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Why Capital does not Migrate to the South: A New Economic Geography Perspective

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Abstract
This paper explains why capital does not flow from the North to the South - the Lucas Paradox - with a New Economic Geography model that incorporates mobile capital, immobile labour, and productively heterogeneous firms. In contrast to neoclassical theories, the results show that even a small difference in the ex-ante productivity distribution between North and South can a have significant impact on the location of firms. Despite differences in aggregate capital to labour ratios, wage and rental rates continue to be the same in both locations. The paper also analyses the effects of risk on industrial locations, and shows why 'low-tech' industries tend to migrate to the South, while 'high-tech' industries continue to locate in the North.

JEL Classifications: F12, F15
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1 Introduction

It has long been a source of consternation among economists as to why there has been considerably less capital flow from the capital rich industrialised economies to the capital poor developing economies [see Robert E. Lucas Jr. (1990)]. Some economists have used differences in fundamentals (production structure, technology, policies, institutions) as explanations for the paradox. For example, Lucas cites the differences in human capital as the key reason why capital does not move to the South. On the other hand, other economists have mainly relied on capital market failures (expropriation risks, sovereign risks, asymmetric information) to resolve the paradox[1].

Nevertheless, it is also clear from empirical research that it is often difficult to distinguish one theory from another. Countries with weak institutions tend to have lower human capital, and weak institutions tend to be associated with greater information asymmetry and expropriation risks. There can be too much or too little capital to the South, depending on which benchmark model is used, what instruments are used, what is defined as capital, and what kind of growth accounting is used[2].

Notwithstanding the various arguments presented, development over the past decade has necessitated a new understanding of the Lucas Paradox. The opening of China, India and other major emerging economies has resulted in increased flow of Foreign Direct Investment (FDI) to what is loosely termed as the South. The flow of capital is however highly uneven [see Stephany Griffith-Jones and Jonathan Leape (2002)]. China attracted a fifth of all private capital flows to developing countries in the 1990s, peaking at \$60 billion in 1997. India's share has been paltry by contrast, with a peak of \$7 billion only in 1994. The latest figures show that China took in \$72 billion in FDI in 2005, while India only received \$6.6 billion[3].

If the Lucas paradox exists for India, it is on the face of it much less of a

[1] See Alfaro et al (2005) for a brief discussion on the various competing hypotheses.

[2] Francesco Caselli and James Feyrer (2007) offer a similar insight by making a distinction between reproducible and non-reproducible capital. The authors argue that the reward to reproducible capital is in fact rather low in the South once proper accounting is done. There is therefore no paradox that capital does not move there.

[3] China's cumulative inward FDI stands at \$318 billion compared to \$45 billion for India (UNCTAD). The difference in the levels of FDI is not due to differences in domestic investment. Inward FDI made up 11.3 per cent of China's gross capital formation between 1990 and 2000, but only 1.9 per cent compared to India. One of the explanations for the big difference is the effect of 'round-tripping' - domestic investment by Chinese firms disguised as FDI in order to gain a tax advantage. A look at the foreign investment position from the US however recorded the following difference: US cumulative investments in China and India (historical price) stand at \$16.9B and \$8.5B respectively. For manufacturing, the respective figures are \$8.8B and \$2.4B (Bureau of Economic Analysis). While 'round-tripping' may well account for some of the difference between the FDI that China and India have received, it is clear that China continues to receive signficantly more bona fide FDI than India. Despite recent headline-grabbing growth rates from India, the FDI gap with China has not closed, although this might change in the near future.

paradox for China. Is it therefore correct to conclude that China somehow has better fundamentals - institutions, technology, human capital, and/or less capital market imperfections? Given the fact that India is a stable parliamentary democracy, has a deeply entrenched English legal system with the associated emphasis on property rights, and a largely free press, it is difficult to turn the argument around and conclude that China has better institutions or better functioning markets that result in the huge difference in observed investment flows[4]. The puzzle is therefore not only why relatively little capital has flowed to the developing economies but also the distribution of the flow of capital to these economies.

New Economic Geography The objective of this paper is to synthesize the New Economic Geography (NEG) understanding of the location of industries with more recent firm-heterogeneity trade models, in order to bring about a new understanding to an old puzzle as well as answer some of these new questions posed.

NEG researchers have had more than a decade of success in demonstrating how industrial agglomeration can result. These models demonstrate how a symmetric fall in trade costs can result in highly asymmetric outcomes (catastrophic agglomeration) [see Paul Krugman (1991); Venables (1996); Krugman and Venables (1995)]. An example of the mechanics is that firms locate where there are workers, and workers locate where there are firms (to reduce cost of living), giving rise to a feedback effect. These models tend to be highly intractable as a result. A more tractable model of industrial location is the 'Footloose Capital' (FC) model due to Philippe Martin and Carol Ann Rogers (1995). The key assumption of the model is that only capital is mobile, while workers and owners of capital are not. Capital income is costlessly repatriated, consumed locally. Since expenditure shares between the locations remain static regardless of the choice of industrial location, there is no agglomerative (or feedback) effect in this class of models.

This paper has chosen to adopt the FC assumption as international economics continue to be dominated by high capital mobility. In essence, the model in this paper assumes mobile capital, immobile labour, and firms with heterogeneous productivity. There are two locations, North and South. Differences between the two regions are characterised not by the aggregate production functions, but by differences in the productivity (pareto) distributions of

[4]The problem with looking at historical data for defaults to explain current allocation, or predict future capital flows, becomes evident here. Historical data do not account for regime changes, changes in investor confidence and perception about the future. Reinhart and Rogoff (2004) duly note that India has never defaulted while China has defaulted on two occasions between 1901 and 2002. Yet it is still the case that China has taken the lion's share of FDI.

firms. The shares of manufacturing firms in each location are then solved for in the equilibrium by equalising the ex-ante value of entry in both locations. Several new results emerge from the exercise.

Explaining the Lack of Capital Flow to South Firstly, while neoclassical models suggest that the productivity differences between North and South have to be very large to explain the lack of capital flow, this paper shows that a small improvement in North's productivity (by changing the mean of the pareto distribution) can have a dramatic impact on the share of firms, while keeping the returns to factors equal in both locations. This therefore provides an alternative resolution to the Lucas paradox. Admittedly, this paper does not explain why the small difference in productivity would arise in the first place. This question is better left to development or political-economy researchers [see James R. Tybout (2000) for a brief discussion].

Resolving The Paradox of Risk The second key result concerns the effect of risk. James R. Tybout (2000) for example notes that it is common to see very large plants existing side by side with very small ones in developing countries, even though there is little evidence to suggest plants in developing countries are inherently less productive. The author therefore suggests that this may be a result of 'uncertainty about policies . . . poor rule of law'. The assumption here is that the South has a riskier productivity draw.

A well known property of the profit function is its convexity. Consider the example of the constant elasticity of substitution (CES) preference function. For whatever the cost of production (inverse of productivity), the firm's revenue is bounded from below by zero - that is, revenue is always positive no matter how high the cost (and price) is. However, there is no upper bound to revenue. Therefore, a mean-preserving spread of productivity actually increases expected profits because of the very convexity of the profit function[5].

If the South were to have greater aggregate productivity risks while keeping its mean productivity equal to the North, this would imply that expected profit is higher there, and mobile capital will flow to the South until the expected return to capital is once again equalised for both locations. This is the 'paradox of risk' for it contradicts commonplace intuition that firms shun locations perceived to have high risks to production. But in principle, the firm is a risk neutral entity. As long as the firm maximises expected profits, why does it care about risk?

It turns out that there is a good reason for this if one thinks of risk as outlined in a firm-specific productive risk in Marc J. Melitz (2003). Each firm

[5] A mean-preserving spread of expenditure will have no such effect since it will still result in the same expected profits since the expenditure is homogeneous of degree one in expenditure.

will have to pay a sunk cost to attempt entry into a market. Upon the payment of this cost, the firm draws a level of productivity specific to itself, from an ex-ante distribution. The firm then makes the decision whether to continue production based on the level of realised productivity. If productivity is high enough, the firm will pay the fixed production cost and produce. Otherwise, the firm 'lets bygones be bygones' and exits.

It turns out that in equilibrium, the level of the sunk cost will have an impact on the location of industries. Suppose one location is riskier than the other while holding the mean of the productivity distribution constant. The riskier distribution will have fatter tails. Ceteris paribus, high sunk cost industries prefer to invest in less risky locations because the higher likelihood of entry dominates (the probability of a really bad draw is low). On the other hand, low sunk cost industries invest in higher risk locations (with fatter right side tails for productivity draws) since the chance of getting a really good productivity draw dominates. Given a particular sunk cost, a firm therefore has to balance these two effects. The model can explain why 'hi-tech' industries - characterised by high sunk costs - cluster in the less risky North while 'low-tech' industries move to the risky South. Trade liberalisation results in North and South specialising in different industries.

2 The Model Setup

2.1 Endowments and Regions

There are two primary factors of production - capital and labour. There are two regions - North and South. The North has K_N units of capital and L_N units of labour while the South has K_S and L_S, all factors in fixed and known quantities. Capital is completely mobile between regions, and capital returns can be costlessly remitted to owners for consumption. Workers (who are also owners of capital) are completely immobile between regions, and their labour is supplied inelastically to the local market.

2.2 Preferences

There are two types of goods - agriculture (a) and manufacturing (m). The motivation is similar to most NEG models, with the agriculture sector equalising wages across economies in an equilibrium characterised by incomplete specialisation and without trade cost in agriculture. The j consumer's utility is given as

$$u_j = c_{mj}^{\mu} c_{aj}^{1-\mu}$$

where $c_{mj} = \left[\int_\Omega c_i^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$ is the consumption of the Ω set of manufactured goods, $\sigma > 1 > \mu > 0$.

2.3 Technology and Firms

2.3.1 Agriculture

The agricultural sector has a constant returns to scale production function. For simplicity, units are chosen such that 1 unit of labour produces 1 unit of output. As per the usual assumption for NEG models, the agricultural good is costlessly traded between countries. This assumption equalises the price of the agricultural good and wages across countries.

2.3.2 Manufacturing

The manufacturing sector requires a composite factor production κ which is produced by the primary factors - capital and labour - with a constant returns to scale Cobb-Douglas production technology

$$\kappa = AK^\alpha L^{1-\alpha}$$

where A is the aggregate technology parameter.

There is a large number of firms, each producing one variety. The firm's technology is homothetic and represented by the familiar increasing returns function

$$C_i = \left[f + \frac{q_i}{\varphi_i} \right] P_\kappa$$

where f is the fixed production cost and q the output. Therefore $\left[f + \frac{q_i}{\varphi_i} \right]$ gives the total input required of the firm in terms of κ and C is the total cost function given P_κ which is the price of the industrial composite. All firms have the same fixed cost but different levels of productivity φ.

Traditionally, the FC model has a disembodied technology - capital inputs for fixed cost and labour inputs for variable cost. Using a standard FC model but incorporating firm heterogeneity, Richard E. Baldwin and Toshihiro Okubo (2005) show how the home market effect can induce more productive firms to relocate to the larger market. That paper takes the ex-post productivity distribution of firms as given and ignores the entry or exit decision of firms. In a subsequent paper, Baldwin and Okubo (2006) introduce the entry and exit process. In that paper, the authors again highlight the home market effect, but further show how instantaneous entry and exit is a perfect substitute for relocation.

To achieve more analytical tractability, Baldwin and Okubo (2006) make some simplifying assumptions. Sunk cost, fixed export cost (beachhead cost), and variable production cost is borne by labour inputs only. Fixed production cost consists of capital only. The production technology is therefore a non-homothetic one, much like the standard FC model. In a firm heterogeneity setup however, there are many types of cost. This paper therefore adopts a more uniform approach towards the various types of costs by assuming a homothetic production technology that is more similar to Andrew B. Bernard, Stephen J. Redding and Peter K. Schott (2007) - known henceforth as BRS - where all costs require the same composition of inputs. There are several advantages with this setup.

Firstly, it is more realistic in that all costs will require capital and labour. The homotheticity of inputs towards manufacturing allows the model to be solved easily as in Melitz (2003) even in the presence of firm heterogeneity by making use of the 'Zero Cutoff Profits' and 'Free Entry' conditions. Secondly, symmetric changes in endowments across countries do not have an impact on firm level aggregates. Changes in endowments only affect the levels of composite as well as the capital-labour ratio. In a homothetic production setting, changes in endowments affect only the number of firms, relative returns of primary factors, and associated welfare, with no additional effect on firm level aggregates[6]. The effect of changing endowments proportionately is just like changing market size.

Finally, though this paper draws inspiration from BRS (2007), there is a key difference. In BRS (2007) both factors of production - skilled and unskilled labour - are immobile. In this paper however, one of the factors - capital - is completely mobile. In essence, the technology function in this paper is a hybrid, combining elements of various research [Martins and Roger; Melitz; BRS] to incorporate various useful properties.

2.3.3 Capital Market

This paper abstracts from any capital market imperfections by assuming that there is a well functioning capital market such that capital is transferred from owners to firms, and rewards are transferred costlessly back to owners for consumption.

[6] Consider the opposite case with a non-homothetic technology, supposing only capital is used for the sunk cost f_e. An increase in capital endowment, relative to labour, will mean that there will be relatively more resources for sunk cost compared to production. In equilibrium, it has to be more difficult to gain entry, and cutoff productivity has to increase. In other words, with a non-homothetic technology, changes in endowment will affect firm level aggregates.

2.3.4 Normalisation of Prices

The cost of the composite input κ - which depends on r and w - will be also equalised between the two regions given free capital mobility and costless agriculture trade. Applying cost minimisation, together setting the P_κ as the numeraire, gives the following equation

$$P_\kappa = \frac{w^{1-\alpha} r^\alpha}{A} \left[\left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left(\frac{1-\alpha}{\alpha} \right)^\alpha \right] \equiv 1 \tag{1}$$

This equilibrium relationship, in the situation of incomplete specialisation, allows the interest rate to be expressed in terms of wage rate and parameters (or vice versa)[7].

Furthermore, an implication of both cost minimisation and the equalisation of factor prices is that the rental-wage ratio can be expressed as

$$\frac{r}{w} = \left(\frac{\alpha}{1-\alpha} \right) \frac{L_M}{K_N + K_S} \tag{2}$$

where L_M is the total labour used in manufacturing[8]. Equation (2) allows r to be expressed as a function of w and parameters. Substituting this into equation (1), one can express the labour to capital ratio in manufacturing as a function of w only.

2.3.5 Pareto Productivity Distributions

All manufacturing firms face an ex-ante distribution of productivity in each location. This paper assumes pareto distributions for productivities in both North and South [Elhanan Helpman, Melitz and Stephen R. Yeaple (2005); BRS (2007); Baldwin and Okubo (2006)][9]. The parameters for the North are $\bar{\varphi}_N$ and k_N, where $\bar{\varphi}_N$ specifies the minimum support and k_N the shape of

[7] The advantage of choosing P_κ as the numeraire (rather than wages) is that it allows all equilibrium conditions for the manufacturing firms to be written in terms of κ only, without having to deal with the cost minimising price function of κ.

[8] In an interior equilibrium, since r and w are common to both economies, they will have the same labour-capital ratios in the differentiated sectors. Hence $\frac{r}{w} K_{MN} = \frac{\alpha}{1-\alpha} L_{MN}$ and $\frac{r}{w} K_{MS} = \frac{\alpha}{1-\alpha} L_{MS}$ where K_{MN} and K_{MS} are the mobile capital deployed to the North and South respectively (whiole L_{MN} and L_{MS} are the labour employed in manufacturing respectively). Since all capital sums up to world endowment, $\frac{r}{w}(K_N + K_S) = \frac{\alpha}{1-\alpha} L_M$, where $L_M = L_{MN} + L_{MS}$.

[9] The relevant cumulative density, probability density, mean and variance are given as

$$G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi} \right)^k \qquad g(\varphi) = \frac{k \varphi_m^k}{\varphi^{k+1}}$$

$$E(\varphi) = \frac{k \varphi_m}{k-1} \qquad Var(\varphi) = \frac{\varphi_m^2 k}{(k-1)^2 (k-2)}$$

where $k > 2$ and $\varphi_m > 0$. For a pareto distribution, both mean and variance is decreasing in k.

the distribution. The corresponding parameters for the South are $\bar{\varphi}_S$ and k_S.

2.3.6 Sunk Cost

Firms trying to enter the manufactured goods market are required to pay a sunk cost of f_e (again in terms of κ) to draw the firm specific productivity φ. As capital is completely mobile, a firm can choose to pay this cost either in the North or in the South, upon which its productivity will be drawn from the respective distribution. The paper assumes that firms are not allowed to relocate their investment once they have selected on the initial location. The reason for this assumption is simple. Firm specific productivity is assumed to be tied to the institutional context in which sunk cost is incurred[10].

2.3.7 Trade Cost

Trade in the manufacturing sector is costly. There is a $\tau > 1$ iceberg trade cost for every unit shipped. In addition, exporters will have to incur a beachhead, or a fixed export cost f_X in order to export. Both costs are in terms of κ, paid in the home country. Selection into the export market will occur if there exist firms with productivity below φ that find it profitable to operate domestically (with domestic revenue r_D) but not export (thereby foregoing revenue r_X).

3 Trade Equilibrium Conditions

As usual, the agriculture sector equalises wages between the two locations

$$w = p_a = p_a^* = w^*$$

where Southern variables are denoted with the asterisk (except for variables related to productivity φ where locations are denoted with the subscript).

3.1 Export Partitioning

With CES preferences, the optimal pricing of a firm with productivity φ_1 is $p(\varphi_1) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_1}$, and the revenue given as $r(\varphi_1) = \frac{p(\varphi_1)^{1-\sigma}}{P^{1-\sigma}}E$, where E is the aggregate expenditure and P is the CES price aggregate. The ratio of revenues between two firms with productivities φ_1 and φ_2 can therefore be expressed as $\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$. Furthermore, one can define a firm with cutoff productivity φ^* as the marginal firm - one that just makes enough operating profits to cover

[10] If both locations have the same ex-ante productivity distribution, no firms will relocate in equilibrium since the cutoffs are the same. An atomistic firm will have the same expected profits in both locations. If the productivity distributions are different, considering the effects of relocation requires an assumption to be made about whether productivity can be transferred across locations.

the fixed cost of production f. This firm therefore satisfies the relationship of net operating profits equalling the fixed cost: $\frac{1}{\sigma}r(\varphi^*) = f$. This allows one to write the revenue of a firm with an average productivity of $\tilde{\varphi}$ (to be defined later) as a function of the cutoff productivity φ^* only

$$r(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} f$$

Average profits from domestic sales become

$$\bar{\pi}_D = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} f - f = \left[\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right] f$$

Analogously, profits from exporting become

$$\bar{\pi}_X = \left[\left(\frac{\tilde{\varphi}_X}{\varphi_X^*}\right)^{\sigma-1} - 1\right] f_X$$

where φ_X^* is the export cutoff (greater than φ^*) because of the exporting partition condition which is assumed to hold (that is, not all firms export), and $\tilde{\varphi}_X$ is the average productivity of exporters.

3.2 Average Profits

Given these standard derivations, the average profits in the North can be written as

$$\bar{\pi} = \left[\left(\frac{\tilde{\varphi}_N}{\varphi_N^*}\right)^{\sigma-1} - 1\right] f + \tilde{p}_X \left[\left(\frac{\tilde{\varphi}_{NX}}{\varphi_{NX}^*}\right)^{\sigma-1} - 1\right] f_X \tag{3}$$

where φ_N^* is the cutoff productivity for entry, $\tilde{\varphi}_N$ the average productivity of all Northern firms above the cutoff, φ_{NX}^* the cutoff productivity into export, and $\tilde{\varphi}_{NX}$ is the average productivity of Northern exporters. Since only those manufacturers with a productivity draw greater than φ_{NX}^* can export, the term $\tilde{p}_X = \left(\frac{\varphi_N^*}{\varphi_{NX}^*}\right)^{k_N}$ gives the conditional probability of having a high enough productivity to export, conditional upon entry.

The analogous expression for the South is

$$\bar{\pi}^* = \left[\left(\frac{\tilde{\varphi}_S}{\varphi_S^*}\right)^{\sigma-1} - 1\right] f + \tilde{p}_X^* \left[\left(\frac{\tilde{\varphi}_{SX}}{\varphi_{SX}^*}\right)^{\sigma-1} - 1\right] f_X \tag{4}$$

where $\tilde{p}_X^* = \left(\frac{\varphi_S^*}{\varphi_{SX}^*}\right)^{k_S}$ is the conditional probability of exporting in the South.

The marginal firms in the North and South, with productivities φ_N^* and φ_S^*, recover only the fixed cost of production f in equilibrium. This gives the following relationship

$$\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_N^*}\right)^{1-\sigma}\left[\frac{\mu E}{P^{1-\sigma}}\right]=\sigma f=\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_S^*}\right)^{1-\sigma}\left[\frac{\mu E^*}{P^{*1-\sigma}}\right] \tag{5}$$

These are effectively zero profit conditions that will help pin down the productivity cutoffs in equilibrium.

3.3 Productivities of Northern and Southern Firms

As with the usual derivations in such models, average productivities of Northern and Southern firms - $\tilde{\varphi}_N$ and $\tilde{\varphi}_S$ - are functions of the respective cutoffs only[11]. The pareto productivity distributions allow the ratios between the average productivities and their respective cutoffs to be written as a function of parameters only $\left(\frac{\tilde{\varphi}_N}{\varphi_N^*}\right)^{\sigma-1}=\left(\frac{\tilde{\varphi}_{NX}}{\varphi_{NX}^*}\right)^{\sigma-1}=\left[\frac{k_N}{k_N+1-\sigma}\right]$, with analogous expressions holding for the South.

Together, these properties give the extremely useful result that $\frac{\tilde{\varphi}_N}{\varphi_N^*}=\frac{\tilde{\varphi}_{NX}}{\varphi_{NX}^*}$ with the pareto productivity distributions. Though exporters have a higher average productivity, the ratio of average productivity of all producers to the entry cutoff is exactly the same as the ratio of average productivity of all exporters to the export cutoff. Plugging these conditions into equations (3) and (4) greatly simplifies these expressions and the characterisation of equilibrium. Finally, a firm with φ_N^* makes zero profits in the domestic market, while a firm with φ_{NX}^* makes zero profits from exporting (with the analogous relationships holding for the South as well)[12].

3.4 Aggregate Productivity and Prices

The aggregate productivity and price level in a location depend not only on domestic firms, but also on foreign firms selling there. Define the total number of varieties in the North by $M=n+\tilde{p}_X^*n^*$. This indicates that the number of varieties in the North is made up of n domestic firms and $\tilde{p}_X^*n^*$ of Southern firms that are successful in exporting to the North. The corresponding expression for the South is $M^*=\tilde{p}_Xn+n^*$.

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[11] These are $\tilde{\varphi}_N=\left[\frac{1}{1-G_N(\varphi_N^*)}\int_{\varphi_N^*}^{\infty}\varphi^{\sigma-1}g_N(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$ and $\tilde{\varphi}_S=\left[\frac{1}{1-G_S(\varphi_S^*)}\int_{\varphi_S^*}^{\infty}\varphi^{\sigma-1}g_S(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$. With pareto productivity distributions, these can be further simplified to $\tilde{\varphi}_N=\left[\frac{k_N}{k_N+1-\sigma}\right]^{\frac{1}{\sigma-1}}\varphi_N^*$ and $\tilde{\varphi}_S=\left[\frac{k_S}{k_S+1-\sigma}\right]^{\frac{1}{\sigma-1}}\varphi_S^*$.

[12] When the countries are symmetric, the respective export cutoffs are a function of production cutoffs and parameters only, with $\varphi_{NX}^*=\varphi_N^*\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}$ and $\varphi_{SX}^*=\varphi_S^*\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}$. When the countries are not symmetric, one can show that $\varphi_{NX}^*=\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}\varphi_S^*$ and $\varphi_{SX}^*=\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}\varphi_N^*$ [see Svetlana Demidova (2006)]

The average productivity of the North becomes the weighted average of productivities of Northern firms and Southern exporters

$$\hat{\varphi} = \left[\frac{1}{M} \left(n\tilde{\varphi}_N^{\sigma-1} + \tilde{p}_X^* n^* \phi \tilde{\varphi}_{SX}^{\sigma-1} \right) \right]^{\frac{1}{\sigma-1}} \tag{6}$$

where $\phi = \tau^{1-\sigma}$ is the freedom of trade index. The corresponding equation for the South can be written as

$$\hat{\varphi}^* = \left[\frac{1}{M^*} \left(\tilde{p}_X n \phi \tilde{\varphi}_{NX}^{\sigma-1} + n^* \tilde{\varphi}_S^{\sigma-1} \right) \right]^{\frac{1}{\sigma-1}} \tag{7}$$

With these definitions of productivities, the aggregate price levels in the North and South are given as

$$P = M^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{1}{\hat{\varphi}} \qquad\qquad P^* = M^{*\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{1}{\hat{\varphi}^*} \tag{8}$$

This completes the characterisation of the aggregate price levels for both locations. The aggregate prices P and P^* in equation (8) can also be substituted into the marginal firm conditions in equation (5), allowing the zero profit conditions to be expressed as function of firm mass and productivity cutoffs only.

3.5 Equalisation of Expected Values of Entry in North and South

Free entry ensures that the ex-ante value of entry must be equal for both locations if there is to be an interior solution (with manufacturing firms in both locations) The condition for an interior equilibrium can be written as $\tilde{p}\bar{\pi}_N = \tilde{p}^*\bar{\pi}_S^* = f_e$, where $\tilde{p} = 1 - G_N(\varphi_N^*) = \left(\frac{\bar{\varphi}_N}{\varphi_N^*}\right)^{k_N}$ and $\tilde{p}^* = 1 - G_S(\varphi_S^*) = \left(\frac{\bar{\varphi}_S}{\varphi_S^*}\right)^{k_S}$ are the entry probabilities of the North and South respectively[13]. With the appropriate substitutions, this expression can be explicitly written as

$$\tilde{p}^* \left(\frac{\sigma-1}{k_S+1-\sigma} \right) (f + \tilde{p}_X^* f_X) = f_e = \tilde{p} \left(\frac{\sigma-1}{k_N+1-\sigma} \right) (f + \tilde{p}_X f_X) \tag{9}$$

where $\tilde{p}_X$ and $\tilde{p}_X^*$ are the conditional probabilities of exporting.

[13] If manufacturing concentrates completely in one location, one of these equalities will not hold. Expected profits in one location do not cover the sunk cost f_e in equilibrium and no manufacturing firms locate there. This can be used to pin down the break/sustain point.

3.6 Market Clearing

There are in equilibrium n successful entrants in the North and n^* in the South. But due to the cutoffs, the number of firms that attempt entry has to be higher. The total number of firms that attempt entry, including those below the cutoffs, is

$$n_e = \frac{n}{\tilde{p}} \qquad\qquad n_e^* = \frac{n^*}{\tilde{p}^*}$$

where n_e and n_e^* are the total number of entry attempts in the North and South respectively.

The composite input κ is used for four purposes - sunk cost (f_e), fixed production cost (f), marginal production cost, and export costs (this is incurred by exporters only). The key to note here is that even unsuccessful entrants will use up industrial inputs. The marginal cost for each firm is $\frac{1}{\varphi}$, a firm-specific variable. The aggregate variable production cost in the North can be written as $n\left(\frac{k_N}{k_N+1-\sigma}\right)(\sigma-1)f$ [see Appendix]. Aggregate composite input used in the North becomes

$$\kappa = n\left\{f + \left(\frac{k_N}{k_N+1-\sigma}\right)(\sigma-1)f + \frac{f_e}{\tilde{p}} + \tilde{p}_X\left[f_X + \left(\frac{k_N}{k_N+1-\sigma}\right)(\sigma-1)f_X\right]\right\} \tag{10}$$

Multiplied by the number of firms, the first term within the brackets on the right hand side is the total fixed production cost. The second term on the right (again multiplied by the number of firms n) is the aggregate variable cost of all firms. The third term (multiplied by the number of firms) is the total sunk cost incurred, including that of the unsuccessful firms. Finally, the terms inside the square brackets (multiplied by the number of firms) are the total beachhead and exporting production costs, which are incurred by exporters only. An analogous term can be written for the South

$$\kappa^* = n^*\left\{f + \left(\frac{k_S}{k_S+1-\sigma}\right)(\sigma-1)f + \frac{f_e}{\tilde{p}^*} + \tilde{p}_X^*\left[f_X + \left(\frac{k_S}{k_S+1-\sigma}\right)(\sigma-1)f_X\right]\right\} \tag{11}$$

The above two expressions therefore give the quantity of the composite input κ demanded in the North and South respectively.

Due to the cost minimisation property, the derived demand for capital is $K = \frac{1}{A}\left(\frac{w}{r}\right)^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\kappa$. By substituting the demands of κ into the appropriate conditional demands, one can derive the demands of the primary factors capital and labour. Since the total demand of capital in the world must be equal to the endowment, the capital clearing condition can be written as

$$\bar{K}_W = \frac{1}{A}\left(\frac{w}{r}\right)^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}(\kappa+\kappa^*) \tag{12}$$

Equation (12) converts the industrial inputs into capital by substituting κ in equations (10) and (11) into the appropriate cost-minimising function. This is the first market clearing equation.

Similarly, since the conditional demand for labour (for manufacturing) can be written as $L = \frac{1}{A}\left(\frac{r}{w}\right)^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\kappa$, the total labour requirement for manufacturing becomes

$$L_M = \frac{1}{A}\left(\frac{r}{w}\right)^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}(\kappa+\kappa^*) \tag{13}$$

As labour is also used for agriculture, the total manufacturing labour does not equal to the total labour endowment. Instead, the amount of labour available for agriculture is whatever labour not used in manufacturing. This has to be equal to the real demand for agricultural goods (nominal expenditure divided by the price of agriculture goods, which is w), giving the agricultural market (or labour market) clearing condition

$$\bar{L}_W - L_M = (1-\mu)\left[\frac{E+E^*}{w}\right] \tag{14}$$

Substituting equation (13) into (14) then provides the second market clearing condition. With CES preferences, the manufacturing goods market clears since the expenditure on each firm is equal to its revenue. With Walras's Law, the agriculture market also clears.

3.7 Aggregate Expenditure

As owners of capital are immobile, all capital returns are remitted to the owners and consumed locally. The aggregate expenditures for the North and South are simply their respective factor endowments multiplied by the rental and wage rates, which are the same across countries in the incomplete specialisation equilibrium

$$E = rK_N + wL_N \qquad\qquad E^* = rK_S + wL_S$$

3.8 Equilibrium Solution

The endogenous variables for equilibrium are $\{w, \varphi_N^*, \varphi_S^*, n, n^*\}$ - although the interest rate is endogenous, it can be recovered by equation (1). For the five endogenous variables, the equilibrium is pinned down (after appropriate substitutions) by (i) two ex-ante free entry conditions in equation (9); (ii) zero profit condition in equation (5); and two market clearing conditions in equation (12) and (14).

3.8.1 Solving for Global Manufacturing Labour

From equation (14), the global production of agriculture is

$$\begin{aligned}\bar{L}_W - L_M &= (1-\mu)\left[\frac{E+E^*}{w}\right] \\ &= (1-\mu)\left[\frac{r\bar{K}_W + w\bar{L}_W}{w}\right] \\ &= (1-\mu)\left[\frac{r}{w}\bar{K}_W + \bar{L}_W\right]\end{aligned}$$

The second equality makes use of the fact that the global expenditure is a function of wage-rental and global endowments $E + E^* = r\bar{K}_W + w\bar{L}_W$. Substituting the rental-wage ratio from equation (2) then allows the world's labour employed in manufacturing to be expressed as a function of endowments and parameters only

$$L_M = \left[\frac{\mu(1-\alpha)}{1-\alpha\mu}\right]\bar{L}_W \tag{15}$$

Note that μ (which is the share of manufacturing in consumption) has to be less than 1 for $\left[\frac{\mu(1-\alpha)}{1-\alpha\mu}\right]$, the share of global labour in manufacturing, to also be less than 1.

3.8.2 Solving for Rental-Wage Ratio

Substituting equation (15) back to equation (2) then allows the rental-wage ratio to be expressed as a function of parameters only

$$\frac{r}{w} = \left[\frac{\alpha\mu}{1-\alpha\mu}\right]\frac{\bar{L}_W}{\bar{K}_W} \tag{16}$$

Note that the rental-wage ratio is also unaffected by any firm level variables. It depends on the endowments ratio and parameters only.

3.8.3 Solving for Total Composite Resource

Equation (13) gives the relationship between L_M and the composite resource $\kappa + \kappa^*$

$$L_M = \frac{1}{A}\left(\frac{r}{w}\right)^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}(\kappa + \kappa^*)$$

Substituting equations (15) and (16) into the above will give

$$\kappa + \kappa^* = A(1-\alpha)^{1-\alpha}\left[\frac{\mu}{1-\alpha\mu}\right]^{1-\alpha}\bar{L}_W^{1-\alpha}\bar{K}_W^{\alpha} \tag{17}$$

The total composite resource available to the manufacturing sector is an increasing function of endowments, aggregate technology A, and share of manufacturing consumption μ (because this reduces the amount of labour required for agriculture). This therefore pins down the total composite factor supply in terms of endowments and parameters only.

4 When the North Is More Productive

From the free entry conditions in equation (9),

$$\tilde{p}\left(\frac{\sigma-1}{k_N+1-\sigma}\right)(f+\tilde{p}_X f_X)=\tilde{p}^*\left(\frac{\sigma-1}{k_S+1-\sigma}\right)(f+\tilde{p}_X^* f_X)$$

Since $k=k_N=k_S$, this condition becomes

$$\tilde{p}\,(f+\tilde{p}_X f_X)=\tilde{p}^*\,(f+\tilde{p}_X^* f_X)$$

Writing this equation more explicitly

$$\left(\frac{\bar{\varphi}_N}{\varphi_N^*}\right)^k\left[f+\left(\frac{\varphi_N^*}{\varphi_{NX}^*}\right)^k f_X\right]=\left(\frac{\bar{\varphi}_S}{\varphi_S^*}\right)^k\left[f+\left(\frac{\varphi_S^*}{\varphi_{SX}^*}\right)^k f_X\right]$$

From here, the paper states a few simplifying relationships. First, $\bar{\varphi}_N=\psi\bar{\varphi}_S$ where $\psi>1$ represents the rightward shift of North's support for the productivity distribution. Second, one can make use of the following relationships $\varphi_{NX}^*=\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}\varphi_S^*$ and $\varphi_{SX}^*=\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}\varphi_N^*$ when solving for cutoffs for asymmetric countries where $Z=\tau\left(\frac{f_X}{f}\right)^{\frac{1}{\sigma-1}}$ [see Demidova (2006)]. Third, let $\eta^k=\frac{f_X}{f}$ be the ratio of the two fixed cost. Using these simple relationships, the above equation can be further simplified to

$$\left(\frac{\psi}{\varphi_N^*}\right)^k\left[\frac{fZ^k\varphi_S^{*k}+\varphi_N^{*k}f_X}{Z^k\varphi_S^{*k}}\right]=\left(\frac{1}{\varphi_S^*}\right)^k\left[\frac{fZ^k\varphi_N^{*k}+\varphi_S^{*k}f_X}{Z^k\varphi_N^{*k}}\right]$$

After cancellations of terms, one can simplify this to

$$\psi^k\left(Z^k\varphi_S^{*k}+\varphi_N^{*k}\eta^k\right)=Z^k\varphi_N^{*k}+\varphi_S^{*k}\eta^k$$

By grouping the terms, one can express one cutoff as a function of another

$$\varphi_N^*=\left[\frac{\eta^k-\psi^kZ^k}{\psi^k\eta^k-Z^k}\right]\varphi_S^*=\nu\varphi_S^* \qquad (18)$$

where $\nu=\left[\frac{\eta^k-\psi^kZ^k}{\psi^k\eta^k-Z^k}\right]>1$ is simply a function of parameters only.

4.1 Solving for Cutoffs

This subsection solves for South's productivity cutoff using the free entry condition

$$f_e = \tilde{p}^* \left(\frac{\sigma - 1}{k_S + 1 - \sigma} \right) (f + \tilde{p}^*_X f_X)$$

Since the probability of entry is given as $\tilde{p}^* = \left(\frac{\bar{\varphi}_S}{\varphi^*_S} \right)^k$ and the conditional probability of export $\tilde{p}^*_X = \left(\frac{\varphi^*_S}{\varphi^*_{SX}} \right)^k = \left(\frac{\varphi^*_S}{Z\varphi^*_N} \right)^k$, the above equation becomes

$$f_e = \left(\frac{\bar{\varphi}_S}{\varphi^*_S} \right)^k \left(\frac{\sigma - 1}{k + 1 - \sigma} \right) \left[f + \left(\frac{1}{\nu} \right)^k f_X \right] \tag{19}$$

This gives the analytical closed-form solution to South's cutoff φ^*_S. The Northern cutoff can be derived from equation (18). With these productivity cutoffs, the export cutoffs can also be derived. The break point - where all firms locate in the North - is characterised in the Appendix.

4.2 Aggregate and Firm-Level Variables

In characterising this equilibrium, a few facts stand out. Firstly, the equilibrium rental-wage ratio in equation (16) is unaffected by any firm level variables. The amount of industrial resources $\kappa + \kappa^*$ in equation (17) available for the differentiated sector is also independent of firm level variables. These are all functions of endowments and other parameters only. As mentioned before, symmetric changes in endowments (relative or absolute), therefore do not have any impact on firm level variables. Secondly, firm level productivity cutoffs are solved through the free entry conditions in equation (9), and are also completely independent from interest or wage rates. The only interaction between firm-level and aggregate variables is how the size of resources $\kappa + \kappa^*$ affect the number of firms in equilibrium.

4.3 A Numerical Example

This subsection provides a simple numerical example to illustrate the equilibrium characterised[14]. This paper does not make any empirical estimates on any parameters. Instead, parameters on preferences and pareto distribution are taken from existing research. The choice of endowment is arbitrary. However, the same level of endowment is chosen for the North and South in

[14]Numerical solutions are obtained through MATLAB. An initial estimate is provided for all the variables. The endogenous variables are then solved through the equilibrium conditions, and incremental updates in each round are carried out by taking the weighted average between the 'old' and 'new' solutions, until there are no further changes (convergence). The solution method is similar to Krugman (1991) and BRS (2007). I am grateful to Stephen Redding for sharing the MATLAB codes.

order not to introduce the home market effect that would otherwise be evident in an Economic Geography model. This assumption will be relaxed later to bring out the home market effect. The list of parameters is provided in Appendix. The set of cost parameters is $\{f, f_e, f_X, \tau\}$. These parameters will also be varied in various numerical solutions to highlight the effects of changes in them.

In the first set of numerical solutions, North and South have the same distribution shape $k_N = k_S = 3.6$. However, North is given a better productivity compared to the baseline scenario, $\bar{\varphi}_N = 0.205 > \bar{\varphi}_S = 0.2$. This shifts the North's distribution rightwards (first degree stochastic dominance). The North is 2.5 per cent more productive than the South on the basis of the unconditional mean.

Even though North and South have the same level of expenditure (given the same level of endowment), the slight perturbation of the pareto distributions results in dramatic differences in industry location. The equilibrium effects on industrial concentration are presented in Table 1 for three different levels of trade cost (the Tomahawk diagram will be presented in a later section).

Table 1: Share of Firms and Capital in More Productive North

$f = 10$ $f_e = 10$ $f_X = 10$	$\tau = 1.40$	$\tau = 1.30$	$\tau = 1.20$
Share of Firms	0.535	0.566	0.661
Share of Capital	0.550	0.590	0.700

The firm-level variables with a relatively low level of trade cost $\tau = 1.10$ are presented in Table 2.

Table 2: Equilibrium Variables with More Productive North

For $\tau = 1.20$	**North**	**South**
Cutoff Productivity	0.3358	0.3104
Probability of Successful Entry	0.1693	0.2054
Average Firms' Productivity, on entry	0.5745	0.5312
Aggregate Price Levels	0.1331	0.1440

The results show that a reasonably small perturbation in the productivity distribution in the North can have a significant impact on the location of firms and capital. A 2.5 per cent increase in the unconditional mean of the productivity distribution creates a high concentration of industrial activity in the North at a intermediate-low level of trade cost of $\tau = 1.20$ (Table 1). The intuition becomes clear in Table 2. The better productivity distribution in the North means that firms there are more productive and profitable. More firms need to move there until the effects of local market competition cancel out any productivity advantages.

Another striking feature of this equilibrium is that in an interior equilibrium r and w are in fact the same in both locations, despite a higher level of capital in the North. The South continues to have a lower aggregate $\frac{K}{L}$ ratio compared to the North, but the marginal returns to capital is the same in both North and South. The Lucas paradox disappears. The superiority of the North is not in the aggregate production function, but is due to an improvement in firm-specific productivity draws.

Thirdly, the fall in trade cost will accentuate the advantages of locating in the North even though the levels of expenditure are the same in each location. In the traditional FC model, if the expenditures of both locations are the same, location of firms will be symmetric at all positive levels of trade cost. The concentration of industry depends on the home market effect. In other words, trade cost is completely 'impotent' in creating asymmetric concentration when the two markets are of equal size.

This is however not the case here. Expenditure is the same in both locations, but the fall in trade cost brings about an increasing concentration of industry to the North. The key to understanding this lies in the inspection of equations (6) and (7). Because the North has a superior productivity distribution, its firms are more productive in equilibrium. In autarky, North and South's CES price indices only reflect the productivities of their domestic firms.

Therefore, with the opening to trade and the fall in trade cost, the increase in ϕ creates a greater increase in weighted average productivity in the South $\hat{\varphi}^*$ compared with $\hat{\varphi}$. Competitive pressure intensifies more quickly in the South with a fall in trade cost, thereby accentuating the advantages of locating in the North. Conversely, Northern firms are less affected by the effects of increased competition as a result of freer trade since they are more productive than their Southern counterparts.

5 The Impact of Risk

In the previous sub-section, the North is more attractive due to its better productivity distribution. However, suppose the South is not less productive but riskier. How will this change the distribution of capital and firms? It is important that the impact of risk is clearly understood since one of the competing hypotheses on why relatively little capital flows to the South is the inherent riskiness in investing there (expropriation risk, political risk etc). In this set of numerical solutions, it is precisely this effect that is being modelled by allowing the two productivity distributions to have the same mean productivity but greater dispersion in the South.

In this set of numerical solutions, the North has the following minimum support $\bar{\varphi}_N = 0.205 > \bar{\varphi}_S = 0.2$. Moreover, the shape of the North's distribution is tighter with $k_N = 3.8 > k_S = 3.6$. The result of this is that the unconditional productivity means in both locations are the same with $c_N = c_S = 0.277$. However, the variance in the North is 16 per cent smaller than the South. The set of parameters in fact creates a 'mean preserving spread' of the productivity distribution in the South. The South is not less productive on average, but has higher risk as characterised by the higher variance. The numerical solution to the equilibrium firm shares, with a moderate level of trade cost $\tau = 1.2$ and different levels of sunk cost f_e, are presented in Table 3.

Table 3: Share of Firms and Capital in Less Risky North with Different Sunk Cost

$\tau = 1.30$ $f_e = 10$ $f_X = 10$	$f_e = 5$	$f_e = 10$	$f_e = 30$
Share of Firms	0.288	0.335	0.396
Share of Capital	0.196	0.244	0.309

The firm-level variables with $\tau = 1.20$ and $f_e = 20$ are presented in Table 4.

Table 4: Equilibrium Variables with Riskier South

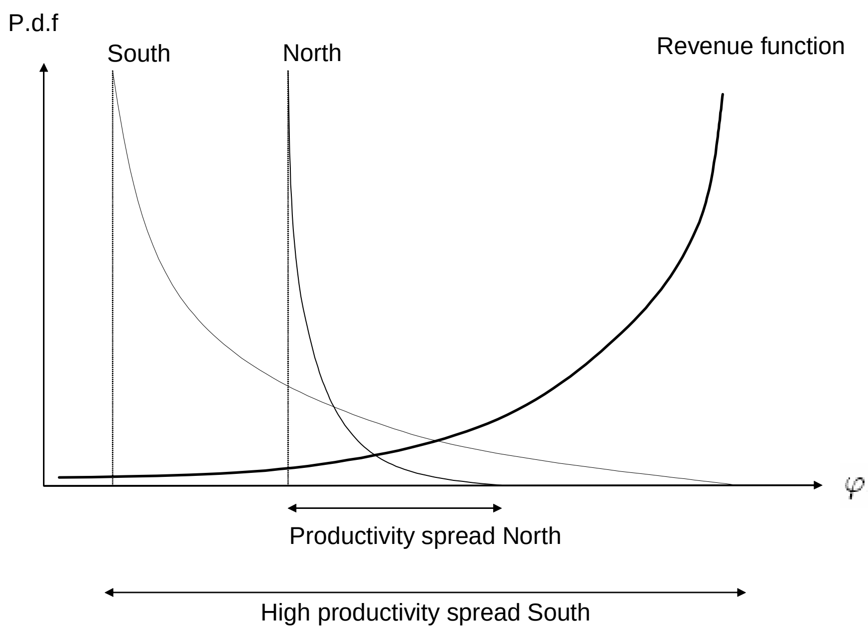
For $\tau = 1.30$ and $f_e = 30$	**North**	**South**
Cutoff Productivity	0.2121	0.2370
Probability of Successful Entry	0.8629	0.5422
Average Firms' Productivity, upon entry	0.3416	0.4057
Aggregate Price Levels	0.2108	0.1886

The results of this sub-section show the effects of greater variance in the productivity distribution. There is a tendency for industrial concentration in the South. The higher variance in the South implies that there is a fatter right side tail for the pareto distribution. As can be seen from Table 4, the effect of this is that although the probability of entry is lower in the South, the average productivity upon successful entry is in fact higher in the South due to the fatter right tail.

What is the economic intuition here? After a firm invests in the sunk cost and discovers its productivity, it can decide whether to incur the fixed production cost f. Incurring the sunk cost creates an option whether to produce, as a firm has a choice of whether to carry out production. At low values of the sunk cost, the South is more attractive since it offers a greater probability of a high productivity draw (and higher average productivity). At higher values of the sunk cost however, this option effectively becomes more expensive and reduces the attraction of the South.

To understand the impact that cutoffs have on distribution of capital, it is useful to first think of ex-ante entry conditions without cutoffs. Suppose a firm has to make a decision to enter either the North or South market in one stage. In other words, there is no separation of f_e and f - a firm discovers its productivity and can begin production without any further investment. Further suppose that the South has a higher productivity spread. With the CES demand, the revenue function is always bounded from below by zero but has no upper bound. A higher productivity spread in the South in fact increases the ex-ante profits, thereby drawing more firms there until any ex-ante difference is equalised. This is the effect seen in Figure 1, where the same CES revenue function is superimposed on the probability densities of the North and South's productivity distribution.

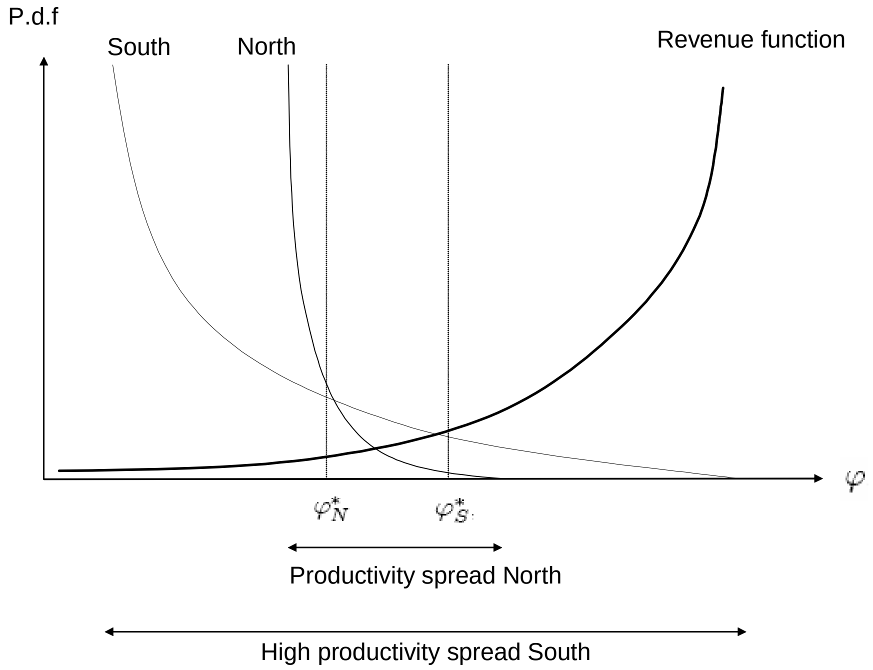
Figure 1: Effects on Expected Revenue with Different North-South Productivity Distributions



The narrow right side tail of the North means that it is giving up the potential for high productivity draws and high profits. Because of the convexity of the revenue function, a firm in the North will have lower expected profits, ceteris paribus. If North and South have similar expenditures, more firms will have to locate to the South until profits are equalised.

However, given a two stage entry game (f_e to discover productivity and f to produce), the riskier location can imply a smaller probability of entry. With the cutoff productivity in a two stage entry decision, revenue functions are now truncated left of the cutoff (see Figure 2). Given that the two locations have different productive distributions, the effect is asymmetric.

Figure 2: Effects of Cutoffs on Expected Revenue (Truncation)



The revenue function is truncated (falls to zero) left of the respective cutoffs. The probability of successful entry can become higher in the North, dominating any foregone probability of an high probability draw in the South. If that happens, more firms will have to locate to the North. It is also possible that potential for high productivity draws in the South to dominate the higher entry probability in the South, and more firms locate to the South in that case. Expected profits are determined by two components - firstly the probability of successful entry and secondly, the expected productivity and profitability post-entry. It is the balance of these two margins that changes the relative attractiveness of each location.

Consider then the effect of the sunk cost f_e. A higher f_e will always shift the cutoffs to the left while a lower f_e shifts cutoff rightwards. As f_e increases and cutoffs shift leftwards, the probability of successful entry always rises faster in the North since it has a narrower productivity distribution. Conversely as f_e falls, North's entry probability falls faster than the South's. The level of f_e therefore changes the balance of the two margins affecting a firm's decision on where to locate. Ignoring the effect of market size for the moment and keeping expenditures the same in both locations, increasing f_e will increase the expected profits of North and result in more firms locating there, and vice versa [see Appendix].

6 Extension to Multi-Industry and Larger North

In this section, the paper further generalises the results to an economy with more than one differentiated industry. As before, there are two regions North and South - where both have the same mean productivity, but South is riskier. The productivity distributions are the same as the previous section[15]. What is different here is that there are two differentiated sectors, A and B. The consumption shares are identical at $\mu = 0.15$ (this is kept small so that the agriculture sector continues to operate in both locations). There are no inter-industry linkages. Furthermore, the North is given an endowment advantage - its capital and labour endowment are 20 per cent more than the South - roughly in line with the idea that developed markets are bigger in size. The paper then shows using numerical solutions how market size can interact with the level of sunk costs to result in different types of specialisation as trade becomes freer.

The two differentiated sectors have exactly the same industrial structure except for one difference. Industry A is a high-tech industry with a sunk cost of $f_e(A) = 30$, while industry B is a low-tech industry with a sunk cost of $f_e(B) = 1$. The two industries have exactly the same cost structure otherwise with $f = 10$ and $f_X = 10$. They also have the same iceberg trade cost. These assumptions are not meant to be realistic. For example, industries with lower sunk cost (low-tech) tend to have higher elasticity of substitution. The assumptions are kept as simple as possible here, only for the purpose of illustrating how two industries with different f_e can end up concentrating at different locations with different ex-ante productivity distributions.

6.1 Tomahawk Diagram

The paper has thus far not presented any Tomahawk diagrams since all intuition will be captured in this section. In the diagram, the level of trade cost falls from the left to the right in the X-axis (ϕ increases from 0 to 1). The Y-axis are the shares of industries located in the North[16]. The Tomahawk

[15] Where $\bar{\varphi}_N = 0.2077 > \bar{\varphi}_S = 0.2$ and $k_N = 4 > k_S = 3.6$.

[16] For industry A, North's share is defined as $\frac{n_A}{n_A + n_A^*}$ where n_A and n_A^* are the number of firms in equilibrium (for zero profits) for the North and South respectively. Similarly for industry B, North's share is defined as $\frac{n_B}{n_B + n_B^*}$.

Figure 3: Tomahawk Diagram with Industries of Two Different Sunk Costs

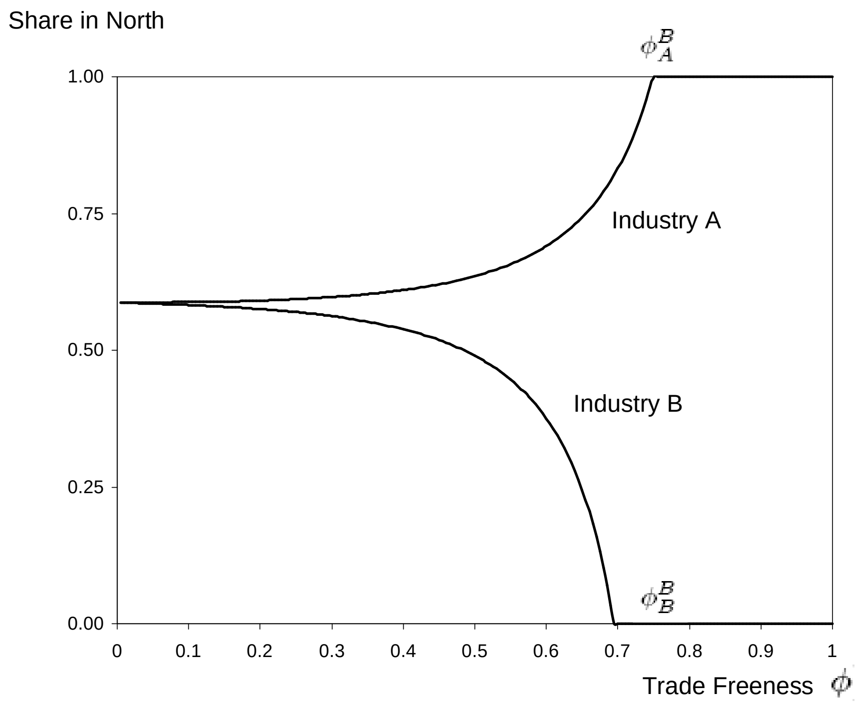


diagram for two industries is presented in Figure 3[17].

As trade becomes freer (from left to right of the diagram), the breakpoints are reached[18]. At ϕ_A^B the break point of industry A, all firms in this industry are located in the North. At ϕ_B^B the break point of industry B, all firms in the industry are located in the South. Again, it is important to emphasize here that North and South will have the same expenditures for each industry. The implication from the analysis is that as trade becomes freer, industries with low sunk costs will migrate to the South while industries with high sunk costs will migrate to the North. The different profiles of the productivity distributions results in different types of specialisation.

7 Conclusion

By synthesising a variant of a New Economic Geography model with recent research into the effects of trade equilibrium under firm heterogeneity, this paper shows that it is possible to rationalise the highly asymmetric allocation of capital between North and South without stipulating large differences in productivity between the two locations.

[17]Note that the shares under autarky are not symmetric since North and South do not have the same productivity distribution.

[18]The break and sustained point are the same for a FC model.

Introducing firm heterogeneity allows the differences between North and South to be modelled by way of firm-level differences rather than through the aggregate production function. With a slight improvement in the North's productivity distribution (first degree stochastic dominance), this paper demonstrates that it is possible to explain the high concentration of firms (and capital) to the North, even though returns to factors of production and expenditures are completely identical between the two regions. The Lucas paradox disappears as a result.

The second key result of the paper demonstrates how the presence of sunk costs in a two-stage entry process can resolve the paradox of risk. 'Hi-tech' or high sunk cost industries tend to locate in the less risky North because it offers them a greater probability of successful entry relative to the South. For 'low-tech' industries with low sunk costs, the North is less attractive since the increase in the probability of entry is offset by the potential of higher post-entry productivity in the South. Capital flows in both directions can be rationalised depending on the level of sunk costs. In a setup with two differentiated sectors, it is possible to show how the high sunk cost industry concentrates in the North and the low sunk cost industry concentrates in the South as trade becomes freer. This result is easily generalised to a multi-industry framework, where the less risky North enjoys a comparative advantage in high sunk cost industries while the South has a comparative advantage in low sunk cost ones. Greater trade liberalisation will lead to both regions specialising in a different set of industries.

The paper also shows how the level of capital flows also depend crucially on the level of trade costs. If trade costs are high, capital will to a large extent be distributed according to expenditure shares. With low trade costs, 'low-tech' industries will locate in the South. This can then explain some stylised differences in the flow of capital to different developing economies. Developing countries with lower trade restrictions will receive more capital particularly from 'low-tech' industries.

8 Appendix

8.1 Calibration of Numerical Simulation

Parameter values are referenced to various research where possible. The list of parameters is given in the table below.

Table 5: Parameters and References

Parameters	Value	Remarks
Preferences		
σ	3.8	Referenced to Bernard et al (2003), Ghironi and Melitz (2004) and BRS (2007) estimate of 3.8.
μ	0.5	Arbitrary, no effect on firm aggregates or distribution of firms between the locations, so long as it is small enough such that agriculture continues to exist in both economies.
Endowment		
K_N L_N K_S L_S	1,000,000	Endowments are kept large relative to the fixed cost in order to have an arbitrarily large number of firms in equilibrium. Endowments are symmetric between North and South except for one set of solutions where the home market effect is modelled by increasing North's endowment by 20 per cent.
Pareto Distribution		
$\bar{\varphi}$	0.2	The baseline support is referenced to BRS (2007). However, in the various sets of simulations, the support is varied.
k	3.6	The baseline shape is referenced to BRS (2007). However, in the various sets of simulations, the shape is varied.
Technology		
A	1	Aggregate productivity is normalised to unity for convenience.
α	0.3	This is the capital share in the production of the composite input. Its effect is only on the wage-rental ratio, and has no effect on distribution of firms.

8.1.1 Deriving Total Resource Cost

This subsection proceeds to solve for the total variable production cost in order to pin down the input requirements for the manufacturing sector [see equations (10) and (11)].

Consider a standard total variable production cost (TC) function. This is the integration of the resources used by each firm $\frac{q(\varphi)}{\varphi}$ (marginal cost $\frac{1}{\varphi}$ multiplied by quantity $q(\varphi)$) over the entire distribution of active firms above the cutoff φ^*

$$TC = \int_{\varphi^*}^{\infty} n\frac{q(\varphi)}{\varphi}\frac{g(\varphi)}{1-G(\varphi^*)}d\varphi = nq(\tilde{\varphi})\int_{\varphi^*}^{\infty}\frac{1}{\varphi}\left(\frac{\varphi}{\tilde{\varphi}}\right)^{\sigma}\frac{g(\varphi)}{1-G(\varphi^*)}d\varphi$$

The second equality makes use of the property that $q(\varphi) = q(\tilde{\varphi})\left(\frac{\varphi}{\tilde{\varphi}}\right)^{\sigma}$. With the pareto distribution and the definition of $q(\varphi)$, the above equation can then be simplified to

$$TC = nq(\tilde{\varphi})\frac{k\varphi^{*k}}{\tilde{\varphi}^{\sigma}}\int_{\varphi^*}^{\infty}\varphi^{\sigma-k-2}d\varphi = n\left[\frac{k}{k+1-\sigma}\right]\frac{q(\varphi^*)}{\varphi^*}$$

Total production cost is a $n\left[\frac{k}{k+1-\sigma}\right]$ factor of the variable production cost of the marginal firm $\frac{q(\varphi^*)}{\varphi^*}$.

Consider $\frac{q(\varphi^*)}{\varphi^*}$. Multiplying the numerator and denominator by $p(\varphi^*)$ will give $\frac{q(\varphi^*)}{\varphi^*} = \frac{p(\varphi^*)q(\varphi^*)}{p(\varphi^*)\varphi^*} = \frac{r(\varphi^*)}{p(\varphi^*)\varphi^*}$. Since the marginal firm's revenue $r(\varphi^*)$ must cover σf in equilibrium, and its optimal price is $p(\varphi^*) = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{\varphi^*}$, it is possible to simplify the equation further to $\frac{q(\varphi^*)}{\varphi^*} = (\sigma-1)f$. This allows the total cost equation to be written as

$$TC = n\left[\frac{k}{k+1-\sigma}\right](\sigma-1)f \tag{B1}$$

Similarly, the total cost to the exporters can be written as

$$TC_X = \tilde{p}_X.n\left[\frac{k}{k+1-\sigma}\right](\sigma-1)f_X$$

These expressions are then used in equations (10) and (11).

8.2 Characterising the Break Point

In FC models, the break points and sustain points are the same since there are no agglomeration effects - whether the initial condition is one of symmetry or asymmetry does not change the outcome. One can begin to solve for the break point by using equation (5). By writing out the aggregate price aggregates explicitly

$$\frac{\varphi_N^{*\sigma-1}}{n\tilde{\varphi}_N^{\sigma-1}+\tilde{p}_X^*n^*\phi\tilde{\varphi}_{SX}^{\sigma-1}} = \frac{\varphi_S^{*\sigma-1}}{\tilde{p}_Xn\phi\tilde{\varphi}_{NX}^{\sigma-1}+n^*\tilde{\varphi}_S^{\sigma-1}}$$

Dividing the numerator and denominator of the LHS by $\varphi_N^{*\sigma-1}$ and the right hand side by $\varphi_S^{*\sigma-1}$, one can further simplify the equation to

$$\frac{1}{n+\tilde{p}_X^* n^* \phi Z^{\sigma-1}} = \frac{1}{\tilde{p}_X n \phi Z^{\sigma-1} + n^*}$$

or

$$\tilde{p}_X n \phi Z^{\sigma-1} + n^* = n + \tilde{p}_X^* n^* \phi Z^{\sigma-1}$$

This equation gives the relationship between n and n^*. When all firms are concentrated in the North, $n^* = 0$. Hence

$$\tilde{p}_X \phi Z^{\sigma-1} = 1$$

Recall that $\tilde{p}_X = \left(\frac{\varphi_N^*}{\varphi_{NX}^*}\right)^k = \left(\frac{\varphi_N^*}{Z\varphi_S^*}\right)^k = \left(\frac{\nu}{Z}\right)^k$ giving

$$\left[\frac{\psi^k \tau^k \eta^{\frac{k}{\sigma-1}} - \eta}{\tau^k \eta^{\frac{k}{\sigma-1}} - \psi^k \eta}\right] \eta^{\frac{1}{k}} = \tau \eta^{\frac{1}{\sigma-1}} \tag{20}$$

which provides the implicit solution to the break point - defined as the smallest level of τ that satisfy the above equation. For simplicity, one can assume $f_X = f$ [such as in Falvey, Greenaway and Yu (2006)] or $\eta = 1$. The above equation reduces to

$$\psi^k \tau^k - 1 = \tau\left(\tau^k - \psi^k\right)$$

The bigger advantage the North is given (higher ψ), the higher the τ that can satisfy this condition.

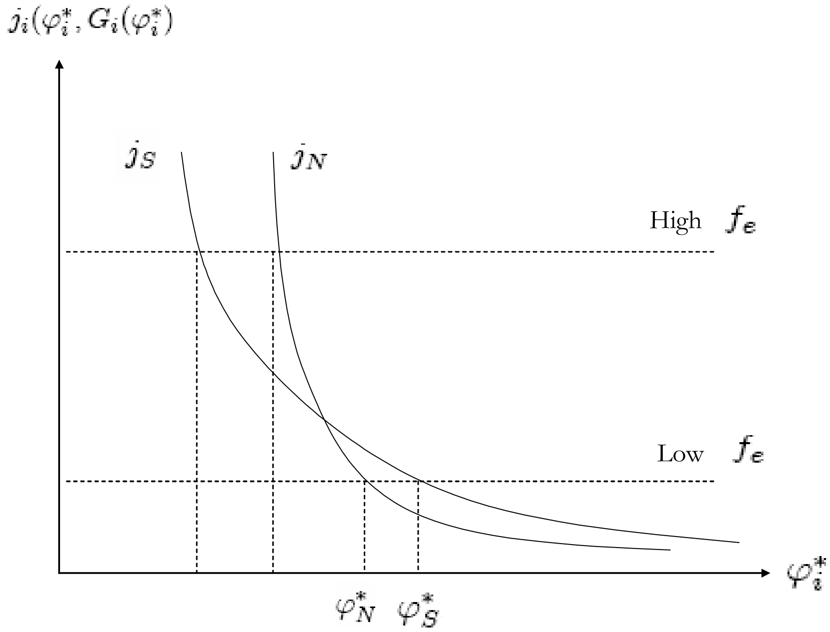
8.2.1 Equilibrium Conditions with Mean Preserving Spread for South

In principle, one can solve for the full equilibrium, including the break/sustain point, in the same manner as when the North is given a productivity advantage. However, because the South's productivity distribution no longer has the same shape as the North $k_N \neq k_S$, it is also not possible for terms to cancel out to arrive at the simple relationship, making it difficult to generalise the marginal firm condition. Instead, this subsection proceeds to provide some comparative static analytical results, while the numerical results are presented in the main text.

If the North has a higher support but a narrow distribution such that the unconditional means are the same, the equilibrium can be depicted by Figure 4.

In the case of low f_e, the average productivity of Southern firms is higher than the North since the cutoff is higher there ($\varphi_S^* > \varphi_N^*$) and that the prob-

Figure 4: Effects of Mean-Preserving Spread of Productivity Distribution



ability mass right of φ_S^* is thicker [see Figure 4]. With low f_e, more firms will have to locate in the South to equalise ex-ante profits between the two locations. Conversely if f_e is high enough, South's cutoff φ_S^* will be low enough relative to φ_N^* such that even the fatter tails cannot compensate. In that case, more firms will locate to the North.

Effects of Increasing Sunk Cost Ignoring the differences between North and South for the moment. Consider only the marginal impact of an increase in the sunk cost f_e. From the ex-ante free entry condition

$$\left(\frac{\bar{\varphi}}{\varphi^*}\right)^k \left(\frac{\sigma-1}{k+1-\sigma}\right)\{f+\tilde{p}_X f_X\} = f_e$$

The mean of a pareto distribution is given as $c = \frac{k\bar{\varphi}}{k-1}$. To keep the mean constant at c while allowing k to vary, the minimum support has to be different. The minimum support can be written as

$$\bar{\varphi} = \frac{c(k-1)}{k}$$

This can be substituted into the previous equation to give

$$\varphi^{*-k}\left[\frac{c(k-1)}{k}\right]^k \left(\frac{\sigma-1}{k+1-\sigma}\right)(f+\tilde{p}_X f_X) = f_e \tag{B4}$$

Partially differentiating φ^* with respect to f_e gives

$$\begin{aligned}\frac{\partial\varphi^*}{\partial f_e} &= \frac{-\varphi^{*k+1}}{k\left[\frac{c(k-1)}{k}\right]^k\left(\frac{\sigma-1}{k+1-\sigma}\right)(f+\tilde{p}_X f_X)} \\ &= \frac{-\varphi^*}{k\varphi^{*-k}\left[\frac{c(k-1)}{k}\right]^k\left(\frac{\sigma-1}{k+1-\sigma}\right)(f+\tilde{p}_X f_X)}\end{aligned} \tag{B5}$$

In equilibrium, equation (B4) will always hold (envelope condition). This allows equation (B5) to be simplified to

$$\frac{\partial\varphi^*}{\partial f_e} = \frac{-\varphi^*}{kf_e} \tag{B6}$$

This result shows that an increase in f_e always reduces the cutoffs φ^* - this is a standard result. But what are the second order effects when one specifically considers the pareto distribution? Equation (B6) shows that $\frac{\partial\varphi^*}{\partial f_e}$ is more negative at lower level of k (higher variance). The cutoff therefore falls relatively more quickly for the location with the lower k.

The probability of entry $\tilde{p}$ is given as

$$\tilde{p} = \left[\frac{c(k-1)}{k\varphi^*}\right]^k$$

The effect of increase in f_e on entry probability can be found by the partial derivative

$$\begin{aligned}\frac{\partial\tilde{p}}{\partial f_e} &= \frac{\partial\tilde{p}}{\partial\varphi^*}\frac{\partial\varphi^*}{\partial f_e} \\ &= \left[\frac{c(k-1)}{k}\right]^k\frac{\varphi^{*-k}}{f_e}\end{aligned} \tag{B7}$$

Since $\left[\frac{c(k-1)}{k}\right]^k$ is increasing in k, the increase in f_e therefore increases the probability of entry relatively more quickly for a location with higher k (lower variance).

Firstly, equation (B6) says that with an increase in f_e, the cutoff φ^* falls relatively faster for a location with higher variance (which is the South in the context of the discussion). Since average productivity is a function of the cutoff only, this implies that the average productivity falls relatively quickly in the South as well. Secondly, equation (B7) says that the probability of entry $\tilde{p}$ is higher when f_e is higher, but this entry probability increases relatively slower for the location with the higher variance (South). This implies that as f_e rises, the average productivity and probability of entry must rise relatively less in the location with the higher variance (South). As the sunk cost f_e increases, the ex-ante profit of the South falls relative to the North and more firms will have to locate in the North to restore the equilibrium.

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