Decomposing the Business Stealing Effect: Impacts on Output and Price

When a firm does more R&D and reduces its cost, it generates a business stealing effect by reducing the profit of other firms. Part of this effect will be due to changes in the output levels of other firms, and part to the change in equilibrium price. In this Appendix, we decompose the total effect on profits into these two components. The objective is to identify the relative size of the two channels. To do this, we analyse a Cournot model with $n+1$ firms with asymmetric costs – firm 1 has unit cost c_1 while the other n firms have unit cost c_2. We analyse the impact of a change in firm 1's unit cost (due to R&D) on the Nash equilibrium levels of output of the other firms and on the price.

Setup:
$n+1$ firms
Demand: $p = Q^{-e}$
Costs: $\{c_1, c_2, c_2, ...c_2\}$, $\Delta c = c_1 - c_2 \gtrless 0$
Output: $Q = \sum_{i=1} q_i$

Each firm solves

$$\max_{q_i} \pi_i = q_i((\sum_{i=1} q_i)^{-e} - c_i)$$

$$\implies (\sum_{i=1} q_i)^{-e} - c_i - e q_i (\sum_{i=1} q_i)^{-(e+1)} = 0$$

It is easy to see from this that there is strategic substitution ($\frac{\partial^2 \pi}{\partial q_j \partial q_i} < 0$), as always in Cournot models. We now impose symmetry on firms $i > 1$ (we label these as firm 2), which gives us the first order conditions in the form

$$i = 1: (q_1 + nq_2)^{-e} - c_1 - eq_1(q_1 + nq_2)^{-(e+1)} = 0$$

$$i = 2: (q_1 + nq_2)^{-e} - c_2 - eq_2(q_1 + nq_2)^{-(e+1)} = 0$$

Multiplying these two equations by $(q_1 + nq_2)^e$ and then dividing one by the other, we get

$$\frac{q_1(1-e) + nq_2}{q_1 + (n-e)q_2} = \frac{c_1}{c_2}$$

Solving for q_2 yields

$$q_2 = \phi q_1$$

$$\phi = \frac{\Delta c + ec_2}{ec_1 - n\Delta c}$$

It is clear that we have to constrain the extent of cost asymmetry Δc in order to ensure both firms operate at strictly positive outputs in equilibrium. We assume this condition holds.

Substituting this expression back into the first order condition for firm 1 and solving, we get the equilibrium outputs

$$\begin{aligned} q_1^* &= c_1^{-1/e}(1+n\phi-e)^{1/e}(1+n\phi)^{-(1+e)/e} \\ q_{2^*} &= \phi q_1^* \end{aligned}$$

For future use, note that we can write

$$\begin{aligned} 1+n\phi &= \frac{e(c_1+nc_2)}{ec_1-n\Delta c} \\ 1+n\phi-e &= \frac{e(1-e+n)c_1}{ec_1-n\Delta c} \end{aligned}$$

Now we want to get the expression for the profits of firm 2 and then analyze the elasticity of both output and profits of firm 2 with respect to a change in the unit cost of firm 1 (which is induced by R&D). The easiest way to get profits for firm 2 is to note that optimization above implies

$$\frac{p-c_2}{p} = \frac{eq_2}{Q} \longrightarrow \quad \pi_2 = (p-c_2)q_2 = epQ(q_2/Q)^2$$

But in equilibrium $q_2/Q = \frac{\phi}{1+n\phi}$ and $pQ = \{q_1^*(1+n\phi)\}^{1-e}$, so we get (hereafter we drop the asterisks denoting equilibrium, to simplify notation)

$$\pi_2 = eq_1^{(1-e)}\phi^2(1+n\phi)^{-(1+e)}$$

Of course, from the earlier expressions, recall that q_1 and ϕ depend on the the unit costs c_1 and c_2, as well as the inverse demand elasticity, e, and the number of firms, n.

We now derive the elasticity of firm 2's output and profit with respect to c_1. Let $\eta_q = \frac{\partial \ln q_2}{\partial \ln c_1}$ and $\eta_\pi = \frac{\partial \ln \pi_2}{\partial \ln c_1}$. We have

$$\eta_q = \frac{\partial \ln \phi}{\partial \ln c_1} + \frac{\partial \ln q_1}{\partial \ln c_1}$$

Log differentiating the expression above for ϕ, we obtain

$$\frac{\partial \ln \phi}{\partial \ln c_1} = \frac{c_1}{\Delta c + ec_2} - \frac{(e-n)c_1}{ec_1 - n\Delta c}$$

Log differentiating the expression above for q_1 and simplifying, we get

$$\frac{\partial \ln q_1}{\partial \ln c_1} = \frac{(e-n)c_1}{ec_1-n\Delta c} - (\frac{1+e}{e})\frac{c_1}{c_1+nc_2}$$

Notice here that a sufficient condition for $\frac{\partial \ln q_1}{\partial \ln c_1} < 0$ is that $n \geq e$ (provided we assume that the elasticity of demand $\frac{1}{e} > 1$ this condition is met).

Using these two results, we get the first elasticity of interest:

$$\eta_q = \frac{c_1}{\Delta c + ec_2} - (\frac{1+e}{e})\frac{c_1}{c_1 + nc_2} \tag{1}$$

This elasticity gives us the effect of a (one percent) change in the marginal cost of firm 1 on the level of output of a representative firm 2 (there are n such firms). Evaluated at the symmetric equilibrium where $\Delta c = 0$, we get $\eta_q = \frac{n-e}{1+n} > 0$. A fall in firm 1's marginal cost reduces each firm 2's equilibrium output.[1]

Next we turn to elasticity of a representative firm 2's profit with respect to the marginal cost of firm 1. From the expression for π_2 above, we have

$$\eta_\pi = (1-e)\frac{\partial \ln q_1}{\partial \ln c_1} + 2\frac{\partial \ln \phi}{\partial \ln c_1} - (1+e)\frac{\partial \ln(1+n\phi)}{\partial \ln c_1}$$

Log differentiating the expression above for $1 + n\phi$, we get

$$\frac{\partial \ln(1+n\phi)}{\partial \ln c_1} = \frac{c_1}{c_1 + nc_2} - \frac{(e-n)c_1}{ec_1 - n\Delta c}$$

Using the results obtained for the log derivatives of q_1, ϕ and $(1+n\phi)$, we get

$$\eta_\pi = \frac{2c_1}{\Delta c + ec_2} - (\frac{1+e}{e})\frac{c_1}{c_1 + nc_2} \tag{2}$$

This elasticity gives us the effect of a (one percent) change in the marginal cost of firm 1 on the total profits (not the *profit margin*) of a representative firm 2 (there are n such firms). Note that in the symmetric case where $\Delta c = 0$, this reduces to $\eta_\pi = \frac{1+2n-e}{1+n} > 0$. A fall in firm 1's marginal cost reduces each firm 2's equilibrium profit level.[2]

Let $\lambda = \frac{c_2}{c_1}$.Using (1) and (2), we can write

$$\frac{\eta_q}{\eta_\pi} = \frac{e(1+n\lambda) - (1+e)(1-(1-e)\lambda)}{2e(1+n\lambda) - (1+e)(1-(1-e)\lambda)}$$

Given assumptions about the (inverse) demand elasticity, the relative unit costs and the number of firms, we can identify the magnitude $\frac{\eta_\pi}{\eta_q}$.

It is natural to evaluate this expression around an initial equilibrium in which firms have symmetric unit costs, $\lambda = 1$. In that case

$$\frac{\eta_q}{\eta_\pi} = \frac{e(1+n) - e(1+e)}{2e(1+n) - e(1+e)} = \frac{1 - \frac{e(1+e)}{1+n}}{2 - \frac{e(1+e)}{1+n}} \tag{3}$$

[1] We cannot sign η_q in the asymmetric case without bounding the degree of cost asymmetry. If we assume $\Delta c < e(n-e)c_2$, then $\eta_q > 0$.

[2] We cannot sign η_π in the asymmetric case without bounding the degree of cost asymmetry. If we assume $\Delta c < ec_1 + e(2n-e)c_2$, then $\eta_\pi > 0$. Note that this upper bound is larger than in the case of η_q. Thus η_π will be positive over a wider range of asymmetry.

Notice that $\frac{\eta_q}{\eta_\pi} \to \frac{1}{2}$ **from below** as $n \to \infty$.That is, for large n, the total effect on profit is divided equally between the effect on output and the effect on price. This is in fact what we assume in the simulations. Moreover, even for finite n, evaluating this expression for various plausible values of e and n yields numbers very close to 2.

Examples:

1. Suppose $e = 1$. Then $n = 4$ implies $\frac{\eta_\pi}{\eta_q} = 8/3$, and $n = 10$ implies $\frac{\eta_\pi}{\eta_q} = 20/9$.

2. Suppose instead $e = .25$ (corresponding to direct demand elasticity of 4).Then $n = 4$ implies $\frac{\eta_\pi}{\eta_q} = 8.75/3.75$ and $n = 10$ implies $\frac{\eta_\pi}{\eta_q} = 20.75/9.75$.

In the computation of the private and social rates of return to R&D, we adjust the estimated impact of $SPILLSIC$ on market value (labelled γ_3 in the model in the paper) downward by multiplying by$\frac{\eta_q}{\eta_\pi}$ (labelled σ in the paper). Since the maximum value of this ratio is $\frac{1}{2}$, our adjustment is conservative in the sense that it overstates the true private return.

It is easy to verify that $\frac{\partial(\frac{\eta_q}{\eta_\pi})}{\partial e} < 0$, so when the direct demand is more elastic (lower e), more of the effect is loaded on output and less on price. Thus the minimum value of $\frac{\eta_q}{\eta_\pi}$in this model occurs when $e = 1$ (its maximum value assuming direct demand elasticity cannot be smaller than one) and $n = 2$ (its minimum value). In this case we get $\frac{\eta_q}{\eta_\pi} = \frac{1}{4}$. Thus in this model we obtain $\frac{\eta_q}{\eta_\pi} \in [\frac{1}{4}, \frac{1}{2}]$.