Panel Data Estimation of Production Functions - cont.

Professor John Van Reenen, LSE, Director of CEP

January 2008
1 Economic Theory Behind Blundell-Bond approach to production functions

What theory justifies the use of lags as IVs for current inputs? Bond and Söderbom (2005) give an example (see also Bond and Van Reenen, 2008).

Value equation

\[ V(K_{t-1}, L_{t-1}) = P_tF_t(K_t, L_t) - P^K_t I_t - W_t L_t \]

\[ -P^K_t G(I_t, K_t) - W_t C_t(H_t, L_t) + \psi E_t[V_{t+1}(K_t, L_t)] \]

\( P^K_t \) = price of capital goods, \( W \) = wage rate, \( I \) = capital investment, \( H \) = labour hiring, \( F(.) \) is the production function, \( G(.) \) is adjustment cost function for capital investment, \( C(.) \) is adjustment cost function for labour hiring, \( \psi \) is the discount factor. Adj costs (last term) are financial, productivity shock is known by firms prior to investment and hiring decisions.
Assume that the accumulation equations are

\[ K_t = (1 - \delta)K_{t-1} + I_t \]

\[ L_t = (1 - q)L_{t-1} + H_t \]

The dynamic first order condition for labour is

\[ P_t \frac{\partial F_t}{\partial L_t} - W_t \frac{\partial C_t}{\partial H_t} = W_t + \lambda_t^L \left[ 1 - (1 - q)\psi_t E_t \left( \frac{\lambda_{t+1}^L}{\lambda_t^L} \right) \right] \]  

(1)

\[ \lambda_t^L = \frac{1}{1 - q} \frac{\partial \psi_t}{\partial L_{t-1}} = W_t \frac{\partial C_t}{\partial H_t}, \text{ the shadow value of inheriting one unit of labour from the previous period. This depends on productivity shocks.} \]

\[ \lambda_t^K = \frac{1}{1 - \delta} \frac{\partial \psi_t}{\partial K_{t-1}}, \text{ the shadow value in period t of inheriting one additional unit of capital.} \]
If there are no labour adjustment costs then (1) becomes the standard static FOC

\[ P_t \frac{\partial F_t}{\partial L_t} = W_t \]  \hspace{1cm} (2)

In (1) the MP of labour is equated to the wage plus a term that depends on current and expected future shadow value of labour. Even if all firms face common wages, idiosyncratic productivity shocks affect the marginal product of labour leading to different optimal levels of the factor inputs across firms.

**Example:** Constant discount factor, prices, forecasting error for hiring is \( \varepsilon_{t+1}^H \) and adjustment cost process is quadratic

\[ C_t(H_t, L_t) = c \frac{H_t^2}{2} \]

So the hiring equation is
\[ H_{t+1} = \frac{1}{(1 - q)\psi} H_t - \left( \frac{1}{c(1 - q)\psi} \right) \left( \frac{P}{W} \frac{\partial F_t}{\partial L_t} - 1 \right) + \varepsilon_{t+1} \]

Current hiring depends on past productivity shocks through past hiring and the previous period’s marginal productivity of labour. Note that the persistence of hiring (and hence employment) depends on the adjustment cost parameter, c. Provided that capital and labour adjustment costs are different this will imply variation in the capital-labour ratio.

Consider a permanent productivity shock. Firm increases both factors but more on the labour side than capital because capital adjustment costs are higher. This leads to a temporary fall in capital-labour ratio.

Monte-Carlo evidence ( Bond and Soderbom (2005) ) suggests that in practice identification is more difficult when:-

- adjustment costs are similar for both factors (collinearity of inputs)
• adjustment costs are ”too low” (too little independent variation of inputs over time)

• adjustment costs are too high (inputs collinear again)

Also show that (a) empirical identification easier when there are stochastic adjustment costs, (b) OP performs poorly.
1.1 Econome "Tricks" of the trade when using GMM-SYS

Programmed in DPD, STATA 9.2 (xtabond2) and OX.

(a) Using all moment conditions for the difference equation is generally **not** a good idea as many of them are uninformative (e.g. instrumenting $\Delta k_{it}$ with $k_{it-7}$ unlikely to be useful). Asymptotically this should not matter, but weak instruments can cause bias in finite samples. One solution is to experiment with different lag lengths and examine sensitivity of results to these.

(b) One step vs. two step GMM. Two-step GMM uses the estimated first step covariance matrix to improve the efficiency of the estimator. Unfortunately, Monte Carlo evidence shows that 2 step standard errors are "too low". Standard practice is to use one step coefficients and a generalized correction for heteroskedacity and autocorrelation a la White (1980). Windemeeijer (2003) has suggested a more efficient procedure.

(c) **Diagnostics.** The Sargan statistic is useful, but has low power to reject the null, especially when the number of over-identification restrictions is
large. The LM tests of serial correlation are better. There should be significant first order correlation (of the first differenced residuals) and no second order correlation for the usual t-2 levels instruments to be valid. If GMM-SYS is used then Sargan Difference tests should be examined to see if there is evidence that the additional moments are valid. If not GMM-DIF may be the best one can do. Finally, the COMFAC restrictions should be tested.

(d) It is good practice to examine the univariate stochastic processes. Estimating AR(q) regressions under the different models is one way to get a feel for the stochastic processes. In particular, we know the biases for the AR(1) model of $y_{it}$, OLS levels upwards, OLS differences downwards. So GMM so give results in between these bounds. If it does not this may signal some problems of the IVs.
2 Olley Pakes (1996)

Estimation of production functions for firm i

\[ y_i = \beta_0 + \beta_k k_i + \beta_l l_i + \epsilon_i \]  \hspace{1cm} (3)

\( y = \ln Y \), etc.

\( Y = \text{output} \)

\( K = \text{capital} \)

\( L = \text{labour} \)

\( \epsilon_i \) could be technology, managerial ability, etc.

Firm knows \( \epsilon_i \) when making an input choice but econometrician does not observe (Marshak and Andrews, 1944).
Example

Capital is fixed, labour is completely variable, input and output markets are perfectly competitive. FOC for labour is

\[ l_i = \frac{1}{1 - \beta_l} \left[ \epsilon_i + \beta_0 + \ln \beta_l + \ln \frac{P}{W} + \beta_k k_i \right] \]

where \( P \) is output price and \( W \) is the wage rate.

The labour input choice is a function of the unobserved shock, \( \epsilon_i \). This means that labour is endogenous in the production function so that OLS will lead to biased results.

In general biases are hard to sign but in 2 factor case where labour is perfectly flexible and capital is subject to adjustment costs there will be an upwards bias on the labour coefficient (reacts to shocks) and a downward bias on the capital coefficient.
The estimators discussed above (within groups, IV, GMM-SYS etc.) suggest themselves

Input prices a possible "external" IV. Problems are:

i) firm level variation input prices needed. Are difficult to observe at the micro level (e.g. movements in the cost of capital determined by aggregate interest rates)

ii) Even when observed input prices contaminated by changes in quality of inputs (e.g. average wage at the firm level may be due to mix of heterogeneous workers - e.g. skilled and unskilled)

2.1 OP Solution

Assumptions:

- timing
• scalar unobservable

• strict monotonicity

\[ y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \eta_{it} \]  \hspace{1cm} (5)

Error term in two parts:

i) A serially uncorrelated idiosyncratic shock (\( \eta_{it} \)) unobserved to econometrician and firm

ii) An "efficiency" term \( \omega_{it} \) unobserved to econometrician, observed to firm. This is assumed to follow a first order Markov Process:

\[ p(\omega_{it+1}|\{\omega_{i\tau}\}_{\tau=0}^t, J_{it}) = p(\omega_{it+1}|\omega_{it}) \]
$J_{it}$ is the information set at time t. $p(\omega_{it+1}|\omega_{it})$ is strictly increasing in $\omega_{it}$. Firms with a higher efficiency level today expect to have a higher efficiency level tomorrow.

Labour has no adjustment costs (perfectly variable). Capital accumulation rule is deterministic

$$k_{it} = (1 - \delta)k_{it-1} + i_{it-1}$$

Important that investment chosen in last period, so that capital is pre-determined in the production function (does not react to $\eta_{it}$ shocks)

Bellman equation considers 2 dimensions:

- **Optimal exit decision/selection.** If flow profits $(\pi(k_{it}, \omega_{it}, \Delta_t) - c(i_{it}, \Delta_t))$ are less than the sell-off value $(\Phi(k_{it}, \omega_{it}, \Delta_t))$, the firm will exit
\[ \chi_{it} = \begin{cases} 1 & \text{if } \omega_{it} \geq \bar{\omega}(k_{it}, \Delta_t) = \bar{\omega}_t(k_{it}) \\ 0 & \text{otherwise (exit)} \end{cases} \] (6)

- **Investment rule** (solves inner maximization problem).

\[ i_{it} = i(k_{it}, \omega_{it}, \Delta_t) = i_t(k_{it}, \omega_{it}, \Delta_t) \]

**Bellman Equation**

\[
V(k_{it}, \omega_{it}, \Delta_t) = \\
\max \left[ \Phi(k_{it}, \omega_{it}, \Delta_t), \pi(k_{it}, \omega_{it}, \Delta_t) - c(i_{it}, \Delta_t) \\
+ \psi E[V(k_{it+1}, \omega_{it+1}, \Delta_{t+1})|k_{it}, \omega_{it}, \Delta_t, i_{it}] \right]
\]

\(k_{it}, \omega_{it}, \Delta_t\) (e.g. market structure) are sufficient to describe the state space.
2.2 Endogeneity of inputs

Ignore selection for the moment and assume \( I > 0 \) (positive investment)

Because (Pakes 1994) investment is strictly monotonic in \( \omega_{it} \), implies that we can invert \( i_t(k_{it}, \omega_{it}, \Delta_t) \) to obtain:

\[
\omega_{it} = h_t(k_{it}, i_{it}) \quad (7)
\]

This depends also on assumption of single scalar unobservable state. Substituting (7) into production function (5) gives

\[
y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + h_t(k_{it}, i_{it}) + \eta_{it} \quad (8)
\]

First stage of OP involves estimating this equation non-parametrically
\[ y_{it} = \beta_l l_{it} + \phi_t(k_{it}, i_{it}) + \eta_{it} \]  \hspace{1cm} (9)

Can use series estimator (i.e. high order polynomials) or kernel methods (e.g. Robinson, 1988)

This will give, \( \hat{\beta}_l \), a consistent estimate of labour coefficient.

Second stage used to estimate capital coefficient. From stage 1 we have \( \hat{\phi}_t \)

\[ \phi_t(k_{it}, i_{it}) = \beta_0 + \beta_k k_{it} + \omega_{it} \]

or in terms of productivity

\[ \widetilde{\omega}_{it}(\beta_0, \beta_k) = \hat{\phi}_{it} - \beta_0 - \beta_k k_{it} \]  \hspace{1cm} (10)
Next decompose $\omega_{it}$ into its expectation at $t-1$ and a residual

$$\omega_{it} = E(\omega_{it}|J_{it-1}) + \xi_{it}$$

Using the Markov assumption

$$\omega_{it} = E(\omega_{it}|\omega_{it-1}) + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$$ (11)

where $g(.)$ is an unknown function that we will estimate non-parametrically. $\xi_{it}$ by definition is uncorrelated with $k_{it}$, because of the timing assumptions.

Using the estimation of $\beta_l$ from the first stage

$$y_{it} - \beta_l l_{it} = \beta_0 + \beta_k k_{it} + \omega_{it} + \eta_{it}$$ (12)

and substituting in $\omega_{it-1}$, (11) gives
\[ y_{it} - \beta l_{it} = \beta_0 + \beta_k k_{it} + g(\omega_{it-1}) + \xi_{it} + \eta_{it} \]  

(13)

then substituting in (10)

\[ y_{it} - \beta l_{it} = \beta_0 + \beta_k k_{it} + g(\phi_{t-1} - \beta_0 - \beta_k k_{it-1}) + \xi_{it} + \eta_{it} \]  

(14)

Or

\[ y_{it} - \beta l_{it} = \beta_k k_{it} + \tilde{g}(\phi_{t-1} - \beta_k k_{it-1}) + \xi_{it} + \eta_{it} \]  

(15)

where \( \tilde{g}(\cdot) \) subsumes the constant.

Key point is that \( \xi_{it} + \eta_{it} \) is uncorrelated with the right hand side variables. If we use a series estimator for \( \tilde{g}(\cdot) \) then (15) can be estimated by non-linear least squares.
An alternative to the two-step OP approach is to do everything in a simple step using GMM (as suggested by Wooldridge, 2005). The moment conditions are

\[ E[\eta_{it} \otimes f_1(k_{it}, i_{it}, l_{it})] = 0 \]

\[ E[(\eta_{it} + \xi_{it}) \otimes f_2(k_{it}, k_{it-1}, i_{it-1})] = 0 \]

where \( f_1(.) \) and \( f_2(.) \) are the vector valued instrument functions. Choices for these lead to similar moments to OP

**Relationship of OP to Blundell-Bond**

The two approaches are not nested within each other and rely on different identification assumptions. Differences include
(a) timing of capital accumulation process

(b) Underlying economic theory.

(c) modelling of $\omega_{it}$. BB parametrically model the $\omega_{it}$ as consisting of an AR(1) process, a fixed effect and a white noise error. In OP $\omega_{it}$ is allowed to take a more general stochastic form so long as it is a first order Markov process. If the Markov process could be approximated by an AR(1), however, then the OP set-up becomes a special case of BB with strong restrictions on the form of the adjustment costs. Under OP (a) labour is perfectly flexible, (b) capital is pre-determined. These timing assumptions are relaxed under BB, but cannot be so easily altered under OP (see below).

**Diagnostics**

- Monotonicity condition. Productivity shock should be increasing with investment (conditional on capital)
• Labor in this period should not be correlated with the innovation in productivity next period (i.e. $E(l_{it} \xi_{it+1}) = 0$)

• If we have more than one proxy available (e.g. Levinsohn-Petrin could use materials or electricity, getting the same results from either is reassuring).
Table 3: Production Function Estimates from Pavcnik (2002)*

<table>
<thead>
<tr>
<th>Factor</th>
<th>OLS</th>
<th>Fixed Effects</th>
<th>Olley-Pakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unskilled Labor</td>
<td>0.178(0.006)</td>
<td>0.210(0.010)</td>
<td>0.153(0.007)</td>
</tr>
<tr>
<td>Skilled Labor</td>
<td>0.131(0.006)</td>
<td>0.029(0.007)</td>
<td>0.098(0.009)</td>
</tr>
<tr>
<td>Materials</td>
<td>0.763(0.004)</td>
<td>0.646(0.007)</td>
<td>0.735(0.008)</td>
</tr>
<tr>
<td>Capital</td>
<td>0.052(0.003)</td>
<td>0.014(0.006)</td>
<td>0.079(0.034)</td>
</tr>
</tbody>
</table>


Figure 1:

[Examples of empirical papers here]
2.3 Extensions to OP

2.3.1 Allowing for endogenous selection

Let exit be determined according to equation (6). Note that since firms with higher capital are less likely to exit, ignoring selection causes us to underestimate the impact of capital on output.

First stage unaffected. The residual $\eta_{it}$ represents factors that are unpredictable by the firm before the exit decision.

Second stage affected because exit depends on $\xi_{it}$ which affects output as seen in equation (13).

Essentially we use the observed exits

\[
Pr(\chi_{it} = 1|J_{it-1}) = Pr(\omega_{it} \geq \bar{\omega}_t(k_{it})|J_{it-1}) \\
= Pr(\chi_{it} = 1|\omega_{it-1}, \bar{\omega}_t(k_{it}))
\]
### ALTERNATIVE ECONOMETRIC ESTIMATES OF THE PRODUCTION FUNCTION

<table>
<thead>
<tr>
<th>Estimation Method</th>
<th>(1) OLS LEVELS</th>
<th>(2) WITHIN GROUPS</th>
<th>(3) OLLEY PAKES</th>
<th>(4) GMM-SYS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(L) ) _u</td>
<td>0.505</td>
<td>0.543</td>
<td>0.426</td>
<td>0.456</td>
</tr>
<tr>
<td>labor</td>
<td>(0.020)</td>
<td>(0.022)</td>
<td>(0.022)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>( \ln(K) ) _u</td>
<td>0.128</td>
<td>0.059</td>
<td>0.156</td>
<td>0.114</td>
</tr>
<tr>
<td>capital</td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.036)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>( \ln(N) ) _u</td>
<td>0.358</td>
<td>0.325</td>
<td>0.412</td>
<td>0.353</td>
</tr>
<tr>
<td>materials</td>
<td>(0.017)</td>
<td>(0.022)</td>
<td>(0.024)</td>
<td>(0.046)</td>
</tr>
</tbody>
</table>

SC(1) p-value: 0.000
SC(2) p-value: 0.195
SARGAN p-value: 0.153
SARGAN-DIF p-value: 0.332
Firms: 709

709 medium sized manufacturing firms, 1994-2004, UK, US, France and Germany

Source: Bloom and Van Reenen (2005). Appendix Table D1

Figure 2:
\[
\begin{align*}
\varphi_t(\omega_{it-1}, k_{it}) &= 
\varphi_t(i_{it-1}, k_{it-1}) \\
&= P_{it}
\end{align*}
\] (16)

Equation (16) can be estimated non-parametrically (e.g. probit model with 4th order polynomial).

If density of \( \omega_{it} \) given \( \omega_{it-1} \) is positive in an area around \( \tilde{\omega}_t \) we can invert (16) and write

\[
\tilde{\omega}_t(k_{it}) = f(\omega_{it-1}, P_{it})
\]

Therefore
\[ E[y_{it} - \beta_l l_{it} | J_{it-1}, \chi_{it} = 1] = \]
\[ \beta_0 + \beta_k k_{it} + g(\omega_{it-1}, f(\omega_{it-1}, P_{it})) + \varsigma_{it} \]  
\[ = \beta_0 + \beta_k k_{it} + \bar{g}(\phi_{t-1} - \beta_k k_{it-1}, P_{it}) + \varsigma_{it} + \eta_{it} \]

This is similar to (15) except now we also have \( P_{it} \) in the nonparametric \( g(.) \) function. \( P_{it} \) controls for the impact of selection on the expectation of \( \omega_{it} \).

Note in selection literature \( P_{it} \) is referred to as the propensity score (Rosenbaum and Rubin, 1983).

In OP need to control for \( P_{it} \) and also \( \omega_{it} \) and \( \bar{\omega}_{it} \).

### 2.3.2 Zero Investment levels

Lots of zeros in micro data on investment (real options from irreversibilities? Bloom, Bond, Van Reenen, 2007, ReStud) - e.g. in Levinsohn
and Petrin (2003) Chilean data 50% are zeros. Consider augmenting the production functions with material inputs, M.

\[ y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \omega_{it} + \eta_{it} \] \hspace{1cm} (19)

The materials demand equation (non-dynamic)

\[ m_{it} = m_t(k_{it}, \omega_{it}) \]

If this is strictly monotonic it can be inverted

\[ \omega_{it} = h_t(k_{it}, m_{it}) \]

\[ y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + h_t(k_{it}, m_{it}) + \eta_{it} \] \hspace{1cm} (20)
\[ y_{it} = \beta_0 + \beta_l l_{it} + \phi_t(k_{it}, m_{it}) + \eta_{it} \] 

(21)

First stage as before - get estimate of \( \hat{\beta}_l \) and \( \hat{\phi}_t \).

\[ y_{it} - \beta_l l_{it} = \beta_0 + \beta_k k_{it} + \beta_m m_{it} + \tilde{g}(\phi_{t-1} - \beta_k k_{it-1}, P_{it}) + \xi_{it} + \eta_{it} \] 

(22)

Note that \( m_{it} \) is not orthogonal to \( \xi_{it} + \eta_{it} \) so LP have to instrument using \( m_{it-1} \). This is rather unsatisfactory as (a) what is the reason why lagged m’s are correlated with current m’s (no adjustment costs by assumption), (b) part of rationale for OP is to avoid having to do IV.

2.3.3 Relaxing assumptions on inputs (e.g. Allowing for adjustment costs for labour)

In standard OP labour is static and variable and capital is fixed and dynamic. But other cases are possible (e.g. adjustment costs for labour
mean labour is dynamic and variable). In this case labour coefficient not identified in first stage (we still get the \( \phi_t(k_{it}, i_{it}, l_{it}) \) estimates. But we do identify labour coefficient at stage 2 using \( l_{it-1} \) as an IV.

<table>
<thead>
<tr>
<th>Is the factor input variable ( (i.e. \text{ correlated with } \xi_{it}) )?</th>
<th>variable</th>
<th>fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is the input static</td>
<td>static</td>
<td>identified from stage 1</td>
</tr>
<tr>
<td>or dynamic</td>
<td>dynamic</td>
<td>identified at 2nd stage</td>
</tr>
<tr>
<td>(i.e. in the state space)</td>
<td></td>
<td>with IVs of ( X_{it}, X_{it-1} )</td>
</tr>
</tbody>
</table>

2.3.4 OP as an example of the control function approach

OP method is actually a version of the "control function" approach (Powell). The endogeneity problem arises from \( \omega_{it} \) so including a "proxy" variable for \( \omega_{it} \) removes the bias. Standard alternative to IV (useful in non-parametric setting - see Blundell and Powell, 2004).
2.3.5 Exact multicollinearity problem

Ackerberg, Caves and Foster (2006), ACF, point out a logical problem with the OP set-up of a Cobb Douglas production function and common prices (input and output) across firms. Any correctly specified control function must be perfectly collinear with the flexible inputs. Re-write the (4)

\[ l_i = \frac{\epsilon_i}{1 - \beta_l} + \ln \frac{\beta_l + \beta_0}{1 - \beta_l} - (w - p) + \frac{\beta_k k_i}{1 - \beta_l} \]  

(23)

If the wage rate is common across firms then there are no valid IVs for labour in the production function that are informative once we condition on capital. The only variation of \( l_i \) after conditioning on capital is due to \( \epsilon_i \), but a valid instrument must be orthogonal to \( \epsilon_i \).

Solutions
i) i.i.d factor price shocks. This goes against the general OP framework. As mentioned before, may be difficult to generate these (e.g. wages reflect labour quality as well as factor price). Minimum wages and unions a possibility. Tax rates also possible for capital (e.g. Bloom, Griffith, Van Reenen, 2002) especially. Problem is generating inter-firm variation (Cummins et al, 2002)

ii) second moment shocks to environment

iii) Timing assumptions. Labour chosen at t-b between t and t-1. So

\[ l_{it} = f_t(\omega_{it-b}, k_{it}) \]

which breaks the exact collinearity problem. But (a) slightly arbitrary, (b) impossible for LP.
2.3.6 Relaxing Scalar Unobservable assumption

1. Productivity not simply a first order Markov process

2. Investment may respond to demand factors independently of productivity so cannot do the inversion.

3. Productivity may partially be controlled by the firm through (e.g.) R&D expenditures (e.g. a controlled Markov process). Doraszelski and Jaumandreu (DJ, 2007) suggest a way to do this. DJ also use albor FOC more structurally: but this relies on exogenous factor price variation between firms.

4. There may be measurement error in investment and this is not allowed for in OP. Nonlinear errors in variable problem.

See ABBP (2005) for discussion
3 Other Issues with production function

3.1 Revenue productivity

The dependent variable is meant to be output, but is actually revenues deflated with an industry price deflator because firm level prices are not observed. This is a problem because the estimated coefficients are a mix of technological parameters from the production function (which we want) and mark-ups (which we don’t necessarily want).

Solution is:

i) get better data on firm prices (Haltiwanger et al, 2005)

ii) to posit more explicit form of demand and jointly estimate mark-ups. Example is Klette and Griliches (See also Mellitz, 2003) who posit monopolistic competition (Tybout et al try a BLP style logit).
Assume that firm specific demand function is

\[ y_i = -\sigma (p_i - p_I) + \nu p_I + d_I \]

where \( p_i \) is firm-specific price (unobservable), \( p_I \) is industry price (observable) and \( d \) are other industry demand shifters. The new parameter, \( \sigma \) is the elasticity of demand. Re-arranging terms

\[ p_i = -\frac{1}{\sigma} [y_i - \nu p_I - d_I] + p_I \]  \hspace{1cm} (24)

What we measure is ”revenue productivity”, \( r \), where revenue of the firm is deflated by an industry price deflator

\[ r_i \equiv (y_i + p_i) - p_I \]  \hspace{1cm} (25)

substituting (24) into (25) :
\[ y_i = \left( 1 - \frac{1}{\sigma} \right) [\beta_l l_i + \beta_k k_i + \epsilon_i] + \frac{\nu}{\sigma} p_I + \frac{1}{\sigma} d_I \]

The term \( \left( 1 - \frac{1}{\sigma} \right) \) is the inverse mark-up (the lower the demand elasticity the greater the mark-up. Estimation of the standard production function will give a coefficient of \( \left( 1 - \frac{1}{\sigma} \right) \beta_l \) which for any positive mark-up will be systematically less than the technological parameter, \( \beta_l \). Klette-Griliches propose using industry output (and time dummies) to proxy \( d \). This enables identification of \( \frac{1}{\sigma} \) and therefore of the production function parameters (strong assumption of course).

3.2 Measurement error

If classical and variables all strictly exogenous, low signal-noise ratio will cause attenuation towards zero. Made worse with fixed effects (see Griliches and Mairesse, 1998)
3.3 Sample Selection

Mundane question of whether the sample itself is representative of the population of interest - e.g. issue of Compustat.

4 Conclusions

Examples from production function, but some more general lessons: (a) endogenous variables, (b) "structural approaches, (c) no free lunch

Basic problem of identification when factors (like labor) are variable

Basic problem of identification when factors (like capital) are persistent