Job Polarization and Structural Change

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Abstract

Job polarization is a widely documented phenomenon in developed countries since the 1980s: employment has been shifting from middle to low- and high-income workers, while average wage growth has been slower for middle-income workers than at both extremes. We document 1) that polarization has started as early as the 1950s in the US, and 2) that this process is closely linked to the shift from manufacturing to services. Based on these observations we propose a structural change driven explanation for polarization. Productivity growth through raising national income leads to a disproportionate increase in the demand for high-end (luxury) services. To attract more workers into the high-skilled services, the wages in this sector have to grow at a faster pace than in the middle. The growing income of the wealthier part of the population in turn increases their demand for low-skilled services, leading to a partial marketization of home production, and a faster growth of the low-skilled service wages.

JEL codes: E24, J22, O41

Keywords: Job Polarization, Structural Change, Home Production

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1 Introduction

The polarization of the labor market in terms of occupations is a widely documented phenomenon in the US and several European countries since the 1980s (Autor, Katz, and Kearney (2006), Goos and Manning (2007), and Goos, Manning, and Salomons (2009)). This phenomenon, besides the relative growth of wages and employment of high earning occupations, also entails the relative growth of wages and employment of low earning occupations. The leading explanation for polarization is the routinization hypothesis, which relies on the assumption that information and computer technologies (ICT) substitute for middle-skill and hence middle-earnings (routine) occupations, whereas they complement the high-skilled and high-earnings (abstract) occupations (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Michaels, Natraj, and Van Reenen (2010), Goos, Manning, and Salomons (2011), Autor and Dorn (2012)).

The contribution of our paper is twofold. First, we document a set of facts which raise flags that routinization, although certainly playing a role from the 1980s onwards, is not the sole driving force behind this phenomenon. Second, based on these facts we propose a novel perspective on the polarization of the labor market, one based on structural change.

Our analysis of US Census data for the period 1950-2000 and American Community Survey (ACS) data for 2008 reveals some novel facts. First, while most of the literature on polarization focuses on occupations, we document that labor market polarization is present also in terms of broadly defined sectors: low-skilled and high-skilled service workers, who are at opposite ends of the earnings distribution have been gaining in terms of wages and employment at the expense of manufacturing workers. Second, we show that the loss in routinizable occupations is not uniform across sectors: routinizable employment only declined in the manufacturing sector. Third, we find that polarization has started as early as the 1950-1960s in the US. This implies that polarization started long before ICT or increased trade flows could have impacted the labor market. Observing a) that polarization seems to be a long-run phenomenon, b) that the middle earning jobs are in manufacturing, c) that manufacturing employment started to fall, while service employment started to increase in the 1950s-1960s, it is natural to investigate whether the structural shift of the economy is driving the polarization of the labor market.

Based on these observations we propose a structural change driven explanation for the joint polarization of wages and employment. In our dynamic general equilibrium model workers with heterogeneous ability select which sector to work in. As technology improves, the demand for both high- and low-skilled services increases disproportionately, which leads to a reallocation of labor from middle-earning manufacturing jobs to high-earning and low-earning service jobs. However, to attract more workers into these two sectors, the wages in these two sectors have to grow at a faster pace than in the middle. Finally, we calibrate the model and quantitatively assess the contribution of structural change – driven by both non-homothetic preferences and unbalanced technological progress – to the polarization of wages and employment.

In the model, there are two types of consumption goods: manufacturing and high-end service goods.
Preferences over these two goods are non-homothetic: high-end services are luxury goods, which are only demanded at high enough income levels. Additionally a subsistence level of home production is required, which can be self-produced or acquired on the market. We model low-skilled service jobs as substitutes for household production. As technology progresses in the consumption goods sector, the employment and production structure of the economy changes: as households gradually become wealthier, the demand for high-skilled services rises over-proportionally due to the non-homotheticity of preferences. This disproportionate demand rise puts an upward pressure on prices and wages in the high-skilled service sector. Consequently, more people acquire education and work in high-end service jobs. As wages increase, more and more people decide to hire low-skilled household workers, instead of doing the housework themselves. This puts an upward pressure on wages at the bottom-end of the distribution, increasing the supply of these workers. Due to the non-homotheticity of preferences, the timing of the expansion of the high- and the low-skilled service sector can be different. While the non-homotheticity is relatively important, the desire to expand the consumption of high-skilled services dominates that of low-skilled services. This can be a potential explanation of the low-skilled service expansion becoming more pronounced later on in the data.

This paper builds on and contributes to the literature both on polarization and on structural change. To our knowledge, these two phenomena until now have been studied separately. However, according to our analysis of the data, polarization of the labor market and structural change are closely linked to each other, and according to our model, industrial shifts can lead to polarization.

The structural change literature has documented for several countries that as income increases resources are shifted away from agriculture and from manufacturing towards services (Kuznets (1957), Maddison (1980)). In particular the employment share of manufacturing has been declining since the 1950s, while the employment share of services has been increasing. The literature has identified two economic forces that lead to structural transformation: preferences and technology. The preferences explanation relies on changes in aggregate income, which if preferences for the output of different sectors are not homothetic lead to a reallocation of resources across sectors (Caselli and Coleman II. (2001), Kongsamut et al (2001)). The technology explanation assumes that productivity growth is different across sectors, which with regular preferences leads to a shift of labor into the lower growth sector (Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008)). The consensus seems to be that both mechanisms together are needed to explain the patterns observed in the data (Buera and Kaboski (2009), Herrendorf et al (2013), Boppart (2011)). Several papers have also established the importance of home production for structural transformation (Ngai and Pissarides (2008), Buera and Kaboski (2009), (2012a), (2012b)). In our model we rely on both mechanisms, and incorporate home production.

We extend the structural change literature in two ways. First, we allow heterogeneous workers to endogenously sort into different skill groups and into jobs in different sectors. In the presence of differential productivity or demand growth, the optimal education decisions naturally change. Since the adjustment of the skill supply takes place gradually, it is possible, that wages increase more initially,
while later on employment reallocation is more pronounced. Moreover, as the supply of skills and the sorting into sectors change, the relative wages and prices are affected. Therefore, we can analyze the effects of structural change on relative sectoral wages, which is not usual in models of structural change.

Second, based on the job polarization phenomenon we distinguish between two types of services: low- and high-skilled. This is an important distinction, due to both the way they enter the utility of agents and the way they are produced. We believe that consumers enjoy these services in different ways. We model low-skilled services as substitutes for household production. In the data we do not find a clear pattern for the total amount of household production – the combined hours of home production and low-skilled service workers. Therefore, we assume that there is a fixed demand for household production, which implies an upper limit on the demand for low-skilled services. The demand for high-skilled services, on the other hand, can increase without bounds. In terms of production, high-skilled services need specialized (educated) workers, while low-skilled services can be supplied by anyone.

Two popular explanations suggested for the polarization of occupations are the routinization hypothesis, and the consumption hypothesis. The routinization hypothesis relies on the assumption that information and computer technologies (ICT) substitute for middle-skill and hence middle-earnings (routine) occupations, whereas they complement the high-skilled and high-earnings (abstract) occupations (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Michaels, Natraj, and Van Reenen (2010), Goos, Manning, and Salomons (2011), Autor and Dorn (2012)). It has been argued that much of the expansion of low-skilled occupations is driven by the expansion of low-skilled service jobs (Autor and Dorn (2012)). The routinization hypothesis suggests, that it is ICT that leads to the substitution of routine workers by machines, and which complements abstract workers. The displaced routine workers find either abstract or manual jobs, increasing the employment share of these occupations (Autor, Levy, and Murnane (2003), Autor, Katz, and Kearney (2006), Michaels, Natraj, and Van Reenen (2010), Goos, Manning, and Salomons (2011)). The routinization hypothesis, linked to ICT, is potentially a convincing explanation for the employment share patterns after the mid-1980s, but without any assumption on demands, it does not provide a mechanism through which the relative wage of manual workers compared to routine workers can increase. It also cannot provide an explanation for the patterns observed before ICT could have taken effect. The consumption spill-over argument, on the other hand, suggests that as the income of high-earners increases, their demand for low-skilled service jobs increases as well, leading to a spillover to the lower end of the wage distribution (Manning (2004), Mazzolari and Ragusa (2007)). Our modeling of low-skilled service jobs as substitutes for home production is reminiscent of this argument.

1 A notable exception is Caselli and Coleman II. (2001).
2 Polarization in the data

In the empirical literature, polarization is mostly represented in terms of occupations. Following the methodology used in [Autor, Katz, and Kearney (2006), Acemoglu and Autor (2011), and Autor and Dorn (2012)], we plot the smoothed changes in employment shares and log real wages for a balanced panel of occupation categories ranked according to their 1950 and 1980 mean wages. The novelty in these graphs is that we show these patterns going back until 1950, whereas most analyses look at data from only 1980 onwards. The top row of Figure 1 show that there has been real wage polarization in all 30-year periods. This polarization is present whether the occupations are ranked according to their 1950 or 1980 mean wages. The polarization of real wages is most pronounced in the first two 30-year intervals, but it is clearly discernible in the following ones as well from the slight U-shape of the smoothed changes. The picture is more mixed in terms of employment polarization (the bottom row of Figure 1): employment polarization is most pronounced in the last period (1980-2008), but it seems to

Figure 1: Wage and employment polarization

Notes: Wages are calculated from US Census data for 1950, 1960, 1970, 1980, 1990, 2000 and American Community Survey (ACS) for 2008. Balanced occupation categories (185 of them) were defined by the authors based on [Meyer and Osborne (2005) and Autor and Dorn (2012)]. The bottom two panels show the 30-year change in employment shares (calculated as hours supplied rather than persons), and the top two panels show the 30-year change in log hourly real wages (again labor supply weighted). In the left panels occupations are ranked based on their 1950 average wage, whereas in the right panels they are ranked according to their 1980 average wage.
be present even in the earlier decades\footnote{This does not necessarily hold for decade-by-decade analysis. Typically in some decades the top gains, whereas in others the bottom gains, but it is never the middle that grows the most in terms of employment shares. See graphs in Appendix.}

These graphs – in line with the literature – plot the change in raw employment shares and in raw log real hourly wages. These changes also include the potential effects of the changing gender, age and race composition of the labor force. These graphs also do not directly relate to the explanations put forward in the literature, as they show the employment share and wage changes for occupations ranked according to their mean wage, not based on their routinizability. Therefore we classify the occupation groups into the following categories: manual, routine, and abstract (as in Acemoglu et al (2011)), and show the patterns for these three broad categories. We also classify industries into three categories: low-skilled services, which are substitutes for household production; manufacturing; and high-skilled services, which are luxury goods. Manufacturing industries are as in the structural change literature (Buera et al (2012a), Kongsamut et al (2001), Ngai et al (2008), Herrendorf et al (2013)). We classify industries to be low-skilled services if they can be viewed as substitutes for household production.

Figure 2: Polarization for industries and occupations

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{polarization.png}
\caption{Polarization for industries and occupations}
\end{figure}

Notes: Employment shares (in terms of hours) are calculated from the same data as in Figure 1 but excluding agricultural occupations/industries. Wages are the residuals from regressing log hourly wages on age, age squared, race, gender, and a dummy for foreign born. For details of the industry and occupation classification see text and the appendix.

Figure 2 shows the patterns of polarization both in terms of employment shares and wages for the above defined occupations and industries between 1950 and 2008. The patterns are strikingly similar between the graphs generated using industries (the left panels) and occupations (right panels). The top two panels show clear employment polarization both in terms of industries and occupations. The
middle earning group (manufacturing/routine occupations) lost significantly in terms of employment share, the top (high-skilled services/abstract) gained, and the bottom (low-skilled services/manual) initially shrunk, but then expanded. The bottom two panels show the change in average industry (or occupation) log hourly wage change in the given decade compared to the mean log hourly wage change in manufacturing (or routine occupations). In most decades average wages in low- and high-skilled services improved relative to manufacturing, while manual and abstract occupations also improved relative to routine occupations. Exceptions are the first and last decade, and the period between 1970-1980, when the high-skilled services and the abstract occupations lost. In this decade there was a secular compression in the skill premium, most probably due to forces outside the scope of this model.\

This striking similarity in the employment share and average wage path of the three broad industry and occupation classifications can be understood when considering the employment shares of the occupation categories in the three industry categories and vice versa. The top panel in Table 1 shows the employment shares in each industry-occupation cell. It can be seen that the largest numbers are on the diagonal, implying there is a tight correspondence between industries an occupations. The middle panel shows for each industry the fraction of employment coming from manual, routine and abstract occupations, while the bottom panel shows the opposite averaged between 1950-2008. The majority of workers in low-skilled services are in manual occupations, the majority in manufacturing are in routine occupations, and the majority in high-skilled services are in abstract occupations. The opposite is also true: the majority of routine workers are employed in manufacturing, and the majority of abstract workers are employed in the high-skilled service industry.

<table>
<thead>
<tr>
<th></th>
<th>low-skilled service</th>
<th>manufacturing</th>
<th>high-skilled service</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>manual</td>
<td>5.90</td>
<td>1.26</td>
<td>5.17</td>
<td>12.33</td>
</tr>
<tr>
<td>routine</td>
<td>3.31</td>
<td>44.61</td>
<td>11.78</td>
<td>59.70</td>
</tr>
<tr>
<td>abstract</td>
<td>1.89</td>
<td>9.05</td>
<td>17.04</td>
<td>27.97</td>
</tr>
<tr>
<td>total</td>
<td>11.09</td>
<td>54.92</td>
<td>33.99</td>
<td>100</td>
</tr>
<tr>
<td>manual</td>
<td>53.66</td>
<td>2.30</td>
<td>15.49</td>
<td>-</td>
</tr>
<tr>
<td>routine</td>
<td>30.07</td>
<td>80.71</td>
<td>35.66</td>
<td>-</td>
</tr>
<tr>
<td>abstract</td>
<td>16.27</td>
<td>16.98</td>
<td>48.84</td>
<td>-</td>
</tr>
<tr>
<td>total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>manual</td>
<td>48.37</td>
<td>10.55</td>
<td>41.08</td>
<td>100</td>
</tr>
<tr>
<td>routine</td>
<td>5.80</td>
<td>74.67</td>
<td>19.53</td>
<td>100</td>
</tr>
<tr>
<td>abstract</td>
<td>6.44</td>
<td>34.72</td>
<td>58.84</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: Employment shares (in terms of hours) as percents are calculated from the same data as in Figure 1, but excluding agricultural occupations/industries. Industry and occupation classification same as in Figure 2. The top panel shows the employment shares in each of the occupation-industry cells. The middle panel shows within each industry the employment share of different occupations, while the bottom panel shows within each occupation group the employment share of different industries.

It is informative to look at the changes in the employment shares in the various industry-occupation cells. Figure 3 shows that the employment share only declined in the routine-manufacturing cell, whereas the routine-high-skilled cell’s employment share was stable, while the routine-low-skilled

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3These forces include the entrance of the baby boomers cohort into the labor market, who had an unusually high college enrollment rate probably in order to postpone the military draft.
cell’s employment share increased between 1950-2008. Therefore it seems that the decline in routinizable occupations is intrinsically linked to the decline in the manufacturing sector.

3 Model setup

Time is infinite and discrete. The demographic structure is an overlapping generations model. Individuals are heterogeneous in their innate ability.

Every individual has to decide whether to acquire education or not. Those who acquire education become high-skilled. In the calibration we identify the high-skilled as having attended college. Those who opt out from education remain low-skilled. Workers with high and low skills are employed in different sectors, and produce different goods. The high-skilled work in high-skilled services, whereas the low-skilled work either in manufacturing or in low-skilled services, which can be substitutes for home production.

Individuals derive utility from consuming high-skilled services and manufacturing goods. Services are luxury products in the sense that as income increases individuals spend an increasing fraction of their total consumption budget on these services. Each individual needs to meet a home production requirement, which can be produced at home, at a utility loss, or can be bought on the market from the low-skilled service workers. We assume that individuals cannot lend or borrow, and hence each
individual spends all income in the period that it is earned.

The economy is in a decentralized equilibrium at all times: individuals make educational decisions and sectoral choices to maximize their expected discounted lifetime utility, where in each period they maximize their utility by optimally allocating their income between low-skilled services, manufacturing goods and high-skilled services. Production is perfectly competitive, wages and prices are such that all markets clear. We analyze the role of technological progress and non-homothetic preferences in explaining the observed wage and employment dynamics since the 1950s.

3.1 Sectors and production

There are three sectors in the model: high-skilled services ($S$), manufacturing ($M$), and low-skilled services ($L$). High-skilled services and manufacturing goods are produced in perfect competition.

The only input in high-skilled service production is high-skilled labor:

$$Y_s = A_s N_s,$$  

where $A_s$ is productivity and $N_s$ is the total amount of efficiency units of labor hired in sector $S$ for production. Sector $S$ firms are price takers, therefore the wage per efficiency unit of labor has to satisfy:

$$w_s = \frac{\partial p_s Y_s}{\partial N_s} = p_s A_s.$$  

Manufacturing goods are produced low-skilled manufacturing workers:

$$Y_m = A_m N_m,$$  

where $A_m$ is productivity, $N_m$ is the total amounts of efficiency units of labor hired in sector $M$. Sector $M$ firms are also price takers, the wage per efficiency unit of labor in sector $M$ has to satisfy:

$$w_m = \frac{\partial p_m Y_m}{\partial N_m} = p_m A_m.$$  

Note that the wage of a worker with $a$ efficiency units of labor working in sector $i \in \{M, S\}$ is $w_i a$.

The low-skilled service sector provides home production hours for households. We assume that each worker is equally talented in providing home production services, i.e. efficiency units of labor do not matter here, it is only the raw amount of hours that a worker can provide, which is crucial. Total amount of low-skilled services provided on the market:

$$Y_L = L_l,$$  

where $L_l$ is the raw units of labor (total amount of people) working in the low-skilled service sector. Note that since everyone has the same amount of raw labor, this implies that everyone working in the
The low-skilled service sector has the same earnings. The unit wage in this sector in equilibrium has to be such that the total amount demanded of low-skilled services is equal to the amount supplied.

3.2 Labor supply and demand for goods

Time is infinite and discrete, indexed by $t = 0, 1, 2...$ The economy is populated by a continuum of individuals who live at most for $T$ periods, but their survival probability declines as they get older, let us denote these survival probabilities by $\Lambda \in \mathbb{R}^T$, with $\Lambda(1) = 1$. Every period a new generation of measure $1/T$ is born. Individuals are heterogeneous in their innate ability (efficiency units of labor), $a$, which is drawn at birth from a time invariant distribution $f(a)$. These assumptions imply that both the size of the population, and the distribution of abilities are constant over time.

Every agent has to decide at birth whether to acquire education or not. Agents choose their education in order to maximize their expected discounted lifetime utility. Acquiring education grants access to the high-skilled service sector ($S$), and the cost is twofold: there is a tuition fee, and a time cost. The tuition fee is $w_s\kappa$, which is proportional to the sectoral unit wage, $w_s$.

There is potentially a study time, $\psi$, during which the worker does not earn any wages. We assume that the study time is less than one period, i.e. $\psi \in [0, 1]$. We calibrate the length of one period in order for this assumption to be reasonable. Those who acquire education have to work in the sector $S$ in the remaining part of the first period.

3.2.1 Sector of work

Each agent in every period of his life has to decide which sector to work in, taking as given his education, his efficiency units of labor, and the sectoral unit wages. Since individuals cannot lend or borrow, they choose their sector to maximize per period utility, which is equivalent to maximizing per period income. Agents with education can work in sector $S$, $M$, or $L$. Agents without education can freely choose between sector $M$ and $L$.

For workers without education it is optimal to work in sector $M$ if

$$w_m(t)a \geq w_l(t) \iff a \geq \frac{w_l(t)}{w_m(t)} \equiv a_{lm}(t).$$  \hspace{1cm} (6)

Therefore in period $t$ among non-educated workers everyone with $a < a_{lm}(t)$ works in sector $L$, and everyone else works sector $M$.

Similarly educated workers prefer to work in sector $S$ rather than $L$ if

$$w_s(t)a \geq w_l(t) \iff a \geq \frac{w_l(t)}{w_s(t)} \equiv a_{ls}(t).$$ \hspace{1cm} (7)

---

4We introduce probabilistic survival in the OLG model to avoid the well known oscillations that arise with deterministic exit. See for example [Abrahám (2008)].

5This assumption is made in order to have a steady state, and it is meant to represent the assumption that somebody working in the same sector has to train the newly entering individual.

6This is assumed both for simplicity, and because we think of the education partly as training on the job.

7In the steady state and the transition that we consider agents with education always optimally work in the sector $S$. 

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10
Therefore all educated workers with $a \geq a_{ls}(t)$ work in sector $S$ in period $t$.

It is not optimal for an educated worker to work in sector $M$ if

$$w_s(t)a \geq w_m(t)a \iff w_s(t) \geq w_m(t).$$

Therefore in period $t$ if $w_m(t) > w_s(t)$ all the workers who are qualified to work in sector $S$ work in sector $M$ instead. This also implies that there are no sector $S$ workers available to train the potential new entrants into sector $S$. Therefore in any such period there is no $S$ production. This cannot be part of an equilibrium unless the highest earner in period $t$ does not demand any good $S$, and none of the newborns wishes to get education.

To summarize if $w_s(t) < w_m(t)$, then the sector of work decision for educated and not educated individuals is identical. If $w_s(t) \geq w_m(t)$, then all educated individuals with $a \geq a_{ls}(t)$ work in sector $S$, and those with $a < a_{ls}(t)$ work in sector $L$.

Since in their education decision individuals consider the lifetime utility from consumption, we have to solve for their indirect utility given income.

### 3.2.2 Demand for consumption goods and low-skilled services

The individual maximizes the following utility in each period:

$$\max_{c_m, c_s, h} \ln \left( \frac{\theta_m c_m^{\gamma_m} + \theta_s (c_s + \gamma_s) c_s^{\gamma_s}}{c} \right) - \phi h^{\nu/\phi}$$

s.t. $p_m c_m + p_s c_s + w_l (\bar{h} - h) \leq m$ ($\lambda_0$)

$$0 \leq c_s (\mu_s), 0 \leq c_m (\mu_m), 0 \leq h (\mu_l), h \leq \bar{h} \equiv \frac{\pi}{A_h} (\mu_h)$$

where $m$ is the individual’s income in the given period, $p_m, p_s$ are the prices of the manufacturing and service goods, $w_l$ is the wage rate for low-skilled service workers. The total number of raw hours needed for household work is $\bar{h}$, which can vary over time. (Specifically we assume that $A_h \bar{h} = \pi$, where $\pi$ is a constant. Therefore if $A_h$ increases over time, then the necessary number of total home production hours declines.) Doing the home production causes disutility to the individual, which is increasing ($\phi > 0$) in the number of hours spent on home production, and potentially convex ($\nu > 1$).

The individual can hire a low-skilled service worker to do some or all of the home production.

The utility of the consumer is non-homothetic in manufacturing goods ($c_m$) and high-skilled service goods ($c_s$). We assume that $\gamma_s > 0$, which is equivalent to assuming that $c_s$ is a luxury good, i.e. the individual only demands a positive amount if it is sufficiently rich.

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8We do not consider such steady states or transitions, as the support for efficiency units of labor is unbounded in the calibration.
The following Corollary summarizes the solution to the consumer’s problem.

**Corollary 1.** The following cases can arise:

1. **At the lowest income levels** \( m < m_{12} \equiv \min\{m_{12a}, m_{12b}\} \) the optimal choices are:

   \[
   \begin{align*}
   h &= \bar{h} \\
   c_s &= 0 \\
   c_m &= \frac{m}{p_m}
   \end{align*}
   \]

   In this case \( \mu_h > 0, \mu_l = 0, \mu_m = 0, \mu_s > 0 \).

2. **For higher income levels** \( m \in \left[ m_{12a}, m_{23} \right] \), where \( m_{23} \equiv \min\{m_{2a3}, m_{2b3}\} \),

   **A.** if \( m \in \left[ m_{12a}, m_{2a3} \right) \) and \( m_{12a} < m_{12b} \), the optimal choices are described by

   \[
   \begin{align*}
   c_s &= 0 \\
   h &= \left( \frac{w_l \theta_m}{p_m \phi} \right)^{\nu - 1} \left( \theta_m \gamma_m^{\nu} + \theta_s \gamma_s^{\nu} \right)^{\nu - 1} - \frac{1}{\nu - 1} c_m^{\nu - 1} \\
   c_m &= \frac{m - w_l (\bar{h} - h)}{p_m}
   \end{align*}
   \]

   In this case \( \mu_h = 0, \mu_l = 0, \mu_m = 0, \mu_s > 0 \).

   **B.** if \( m \in \left[ m_{12b}, m_{2b3} \right) \) and \( m_{12b} < m_{12a} \), then the optimal choices are:

   \[
   \begin{align*}
   h &= \bar{h} \\
   c_m &= \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\nu} (c_s + \gamma_s) \\
   c_s &= \frac{m + p_s \gamma_s}{p_m \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{-\nu} + p_s} - \gamma_s
   \end{align*}
   \]

   In this case \( \mu_h > 0, \mu_l = 0, \mu_m = 0, \mu_s = 0 \).

3. **For the highest income levels, if** \( m > m_{23} \) the optimal choices are:

   \[
   \begin{align*}
   c_s &= \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\nu} c_m - \gamma_s \\
   h &= \left( \frac{w_l \theta_m}{p_m \phi} \right)^{\nu - 1} \left( \theta_m + \theta_s \left( \frac{p_m \theta_s}{p_s \theta_m} \right)^{\nu - 1} \gamma_s \right)^{\nu - 1} - \frac{1}{\nu - 1} c_m^{\nu - 1} \\
   c_m &= \frac{m - p_s c_s - w_l (\bar{h} - h)}{p_m}
   \end{align*}
   \]

   In this case \( \mu_h = 0, \mu_l = 0, \mu_m = 0, \mu_s = 0 \).
The cutoff income levels are defined by the following equations: The cutoff $m_{12a}$ is the income level, where the supply of $h$ exactly equals $\bar{h}$ according to case 2a. We can find the $c_m$ for which $h = \bar{h}$ under case 2a. Then

$$m_{12a} = p_m c_m (h = \bar{h}).$$  \hspace{1cm} (20)

The cutoff $m_{12b}$ is the income level, where the demand for $c_s$ is exactly zero according to case 2b:

$$m_{12b} = p_m \left( \frac{p_m}{p_s} \frac{\theta_s}{\theta_m} \right)^{-\varepsilon} \gamma_s.$$  \hspace{1cm} (21)

The cutoff $m_{2a3}$ is defined as the income level where the demand for $c_s$ is exactly zero under case 3 (therefore it can only follow case 2a). Using that $c_s = 0$ we get that $c_m = \left( \frac{p_m}{p_s} \frac{\theta_s}{\theta_m} \right)^{-\varepsilon} \gamma_s$. Using this value we can express the optimal $h$ under case 3, and can calculate the income level that way:

$$m_{2a3} = p_m \left( \frac{p_m}{p_s} \frac{\theta_s}{\theta_m} \right)^{-\varepsilon} \gamma_s + w_l \left( \bar{h} - \left( \frac{w_l}{p_m} \frac{\theta_m}{\theta_s} \right)^{\frac{1}{\varepsilon}} \left( \theta_m + \theta_s \left( \frac{p_m}{p_s} \frac{\theta_s}{\theta_m} \right)^{\varepsilon-1} \right)^{-\frac{1}{\varepsilon \theta_s}} \gamma_s \right)^{-1} \left( \frac{p_m}{p_s} \frac{\theta_s}{\theta_m} \right)^{-\varepsilon} \gamma_s.$$  \hspace{1cm} (22)

The cutoff $m_{2b3}$ is the income level where the supply of $h$ is exactly $\bar{h}$ under case 3 (therefore it can only follow case 2b). Using that $h = \bar{h}$ we can express $c_m = \left( \frac{w_l}{p_m} \frac{\theta_m}{\theta_s} \right) \left( \theta_m + \theta_s \left( \frac{p_m}{p_s} \frac{\theta_s}{\theta_m} \right)^{\varepsilon-1} \right)^{-1} \bar{h}^{-\left(\nu-1\right)}$, which therefore gives the value of $c_s$. The income that allows the purchase of that bundle is:

$$m_{2b3} = \frac{w_l}{\phi} \bar{h}^{\left(\nu-1\right)} - p_s \gamma_s.$$  \hspace{1cm} (23)

Proof. The Lagrangian of the problem is:

$$\mathcal{L}(c_m, c_s, h) = \ln C \frac{C}{\phi} \phi \frac{h^{\nu}}{\nu} - \lambda (p_m c_m + p_s c_s + w_l (\bar{h} - h) - m) + \mu_s c_s + \mu_m c_m + \mu_l h - \mu_h (h - \bar{h})$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_m} = \frac{1}{C} \theta_m c_m^{-\frac{1}{\varepsilon}} - \lambda p_m + \mu_m = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_s} = \frac{1}{C} \theta_s (c_s + \gamma_s)^{-\frac{1}{\varepsilon}} - \lambda p_s + \mu_s = 0$$

$$\frac{\partial \mathcal{L}}{\partial h} = -\phi h^{\nu-1} + \lambda w_l - \mu_h + \mu_l = 0$$

The solve the consumer’s problem we have to consider which constraints can be binding for which income levels. First notice, that the non-negativity constraint on $h$ never binds due to the specification of the disutility from housework, if prices $(p_m, p_s)$ and wages $(w_l)$ are positive and finite. Since the disutility from housework goes to zero as $h \to 0$, while the utility from consumption only goes to zero as consumption goes to infinity, for finite income levels the household is always better off by increasing $c$ than reducing $h$ to zero. The constraint on the non-negativity of $c_m$ can also never be binding, since the
marginal utility from consumption at \( c_m = c_s = 0 \) is infinity, while the marginal disutility of labor is a positive, finite number. Therefore \( \mu_m = 0 \) and \( \mu_l = 0 \) always hold. By considering the other possibilities the statement of the Corollary follows.

This implies that given parameters and prices the economy in a given period will be either in case A or in case B. In both cases the poorest people \((m < m_{12})\) only consume manufacturing goods, and the highest earners \((m > m_{23})\) consume everything. In case A middle income earners \((m \in [m_{12}, m_{23})\) consume manufacturing goods and low-skilled services, whereas in case B they do all of their own household work, but consume manufacturing goods and some high-skilled services. Each consumer can be considered poor, middle income, or rich depending on which sets of goods and services they consume optimally. Given sectoral wage rates, we can determine the demand coming from each type of worker.

All low-skilled service workers consume the same consumption bundle, as they all receive the same income, \(w_l\). Depending on the position of \(w_l\) relative to \(m_{12}\) and \(m_{23}\), they are all either poor, middle income, or rich consumers.

Since the manufacturing sector workers’ earnings depend on their ability, \(aw_m\), those who have a different ability have a different consumption bundle, and potentially they are different types of consumers. Those with \(a < m_{12}/w_m\) are considered poor\(^9\), those with \(a \in [m_{12}/w_m, m_{23}/w_m)\) are middle income, and those with \(a > m_{23}/w_m\) are rich consumers\(^10\) they consumer everything.

The high-skilled service workers can also be categorized into poor, middle income or rich consumers, where the cutoff ability for belonging to the above categories are respectively, \(m_{12}/w_s\) and \(m_{23}/w_s\)\(^11\).

Given optimal sector of work choice and education decisions, the total demand for each of the services and the manufacturing goods can be calculated by aggregating the demand coming from each type of worker from each cohort who is alive in the given period.

To determine the optimal educational decision, we need to calculate the expected discounted lifetime utility from each choice, for which we need to calculate the individual’s per period utility. Based on Corollary \([1]\) for a given vector of prices \(p \equiv [p_m, p_s, w_l]\), the indirect utility function of a consumer with income \(m\) can be easily constructed. Let \(VU(m; p)\) denote this per period indirect utility function.

### 3.2.3 Choice of education

Given the possibility of workers to choose sectors and the above defined indirect utility function, the expected discounted lifetime utility of a worker with education \(S\), efficiency units of labor \(a\), born at

---

\(^9\)There can only be manufacturing workers like this if \(m_{12}/w_m > a_{lm}\), otherwise all manufacturing workers are at least considered middle income consumers.

\(^10\)There can only be manufacturing workers like this if at least some past education cutoff \(a_s < m_{23}/w_m\).

\(^11\)It is again possible that there are no poor, or middle income high-skilled service workers, i.e. there are no high-skilled workers with ability below \(m_{12}/w_s\) or \(m_{23}/w_s\).
time $t$ can be written as:

$$
V_s(a, t) = VU((1 - \psi)w_s(t)a - w_s(t)\kappa; \mathbf{p}(t)) \\
+ \sum_{i=t+1}^{T-t} \beta^{i-t} \Delta(i - t + 1) VU(\max\{w_s(i)a, w_m(i)a, w_l(i)\}; \mathbf{p}(i)).
$$

Expected discounted lifetime utility of a worker without education, with efficiency units of labor $a$, born at time $t$ is:

$$
V_n(a, t) = T - t \sum_{i=t}^{T-t} \beta^{i-t} \lambda(i - t + 1) VU(\max\{w_m(i)a, w_l(i)\}; \mathbf{p}(i)).
$$

Note that in both cases ($N, S$) the expected present value of lifetime utility is increasing in ability, $a$, of the individual. This implies that between any two options, the optimal decision rule can be summarized by a set of cut-off ability levels, which determine ranges of abilities, $a$, where one decision is optimal, while for other ability levels, the other decision is optimal.

**Lemma 1.** The optimal educational choice in period $t$ can be described by a unique cutoff ability level $a_s(t)$, defined as the solution of:

$$
V_s(a_s(t), t) = V_n(a_s(t), t).
$$

For individuals born in period $t$ with $a < a_s(t)$ it is optimal to remain low-skilled, while for all individuals born in period $t$ with $a \geq a_s(t)$ it is optimal to acquire education.

**Proof.** It is optimal to acquire education if the following difference is positive:

$$
V_s(a, t) - V_n(a, t) = VU((1 - \psi)w_s(t)a - w_s(t)\kappa; \mathbf{p}(t)) - VU(\max\{w_m(t)a, w_l(t)\}; \mathbf{p}(t)) \\
+ \sum_{i=t+1}^{T-t} \beta^{i-t} \Delta(i - t + 1) VU(\max\{w_m(i)a, w_l(i)\}; \mathbf{p}(i)) - VU(\max\{w_m(i)a, w_l(i)\}; \mathbf{p}(i)).
$$

In the Appendix we show that this function is continuous, negative for $a = 0$, and crosses zero only once.

Deriving the labor supply in period $s$ of a cohort born in period $0 < t \leq s$ is straightforward given Lemma 1 and given the unit wages in each sector in period $s$.

**Corollary 2.** The cohort born in period $t > 0$ supplies labor to each sector in period $s$ in the following way.
1. If the period $s$ unit wages satisfy $w_m(s) \leq w_s(s)$, and $a_s(t) \leq a_{lm}(s)$, then

$$L_l(t, s) = \frac{\lambda(s-t+1)}{T} \int_0^{\max\{a_{ls}(s), a_{ls}(t)\}} f(a)da$$  \hspace{1cm} (25)

$$N_m(t, s) = 0$$ \hspace{1cm} (26)

$$N_s(t, s) = \frac{\lambda(s-t+1)}{T} \int_0^{\max\{a_{ls}(s), a_{ls}(t)\}} af(a)da$$ \hspace{1cm} (27)

2. If the period $s$ unit wages satisfy $w_m(s) \leq w_s(s)$, and $a_{lm}(s) < a_s(t)$, then

$$L_l(t, s) = \frac{\lambda(s-t+1)}{T} \int_0^{a_{lm}(s)} f(a)da$$ \hspace{1cm} (28)

$$N_m(t, s) = \frac{\lambda(s-t+1)}{T} \int_{a_{lm}(s)}^{a_s(t)} af(a)da$$ \hspace{1cm} (29)

$$N_s(t, s) = \lambda(s-t+1) T \int_{a_s(t)}^{\infty} af(a)da$$ \hspace{1cm} (30)

3. If $w_m(s) > w_s(s)$, then

$$L_l(t, s) = \frac{\lambda(s-t+1)}{T} \int_0^{a_{lm}(s)} f(a)da$$ \hspace{1cm} (31)

$$N_m(t, s) = \frac{\lambda(s-t+1)}{T} \int_{a_{lm}(s)}^{a_s(t)} af(a)da$$ \hspace{1cm} (32)

$$N_s(t, s) = 0$$ \hspace{1cm} (33)

Proof. It is easy to see that we have covered each possible configuration of cutoffs. See Appendix for details.

Corollary 3. The effective labor supply in period $s$ of workers born in period $s$:

$$L_l(s, s) = \frac{1}{T} \int_0^{a_{ls}(s)} f(a)da$$ \hspace{1cm} \hspace{1cm} (34)

$$N_m(s, s) = \frac{1}{T} \int_{a_{ls}(s)}^{a_{lm}(s)} af(a)da;$$ \hspace{1cm} (35)

$$N_s(s, s) = \frac{1}{T} (1 - \psi_s) \int_{a_s(s)}^{\infty} af(a)da - \frac{1}{T} \kappa \int_{a_s(s)}^{\infty} f(a)da. \hspace{1cm} (36)$$

Proof. Using the fact that education takes time, those born in period $s$, who get educated only work in a fraction $(1 - \psi)$ of period $s$. On top of this, those who acquire education require training, in the amount $\kappa$ for each individual. This amount of labor is used for their training, rather than for production, therefore they reduce the effective labor supply by this amount per person.

Assumption 1. We assume that the economy starts in period 0, with $T$ cohorts, one from each generation, and each generation made education decisions according to $0 < a_s(0) < \infty$. 

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Note that this assumption is equivalent to assuming that the economy from the start of time \((t = -\infty)\) has been in a steady state where the optimal educational decision is given by cutoffs \(0 < a_s(0) < \infty\), and whatever change (if any) happened in period 1 was unanticipated by all agents born until period 0.

**Corollary 4.** Assume that the period \(s\) unit wages satisfy \(w_m(s) \leq w_s(s)\). The cohort born in period \(t = 0\) supplies labor to each sector in period \(s \leq T - 1\) in a similar way as in Corollary 2 except the multiplier is \(\sum_{i=1}^{s+1} \Delta(i)/T\) instead of \(\lambda(s + 1)/T\) which would be indicated by applying Corollary 2 to \(t = 0\).

Given the labor supplies by each cohort in Corollaries 2 and 3 the total effective labor supply in period \(s\) is given by:

\[
L_l(s) = \sum_{j=s-T+1}^{s} L_l(s-j,s) \tag{37}
\]

\[
N_m(s) = \sum_{j=s-T+1}^{s} N_m(s-j,s) \tag{38}
\]

\[
N_s(s) = \sum_{j=s-T+1}^{s} N_s(s-j,s) \tag{39}
\]

### 4 Competitive Equilibrium

A competitive equilibrium is a sequence of cutoff education abilities \(\{a_s(t)\}_{t=1}^{\infty}\), labor supply abilities \(\{a_{lm}(t), a_{ls}(t)\}_{t=1}^{\infty}\), wages \(\{w_l(t), w_m(t), w_s(t)\}_{t=1}^{\infty}\), prices \(\{p_m(t), p_s(t)\}_{t=1}^{\infty}\), given the path of productivities \(\{A_h(t), A_m(t), A_s(t)\}_{t=0}^{\infty}\) and initial education decisions \(a_s(0)\) which satisfy:

1. \(a_s(t)\) is the solution to equation (24);
2. \(a_{lm}(t), a_{ls}(t)\) are defined as in (6), and (7);
3. the unit wage rates are such that the market for \(L\), \(M\) and \(S\) labor clears;
4. \(p_m\) and \(p_s\) are such that the market for \(M\) goods and \(S\) goods clears.

The economy is always in a competitive equilibrium, where newborns choose their education optimally, and older cohorts choose their sector of work optimally. Firms maximize their profits. Markets clear.

For any initial condition, there is a unique stationary competitive equilibrium, which features constant education and sector-of-work decisions. In a steady state the following holds for the education and sector-of-work decisions:

\[
a_{lm} = \frac{w_l}{w_m}.
\]
while $a_s$ satisfies

$$VU(w_s((1 - \psi)a_s - \kappa); p) - VU(w_m a_s; p) + (VU(w_s a_s, p) - VU(w_m a_s, p)) \sum_{i=2}^{T} \beta^i \lambda(i) = 0.$$ 

In the steady state the effective and raw labor supplies are given by:

$$L_l = \int_{0}^{a_{lm}} 1dF(a);$$

$$N_m = \int_{a_{1m}}^{a_m} adF(a);$$

$$L_m = \int_{a_{lm}}^{a_m} dF(a);$$

$$N_s = \frac{\left(\sum_{i=2}^{T} \lambda(i) + (1 - \psi)\right)}{T} \int_{a_{s}}^{\infty} dF(a) - \frac{1}{T} k \int_{a_{s}}^{\infty} 1dF(a);$$

$$L_s = \int_{a_{s}}^{\infty} 1dF(a).$$

Therefore across different steady states it holds that $L_l$ increases with $w_l/w_m$, and $N_s(L_s)$ increases with $w_s/w_m$. This is similar to some kind of polarization: if the relative unit wages increase at the top and at the bottom, then the raw labor supplies move the same way. However, in reality we do not observe the relative unit wages, but we observe the relative average wages.

### 5 Quantitative results (Incomplete)

In this section we quantitatively assess the contribution of structural transformation to the polarization of employment and wages across industries. To do this we consider the economy to be on a full foresight transition to the steady state. The model economy starts in 1950, and all workers/consumers are aware that there will be 120 years of unequal productivity growth. We calibrate the first period of the transition to match some 1950 data moments. We first describe the data targets and the calibration strategy, and then discuss the quantitative importance of our mechanism.

#### 5.1 Data targets

We calibrate our model to replicate six key moments of the US economy in 1950. These moments are the relative average industry wages, the industry employment shares and the sectoral value added shares. Data for the average industry wages and the industry employment shares come from the 1950 US Census data. Each employed individual is categorized as a high-skilled service, manufacturing or low-skilled service worker, based on their industry code (ind1990). Employment shares are calculated as share of hours worked. We regress log hourly wages on worker age and its square, a female dummy, a born-abroad dummy and a non-white dummy. The relative industry wages are calculated as the ratio of the average of the exponential of the residual wages across industries. For sectoral value added shares we rely on the National Income and Product Accounts (NIPA), but adjust them for intermediaries using the input-output tables published by the Bureau of Economic Analysis (BEA), similar to Herrendorf, Rogerson, and Valentinyi (2013).
5.2 Calibration

All parameters are time-invariant, and the only exogenous change over time is productivity growth. Herrendorf, Herrington, and Valentinyi (2012) estimate a labor-augmenting technological progress on value added output, with annual growth rates of 2% for manufacturing, and 1.1% for the service sector.

We set a model period to be 10 years. The usual yearly discount rate in the literature is 0.95, therefore we use a discount rate over 10 years of $\beta = 0.95^{10} = 0.5987$. We consider the potential working life of individuals from the age of 18. Szafran (2002) documents that labor force participation tapers off only above the age of 70, therefore we set $T = 6$, which implies that we allow people to potentially work until the age of 78. In this model, since there are no savings, labor force participation coincides with positive consumption. Therefore in terms of the model death and exit from the labor market are equivalent, and we consider them exogenous, as this decision is not modeled. To capture this in the survival/participation probability vector, we only consider exit due to death up to the age of 58, and between the age of 58-68 and 68-78 we reduce the probability of survival by the trend drop in labor force participation[12]. The following vector for survival/participation probabilities emerges: $\lambda = [1, 0.991, 0.98, 0.961, 0.512, 0.102]$ Since college education takes about 4 years, and one period lasts 10 years in our model, we set the fraction of a period needed for education to $\psi = 0.4$.

We normalize all initial productivity values to 1, i.e. $A_h(0) = A_m(0) = A_s(0) = 1$. We normalize the mean of the ability distribution to be 1, as this cannot be separately identified from other parameters, implying $\mu = -\frac{1}{2} \sigma^2$. We normalize the weight on the disutility from home production to be unity, $\phi = 1$. In the literature, when sectoral output is measured in value added terms, a very low value of $\epsilon$ is estimated and used (Herrendorf, Rogerson, and Valentinyi (2013)). In the calibration exercise we test the sensitivity of our results to this parameter, but in the baseline case we set it to 0.1.

6 Quantitative Results (Incomplete)

We simulate the transition path of the model to its steady state. The economy is not in a steady state initially, and workers have perfect foresight of the exogenous path of productivity growth. We study the endogenous response of employment and wages. Our baseline is, as in the data, that productivity growth is higher in manufacturing than in services. Taking estimates by Herrendorf, Rogerson, and Valentinyi (2013) we let $M$’s productivity grow by 2% per annum and $S$’s productivity grow by 1.1%.

Figure 4 plots the resulting transition[13]. Productivity growth implies that all the consumption goods, $M$ and $S$ become relatively cheaper. Since $S$ is a luxury good, the demand for it increases more than the demand for $M$. This implies that more labor needs to be employed in sector $S$, which is exacerbated by the higher productivity growth in $M$ than in $S$. To satisfy demand, employment in the $S$ sector has to increase, and firms have to pay higher unit wages to attract workers. The highest ability workers who

[12] Data on survival probabilities come from the 2006 Life Tables of CDC/National Center for Health Statistics, whereas the trend drop in labor force participation comes from Chart 3 in Szafran (2002).

[13] The following parameters were used: $\kappa = 0.6$, $\varpi = 0.5$, $\theta_M = 0.4$, $\gamma_S = 0.8$, $\nu = 2$, $\sigma = 0.3$. 

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would not have acquired education and worked in manufacturing before, now acquire education and work in sector $S$. Since only the new generations can acquire more education, increasing the supply of high-skilled service workers takes time. $S$ sector unit wages have to increase steeply to attract a sufficient amount of new entrants. As a consequence, the cutoff ability of workers sorting into $S$ falls, which tends to decrease the average wage paid in the sector. However, the effect of higher unit wages is the dominating one, and on average $S$ workers wages increase relative to $M$. This is reflected in the gradual increase of the employment share in sector $S$, and the steep initial rise in the relative $S$ wages, as can be seen in the two right panels in the bottom row of Figure 4. Hence, both wages and employment at the top-end of the distribution increase.

Moreover, with national income the demand for services substituting for home production increases, time spent on home production falls, and instead the employment of $L$ labor increases. The steep rise in $S$ wages implies that the increase in national income goes disproportionately to the highest earners, which amplifies the increase in demand for low-skilled services. As the demand for $L$ services increase, also the wage for $L$ workers rises, in order to pull workers into the low-skilled services sector. As a consequence, employment and wages rise at both ends of the distribution.
References


Appendix

Figure 5: Wage and employment polarization II.

Notes: Wages are calculated from US Census data of 1950, 1960, 1970, 1980, 1990, 2000 and American Community Survey (ACS) of 2008. Balanced occupation categories (185 of them) were defined by the authors based on Meyer and Osborne (2005) and Autor and Dorn (2012). The bottom two panels show the 10-year change in employment shares (calculated as hours supplied rather than persons), and the top two panels show the 10-year change in log hourly real wages (again labor supply weighted). In the left panels occupations are ranked based on their 1950 average wage, whereas in the right panels they are ranked according to their 1980 average wage.