Downward Wage Rigidities and Optimal Monetary Policy in a Monetary Union

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Abstract

This paper analyses the effects of downward nominal and real wage rigidities on the transmission of technology shocks in a monetary union under different monetary policy regimes, including optimal policy. Downward rigid wages imply higher adjustment costs for wage cuts but lower costs for wage increases compared to a symmetric situation. In the presence of downward nominal rigidity aggregate volatility leads to a strictly positive inflation rate. Monetary policy is therefore more responsive following a negative productivity shock compared to a positive one. These effects are stronger the more flexible prices are, but are absent in the case of real wage rigidity (indexation). Following a union-wide shock, in which regions differ in the type of rigidity, terms of trade always improve over the short-run for the country with real rigidity independently of the direction of the shock. Following asymmetric shocks, relative price adjustments dominate over wage asymmetry leaving responses nearly unaltered to symmetric wage adjustments. We find a small role for monetary policy even in the case of purely asymmetric shocks due to curvatures in the production and the utility function. Finally, compared to optimal monetary policy, an inflation targetter generates large inflation differentials and terms of trade between countries with the adjustment burden mostly on the nominal rigid country. Commonly used Taylor rules are not aggressive enough during the initial phase of adjustment compared to the optimal policy.

JEL classification: E31; E52; C61.

Key words: Monetary Union, Optimal inflation, Downward wage rigidity, asymmetric adjustment costs, non-linear dynamics.

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1 Introduction

Flexible labour markets are essential for the well-functioning of the European Monetary Union (EMU). In the absence of nominal exchange rate adjustments, a flexible response of wages to both symmetric and asymmetric shocks is important to ensure a smooth and efficient economic adjustment within a monetary union. Although various labour market reforms have taken place in a number of euro area countries over the past few decades, ten years after EMU labour markets are still characterised by large differences in collective bargaining and other labour market institutions as, for example, described by Du Caju et al. (2008c). The aim of this paper is to analyse the implications of one particular friction, downward wage rigidities, in a monetary union. More particularly, we focus on the transmission of productivity shocks within a monetary union characterised by labour markets with differences in the degree of downward rigidities either in nominal or real wages and on the implications of such rigidities for optimal monetary policy.

Recent empirical research has highlighted the presence of downward wage rigidities in a large number of countries. Dickens et al. (2007) summarise the findings of the so-called International Wage Flexibility project, which uses micro-economic wage data to investigate the extent to which nominal and real wages are downwardly rigid across countries. Investigating the shape and skewness of wage change distributions, this paper finds clear evidence of both nominal and real downward wage rigidity in a number of countries. The extent of downward nominal versus real wage rigidity differs, however, across countries and is related to differences in labour market institutions such as the presence of indexation, the incidence of unemployment benefits and other labour market regulations. Within the context of the Eurosystem Wage Dynamics Network Du Caju et al. (2008a) confirms and updates some of these findings, quantifying the extent of downward wage rigidity across a number of European countries. Within EMU, one interesting case characterised by prevalent downward real wage rigidity is Belgium, which still has a system of widespread indexation of wages to changes in consumer prices (see Du Caju et al. (2008b)). Another case characterised more by downward nominal wage rigidity is Portugal, where wage cuts for job stayers are prohibited by law (see Portugal (2008)). Although the micro evidence indicates the existence of downward wage rigidities, it is not obvious to what extent these rigidities are also relevant for employment adjustment over the business cycle at a more aggregate level.

Most studies using microeconomic data are restricted to job stayers, while adjustments of labour costs could take place through wage adjustments of new hires and/or compositional changes in the age and skill composition over the business cycle. Holden and Wulfsberg (2007a, 2007b) analyse wage changes at the industry level for 19 OECD countries over the period 1973–1999 and confirm the existence of both nominal and real downward wage rigidities of different extent across different countries even at this more aggregate level.

In this paper we address the following questions: How does the monetary union and its regions adjust to symmetric productivity shocks in the presence of asymmetric wage adjustments? How do asymmetric shocks influence wages and prices in the union? How should monetary policy take these asymmetries into account? What are the effects on steady-state inflation? With respect to the latter question, our analysis relates to the debate on the greasing effects of inflation initiated by Tobin (1972). In his Presidential address, he argued that inflation helps to improve the adjustment within the labour market as it reduces the necessity for nominal wage cuts.

To address these questions, we set up a two-region monetary union DSGE model with costs of adjusting prices and wages in the spirit of Rotemberg (1982). Following Kim and Ruge-Murcia (2007), the rigidities can be symmetric or asymmetric for upward and downward adjustments. Moreover, in view of the empirical evidence for some of the euro area countries, we also allow for real wage rigidity, in which case

nominal wages are indexed to current inflation. Using a closed-economy model estimated on US data, Kim and Ruge-Murcia (2007) find that a low, but positive inflation objective does grease the wheels of the economy in the face of downward nominal wage rigidity and that price adjustments compensate for the lower flexibility in the wage setting. We extend their model to a currency area along the lines of Benigno (2004), Gali and Monacelli (2008) and Poilly and Sahuc (2008).

Our main contribution is therefore that we extend the analysis of asymmetric wage adjustment to a two-country set-up and to include real rather than nominal wage rigidity. This allows us to investigate how asymmetric wage adjustment affect the transmission process within the monetary union. Regarding the modelling of the labour market, we follow the monopolistically competitive model of Erceg et al. (2000), which is different from the recent focus on frictional labour markets as, for example, analysed in Poilly and Sahuc (2008), Abbritti and Mueller (2007), Andersen and Seneca (2007), Campolmi and Faia (2006), or Christoffel et al. (2008). As a result, we ignore matching frictions that govern the transition from unemployment to employment and focus exclusively on rigidities in wage adjustment. Our paper is also related to Fagan and Messina (2008), who use a flexible menu cost model to match wage change distributions at the micro level in a number of countries and investigate the implications for optimal steady-state inflation.¹ Finally, in terms of the monetary policy analysis, our paper is related to the literature on optimal monetary policy using second-order approximations of the objective function as in Benigno and Woodford (2006), but our methodology uses the second-order perturbation methods as, for example, those by Schmitt-Grohe and Uribe (2005) and Darracq Paries et al. (2007).

In the rest of this paper, we first outline the monetary union model with costly wage and price adjustments in Section 2. Section 3 discusses the baseline calibration. The results are described in Section 4 and Section 5 contains the main conclusions. Three of those are worth highlighting here. First, downward nominal wage rigidity leads to a positively skewed response in nominal wage changes and a sizeable positive optimal steady-state inflation rate. This effect is stronger the lower the degree of price rigidity. To the extent that higher price stickiness in the euro area than in the United States is due to greater price rigidity as found by Altissimo et al. (2007), this would suggest a lower optimal inflation rate in the euro area. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if wage adjustment costs are symmetric. Second, regarding the dynamic responses to productivity shocks in the face of downward wage rigidities, we find that beyond the direct effects on wage changes also price changes are altered and become asymmetric, highlighting the strong link between prices and wages. In addition, when real wage adjustments are costly and asymmetric, the effects on hours worked are also highly asymmetric. In the case of two regions with different types of asymmetries (nominal and real), terms of trade effects are sizeable with the more rigid-wage region always adjusting with higher prices for both positive and negative shocks. Third, in the simple version of the two-country model the main burden of adjustment in response to asymmetric technology shocks is through relative price adjustments. Asymmetries in the way nominal wages are set in the two regions affect the transmission to a much smaller degree than the terms of trade channel governed by relative prices. Nevertheless, the model reveals that asymmetric shocks bear potential deflationary pressures for the union as a whole. To what degree other setups for international risk sharing and asymmetries in price adjustments lead to more profound effects in current account remains an area for further research.

¹Another related paper is Benigno and Ricci (2008), who explicitly address downward wage rigidity in a different type of model, but focus on the resulting non-linearities of the Philips curve at low inflation levels.

2 The Model

We present the model in four parts. The first part regards the aggregation of goods into consumption bundles and the definitions for prices and terms of trade. The second and third part state the maximisation problem households and firms face taking into account that both types of agents are monopolistic suppliers of labour and goods respectively. The fourth part defines the monetary policy rules and the equilibrium. The model follows mainly Benigno (2004) and Poilly and Sahuc (2008) for the setup of the monetary union, Erceg et al. (2000) for the labour markets and Kim and Ruge–Murcia (2007) regarding adjustment costs for prices and wages.

2.1 Aggregation and prices

The economy consists of a continuum of infinitely lived households distributed on the unit interval $[0, 1]$ and living in two regions, home $\mathcal{H}$ and foreign $\mathcal{F}$. Households $[0, n)$ live and work in region $\mathcal{H}$ and households $[n, 1]$ live in the foreign region $\mathcal{F}$, labour is immobile across regions. The households consume Dixit–Stiglitz aggregates of domestic and imported goods and supply labour services monopolistically. For goods produced in the two regions we use subscript $\mathcal{H}$ and $\mathcal{F}$ respectively, and for consumption we use no index for the home country and a star for the foreign country (C_t and C_t^*).

The composite consumption bundles C_t , C_t^* in the two regions consist of home and foreign produced goods with an elasticity of substitution of ω between the two production countries:

$$C_t = \left[\eta^{1/\omega} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-\eta)^{1/\omega} C_{Ft}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad \text{and} \quad C_t^* = \left[(1-\eta^*)^{1/\omega} C_{Ht}^{*\frac{\omega-1}{\omega}} + \eta^{*1/\omega} C_{Ft}^{*\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (1)$$

where η, η^* are the degrees of preference for goods produced in the consumer's region of residence. With $\eta > 1/2$ consumers in the home country prefer home produced goods and with $\eta^* > 1/2$ foreign consumers prefer goods produced in country $\mathcal{F}$. From cost minimisation of (1) the demand for the composite home and foreign produced goods by the representative consumers in each region is

$$\begin{aligned} C_{Ht} &= \eta \left(\frac{P_{Ht}}{P_t} \right)^{-\omega} C_t \quad \text{and} \quad C_{Ft} = (1-\eta) \left(\frac{P_{Ft}}{P_t} \right)^{-\omega} C_t, \\ C_{Ht}^* &= (1-\eta^*) \left(\frac{P_{Ht}}{P_t^*} \right)^{-\omega} C_t^* \quad \text{and} \quad C_{Ft}^* = \eta^* \left(\frac{P_{Ft}}{P_t^*} \right)^{-\omega} C_t^*, \end{aligned} \quad (2)$$

where P_{Ht} and P_{Ft} are respectively the prices for home and foreign produced bundles and the law of one price holds between regions. Each of the consumption goods C_{Ht} and C_{Ft} is itself a composite of goods produced monopolistically by the firms in the two regions. The aggregators for the region-specific production earn

$$Y_{Ht} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n (y_t(h))^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad Y_{Ft} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 (y_t(f))^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}} \quad (3)$$

where Y_{Ht} is the total production of goods at home, Y_{Ft} is the total production of goods in the foreign country, and σ is the elasticity of substitution between goods, identical across regions, and n characterizes the size of the region. For a given level of consumption we can define through cost minimisation the demand towards single home and foreign firms:

$$y_t(h) = \frac{1}{n} \left(\frac{p_t(h)}{P_{Ht}} \right)^{-\sigma} Y_{Ht} \quad \text{and} \quad y_t(f) = \frac{1}{1-n} \left(\frac{p_t(f)}{P_{Ft}} \right)^{-\sigma} Y_{Ft}, \quad (4)$$

where the corresponding producer price indices (*PPI*) P_{Ht} and P_{Ft} are defined as the unit price for a home and foreign production bundle

$$P_{Ht} = \left[\frac{1}{n} \int_0^n p_t(h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_{Ft} = \left[\frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

We assume that the law of one price holds, $p_t^F(h) = p_t^H(h) = p_t(h)$ and $p_t^F(f) = p_t^H(f) = p_t(f)$, implying that the price for the bundles Y_{Ht} and Y_{Ft} is identical across regions. Instead, the consumption price index (*CPI*) is determined by the cost-minimisation problem of the household when splitting expenditure between home and foreign produced goods. While producer prices are equated between regions, the utility-based consumer price index is not necessarily identical across regions due to the home bias of consumers. The CPIs in the home and foreign country, P_t and P_t^* respectively, are

$$P_t = [\eta P_{Ht}^{1-\omega} + (1-\eta) P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}} \quad \text{and} \quad P_t^* = [(1-\eta^*) P_{Ht}^{1-\omega} + \eta^* P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}} \quad (5)$$

In addition to the preference parameters η, η^* the size of the two countries is important for determining demand for home and foreign produced goods. With equally sized regions the case for perfect risk sharing with the equation of consumption levels across regions is $\eta = \eta^* = 1/2$. In the case of different sized countries this condition generalizes to $\eta = \eta^* = n$, where the economic size is equated to the preference structure and perfectly identical consumption patterns prevail as found by Benigno (2004).

In the case of identical producers in each region total demand depends on preferences by the home and foreign households, η and η^* , the region's sizes $n, (1-n)$, the level of consumption in the two regions and the terms of trade defined as the relative price of foreign to home producer prices, $S_t = P_{Ft}/P_{Ht}$. The demand directed towards a single producer in the home and foreign country stems from the n households in the home country and $(1-n)$ households in the foreign country by combining equations (2) and (4), employing the definition for the CPI (5) and exploiting the fact that all firms in one region are identical.²

$$c_t(h) = \eta \left(\frac{P_{Ht}}{P_t} \right)^{-\omega} C_t + \frac{1-n}{n} (1-\eta^*) \left(\frac{P_{Ht}}{P_t^*} \right)^{-\omega} C_t^* \quad (6)$$

$$= \eta [\eta + (1-\eta) S_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t + \frac{1-n}{n} (1-\eta^*) [1-\eta^* + \eta^* S_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (7)$$

$$c_t(f) = \frac{n}{1-n} (1-\eta) \left(\frac{P_t}{P_{Ft}} \right)^{\omega} C_t + \eta^* \left(\frac{P_t^*}{P_{Ft}} \right)^{\omega} C_t^* \quad (8)$$

$$= \frac{n}{1-n} (1-\eta) S_t^{-\omega} [\eta + (1-\eta) S_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t + \eta^* S_t^{-\omega} [1-\eta^* + \eta^* S_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (9)$$

To recap, the aggregation structure is as follows: the goods from the monopolistic producers in the home and foreign country, $y_t(h)$ and $y_t(f)$ respectively, are aggregated to country-specific bundles Y_{Ht} and Y_{Ft} using (3). Households aggregate home and foreign bundles to consumption goods C_t and C_t^* by (1) and decide on the respective expenditure for home and foreign goods.

Terms of trade and international risk sharing. The terms of trade are the relative prices of foreign to home produced goods

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ft}^*}{P_{Ht}^*},$$

where the second equality holds due to the law of one price. They represent an index of competitiveness and play a crucial role in the transmission of asymmetric shocks in the two regions. We can express the

²The demand for a single firm denoted by $c_t(h)$ and $c_t(f)$ respectively is gross production $y_t(h), y_t(f)$ net of possible adjustment costs in prices and wages that firms and households incur.

CPI (5) in the two regions employing the terms of trade:

$$\begin{aligned} P_t &= P_{Ht} [\eta + (1 - \eta) S_t^{1-\omega}]^{\frac{1}{1-\omega}} \\ P_t^* &= P_{Ft} [(1 - \eta^*) S^{- (1-\omega)} + \eta^*]^{\frac{1}{1-\omega}} \\ &= P_{Ht} [1 - \eta^* + \eta^* S^{1-\omega}]^{\frac{1}{1-\omega}}, \end{aligned} \quad (10)$$

linking in this way the consumer prices to the producer prices.

The real exchange rate, the ratio of relative consumption price indices $RER_t = P_t^*/P_t$, can be expressed as ratio of the home and foreign producer prices using (5)

$$RER_t = \frac{P_t^*}{P_t} = \left[\frac{1 - \eta^* + \eta^* S_t^{1-\omega}}{\eta + (1 - \eta) S_t^{1-\omega}} \right]^{\frac{1}{1-\omega}} \quad (11)$$

2.2 Households

The economy consists of a continuum of infinitely lived households distributed on the unit interval $[0, 1]$ and living in two regions, home $\mathcal{H}$ and foreign $\mathcal{F}$. Households $[0, n)$ live and work in region $\mathcal{H}$ and households $[n, 1]$ live in the foreign region $\mathcal{F}$ and they are immobile across regions. Households consume Dixit–Stiglitz aggregates of domestic and imported goods and supply labour services monopolistically following Erceg et al. (2000). All households have identical preferences and initial endowments. As the household’s problem is identical for the two regions we describe here only one region to save on notation. The representative household for home and foreign region maximises

$$E_0 \sum_{t=0}^{\infty} \beta^{t-\tau} \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{h_t^{1+\chi}}{1+\chi} \right], \quad (12)$$

where C_t represents the consumption bundle consisting of home and foreign goods as defined in (1) with ρ being the inter-temporal elasticity of substitution for consumption goods, h_t represents hours worked by the household and χ the disutility of work. The sequence of household’s budget constraints in real terms is

$$C_t + \frac{B_t - I_{t-1} B_{t-1}}{P_t} + \frac{Q_t B_t^D - B_{t-1}^D}{P_t} = \frac{W_t h_t}{P_t} (1 - \Phi_t) + \frac{D_t}{P_t} \quad \text{for } t = 1, 2, \dots \quad (13)$$

Households purchase a composite consumption good C_t and acquire non-contingent bonds B_t that earn nominal interest rates I_t . Labour income stems from working h_t hours for a nominal wage rate W_t net of wage adjustment costs Φ_t , dividends D_t are paid from firms to the households being owners of firms. We will analyse three different specifications for the international risk sharing: The first one assumes access to a full set of state-contingent assets, the second one assumes autarky, whereby the country’s production equates consumption, and the third assumption allows for an internationally traded bond B_t^D between the two countries which costs Q_t nominal units and earns one nominal unit next period.

The household is a monopolistic supplier of labour services following Erceg et al. (2000), which are aggregated for each country in a Dixit–Stiglitz fashion

$$h_{Ht} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n h_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad h_{Ft} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 h_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \quad (14)$$

with n representing the relative size of the home region and θ being the elasticity of substitution for differentiated labour services, identical across regions. In order to adjust wages, households incur adjustment costs Φ_t which take the form of a linex cost function following Kim and Ruge–Murcia (2007).

Figure 1: Different specifications of adjustment cost curves. The dotted line represents a symmetric adjustment cost function as used for prices, the continuous line depicts an asymmetric cost function as used for nominal asymmetric wage adjustments and the dashed-dotted line is an asymmetric adjustment cost function as used for asymmetric real wages with an underlying indexation of 2%.

These costs are with respect to nominal wage increases that may be indexed to an inflation measure:

$$\Phi_t(\Omega_t^{ind}) = \frac{\phi}{\psi^2} \left\{ \exp[-\psi(\Omega_t^{ind} - 1)] + \psi(\Omega_t^{ind} - 1) - 1 \right\}, \quad (15)$$

with $\Omega_t^{ind} = \frac{W_t}{(\Pi_t^{ind})^\nu W_{t-1}}$ being the wage increase of indexed wages. The cost function is a continuous and differentiable function in wage inflation. The parameter ϕ determines the degree of asymmetry, while ψ determines the degree of convexity of the function. If $\psi = 0$ the specification nests the standard quadratic case as in Rotemberg (1982), $\Phi_t(\Omega_t^{ind}) = \phi(\Omega_t^{ind} - 1)^2$. In addition to adjustment costs with respect to nominal wages it incorporates also differing degrees of real wage rigidity determined by $\nu \in [0, 1]$. Nominal downward wage rigidity, characterised by $\nu = 0$, implies higher costs for nominal wage cuts as opposed to wage increases, while downward real wage rigidity, $\nu = 1$, implies asymmetric costs if wage growth is stronger or smaller than the inflation measure used for indexation. This inflation measure may be past, contemporaneous or contemporaneous inflation of the single country or the monetary union as a whole. Figure (1) presents a visual impression for a symmetric, and asymmetric adjustment cost functions with and without indexation Π_t^{ind} .

The functional specification bears two main advantages for our purpose: first, it includes symmetric and asymmetric adjustment costs within a single functional form to be altered smoothly governed entirely by the parameter ϕ , and second, it is differentiable for all positive real numbers, especially around the point of zero wage inflation, i.e. $W_t/W_{t-1} = 1$ with the possibility of indexation to inflation measures. The extent of downward nominal and real rigidities in the euro area has been documented by the International Wage Flexibility Project in Dickens et al. (2007) and the Eurosystem Wage Dynamics Network, e.g. Du Caju et al. (2008b). The euro area faces downward nominal and real wage rigidities due to explicit wage indexation either to national inflation (e.g. Belgium) or to an index of inflation and wage inflation in the home and in neighbouring countries. In order to account for real wage rigidities we allow the inflation rate Π_t^{ind} to which indexation takes place to be the area-wide rate $\Pi_t^w = P_t^n (P_t^*)^{1-n}$ in order

to capture inflation evolutions of the country.

2.2.1 Household optimization

The household maximises (12) under condition (13) and (15) with respect to consumption C_t , bonds B_t , B_t^D and wages W_t .

$$\lambda_t = u'(C_t) = C_t^{-\rho} \quad (16)$$

$$\frac{\lambda_t}{P_t} = I_t \beta \mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}} \quad (17)$$

$$Q_t \frac{\lambda_t}{P_t} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}} \quad (18)$$

$$\begin{aligned} & \frac{\eta_t}{P_t} \left[\frac{\theta}{\theta-1} h_t (1 - \Phi_t^i) + \left(\frac{W_t}{W_{t-1}} \right) (\Pi_t^w)^{-\nu} h_t \Phi_t' \right] \\ = & h_t (1 - \Phi_t) + \left(\frac{\theta}{\theta-1} \right) \frac{h_t^{1+\chi}}{W_t} + \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \left(\frac{W_{t+1}}{W_t} \right)^2 (\Pi_{t+1}^{ind})^{-\nu} h_{t+1} \Phi_{t+1}' \right] \end{aligned}$$

The first equation states the valuation of consumption in form of the Lagrange multiplier, the second equation states consumption Euler equation in nominal terms for households, the third reflects the optimal holding of internationally traded bonds and the last equation states the wage equation which we can simplify to

$$\frac{\Omega_t}{(\Pi_t^{ind})^\nu} \Phi_t' = \left[\theta \frac{h_t^\chi}{\lambda_t w_t} - (1 - \Phi_t) \right] / (\theta - 1) + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{h_{t+1}}{h_t} \frac{\Omega_{t+1}^2}{(\Pi_{t+1}^{ind})^\nu} \Phi_{t+1}' \right], \quad (19)$$

where $\Omega_t = W_t/W_{t-1}$ is the nominal wage growth in the region and Π_t^{ind} is the inflation rate used for indexation. We use a specification with contemporaneous inflation. Indexation is often not a purely backward looking phenomenon, but also inflation expectations are taken into account. To synthesize the backward and forward-looking elements we simplify and settle for a contemporaneous specification. In order to better understand the equation for wage growth we state (19) the case of flexible wages in which adjustment costs Φ_t are zero:

$$\frac{W_t}{P_t} = w_t = \theta h_t^\chi / u'(C_t) = \theta MRS_t.$$

Wages are a mark-up θ over the marginal rate of substitution because households supply differentiated labour services. In the case of convex adjustment costs, households face the loss of the adjustment cost Φ_t and wages become rigid as households have an incentive to smooth wage adjustments over time.

2.2.2 Fundamental risk sharing condition

The three different specifications of the international risk sharing, i.e. full set of contingent assets, autarky, and internationally traded bond leads to the following three conditions for risk sharing. With complete markets the Euler equations (16) of the two regions are linked and lead to a relationship between the real exchange rate and the consumption levels

$$RER_t = \psi_0 \frac{u'(C_t^*)}{u'(C_t)} = \psi_0 \left(\frac{C_t^*}{C_t} \right)^{-\rho}, \quad (20)$$

where $\psi = RER_0 \frac{u'(C_0^*)}{u'(C_0)} = \frac{P_0^*}{P_0} \frac{u'(C_0^*)}{u'(C_0)}$ is a constant including only initial conditions for asset holdings. Changes in real exchange rates reflect differing consumption levels in the two countries. Complete markets

therefore do not lead to identical consumption, but to an efficient consumption differential stemming from the home bias by consumers. We assume that the initial wealth distribution and price levels between the countries is identical across the countries, implying $\psi_0 = 1$.

In the case of autarky, the value of net home production and consumption are identical, $PC = P_H C_H$ and $P^* C^* = P_F C_F$. The current account is closed in every period and only the non-contingent bond paying nominal interest rates serves for smoothing consumption.

2.3 Firms

The final good in each region is an aggregate of differentiated products, each produced by a continuum of firms distributed on the unit interval $j \in [0, 1]$. Each firm produces with labour as unique input with decreasing returns to scale and stochastic technology in the two countries

$$y_t = x_t A_t h_t^{1-\alpha}.$$

The firm's objective is to maximise profits in real terms over an infinite horizon subject to the labour market imperfections consisting of price adjustment costs, imperfect labour substitutes and the demand function by households. The firm maximises profits consisting of the production net of price adjustment costs and wage payments to the households:

$$E_\tau \sum_{t=\tau}^{\infty} \frac{\Lambda_{\tau,t}}{P_t} \left[(1 - \Gamma_t) P_t y_t - \int_0^1 W_t^n h_t(n) dn \right],$$

where $\Lambda_{\tau,t} = \beta^{t-\tau} \frac{\lambda_t}{\lambda_\tau}$ is the firm's stochastic discount factor consisting of the household's discount factor β and the Lagrange multiplier λ_t . The per period profits consist of revenue from goods sold to the households net of price adjustment costs Γ_t that may differ between countries and net of wage payments to the employed workers, where h_t represents aggregate labour. All firms act individually, but are identical. The demand for labour of the two countries is obtained from cost minimisation of the labour service aggregator (14)

$$h_t(h) = \frac{1}{n} \left(\frac{w_t(h)}{W_{Ht}} \right)^{-\frac{\theta}{\theta-1}} h_{Ht} \quad \text{and} \quad h_t(f) = \frac{1}{1-n} \left(\frac{w_t(f)}{W_{Ft}} \right)^{-\frac{\theta}{\theta-1}} h_{Ft},$$

where h_{Ht} and h_{Ft} denote the composite labour service from home and foreign workers, $w_t(h)$, $w_t(f)$ are the real wages for the single labour services and W_H and W_F are the nominal wage indices for the composite labour service defined as

$$W_{Ht} = \left[\frac{1}{n} \int_0^n (W_t(h))^{\frac{1}{1-\theta}} dh \right]^{1-\theta} \quad \text{and} \quad W_{Ft} = \left[\frac{1}{1-n} \int_n^1 (W_t(f))^{\frac{1}{1-\theta}} df \right]^{1-\theta}.$$

Firms maximize with respect to the product price and hours worked while taking into account the demand function from households (??), the labour aggregator (14) and the price adjustment cost function:

$$\Gamma_t \left(\frac{P_t}{P_{t-1}} \right) = \frac{\gamma}{\varsigma^2} \left(\exp \left[-\varsigma \left(\frac{P_t}{P_{t-1}} - 1 \right) \right] + \varsigma \left(\frac{P_t}{P_{t-1}} - 1 \right) - 1 \right). \quad (21)$$

Price adjustment costs have the same functional form as wage adjustment costs for households (15). We stick to the same specific form in order to keep price and wage adjustments comparable in the degree of convexity. The parameter γ affects the convexity of the cost function while ς affects its asymmetry. As our focus here is on the effects of asymmetric wages, we consider only symmetric price adjustments, implying $\varsigma = 0$ and $\Gamma_t \left(\frac{P_t}{P_{t-1}} \right) = \gamma \left(\frac{P_t}{P_{t-1}} - 1 \right)^2$.

2.3.1 Firm optimization

The firm's first order conditions are composed of demand for labour and the price setting decision for final goods. The labour demand equates marginal product of labour to the real wage

$$MPL_t = (1 - \alpha) x_t h_t^{-\alpha} = w_t = \frac{W_t}{P_t},$$

and the optimal pricing equation, the Philips curve, of the firm takes the form

$$\Pi_t \Gamma'_t = \left[\frac{\sigma}{\sigma - 1} \frac{h_t W_t}{(1 - \alpha) P_t C_t} - (1 - \Gamma_t) \right] (\sigma - 1) + E_t \left[\Lambda_{t,t+1} \frac{C_{t+1}}{C_t} \frac{P_{i,t}}{P_{t+1}} \Pi_{t+1}^2 \Gamma'_{t+1} \right]. \quad (22)$$

The price equation (22) includes the case of flexible prices if $\Gamma'_t = \Gamma'_{t+1} = 0$:

$$P_t = \frac{\sigma}{\sigma - 1} \frac{W_t}{(1 - \alpha) Y_t / h_t} = \frac{\sigma}{\sigma - 1} MC_t.$$

Prices are set as a mark-up over nominal marginal costs³. The price setting equation (22) is a forward-looking Phillips-curve.

2.3.2 Resource constraint

Production in the economy is used for two different purposes, either it is demanded by households in the two regions for consumption or it is used for wage and price adjustments by the continuum of firms and households. These adjustment costs lead to a loss in aggregate value added. Consumption within the union is therefore production by all firms for each country net of price and wage adjustment costs, Γ, Φ respectively. The net supply of goods is

$$\begin{aligned} c_t(h) &= x_{Ht} A_t h_{Ht}^{1-\alpha} (1 - \Gamma_t^H) - \frac{W_{Ht}}{P_{Ht}} h_{Ht} \Phi_{Ht} \\ c_t(f) &= x_{Ft} A_t h_{Ft}^{1-\alpha} (1 - \Gamma_t^F) - \frac{W_{Ft}}{P_{Ft}} h_{Ft} \Phi_{Ft}. \end{aligned} \quad (23)$$

whereby the price adjustment costs are born by the firm while the wage adjustment costs are born by the household.

2.3.3 Shocks

The model exhibits two types of technology shocks. The aggregate technology shock affects both regions alike and follows an AR(1) process. The asymmetric shock is a perfectly negatively correlated shock between the regions. This specification allows us to analyse symmetric and asymmetric shocks entirely separately. The autoregressive structure is identical to the aggregate shock

$$\begin{aligned} A_t &= \exp(\epsilon_{at}) A_{t-1}^{\chi_a}, \\ x_{Ht} &= \exp(\epsilon_{ht}) x_{Ht-1}^{\chi_h}, \\ x_{Ft} &= x_{Ht}^{-1}, \end{aligned}$$

where $\chi_a = \chi_h$. The innovations ϵ_{at} and ϵ_{ht} are iid and drawn from a normal distribution, $\mathcal{N}(0, \sigma_a)$, $\mathcal{N}(0, \sigma_h)$ respectively.

³With flexible prices no adjustment costs Γ_t are incurred by the firm and hence we use the equality $C_t = Y_t$ in this case.

2.4 Monetary policy

The analysis takes into account three different types of monetary policy which sets interest rates for the monetary union as a whole. The benchmark specification assumes an optimal monetary policy which takes the first order conditions of households and firms into consideration when maximising its objective function

$$\max E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[n \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{h_t^{1+\chi}}{1+\chi} \right) - (1-n) \left(\frac{(C_t^*)^{1-\rho}}{1-\rho} - \frac{(h_t^*)^{1+\chi}}{1+\chi} \right) \right]. \quad (24)$$

The optimization attributes the relative size of the regions as weights in the aggregation of home and foreign utility. The constraints taken into consideration are the first order conditions of the household for wage setting (19) and the firm's price setting equation (22) as well as the resource constraints (23) and (??). The monetary policy optimises with respect to a sequence of the variables $\{I_t, \Pi_t^H, \Pi_t^F, \Omega_t^H, \Omega_t^F, C_t, C_t^*, h_t, h_t^*, w_{Ht}, w_{Ft}\}$.

As an alternative to the utility-based monetary policy, monetary authorities have as objective to minimize a weighted average of price and wage inflation volatility in the following manner:

$$\max E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[\left(\frac{\Pi_t}{\Pi^{obj}} - 1 \right)^2 + \omega \left(\frac{\Omega_{ht}^n \Omega_{ft}^{1-n}}{\Pi^{obj}} - 1 \right)^2 \right], \quad (25)$$

where Π_t is the union-wide inflation rate, Π^{obj} is the objective interest rate of the monetary authority, Ω_{ht} and Ω_{ft} are Home and Foreign nominal wage inflations respectively and ω is the weight given to the stabilisation of nominal wage growth.

The third approach consists of Taylor rules in the form:

$$I_t = I_{t-1}^{\lambda_I} \left[\beta^{-1} (\Pi^{obj})^{1-\lambda_\pi} \Pi_t^{\lambda_\pi} \right]^{(1-\lambda_I)} \quad (26)$$

where λ_I is the weight on past interest rates, reflecting interest rate smoothing

3 Calibration

The model employs different data sources for the calibration to the euro area. We distinguish three types of parameters, those that we take as standard in the literature, others that match euro area volatility and a third set dealing with the asymmetry and persistence in wage and price adjustments. The model is calibrated to quarterly data. The two regions have identical size $n = 0.5$ in order to concentrate on the wage adjustment mechanism. All parameters are resumed in table 1.

Preferences. The household's discount factor is set to $\beta = 0.992$, reflecting a real interest rate of 3.2%. The elasticity of inter-temporal substitution is $\rho = 1.1$ and the elasticity of labour disutility takes the value $\chi = 1.5$ both from estimations by Smets and Wouters (2003) and in line with common other calibrations. The mark-up for goods is $\mu = 1.2$, or alternatively the elasticity of substitution between goods $\sigma = 6$. For labour services, instead, we apply an elasticity of substitution of $\theta = 3.5$ also used in the estimation by Rabanál and Rubio-Ramirez (2008). Both regions are characterised by the same degree of home bias in consumption preferences, namely $\eta = 0.8$ ($\eta^* = 0.2$) following Poilly and Sahuc (2008) and are characterised by an elasticity of substitution between home and foreign goods of 1.5.

Production. Production consists exclusively of labour input characterised by decreasing returns to scale with an elasticity for labour of 0.64, i.e. $\alpha = 0.36$. The aim is to reproduce increasing marginal costs when expanding production through more hours worked.

Param.	Value	Description
α	0.36	Capital income share
β	0.992	Discount factor, annual rate of 3.3%
ρ	1.1	Inter-temporal elasticity of substitution, reflecting near log-utility
χ	1.5	Labour supply elasticity, mode in Smets and Wouters (2003)
σ	6.0	Elasticity of product market goods, implies a mark-up of 1.2, RRR2008.
θ	3.5	Labour market mark-up, implies a mark-up of 1.4, RRR2008.
ω	1.5	Elasticity of substitution between foreign and home goods
n	0.5	Size of home country
$\eta = \eta^*$	0.8	Home bias of the home country reflecting openness, Poilly and Sahuc (2008)
$\gamma^H = \gamma^F$	45	Convexity in price adj.cost function, implies price autocorr. of 0.48
$\phi^H = \phi^F$	45	Convexity in wage adj.cost function, implies wage autocorr. of 0.71
$\varsigma^H = \varsigma^F$	0	Asymmetry in the price adjustment cost function: no asymmetry
$\psi^H = \psi^F$	0, 800	Asymmetry parameter in the wage adjustment cost function
ν_H, ν_F	0, 1	0 no wage index. (foreign), 1 full indexation area-wide inflation (home)
$\rho_a = \rho_h$	0.92	Autocorr. of both technology shocks, estimation by RR-R2008
σ_a	0.01	Std. deviation of aggregate technology
σ_h	$\frac{1}{2}\sigma_a$	Std.deviation of asymm. technol., cross-country correlation is 0.66

Table 1: The table reports calibrated parameter values for identical countries and symmetric adjustment costs.

Price and wage rigidity. Regarding price and wage rigidities in the two regions we first calibrate these by using symmetric adjustment costs for prices and wages and address the asymmetry in a second step. The cost functions for wage and price adjustment (21), (15) consist of one parameter describing the convexity of the cost function (γ for prices and ϕ for wages) and a second parameter specifying the asymmetry between upward and downward adjustments (ς for prices and ψ for wages). The latter set of parameters is fixed to 0 in the first step. We calibrate the convexity identically between countries ($\gamma^H = \gamma^F$ for prices and $\phi^H = \phi^F$ for wages), in this way we can focus on wage asymmetry as the sole difference between regions. Using the estimations by Rabanál and Rubio-Ramirez (2008) for the euro area, we replicate their findings on price and wage persistence by setting γ^H and γ^F . Obviously there is a strong interplay in persistence of price rigidities and nominal wages to be taken into account and the presence of aggregate and asymmetric shocks reduces inflation persistence in our model compared to Erceg et al. (2000) which already has difficulties matching empirical inflation persistence. Tables 2 and 3 in the appendix summarize different moments of the baseline symmetric and asymmetric calibration.

The parameter ruling asymmetric adjustments to wages is ψ . Symmetric wage setting is characterised by $\psi = 0$ while we set $\psi = 800$ for the asymmetric case, generating in this way the skewness of wage-change distributions for nominally rigid and real rigid countries. Instead of fitting the model to a specific value of skewness for some country, we want to highlight the implications of asymmetric wage adjustments on other variables and on the transmission of productivity shocks. The International Wage Flexibility Project summarized by Dickens et al. (2007) has demonstrated that nominal and real wage rigidities wide spread in the euro area. The studies on downward nominal and real wage rigidities by Holden and Wulfsberg (2007a) and Holden and Wulfsberg (2007b) using industry data for 19 OECD countries between 1973 and 1999 gives us a reference point on the extent of nominal and real wage rigidities. The euro area may be characterised consisting of a large number of countries with downward real rigidity (Austria, Belgium, Germany, the Netherlands as well as Finland), some with downward nominal rigidity

(Denmark, Sweden, Portugal and Italy) and few countries with no apparent downward rigidity (Spain, France and Greece, UK and Ireland). The effects downward rigidities have on other variables within the model become visible mainly in the mean and skewness

The parameter responsible for generating real wage rigidities is ν that appears in the cost function for wage changes (15). Nominal wages are rigid with $\nu = 0$, while $\nu = 1$ reflects a situation of full contemporaneous wage indexation with area-wide inflation.

Shock size. In order to match volatility and persistence of output we can vary volatility and persistence of symmetric and asymmetric shocks in the union. We set persistence of both shocks to be identical, $\rho_a = \rho_h$ and set the standard deviation of the asymmetric shocks to be half that of the aggregate shock $\sigma_h = \frac{1}{2}\sigma_a$. Due to the fact that asymmetric shocks imply a perfectly negatively correlated shock between the two countries, setting the standard deviation in this way makes the difference between the two countries of same magnitude as a symmetric shock although the transmission is very different. With our choice of volatility we obtain a cross-correlation of output between the regions of 0.66. The asymmetric shocks strongly affect price volatility as it is the main adjustment channel for asymmetric shocks, reducing thereby the inflationary persistence induced by aggregate shocks.

3.1 Computational solution

The model has been set up to analyse the asymmetric behaviour within a monetary union to symmetric and asymmetric productivity shocks. In order to adequately address this question the solution algorithm needs to account for such asymmetry. We use a second-order Perturbation implemented in Dynare to approximate the solution around the steady state following the algorithm by Schmitt-Grohe and Uribe (2004). The Ramsey optimization employs the algorithm implemented in Dynare by Lombardo and Sutherland (2007), which itself is similar to the one by Levin et al. (2005). In addition we counter-checked steady state results using the implementation by Ondra Kamenik in Dynare++ which uses alternative solution procedures.⁴

Simulations with second-order approximations exhibit the possibility of explosive paths due to the existence of an approximation-induced second steady state which is unstable. In order to make sure to remain on the stable path we implement in Dynare the pruning algorithm proposed by Kim et al. (2008). The code for the pruning algorithm is made available on the Dynare website and is readily implementable in other models that use simulations in a second-order approximation⁵.

4 Results

4.1 Steady state effects

Asymmetric wage adjustment costs reflecting downward wage rigidity may have effects on steady state values. Tobin (1972) stated the importance of a positive inflation rate to ease real wage adjustments due to the presence of nominal illusion and an aversion for nominal wage cuts. The International Wage Flexibility project has found a large extent of downward wage rigidity at the microeconomic level for job stayers, see Dickens et al. (2007). Indeed, Bewley (2007) has found substantial resistance for downward nominal wage adjustments from managers for its detrimental impact on motivation by employees. For a

⁴ All these codes can be found on the Dynare website <http://www.cepremap.cnrs.fr/dynare/>.

⁵ The pruning code can be found in the Forum section under "DYNARE contributions and examples". The exact URL is

<http://www.cepremap.cnrs.fr/juillard/mambo/index.php?Itemid=95&page=viewtopic&t=1669>

Model	y-o-y	Mean μ (%)		Std. dev. σ (%)				Skewness γ_1			
		$\Pi = \Omega$	I	Π	Ω_H	Ω_F	I	Π	Ω_H	Ω_F	I
Baseline		0.19	3.40	0.22	0.71	0.42	0.29	1.11	1.31	1.68	1.13
Symm.nom. $\psi = 0, \nu = 0$		0.00	3.20	0.21	0.22	0.22	0.24	0.04	-0.03	-0.03	0.16
Asymm.nom. $\psi = 800, \nu = 0$		0.26	3.44	0.22	0.34	0.34	0.27	0.97	1.83	1.83	0.96
Asymm.real $\psi = 800, \nu = 1$		0.00	3.16	0.29	0.81	0.81	0.26	0.81	1.10	1.10	0.79
Asymm.nom.w / flex p., $\gamma = 1$		0.44	4.48	0.22	0.26	0.26	0.06	2.11	1.97	1.97	-1.84
Asymm.nom.w / rigid p., $\gamma = 100$		0.13	3.32	0.16	0.19	0.19	0.25	0.57	1.64	1.64	0.62

Table 2: Steady state effects on mean, standard deviation and skewness of quarter-to-quarter price and wage inflation and optimal interest rates for different model specifications. The baseline model is characterised by real asymmetric wage adjustments in the home country, and nominal adjustments in the foreign country. The other models consider identical regions with symmetric, and asymmetric adjustment costs.

more detailed discussion on possible adjustment margins at the sectoral level beyond merely wages for job-stayers see Holden and Wulfsberg (2007a).

The results are presented for our reference model in which the monetary union consists of a home region with downward real rigid wages and a foreign region with downward nominal rigid wages. In addition, to better understand the underlying mechanism of the indexation and the asymmetry we then also present special cases changing the degrees and types of rigidities in order to isolate specific mechanisms. In our model the positive greasing effects of inflation are generated through the asymmetric adjustment cost structure for nominal wages as already pointed out in Kim and Ruge-Murcia (2007). A positive inflation rate, although permanently causing wage and price adjustment costs, bears the advantage that negative shocks do not generate as large costs for downward nominal wage adjustments as they would at lower steady state inflation rates. The greasing effect is therefore of precautionary nature to reduce adjustment costs after negative shocks compared to a zero inflation steady state.

Table 2 presents steady state wage and price inflation for our reference calibration and other specifications. The reference setup for the euro area includes one region with asymmetric real wage adjustment costs and a second region with nominal asymmetries, while prices are characterized by symmetric adjustment costs for both countries. The choice for this specific combination is driven by the findings of Holden and Wulfsberg (2007b) of a large extent of nominal and real downward rigidity within OECD countries including many euro area countries. At this stage, and especially due to the fact that we consider only productivity shocks as drivers of fluctuations, the calibration is indicative of the effects, but nevertheless it is helpful to understand how rigidities in one region affect the other region and the monetary union as a whole.

The baseline model with nominal and real downward rigidities exhibits positive inflation rates of 19 basis points per year. Inflation volatility remains relatively modest compared to wage inflation. The aggregate technology shocks have a 1.1% standard deviation and asymmetric shocks have half that size. The region with real asymmetric wage rigidity is characterised by higher volatility in nominal wage changes than the nominal rigid region. An interesting finding is that the right skewed distribution in wage adjustments also shifts the distribution of price adjustments seen in the results for the skewness.

To compare different alternative model specifications, we compute the same set of steady state values for a monetary union with identical regions with either symmetric ($\psi = 0$) or asymmetric ($\psi = 800$) adjustment costs either in nominal ($\nu = 0$) or in real ($\nu = 1$) terms. The last two rows highlight the effects of more flexible ($\gamma = 1$) or more rigid ($\gamma = 100$) prices, the reference model employed $\gamma = 45$ for both regions.

Grease inflation is most important for asymmetric nominal wage adjustment costs and relatively flexible prices. If adjustment costs are symmetric, either in real or nominal terms, positive and negative shocks generate the same degree of costs and therefore no benefits are obtained from a positive as opposed to a zero inflation rate. Likewise, if adjustment costs are asymmetric with respect to real wages, i.e. indexed wages to current inflation, higher inflation does not reduce adjustment costs for negative shocks as the indexation does not ease real wage adjustments and the inflationary bias vanishes.

In addition, the relative costs for wage and price adjustments are important (last two rows of table 2). If prices are strongly rigid due to larger convexities in adjustment costs, a positive inflation rate in steady state generates large costs which may outweigh the precautionary motive offsetting and ultimately cancelling the inflationary bias. The greasing effect is largest in the case of strongly asymmetric nominal wages combined with low rigidity in prices. With regions characterised by different types of asymmetric wage adjustments, i.e. if only one region exhibits asymmetric costs, the effects are smaller. No effect is present if adjustments for both countries are either symmetric, asymmetric in real terms or a combination of the two.

Beyond the effects on the first moment, our asymmetric wage adjustments affect second and third moments. Especially volatility in wage inflation increases with the degree of asymmetry in wage adjustments in either nominal or real terms. The reason being that asymmetry implies higher costs for downward adjustments but lower costs for wage increases compared to the symmetric adjustment cost function as depicted in figure 1. This implies that downward movements in nominal wages are strongly squeezed and do not exhibit large movements, wage increases do move by large margins due to the comparatively low costs for adjusting upwards.

As a consequence, the shape of the ergodic distribution of wage and price adjustments is also strongly affected by downward rigidity as found in the value for skewness. Especially nominal wage changes and to a lower degree price changes become skewed due to asymmetric adjustment costs. The asymmetric wage adjustments affect the shape of interest rate responses to different degrees. In the case of flexible prices, the interest rate adjustments go in the opposite direction. With real rigid wages the interest rate adjustments are less skewed due to the fact that optimal monetary policy takes the stronger detrimental effects into account when setting interest equilibrating thereby the responses to positive and negative shocks.

Results reported in the appendix reveal that wages and hours exhibit skewness in opposite directions and are smaller in absolute size. Real wages are positively skewed like nominal wage changes and inflation, whereas hours worked are negatively skewed. As will become clearer with the analysis of impulse responses, real wages increase strongly after positive shocks but with only small movements in hours worked. In the opposite case of a negative impact on real wages, these resist to fall which implies that employment drops by a large magnitude leading to the type of shape observed.

The effects of asymmetries on the persistence of variables is mixed. While on the one hand the stronger rigidity induces more persistence especially for real wage rigidity, the asymmetry shortens persistence at longer horizons due to the variability of nominal wages following positive shocks. Table 3 reports coefficients of autocorrelation at one and four lags indicating no large variations except for the situation with more flexible prices.

The steady state results depend importantly on the volatility of shock sizes. In linear models shock volatility affects exclusively the volatility of variables but has no effects on their mean or skewness, while in non-linear approximations the shock size may affect both of them. We are especially interested how the greasing effect varies with the volatility of aggregate shocks. Figure 2 depicts the relationship between volatility in technology shocks and inflation as well as nominal and real interest rates for the baseline

Model	AR 1				AR 4			
	Π	Ω_H	Ω_F	I	Π	Ω_H	Ω_F	I
Baseline	0.57	0.46	0.45	0.78	-0.14	-0.19	-0.11	0.44
Symm.nom.	0.47	0.69	0.69	0.87	-0.10	0.12	0.12	0.62
Asymm.nom.	0.58	0.45	0.45	0.79	-0.03	-0.07	-0.07	0.51
Asymm.real	0.59	0.48	0.47	0.79	-0.20	-0.20	-0.19	0.36
Asymm.nom.w/flex p	0.005	-0.02	-0.02	0.91	-0.01	-0.012	-0.011	0.71
Asymm.nom.w/rigid p	0.66	0.62	0.62	0.86	0.07	0.04	0.05	0.57

Table 3: Autocorrelations for one and four quarters

Figure 2: Effect of volatility on mean inflation and nominal interest rate. Aggregate standard deviation is varied from zero to 4% and at the same time asymmetric shocks are jointly increased in size.

model⁶. As expected, steady state inflation increases due to the aforementioned precautionary motive, but the greasing effect is not linear in volatility. The increase in interest rates, though parallel to the one by inflation is less strong, reflecting that the welfare based optimal rule takes the detrimental real effects on hours and output into consideration by reducing the real interest rate.

4.2 Dynamic effects

The introduction of asymmetric adjustment costs in nominal wage setting generates differing dynamic responses to positive and negative shocks. We compute the impulse responses to an aggregate productivity shock to the entire monetary union and a shock to the . The impulse response functions presented depict the dynamics after the economy has been hit by 2 different shock sizes to aggregate productivity, either a positive or negative shock of one and a half standard deviations. We compute both shocks to illustrate the asymmetric behaviour for positive and negative impulses especially if these are larger than average shock sizes. If a large technology impulse shocks the economy, the asymmetries are more apparent and the non-symmetric dynamics come into play. In order to force the model to explore that region the impulse size needs to be larger than the general level of fluctuations, this is achieved by using 1.5σ shocks.

⁶In this exercise we vary both types of volatility: aggregate productivity shocks become larger and at the same time asymmetric shocks are increased proportionally in order not to distort the shock composition.

Figure 3: Response in percentage deviations (for nominal variables in percentage points) to positive and negative shocks of one and two standard deviations to an aggregate technology in a model with identical countries and symmetric nominal wage rigidity. Parameter values are: $\gamma = \phi = 45$, $\psi = 0$, $\nu = 0$, $\chi = 1.5$, $\rho = 1.1$.

To analyse the response to an aggregate productivity shock we use a model specification with symmetric regions and symmetric nominal wage adjustment costs as reference to understand the mechanisms. The impulse responses are stated for a quarterly time frame in percentage deviations to a shock of one and half positive and negative standard deviations. We first discuss aggregate productivity shocks and thereafter asymmetric shocks.

4.2.1 Aggregate shocks

Following an aggregate shock the two regions have identical adjustments and do not exhibit any relative wage or price differentials and therefore no terms of trade effects. For simplicity we report only the responses in one region, the other region's response being identical. In the case of symmetric adjustment costs nominal wages and prices share the burden to adjust real wages to the changed productivity. After a positive shock nominal wage inflation increases in both regions and price inflation falls, in this way real wages increase. The observed increasing nominal wages reflect higher productivity, and decreasing prices relate to lower marginal costs incurred by firms. The presence of rigid prices and nominal wages leads to rigid real wages which exhibit a strong and persistent hump-shaped adjustment pattern. Optimal nominal interest rates are reduced after a positive shock to counter deflationary pressures and are persistent although the response of price inflation is short-lived. Note that interest rates respond strongly after a negative productivity shock to counter inflationary pressures, while response to a positive shock is slightly more muted. Hours worked combines substitution and wealth effects leading initially to an increase in hours worked.

Adding downward nominal wage rigidity implies higher adjustment costs for nominal wage cuts and lower costs for wage increases. This hinders nominal wage deflation and shifts the adjustment margin for real wage adjustments towards price inflation. Figure 4 illustrates the response to different shocks in a monetary union characterized by identical regions with downward nominal wage rigidities and restates the results of Kim and Ruge-Murcia (2007). As expected, the response of wage inflation is muted for

Figure 4: Response in percentage deviations (for nominal variables in percentage points) to positive and negative shocks of one and two standard deviations to an aggregate technology shock in an economy with identical regions and asymmetric nominal wage rigidity. Parameter values are: $\gamma = \phi = 45, \psi = 800, \nu = 0, \chi = 1.5, \rho = 1.1$.

negative shocks compared to positive shocks and instead price inflation helps to make the real wage adjustment. With a positive shock, price deflation is muted because nominal wages adjust easily, instead after a negative shock inflation is strong to compensate for the downward nominal wage rigidity. Price inflation after a negative shock induces monetary policy to firmly react by increasing the interest rate more strongly than in the symmetric case of figure 3 while it is muted for negative shocks. The response of consumption is altered only marginally, while hours worked decline strongly in the case of real wage rigidity reflecting the highly detrimental effects of high wages on employment.

To further understand the quantitative effects of different types of rigidities figure 5 reports the response of different variables to a negative shock of two standard deviations to aggregate technology. All situations combine identical countries with either flexible or rigid wages with symmetric adjustment costs, downward rigid wages with nominal and real rigid wages.

When moving from flexible to nominal rigid models and to the model with downward real wage rigidity, the response of wage inflation becomes more and more positive while at the same time the response of price inflation magnifies to compensate for the inability of wage adjustments. The downward adjustment of real wages becomes more and more difficult which leads to highly detrimental effects on hours worked and creates persistence in the response of consumption. Although the overall responses change strongly in nominal and real variables, the optimal monetary policy exhibits similar responses in all four models.

At this stage we are ready to analyse our baseline scenario of a monetary union consisting of the home region with downward real rigidities and the foreign region with downward nominal rigidities. Figure 6 illustrates the response to an aggregate shock in such a monetary union. The two regions characterised by different types of asymmetries lead to terms of trade effects even for symmetric shocks. The home region with real rigid wages is characterised by a more positive response of both wage and price inflation: after a negative shock, wage inflation remains positive due to indexation and price inflation is strong to reduce real wages, while these effects are smaller in the downward nominal rigid (foreign) region. For a

Figure 5: Response in percentage deviations (for nominal variables in percentage points) to a negative aggregate technology of two standard deviations. The bold line characterizes a response in a model with wage flexibility ($\phi = 0, \psi = 0, \nu = 0, \cdot$), the dotted line with symmetric adjustment costs ($\phi = 45, \psi = 0, \nu = 0, \cdot$), the dashed line a response with downward nominal rigidity ($\phi = 45, \psi = 800, \nu = 0, \cdot$) and the starred line with real wage rigidity (wage indexation, $\phi = 45, \psi = 800, \nu = 1$).

positive shock wage inflation is larger for the real rigid region and inflation deceleration is more muted. In both cases prices in the real rigid country are larger than in the nominal rigid region which leads to persistent terms-of-trade effects, compensated only in the long-run.

For negative shocks the response of hours worked is highly negative for the real rigid region, but is not strongly positive after a positive shock highlighting again a strong asymmetry in response due to the concavity in the labour part of the utility function determined by the parameter χ . This illustrates that productivity increases do not lead to an increase in hours worked, while negative shocks combined with real wage rigidity are highly detrimental for employment. Finally, the response of monetary policy remains similar to situations with identical regions: a strong movement after negative shocks to counter inflationary pressures and a muted response for positive technology shocks.

4.2.2 Asymmetric shocks

The asymmetric shock is modelled as a perfectly negatively correlated shock to the home and foreign region, i.e. a zero-sum productivity shock. In this way we assess the transmission of this shock within the union without being affected by productivity gains or losses at the union level. In addition, in a linear world such an asymmetric shock leads to no aggregate effects, but through asymmetries in the transmission stemming from the second order approximation aggregate variables may nevertheless be affected.

Comparing the different impulse responses, the most striking feature is the fact that relative prices carry the biggest burden for the adjustment across regions, while wage adjustments play a minor role. Hereby the international risk sharing equation (20) and the elasticity of substitution between home and foreign goods for the aggregation creates a strong tie between terms of trade, i.e. the relative price of the two regions, and net production of the two regions as seen in (10). The risk sharing and the consumption

Figure 6: Response to different sized aggregate technology shocks in a monetary union with downward nominal wage rigidities in the home region and downward real rigidities in the foreign region.

link are important in shaping the relative relative adjustment of the two regions working mainly through the relative price levels and less through the relative wage setting mechanisms. Figure 7 illustrates the strong response of terms of trade and the smaller importance of wage inflation.

Differently to a linear approximation, the second order approximation leads after a purely asymmetric shock between the regions to a positive response of interest rates by optimal monetary policy. The curvatures in the preference and the production function are sufficient to generate this positive response. These elements lead to different sized effects between the regions triggering an increase in nominal and real interest rates. Indeed, due to the fact that households set wages as mark-up over the marginal rate of substitution, wages increase by less in one region than they decrease in the other region leading to area-wide disinflationary tendencies and making an increase in the interest rate optimal in order to postpone and smooth consumption of households. With perfectly functioning capital markets, asymmetric shocks have beneficial effects for the union as a whole.

By comparing the response in four different models (flexible wages, symmetrically nominal rigid, nominal and real downward rigid) for an asymmetric shock affecting the home country negatively, the importance of the risk sharing mechanism is further underlined. A negative shock at home and a positive shock in the foreign region lead to an initial increase in wage inflation at home and negative wage inflation in the foreign country. Contrasting this, prices react nearly identically to all specifications, indicating the strength of the international transmission through relative prices, which are tightly linked to production in the two regions. Terms of trade are beneficial to the home country and especially so, if wages are indexed to inflation. With higher wage rigidity the deflationary pressures are more persistent, also monetary policy reacts more firmly. With this understanding for asymmetric shocks we now turn to our reference model consisting of a real downward rigid home region and a nominally downward rigid foreign region. The effects are again dominated by the terms of trade effects, while the asymmetric wage adjustments play a minor role. For the international channel to be of higher asymmetric impact it is necessary to alter

Figure 7: Response in percentage deviations (for nominal variables in percentage points) to positive and negative shocks of one and two standard deviations to an asymmetric technology shock in a model with identical countries and symmetric nominal wage rigidity. Parameter values are: $\gamma = \phi = 45$, $\psi = 0$, $\nu = 0$, $\chi = 1.5$, $\rho = 1.1$.

either the type of risk sharing, the aggregation of home and foreign consumption bundles as is done in the sensitivity analysis, or even introduce asymmetric adjustments of prices, but the empirical evidence for the last possibility is scarce.

4.3 Sensitivity analysis

A core parameter for our results is the degree of asymmetry in wage adjustments, specified by ψ . We therefore undertake a sensitivity analysis for this represented in figure ?? in the appendix. Nominal wage inflation changes in a highly non-linear way, the skewness increases strongly from zero to above two for values between 0 and 400 and is flat thereafter. While the skewness of prices increases linearly, the skewness in interest rates become steeper once the wage inflation skewness has reached its upper level. This indicates that from that point on, real effects are taken into consideration more strongly. This is also highlighted by the fact that skewness in real wages picks up at roughly the same level. Overall, the sensitivity analysis indicates that asymmetric wage adjustments first influence the nominal side alone, and at higher asymmetries it also affects the real side through real wages.

A similar picture to the changes induced by asymmetric wage adjustments stems from volatility in technology shocks as seen in figure ?. The larger these become, the larger the skewness of the variables but the relationship is not linear for nominal wages and prices. Again, nominal wages face an early satiation level, but now price adjustments also tend to reach a maximum level of skewness if volatilities are raised further. Instead, interest rates, real wages and hours worked exhibit linear relationships for the ranges of volatility considered.

We have chosen a value for the elasticity of substitution $\omega = 1.5$, in line with the literature. The substitution between home and foreign goods also affects the importance of wage rigidities for the international transmission of shocks. If the goods are highly complementary, price changes in one country have

Effect of a negative 1σ technology shock under different wage setting specifications: asymmetric shock

Figure 8: Response in percentage deviations (for nominal variables in percentage points) to a negative asymmetric technology shock of two standard deviations in the home region. The bold line characterizes a response in a model with wage flexibility ($\phi = 0, \psi = 0, \nu = 0$), the dotted line with symmetric adjustment costs ($\phi = 45, \psi = 0, \nu = 0$), the dashed line a response with downward nominal rigidity ($\phi = 45, \psi = 500, \nu = 0$) and the starred line with real wage rigidity (wage indexation, $\phi = 45, \psi = 800, \nu = 1$).

large effects on the other region's expenditure, and the price adjustment dominates the overall adjustment process compared to wage adjustments. In such a case the specific wage rigidity is of little importance for the overall adjustment following asymmetric shocks. Instead, with high substitutability, already small price changes lead to large swings in consumption and the price rigidity is overall of lower importance. In such a setup the asymmetry in wage setting dominates and affects the transmission of shocks between the regions. We highlight this finding for a monetary union with indexed wages in the home country and nominal wage rigidity in the foreign country in figure 10. With higher substitutability terms of trade are reduced. The adjustment of home and foreign nominal wages is very different due to the indexation. While wages in the home country follow the inflation rate, foreign wages reflect the higher productivity of the region the more substitutable the home and foreign goods are. Differences following a union wide shock are minimal.

Regarding the role of risk-sharing in the Euro area, it is arguably far from being perfect. We therefore also analyse the situation with autarkic countries, in which the value of home produced goods equates the value of total consumption in every period in order to exhibit a closed trade account, i.e. $P_t C_t = P_{Ht} C_{Ht}$. The risk-sharing setup will be especially important following asymmetric productivity shocks, we therefore focus only on those in a setup where the home country exhibits indexation and the foreign country is characterized by nominal wage rigidity and analyse a version with complete markets and a second oen with autarky. The biggest change between the two setups lies in the response of foreign wage inflation and nominal interest rates. Following a negative shock in the home country, and due to the fact that the trade balance is closed in the autarky case, the foreign country is not expanding its production to compensate for the reduced productivity in the home country. This leads to reduced nominal wage growth. In the home country, the general pattern of adjustment remains intact, dominated by the adjustment in

Figure 9: Response to asymmetric technology shocks of different sizes in a monetary union with real (home) and nominal (foreign) downward rigid wages. The shocks are of one and two standard deviations, both positive and negative.

prices and through indexation also of prices. In the autarky case, nominal interest rates have only a small role, while optimal policy with complete markets allows for expansion in the higher productive country. Any intermediate setup with uncovered interest rate parity would thereby also have little effects on that margin.

4.3.1 Alternative Monetary Policy

The analysis up to now has been made using Ramsey optimal monetary policy. In this section we analyse other objectives and ask the question how a pure inflation targetter performs in terms of welfare over a large number of simulations. In a second step we then compare the dynamic response of the inflation targetter with the optimal policy considering again symmetric and asymmetric shocks, whereby our interest focuses on how symmetric shocks are taken into consideration.

The transmission of an asymmetric technology shock in the presence of real wage rigidity in the home country and nominal rigidity in the foreign country leads to strong differences especially for the nominal rigid country under different monetary policies. It seems that it is the nominal rigid country that is required to adjust more if monetary policy is focussed exclusively at targetting inflation due to the fact that the real wage rigidity is dominating the adjustment process in the home country. Real wages strongly decline as well as hours and consumption due to a strong hike in interest rates. In this way, terms of trade and inflation differentials are increased, although it is an symmetric shock hitting the monetary union. A Taylor rule with persistence in interest rate setting of $\lambda_r = 0.85$ and a sensitivity to inflation of $\lambda_\pi = 1.5$ is too weak initially, letting inflation slip for too long. Real wages and hours worked for the foreign country strongly differ across the three monetary policy setups.

Effect of a negative 1.5σ asymmetric technology shock under different wage setting specifications

Figure 10: Effect of a negative technology shock in the home country in the baseline setup for different degrees of substitutability between home and foreign goods. The home country exhibits wage indexation while the foreign country is characterised by nominal rigidity.

Effect of a negative 1.5σ asymmetric technology shock: autarky vs. perfect risk sharing

Alternative monetary policies under perfect risk sharing: negative union-wide shock

5 Conclusion

Downward nominal and real wage rigidities introduce distortions in the adjustment of nominal and real wages to technology shocks. In this paper we highlighted to what degree these rigidities become important in a monetary union and have identified three distinct areas, the effects on steady state, the response to an aggregate shock with potential effects on terms of trade and finally the distortions following an asymmetric technology shock in which terms of trade adjustments are a central adjustment mechanism. In addition we further highlighted the mechanism by sensitivity analyses.

Downward nominal wage rigidities lead to a positively skewed response in nominal wage changes and a positive optimal inflation rate. This effect is stronger the larger the asymmetry in the wage adjustment costs and the lower price rigidity, but for reasonable calibrations it is small and indicating that Tobin's claim of the positive greasing effects of inflation are negligible. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if adjustment costs are symmetric. It is important to notice that our setup does not consider cross sectoral adjustments, but instead uses representative firms in each country. The greasing effects may therefore be larger once idiosyncratic firm-level shocks are taken into account.

Regarding the dynamic responses in the presence of downward wage rigidities we find that beyond the direct effects on wage changes also price changes are altered and become asymmetric, highlighting the strong link between prices and wages. Wage indexation (downward real wage rigidity) has also strong dynamic effects on hours worked and generate highly asymmetric responses. In the case of two regions with different types of asymmetries (nominal and real), terms of trade effects are sizeable. The region characterized by stronger asymmetries adjusting to positive and negative technology shocks always with higher inflation. Terms of trade are larger, the more complementary home and foreign goods are, while wage asymmetries are more important the more substitutable the goods are. Considering other setups for international risk sharing does not alter the main findings for aggregate shocks, but strongly alters adjustments for the nominal rigid country. Differing Monetary policies. Instead, higher substitutability between home and foreign goods does increase the role of the regional wage setting asymmetries while reducing the effects on terms of trade.

The effects of asymmetric shocks between regions of a monetary union have been at the heart of debates regarding the optimality of currency areas. The simple version presented here uses as main adjustment mechanism between the regions the relative price adjustments of the concerned regions. Asymmetries in the way nominal wages are set in the two regions affect the transmission to a much smaller degree than the terms of trade channel governed by relative prices.

From here, different strands of research may be further fruitful to investigate. A project already started addresses the question how asymmetric wage changes affect wages and employment within a matching model. Due to the opposite effects on skewness on wages and hours worked a matching framework may transmit and magnify this effect through the delay in re-matching workers that have become unemployed.

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6 Appendix

6.1 Derivation of the equations

The composite consumption bundles C_t , C_t^* in the two regions consist of home and foreign produced goods with an elasticity of substitution of $\omega = (1 - \rho)^{-1}$:

$$C_t = \left[a_H^{1-\rho} C_{Ht}^\rho + a_F^{1-\rho} C_{Ft}^\rho \right]^{1/\rho} \quad \text{and} \quad C_t = \left[\eta^{1/\omega} C_{Ht}^{\frac{\omega-1}{\omega}} + (1-\eta)^{1/\omega} C_{Ft}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (27)$$

$$C_t^* = \left[a_H^{*1-\rho} C_{Ht}^{*\rho} + a_F^{*1-\rho} C_{Ft}^{*\rho} \right]^{1/\rho} \quad \text{and} \quad C_t^* = \left[(1-\eta^*)^{1/\omega} C_{Ht}^{*\frac{\omega-1}{\omega}} + \eta^{*1/\omega} C_{Ft}^{*\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (28)$$

$$a_H, \eta > 1/2 : \text{home bias in the home country} \quad (29)$$

$$a_F^*, \eta^* > 1/2 : \text{home bias in the foreign country} \quad (30)$$

$$C_t = \frac{(C_{Ht})^\eta (C_{Ft})^{1-\eta}}{\eta^\eta (1-\eta)^{1-\eta}} \quad \text{and} \quad (31)$$

$$C_t^* = \frac{(C_{Ht}^*)^{1-\eta^*} (C_{Ft}^*)^{\eta^*}}{(\eta^*)^{\eta^*} (1-\eta^*)^{1-\eta^*}} \quad (32)$$

$$\eta > 1/2 : \text{home bias in the home country} \quad (33)$$

$$\eta^* > 1/2 : \text{home bias in the foreign country} \quad (34)$$

Cost minimisation leads to the following demand functions:

$$C_{Ht} = \eta \left(\frac{P_{Ht}}{P_t} \right)^{-\omega} C_t \quad \text{and} \quad C_{Ft} = (1-\eta) \left(\frac{P_{Ft}}{P_t} \right)^{-\omega} C_t, \quad (35)$$

$$C_{Ht}^* = (1-\eta^*) \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-\omega} C_t^* \quad \text{and} \quad C_{Ft}^* = \eta^* \left(\frac{P_{Ft}^*}{P_t^*} \right)^{-\omega} C_t^*$$

$$C_{Ht} = \eta \left(\frac{P_{Ht}}{P_t} \right)^{-1} C_t \quad \text{and} \quad C_{Ft} = (1-\eta) \left(\frac{P_{Ft}}{P_t} \right)^{-1} C_t, \quad (36)$$

$$C_{Ht}^* = (1-\eta^*) \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-1} C_t^* \quad \text{and} \quad C_{Ft}^* = \eta^* \left(\frac{P_{Ft}^*}{P_t^*} \right)^{-1} C_t^*$$

where P_{Ht} and P_{Ft} are respectively the prices for home and foreign produced bundles and the law of one price holds between regions.

Each of the consumption goods C_{Ht} and C_{Ft} is itself a composite of goods produced monopolistically by the firms in the two regions:

$$C_{Ht} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n (c_t(h))^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad C_{Ft} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 (c_t(f))^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}, \quad (37)$$

where μ is the price mark-up, identical across regions, implying an elasticity of substitution of $\sigma = \mu/(\mu - 1)$ between different goods, and n characterizes the size of the region. For a given level of consumption we can define through cost minimisation the demand towards single home and foreign firms:

$$c_t(h) = \frac{1}{n} \left(\frac{p_t(h)}{P_{Ht}} \right)^{-\sigma} C_{Ht} \quad \text{and} \quad c_t(f) = \frac{1}{1-n} \left(\frac{p_t(f)}{P_{Ft}} \right)^{-\sigma} C_{Ft},$$

where the corresponding price levels P_{Ht} and P_{Ft} are defined as the unit price for a home and foreign consumption bundle

$$P_{Ht} = \left[\frac{1}{n} \int_0^n p_t(h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_{Ft} = \left[\frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The consumption price index is determined by the cost-minimisation problem of the household when splitting expenditure between home and foreign produced goods. We assume that the law of one price holds, $p_t^F(h) = p_t^H(h) = p_t(h)$ and $p_t^F(f) = p_t^H(f) = p_t(f)$, implying that the price for the bundles C_{Ht} and C_{Ft} is identical across regions. While producer prices are equated between regions, the utility-based consumer price index (CPI) is not necessarily identical across regions due to the home bias of consumers. The CPIs in the home and foreign country, P_t and P_t^* respectively, are

$$P_t = [\eta P_{Ht}^{1-\omega} + (1-\eta) P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}} \quad (38)$$

$$P_t^* = [(1-\eta^*) P_{Ht}^{1-\omega} + \eta^* P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}} \quad (39)$$

$$P_t = P_{Ht}^\eta P_{Ft}^{1-\eta} \quad (40)$$

$$P_t^* = P_{Ht}^{1-\eta^*} P_{Ft}^{\eta^*} \quad (41)$$

In addition to the preference parameters η, η^* the size of the two countries is important for determining demand for home and foreign produced goods. With equally sized regions the case for perfect risk sharing with consumption equation across regions is $\eta = \eta^* = 1/2$. In the case of different sized countries this condition generalizes to $\eta = \eta^* = n$, where the economic size is equated to the preference structure and perfectly identical consumption patterns prevail as found by Benigno (2004). The demand for home and foreign produced goods as a function of preference and the region's size is

$$c_t(h) = \eta \left(\frac{P_{Ht}}{P_t} \right)^{-\omega} \frac{1}{n} \left(\frac{p_t(h)}{P_{Ht}} \right)^{-\sigma} C_t \quad (42)$$

$$= \frac{\eta}{n} \left(\frac{P_{Ht}}{P_t} \right)^{-\omega} C_t \quad (43)$$

$$= \frac{\eta}{n} [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t \quad (44)$$

$$c_t(f) = \frac{1}{1-n} \left(\frac{p_t(f)}{P_{Ft}} \right)^{-\sigma} (1-\eta) \left(\frac{P_{Ft}}{P_t} \right)^{-\omega} C_t, \quad (45)$$

$$= \frac{1-\eta}{1-n} \left(\frac{P_{Ft}}{P_t} \right)^{-\omega} C_t \quad (46)$$

$$= \frac{1-\eta}{1-n} [\eta T_t^{-(1-\omega)} + 1 - \eta]^{\frac{\omega}{1-\omega}} C_t \quad (47)$$

$$c_t^*(h) = \frac{1}{n} \left(\frac{p_t(h)}{P_{Ht}} \right)^{-\sigma} (1-\eta^*) \left(\frac{P_{Ht}}{P_t^*} \right)^{-\omega} C_t^* \quad (48)$$

$$= \frac{1-\eta^*}{n} [1 - \eta^* + \eta^* T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (49)$$

$$c_t^*(f) = \frac{1}{1-n} \left(\frac{p_t(f)}{P_{Ft}} \right)^{-\sigma} \eta^* \left(\frac{P_{Ft}}{P_t^*} \right)^{-\omega} C_t^* \quad (50)$$

$$= \frac{\eta^*}{1-n} \left(\frac{P_{Ft}^*}{P_{Ft}} \right)^{\omega} C_t^* \quad (51)$$

$$= \frac{\eta^*}{1-n} [(1-\eta^*) T_t^{-(1-\omega)} + \eta^*]^{\frac{\omega}{1-\omega}} C_t^* \quad (52)$$

Total demand for the producer of a good is:

$$c_t(h) = n \frac{\eta}{n} [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t + (1-n) \frac{1-\eta^*}{n} [1 - \eta^* + \eta^* T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (53)$$

$$= \eta [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t + \frac{1-n}{n} (1-\eta^*) [1 - \eta^* + \eta^* T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (54)$$

$$c_t(f) = \frac{n}{1-n} (1-\eta) T_t^{-\omega} [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t + \eta^* T_t^{-\omega} [1 - \eta^* + \eta^* T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (55)$$

Total demand for monopolist's products:

$$c_t(h) = n \frac{\eta}{n} T_t^{1-\eta} C_t + (1-n) \frac{1-\eta^*}{n} T_t^{\eta^*} C_t^* \quad (56)$$

$$= \eta T_t^{1-\eta} C_t + \frac{1-n}{n} (1-\eta^*) T_t^{\eta^*} C_t^* \quad (57)$$

$$c_t(f) = n \frac{1-\eta}{1-n} T_t^{-\eta} C_t + (1-n) \frac{\eta^*}{1-n} T_t^{-(1-\eta^*)} C_t^* \quad (58)$$

$$= \frac{n}{1-n} (1-\eta) T_t^{-\eta} C_t + \eta^* T_t^{-(1-\eta^*)} C_t^* \quad (59)$$

$$c_t(h) = \eta [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t + \frac{1-n}{n} (1-\eta^*) [1-\eta^* + \eta^* T_t^{1-\omega}]^{\frac{\omega}{1-\omega}} C_t^* \quad (60)$$

$$c_t(f) = \frac{n}{1-n} (1-\eta) [\eta T_t^{-(1-\omega)} + 1-\eta]^{\frac{\omega}{1-\omega}} C_t + \eta^* [(1-\eta^*) T_t^{-(1-\omega)} + \eta^*]^{\frac{\omega}{1-\omega}} C_t^* \quad (61)$$

The aggregation structure is as follows: the goods from the monopolistic producers in the home and foreign country, $c_t(h)$ and $c_t(f)$ respectively, are aggregated to country-specific bundles C_{Ht} and C_{Ft} using (3). Households aggregate home and foreign bundles to consumption goods C_t and C_t^* by (1) and decide on the respective expenditure for home and foreign goods and the demands for the single producers is the aggregate demand from the two regions stated by (??) following the cost minimisation procedure by households.

$$T_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ft}^*}{P_{Ht}^*},$$

where the second equality holds due to the law of one price. They represent an index of competitiveness and play a crucial role in the transmission of asymmetric shocks in the two regions. We can express the CPI (??) in the two regions employing the terms of trade:

$$P_t = [\eta P_{Ht}^{1-\omega} + (1-\eta) P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}} \quad (62)$$

$$= P_{Ht} [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{1}{1-\omega}} \quad (63)$$

$$P_t^* = [(1-\eta^*) P_{Ht}^{1-\omega} + \eta^* P_{Ft}^{1-\omega}]^{\frac{1}{1-\omega}} \quad (64)$$

$$= P_{Ft} [(1-\eta^*) T_t^{-(1-\omega)} + \eta^*]^{\frac{1}{1-\omega}} \quad (65)$$

linking in this way the consumer prices to the producer prices.

The real exchange rate, the ratio of relative consumption price level $RER_t = P_t^*/P_t$, can be expressed as ratio of the home and foreign producer prices using (5)

$$RER_t = \frac{P_t^*}{P_t} = \quad (66)$$

$$= \frac{P_{Ft} [(1-\eta^*) T_t^{-(1-\omega)} + \eta^*]^{\frac{1}{1-\omega}}}{P_{Ht} [\eta + (1-\eta) T_t^{1-\omega}]^{\frac{1}{1-\omega}}} \quad (67)$$

$$= T_t \left[\frac{(1-\eta^*) T_t^{-(1-\omega)} + \eta^*}{\eta + (1-\eta) T_t^{1-\omega}} \right]^{\frac{1}{1-\omega}} \quad (68)$$

$$RER_t = \left[\frac{(1-\eta^*) + \eta^* T_t^{1-\omega}}{\eta + (1-\eta) T_t^{1-\omega}} \right]^{\frac{1}{1-\omega}} \quad (69)$$