (1-\zeta)/\zeta$. Hence:

$$\frac{\partial l}{\partial g} \cdot \left[1 + \frac{\varepsilon^d}{\varepsilon^s + \varepsilon^d} \cdot u \cdot (\psi - 1) \cdot \sigma\right] = -\frac{\varepsilon^d}{\varepsilon^s + \varepsilon^d} \cdot \left[1 - u \cdot (\psi - 1) \cdot \sigma \cdot \frac{1-\zeta}{\zeta}\right].$$

Using expressions (A8) and (A10) for ε^s and ε^d, I obtain the effect of public employment on private employment:

$$\frac{\partial l}{\partial g} = -\frac{1-u\cdot(\psi-1)\cdot\sigma\cdot[(1-\zeta)/\zeta]}{1+\left(\varepsilon^s/\varepsilon^d\right)+u\cdot(\psi-1)\cdot\sigma}. \tag{A14}$$

Part 2. I assume $\psi \geq 1$. Equation (19) implies that in equilibrium the share σ of public unemployment satisfies $\sigma = [1+\psi\cdot(1-\zeta)/\zeta]^{-1}$. In equilibrium σ remains constant over the business cycle: it does not depend on technology a. Labor supply $n^s(\theta)$ is defined by (20). $\partial n^s/\partial\theta > 0$ because $1+\sigma\cdot(\psi-1)\geq 0$ for all $\sigma \geq 0$. Therefore the results from Lemma A2 also apply in this extension. As in the basic model, the elasticity ε^d of labor demand satisfies (A10), and the elasticity ε^s of labor demand satisfies (A8). Therefore, $d\varepsilon^d/da > 0$, $d\varepsilon^s/da < 0$, $du/da < 0$. Using (A14), I conclude that $d[\partial l/\partial g]/da > 0$ and $d[\partial n/\partial g]/da > 0$ (note that $\partial l/\partial g < 0$). The marginal cost of public employment is now $mc = \pi\cdot w + [1-\delta\cdot(1-s)]\cdot r/q(\theta)$, but its cyclicality remains unchanged: $dmc/da > 0$. Therefore $d\lambda/da < 0$.

C Expressions for the fiscal multiplier

Here I collect results derived in Section B. Let $mc = (w/a)+[1-\delta\cdot(1-s)]\cdot r/q(\theta)$ be the marginal cost of hiring a worker. In the basic model, the fiscal multiplier is:

$$\lambda = \frac{1-\alpha}{1-\alpha+(1-\zeta)\cdot\frac{\eta}{(1-\eta)}\cdot\frac{\Omega}{u}}\cdot\frac{1}{mc}.$$

In the extension with public capital, the fiscal multiplier is:

$$\lambda = \left[(1-\alpha)+\xi\cdot\alpha\cdot\frac{1-\zeta}{\zeta}\right]\cdot\frac{1}{(1-\alpha)+(1-\zeta)\cdot\frac{\eta}{(1-\eta)}\cdot\frac{\Omega}{u}}\cdot\frac{1}{mc}.$$

In the extension with endogenous labor supply, the fiscal multiplier is:

$$\lambda = \frac{1-\alpha}{1-\alpha+(1-\zeta)\cdot\frac{\eta}{(1-\eta)}\cdot\frac{\Lambda}{1+\Lambda}\cdot\frac{\Omega}{u}}\cdot\frac{1}{mc}.$$

In the extension with public-sector wages, the fiscal multiplier is:

$$\lambda = \frac{u\cdot\left[(1-\alpha)\cdot\frac{1-\eta}{\eta}\cdot\Omega^{-1}+(\psi-1)\cdot\sigma\cdot\frac{1-\zeta}{\zeta}\right]}{(1-\zeta)+u\cdot\left[(1-\alpha)\cdot\frac{1-\eta}{\eta}\cdot\Omega^{-1}+(\psi-1)\cdot\sigma\cdot(1-\zeta)\right]}\cdot\frac{1}{mc},$$

where the marginal cost is $mc = \pi\cdot(w/a)+[1-\delta\cdot(1-s)]\cdot r/q(\theta)$ since public-sector wages differ from private-sector wages.

D Efficient Allocation

An *allocation* is a collection $\{g_t, l_t, \theta_t, c_t^l, c_t^g, c_t^u, y_t, p_t\}_{t=0}^{+\infty}$ for public and private employment; labor market tightness; consumption in private jobs, public jobs, and unemployment; and output of private and public good. A *feasible allocation* is an allocation that satisfies (a) the production constraint (1) for the private good; (b) a production constraint (4) for the public good; (c) the resource constraint in the economy, which imposes that the private good be either consumed or allocated to recruiting:

$$y_t = \left[l_t \cdot c_t^l + g_t \cdot c_t^g + (1 - n_t) \cdot c_t^u\right] + \frac{r \cdot a}{q(\theta_t)} \cdot h_t;$$

and (d) the law of motion for aggregate employment $(1-s) \cdot n_{t-1} + u_t \cdot f(\theta_t) = n_t$, where I simplify notations by using total employment $n_t = l_t + g_t$, unemployment $u_t = 1 - (1-s) \cdot n_{t-1}$, and new hires $h_t = n_t - (1-s) \cdot n_{t-1}$. The *efficient allocation* is the feasible allocation that maximizes

$$\sum_{t=0}^{+\infty} \delta^t \cdot \left\{\omega_v \cdot v(p_t) + \left[l_t \cdot v(c_t^l) + g_t \cdot v(c_t^g) + (1 - n_t) \cdot v(c_t^u)\right]\right\}.$$

Since workers are risk-averse, the efficient allocation provides full insurance against unemployment: $c_t^l = c_t^g = c_t^u = c_t$. Proposition A1 establishes that the efficient allocation is invariant to technology:

PROPOSITION A1. *In the steady-state efficient allocation, labor market variables g, l, θ and ratios c/a, y/a, p/a are independent of technology a.*

Proof. In steady state, the resource and production constraints imply that the ratios of private consumption, private output, and public output to technology are only function of labor market variables: $c/a = x(l) - [r/q(\theta)] \cdot s \cdot n$, $y/a = x(l)$, $p/a = \omega_p \cdot x(g)$. Therefore, these ratios are independent of technology a if labor market variables are.

Labor market variables $\{g, l, \theta\}$ in the efficient allocation are characterized by three equations, which I present below. When the labor market is in steady state, employments g and l, and labor market tightness θ are related by a Beveridge curve

$$l + g = \frac{f(\theta)}{s + (1-s) \cdot f(\theta)}. \tag{A15}$$

To derive the two other relationships, I derive first-order conditions of the planner's problem with respect to $\{c_t, \theta_t, n_t, l_t, g_t\}_{t=0}^{+\infty}$. The Lagrangian of the planner's problem is

$$L = \sum_{t=0}^{+\infty} \delta^t \cdot \left\{\omega_v \cdot v(a_t \cdot \omega_p \cdot x(g_t)) + v(c_t) + \mu_t^1 \cdot \left[a_t \cdot x(l_t) - c_t - \frac{r \cdot a}{q(\theta_t)} \cdot \{n_t - (1-s) \cdot n_{t-1}\right]\right.$$
$$\left. + \mu_t^2 \cdot [(1-s) \cdot (1 - f(\theta_t)) \cdot n_{t-1} + f(\theta_t) - n_t] + \mu_t^3 \cdot [n_t - l_t + g_t]\right\}.$$

where $\{\mu_t^1, \mu_t^2, \mu_t^3\}_{t=0}^{+\infty}$ are Lagrange multipliers. The first-order conditions are

$$v'(c_t) = \mu_t^1 \tag{A16}$$

$$\mu_t^2 \cdot q(\theta_t) \cdot u_t = r \cdot a \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{f(\theta_t)} \cdot \mu_t^1 \cdot h_t \tag{A17}$$

$$\mu_t^1 \cdot \frac{r \cdot a}{q(\theta_t)} + \mu_t^2 = \mu_t^3 + \delta \cdot (1-s) \cdot \left[\mu_{t+1}^1 \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} + \mu_{t+1}^2 \cdot [1 - f(\theta_{t+1})]\right] \tag{A18}$$

$$\mu_t^3 = \mu_t^1 \cdot a \cdot x'(l_t) \tag{A19}$$

$$\mu_t^3 = \omega_v \cdot v'(p_t) \cdot a \cdot \omega_p \cdot x'(g_t). \tag{A20}$$

Since hiring a worker has the same marginal cost in the private and public sector, the marginal benefit from a worker must be equal in both sectors. In steady state, combining the first-order conditions (A19) and (A20) with respect to l and g yields:

$$\omega_v \cdot \omega_p \cdot \frac{v'(p)}{v'(c)} = \omega_v \cdot \omega_p \cdot \left[\frac{c}{p}\right]^{\nu} = \frac{x'(l)}{x'(g)} = \left[\frac{g}{l}\right]^{1-\alpha}. \tag{A21}$$

In particular, it is always optimal to employ some workers in the public sector as long as the public good is valuable ($\omega_v > 0$) and the government is productive ($\omega_p > 0$). The ratios p/a and c/a are only function of labor market variables, so p/c is only a function of labor market variables, so the efficient ratio of public to private employment g/l is only function of labor market variables. The trade-off between private and public employment remains unchanged when technology falls because the productivities of public and private jobs fall in concert.

Noting that $f(\theta_t) = h_t/u_t$, the first-order condition (A17) with respect to θ_t yields

$$\frac{\mu_t^2}{\mu_t^1} = \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta_t)}.$$

I combine first-order conditions (A18) and (A19) with respect to n_t and l_t, and divide by $a \cdot \mu_t^1$:

$$x'(l_t) = \frac{r}{q(\theta_t)} + \frac{\mu_t^2}{\mu_t^1 a} - \delta \cdot (1-s) \cdot \left[\frac{\mu_{t+1}^1}{\mu_t^1 \cdot a} \cdot \left[\frac{r \cdot a}{q(\theta_{t+1})} + \frac{\mu_{t+1}^2}{\mu_{t+1}^1} \cdot [1 - f(\theta_{t+1})]\right]\right]$$

I combine these two relationships, multiply by $(1-\eta)$, and impose steady state:

$$(1-\eta) \cdot x'(l) = [1 - \delta \cdot (1-s)] \frac{r}{q(\theta)} + \delta \cdot (1-s) \cdot \eta \cdot r \cdot \theta. \tag{A22}$$

This relationship says that the marginal benefit from having workers search for jobs must equal the marginal benefit from having them produce goods. The system of three equations { (A15), (A21), (A22)}, and the efficient allocation $\{g, l, \theta\}$, are independent of technology a. □

E Generalization to Other Fiscal Policies

To show how the framework and results of Section 4 extend to other fiscal policies, I study the effect of government spending on a wage subsidy T. If pre-tax wages are rigid (as assumed in this paper), a wage subsidy paid to firms, implementable as a payroll tax cut on the firm side, necessarily reduces unemployment by reducing the marginal cost of labor. All equilibrium conditions of Section 2 are unchanged, but for the firm's profit-maximizing condition, which becomes

$$x'(l) = (1-T)\cdot\frac{w}{a} + [1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)}, \tag{A23}$$

and which defines labor demand $l^d(\theta, a, T)$. I define the wage-subsidy multiplier $\lambda_T \equiv (dn/dT)/mc_T$. λ_T is the increase in employment achieved by spending one unit of private good on a wage subsidy. $mc_T = w\cdot l$ is the per-period marginal cost of a wage subsidy. Since a wage subsidy stimulates aggregate labor demand $n^d(\theta, a, g, T)$ as public employment, the methodology of Section 4 remains valid and:

$$\lambda_T = \frac{1}{mc_T}\cdot\frac{\partial n^d}{\partial T}\cdot\frac{1}{1+\left(\varepsilon^d/\varepsilon^s\right)},$$

where equations (A8) and (A10) from the model with public employment relating elasticities ε^s and ε^d to equilibrium variables remain valid. Equation (A23) implies that $\left(\partial n^d/\partial T\right) = \partial l^d/\partial T = 1/(1-\alpha)\cdot(w\cdot l)/mpl = 1/(1-\alpha)\cdot mc_T/mc$. Thus:

$$\lambda_T = \frac{1}{1-\alpha}\cdot\frac{1}{1+\left(\varepsilon^d/\varepsilon^s\right)}\cdot\frac{1}{mc},$$

where $mc = (1-T)\cdot(w/a) + [1-\delta\cdot(1-s)]\cdot r/q(\theta)$ is the marginal cost of hiring a worker. A constant wage subsidy does not modify the cyclicality of equilibrium variables. Thus the cyclical fluctuations of the wage-subsidy multiplier λ_T are identical to those of the public-employment multiplier λ. Proposition A2 establishes that the wage-subsidy multiplier λ_T is countercyclical:

PROPOSITION A2 (Wage-subsidy multiplier). *Under Assumptions 3, 4, and 5,* $d\lambda_T/da < 0$.

Proof. Identical to the proof of Part 2 of Proposition 1. □

A wage subsidy reduces the marginal cost of labor, which leads firms to increase employment; higher aggregate employment increases tightness and recruiting costs until a new equilibrium is reached, at which point the new marginal cost of labor equals the marginal product of labor. In recessions when jobs are rationed, recruiting costs are low and do not vary much with employment so a wage subsidy triggers a large increase in employment; in expansions, recruiting costs are high and increase rapidly with employment so a wage subsidy only achieves a small increase in employment; thus, a wage subsidy is more effective in recessions.

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