Panel Data Estimation of Production Functions

Professor John Van Reenen, LSE, Director of CEP

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1 Lecture 4: Panel data estimation of production functions

Estimation of production functions has a long history closely related with panel data techniques (e.g. farm production functions)

Why interesting?

1. Aggregate productivity growth related to shifts in the social production possibility frontier (welfare metric due to increases in technical efficiency under reasonable assumptions- e.g. Hulten, 1978)

2. Economic growth mainly determined by productivity growth rather than the accumulation of inputs (Solow, 1957)

3. (Original) Are factors really paid their marginal products?

4. What is the dispersion of (total factor) productivity? Appears very wide -why? Human capital, innovation (R&D, patents), Information technology

5. How does this dispersion change over time (and why)?. Trade Opening (Pavnik, 2002), Deregulation (Olley-Pakes, 1995). Are the changes due to incumbent plants becoming more productive ("within plant"), or market share being reallocated to more productive firms, or to entry/exit ("between plant")?

6. Economies of scale

Interest revitalized in last 10 year by growth in availability of micro-panel data on firms (company accounts like Compustat and Datastream) and plants from the Census Bureau of many countries (e.g. LRD in US, ARD in UK). Bartelsman, Haltiwanger and Scarpetta (2005) useful introduction to data availability

We want to estimate

$$Y_i = F(A_i, K_i, L_i)$$

But we know that:

1. Many of the inputs (K,L) are chosen by the firm, so are endogenous (Marshak and Andrews, 1944)

2. There is likely to be unobserved heterogeneity (A) correlated with inputs and outputs (Mundlak, 1961). Unless we measure this "managerial ability" (Bloom and Van Reenen, 2007) we have unobserved heterogeneity

A good overview of the issues is Griliches and Mairesse (1998)

Chambers Book on production theory

Solutions?

- Ignore and do OLS

- Give up and do growth accounting exercises (or estimate in TFP terms)

- Implement some of suggested methods (and/or some new ones suggested below)

2 Blundell Bond (1998, 2000)

Based on the "system GMM" estimators described in previous lectures.

Consider basic production function

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \epsilon_{it} \tag{1}$$

y = lnY, etc.

Y = output

K =capital

L = labour

The "error" term takes the form

$$\epsilon_{it} = \eta_i + \nu_{it} + \tau_t$$

η_i is the time invariant fixed effect

τ_t are time dummies

ν_{it} is an AR(1) error term:

$$\nu_{it} = \rho\nu_{it-1} + \varepsilon_{it}$$

The firm inputs are endogenous wrt ε_{it}. This maps directly back into the previous discussion. We have no instruments for (1) in first differences because lags of y_{it} and the factor inputs are correlated with past ε_{it} shocks and the autoregressive error term in related to these.

But we can transform the model through substitution as before to obtain

$$y_{it} = \pi_1 y_{it-1} + \pi_2 k_{it} + \pi_3 k_{it-1} + \pi_4 l_{it} + \pi_5 l_{it-1} + \eta_i^* + \tau_t^* + \varepsilon_{it} \quad (2)$$

where

$$\begin{aligned} \pi_1 &= \rho, \pi_2 = \beta_k, \pi_3 = -\rho\beta_k, \pi_4 = \beta_l, \pi_5 = -\rho\beta_l, \\ \eta_{it}^* &= (1-\rho)\eta_i, \tau_t^* = \tau_t - \rho\tau_{t-1} \end{aligned} \quad (3)$$

This is now a dynamic model with serially uncorrelated shocks and can be estimated in the standard way using the levels and differenced equation. The capital terms are likely to be highly autocorrelated (close to a random walk) so using just the difference equations alone are going to be problematic because of the weak instrument problem. Consequently using the extra moment conditions in levels has been shown to be very informative (Blundell and Bond, 2000). In other words if k and l are both endogenous.

For the first differenced equations the Arellano-Bond (1991) moment conditions are:

$$\begin{aligned} E(y_{it-s}\Delta\varepsilon_{it}) &= 0 \text{ for s} \geq 2 \\ E(k_{it-s}\Delta\varepsilon_{it}) &= 0 \text{ for s} \geq 2 \\ E(l_{it-s}\Delta\varepsilon_{it}) &= 0 \text{ for s} \geq 2 \end{aligned} \tag{4}$$

If we were prepared to assume capital was pre-determined rather than endogenous the extra moment conditions are

$$E(k_{it-s}\Delta\varepsilon_{it}) = 0 \text{ for s} \geq 1$$

For the levels equations we use the (Blundell-Bond/Arellano-Bover) moments

$$\begin{aligned} E(\Delta k_{it-s}(\varepsilon_{it} + \eta_i^*)) &= 0 \text{ for s} \geq 1 \\ E(\Delta l_{it-s}(\varepsilon_{it} + \eta_i^*)) &= 0 \text{ for s} \geq 1 \end{aligned} \tag{5}$$

It can be shown (Blundell-Bond, 1998) that after exploiting all the moments in (4)and (5)the only non-redundant remaining moments in the levels equations are of the form $E(\Delta l_{it-1}(\varepsilon_{it} + \eta_i^*)), etc.$

Loosely speaking, this means that we are using lagged differences as instruments for the levels equations (in addition to the lagged levels as instruments for the differenced equation). Again, this assumes ε_{it} is uncorrelated. If ε_{it} is MA(1) for example, then we would use moments of the form:

$E(l_{it-s}\Delta\varepsilon_{it})$, s$\geq$ 3 for the differenced equations and

$E(\Delta l_{it-s}(\varepsilon_{it}+\eta_i^*)), s=2$ for the levels equations.

Note that once (2) is estimated consistently we can recover the underlying structural parameters by imposing the nonlinear restrictions of (3) by Minimum Distance.

Table 4. Production Function Estimates

	OLS LEVELS	WITHIN GROUPS	GMM DIF $t-2$	GMM DIF $t-3$	GMM SYS $t-2$	GMM SYS $t-3$
n_t	0.479	0.488	0.513	0.499	0.629	0.472
	(0.029)	(0.030)	(0.089)	(0.101)	(0.106)	(0.112)
n_{t-1}	-0.423	-0.023	0.073	-0.147	-0.092	-0.278
	(0.031)	(0.034)	(0.093)	(0.113)	(0.108)	(0.120)
k_t	0.235	0.177	0.132	0.194	0.361	0.398
	(0.035)	(0.034)	(0.118)	(0.154)	(0.129)	(0.152)
k_{t-1}	-0.212	-0.131	-0.207	-0.105	-0.326	-0.209
	(0.035)	(0.025)	(0.095)	(0.110)	(0.104)	(0.119)
y_{t-1}	0.922	0.404	0.326	0.426	0.462	0.602
	(0.011)	(0.029)	(0.052)	(0.079)	(0.051)	(0.098)
m1	-2.60	-8.89	-6.21	-4.84	-8.14	-6.53
m2	-2.06	-1.09	-1.36	-0.69	-0.59	-0.35
Sargan	-	-	.001	.073	.000	.032
Dif-Sar	-	-	-	-	.001	.102
β_n	0.538	0.488	0.583	0.515	0.773	0.479
	(0.025)	(0.030)	(0.085)	(0.099)	(0.093)	(0.098)
β_k	0.266	0.199	0.062	0.225	0.231	0.492
	(0.032)	(0.033)	(0.079)	(0.126)	(0.075)	(0.074)
α	0.964	0.512	0.377	0.448	0.509	0.565
	(0.006)	(0.022)	(0.049)	(0.073)	(0.048)	(0.078)
Comfac	.000	.000	.014	.711	.012	.772
CRS	.000	.000	.000	.006	.922	.641

See notes to Tables 1 and 3.
Comfac is a minimum distance test of the non-linear common factor restrictions imposed in the restricted models. P-values are reported.
CRS is a Wald test of the constant returns to scale hypothesis $\beta_n + \beta_k = 1$ in the restricted models. P-values are reported.

Source: Blundell and Bond (2000).