Apologies for sending around an unfinished model. But I hope there is enough in here to make for an interesting discussion. Obviously, all comments are most welcomed.

1 Empirical Trends

- Mexico has adopted patent laws of highest global standard in 1991, and tightened them further in the context of NAFTA.
- From early 1990s we see an explosion in patent applications in Mexico—very much as expected from standard theory: greater incentives to innovate, hence more innovation. However, this increase in patent applications is very much limited to foreigners. The count of patent applications by residents remains stable, and even decreases slightly (cf. Figure 1).

![Figure: "Chart app" in mexico.xls]

- One should expect the patenting behaviour of Mexican residents to follow a broadly similar trend in the US. However, the patent applications to the USPTO actually experiences a marked increase in the late 1980s, early 1990s. Looking at the share in total applications puts that somewhat into perspective. Patent applications have increased dramatically in the US from the mid-eighties (which can mainly be explained by various landmark court rulings that opened the door to patenting a much broader class of “inventions”). Many firms have used the existence of such patents to enact barriers to entry by indiscriminately patenting all sorts of things: in the words of Bill Gates in an internal memo: ”patenting as as much as we can”). Therefore, the share of Mexican patent applications in all applications to the USPTO may serve as a better indicator of Mexican performance. The share of Mexican patents has not significantly changed in the 1990s and is, historically seen, on a fairly low level. (cf. Figure 2).

![Figure: "chart US app" in mexico.xls]

- Effect on R&D: unfortunately only short time series (has UNESCO longer series?). As can be seen in figure 3 gross expenditure on R&D has broadly increased over the 1990s, in real terms, as well as a share of GDP. Looking at business expenditure (figure 4) that trend is confirmed. Although more volatile, R&D seems to be on an upward trend. How can this be explained in the face of the poor performance in patenting over the same time period?

- The aim of this paper is to shed light on the likely effects of stronger patent protection under the TRIPs agreements on developing countries. Mexico is a good example as it already introduced tougher IPR protection under the NAFTA accords. In particular this paper will explore the implications of the possibility to imitate Northern products in countries with low IPR protection. Typically this imitative industry is very strong in developing
countries. The next part of the paper, the very simple model, explores what the outside option of imitating implies for the behaviour of firms. The third part assumes that firms may gain insights from producing a product. So the more they produce, the better they know the production process, and the likelier they are to make an invention that improves the production technique.

2 Model 1: very simple

Assume two regions N, and S. Each houses a firm that may decide to invest some money on order to innovate. For the moment consider only process innovation. When a firm makes a discovery that will reduce its MC from $c_h$ to $c_l$. The probability of any one firm to make an innovation in a time period $dt$ is given by $h_i(R_i) dt$, $i = N, S$. $R_i$ is the spending on research and development of the firm. We will assume that $h_i(.)$ is concave.

2.1 Autarky

Assume that both countries are perfectly closed, so that no trade, and no information flows between the two. The firms in the market then holds a monopoly. As should be evident, profits are nonincreasing in MC. Therefore there exists an intertemporal tradeoff for the firm: Decrease profits today by investing in R&D, but reap the benefit (higher profits) tomorrow. The probability at time $t$ that there has been no discovery so far is $e^{-h(R)it}$.

that follows from the Poisson distribution (note a random variable $X$ that follows Poisson distribution, has probability function:

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!},$$

therefore for $x = 0$ we obtain $P(X = 0) = e^{-\lambda}$.

Between $t$ and $dt$ when no innovation has occurred, the profits of the monopolist will be

$$(\Pi(c_h, q) - R) dt.$$

With probability $h(R) dt$ innovation occurs during time interval $dt$, lowering costs to $c_l < c_h$ and he gets profits stream $\frac{\Pi(c_l, q)}{\rho}$ (since he will get $\Pi(c_l, q)$ forever).

The firms will maximise:

$$V = \int_0^\infty e^{-\rho t} e^{-h(R)t} \left( \Pi^h - R + h(R) \frac{\Pi^l}{\rho} \right) dt = \frac{\Pi^h - R + h(R) \Pi^l}{\rho + h(R)},$$

where $V$ is the discounted value of profits and investment when costs are $c_h$.

The firm will pick its optimal level of R&D expenditure by setting:

$$\frac{dV}{dR} = 0$$

After some transformation and making use of the definition of $V$ from above, this can be rewritten as:

$$\frac{\partial R}{\partial h} = \frac{\Pi^l}{\rho} - \frac{\Pi^h - R + h(R) \Pi^l}{\rho + h(R)}$$

$$\frac{\partial R}{\partial h} = \frac{\Pi^l}{\rho} - V$$
Remember that $V$ is the current value of the firm and $\Pi / \rho$ is the potential gain. So this equation is relating the R&D expenditure of the firm to the capital gain it can make.

### 2.2 Trade Equilibrium

What happens once we open the economy to international trade and competition? There are essentially two effects to be considered. Firstly, the effect of a larger market, be it on current profits and output, or on future profits. Secondly, competition will arise between firms to make the invention. I assume that each country houses one firm. Whoever is first to innovate, can patent the invention. I am going to consider two scenarios. Under one, patent rights are not enforced in the foreign country. The other scenario is complete and perfect patent protection in the whole world. I assume that the firm in $N$ is the current holder of the patent. We will assume that firms pick their optimal level of $R$ taking the R&D expenditure of the competitor as given.

#### 2.2.1 No Imitation

Since the firm in $N$ is the current patent holder, it is the only producer in the world. It makes a profit $\Pi_{NN}$, and $\Pi_{NS}$ in region $N$, and $S$ respectively. The firm in $S$ is a competitor for the innovation, if its expected discounted profit is positive. For firm $S$ we have:

$$V_S = \left\{ \begin{array}{ll}
\int_0^\infty e^{-\rho t} e^{-|h_N(R_N)+h_S(R_S)|t} \left(-R_S + h_S(R_S) \frac{\Pi_{NN} + \Pi_{NS}}{\rho}\right) dt & \text{if } -R_S + h_S(R_S) \frac{\Pi_{NN} + \Pi_{NS}}{\rho} > 0, \\
0 & \text{if } -R_S + h_S(R_S) \frac{\Pi_{NN} + \Pi_{NS}}{\rho} < 0
\end{array} \right. ,$$

So if $h_S$ were low enough, firm $S$ would not enter the market at all, and $N$ will be a monopolist before and after innovation. The model is then the same as in the previous section, except that the total profit levels are now higher, since the firm can make an additional profit in the $S$ market. If $S$ decides to enter we have:

$$V_S = \frac{-R_S + h_S(R_S) \frac{\Pi_{NN} + \Pi_{NS}}{\rho}}{\rho + h_N(R_N) + h_S(R_S)}$$

the FOC gives

$$\frac{\partial R_S}{\partial R_S} = \frac{\Pi_{NN} + \Pi_{NS}}{\rho} - \frac{-R_S + h_S(R_S) \frac{\Pi_{NN} + \Pi_{NS}}{\rho}}{\rho + h_N(R_N) + h_S(R_S)}$$

$$\frac{\partial R_S}{\partial R_N} = \frac{\Pi_{NN} + \Pi_{NS}}{\rho} - V_S$$

Similarly to the autarky model, this equation can be interpreted as giving a relation between the research effort of a firm (the LHS) and the total gain (the RHS).

For a firm in $N$ we have

$$V_N = \int_0^\infty e^{-\rho t} e^{-|h_N(R_N)+h_S(R_S)|t} \left(\Pi_{NN}^N + \Pi_{NS}^N - R_N + h_N(R_N) \frac{\Pi_{NN} + \Pi_{NS}}{\rho}\right) dt$$

$$= \frac{\Pi_{NN}^N + \Pi_{NS}^N - R_N + h_N(R_N) \frac{\Pi_{NN} + \Pi_{NS}}{\rho}}{\rho + h_N(R_N) + h_S(R_S)}$$

From the FOC get:
has a monopoly in market \( S \). We shall assume that the competition takes the Cournot form.

For firm \( N \) we get:

\[
\frac{\partial R_N}{\partial N} = \frac{\Pi_{SS}^r - R_N + h_N(R_S) \frac{\Pi_{NS}^r}{\rho}}{\rho + h_N(R_S) + h_S(R_S)}
\]

That expression lends itself to the usual interpretation of giving a relationship between R&D and total gain.

Note, so far I have implicitly assumed that whoever makes the invention will then become a monopolist. However, the firm in \( N \) is already in the market, so it can remain in the market and sell at MC \( c_h \), if firm \( S \) is the first to innovate. Optimal reply of firm \( S \) to that would be to set \( p = c_h - \varepsilon \), thereby being the only firm in the market. That will then imply that \( \Pi_S^r \) is lower, than when the innovation is drastic, i.e. when the reduction in \( c \) is so big that the monopoly price \( p_S^M (c_l) < c_h \).

### 2.2.2 Imitation

Assume that IPR protection in \( S \) is very low, or non-existent. While firm \( N \) has a monopoly in market \( N \), it has to face the competition of firm \( S \) in market \( S \). We shall assume that the competition takes the Cournot form.

Firm \( N \) will then maximise:

\[
V_N = \int_0^\infty e^{-\rho t} e^{-[h_N(R) + h_S(R)]t} \left( \Pi_{NN}^r + \Pi_{NS}^r - R_N + h_N(R_N) \frac{\Pi_{NS}^r + \Pi_{SS}^r}{\rho} \right) dt
\]

\[
= \frac{\Pi_{NN}^r + \Pi_{NS}^r - R_N + h_N(R_N) \frac{\Pi_{NS}^r + \Pi_{SS}^r}{\rho}}{\rho + h_N(R_N) + h_S(R_S)}
\]

This looks the same as under the no imitation scenario, except that I replaced the \( \Pi_{NS} \) by \( \Pi_{NS}^r \). This is just to remember that firm \( N \)’s profit in market \( S \) is now the Cournot profit, instead of monopoly profit. The FOC then gives:

\[
\frac{\partial R_N}{\partial N} = \frac{\Pi_{NN}^r - R_N + h_N(R_S) \frac{\Pi_{NS}^r}{\rho}}{\rho + h_N(R_S) + h_N(R_S)}
\]

For firm \( S \) we get:

\[
V_S = \int_0^\infty e^{-\rho t} e^{-[h_N(R_N) + h_S(R_S)]t} \left( \Pi_{SS}^r - R_S + h_S(R_S) \frac{\Pi_{NS}^r + \Pi_{SS}^s}{\rho} + h_N(R_N) \frac{\Pi_{SS}^s}{\rho} \right) dt
\]

\[
= \frac{\Pi_{SS}^r - R_S + h_S(R_S) \frac{\Pi_{NS}^r + \Pi_{SS}^s + h_N(R_N) \frac{\Pi_{SS}^s}{\rho}}{\rho + h_N(R_N) + h_S(R_S)}}{\rho + h_N(R_N) + h_S(R_S)}
\]

Note the difference between \( \Pi_{SS}^r \) and \( \Pi_{SS}^s \). If firm \( S \) is the first to innovate it becomes a monopolist in \( N \) and \( S \). However, if firm \( N \) is the first to make the invention, \( S \) can copy the production technology, so that once again firms \( N \) and \( S \) form a duopoly in market \( S \). (Note that firm \( N \) can not choose to become an imitator if \( S \) is first to innovate, since it is based in \( N \) and it cannot legally copy there, even if the goods are to be exported and sold in \( S \).) That means that \( S \) has an outside option. If it fails to be the first to innovate, it can still copy the product in its own market and make a Cournot profit. Its FOC gives:

\[
\frac{\partial R_S}{\partial S} = \frac{\Pi_{SS}^r - R_S + h_S(R_S) \frac{\Pi_{NS}^r + \Pi_{SS}^s + h_N(R_N) \frac{\Pi_{SS}^s}{\rho}}{\rho + h_N(R_N) + h_S(R_S)}}{\rho + h_N(R_N) + h_S(R_S)}
\]

\[
\frac{\partial R_S}{\partial S} = \frac{\Pi_{SS}^s}{\rho} - V_N
\]
Comparing this with the FOC under the no-imitation scenario, we see that the RHS is now smaller (for the same $h_N$), since the $V_S$ is larger. The expected discounted profit stream of $S$ increased, since it is making a current profit ($\Pi_{SS}^S$), and because of the outside option in case $N$ innovates first.

2.2.3 Equilibrium and the case of Mexico

To find the equilibrium we are looking for a Nash equilibrium in which each firm chooses its level of R&D, taking the R&D expenditure of its rival as given. Obviously we require that the present value $V$ be nonnegative for a firm to operate in the market and commit resources to research.

This paper should try to offer a tentative explanation of the empirical facts presented above. Namely, on how the number of patent applications fails to increase, despite an apparent increase in R&D expenditure. The "imitation" scenario presented above may be thought of as the situation in Mexico before 1991, while the "no imitation" scenario should reflect the stronger patent protection after the reforms. So the question is how R&D expenditure of $N$, and $S$ change as we move from one to the other, and how the respective probabilities of invention $h(R)$ are affected. It is possible that $R_S$ increases, but that the relative increase of $h_N$ as compared to $h_S$ is more important, so that the likelihood of $S$ making the invention first decreases.

To summarise the results so far, consider the table below. It shows current (pre-invention) payoffs for firms $N$, and $S$.

<table>
<thead>
<tr>
<th></th>
<th>Imitation</th>
<th>No Imitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ to innovate first</td>
<td>$(\Pi_{NN}^N + \Pi_{NS}^N, \Pi_{SS}^N)$</td>
<td>$(\Pi_{NN}^N + \Pi_{NS}^N, 0)$</td>
</tr>
<tr>
<td>$S$ to innovate first</td>
<td>$(0, \Pi_{SN}^N + \Pi_{SS}^N)$</td>
<td>$(0, \Pi_{SN}^N + \Pi_{SS}^N)$</td>
</tr>
</tbody>
</table>

The next table shows the respective post-invention payoffs, depending on which firm was first to innovate (note that for the moment I am only considering drastic innovation).

<table>
<thead>
<tr>
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<th>Imitation</th>
<th>No Imitation</th>
</tr>
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</tr>
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<td>$(0, \Pi_{SN}^N + \Pi_{SS}^N)$</td>
<td>$(0, \Pi_{SN}^N + \Pi_{SS}^N)$</td>
</tr>
</tbody>
</table>

So as we move from the imitation regime to the no imitation regime, we have to ask the questions: how does $R_N$ change as its pre and post invention profits increase? How does $R_S$ change as its pre and post invention profits decrease? How will the respective $R$ change, given the change in the competitors $R$? And finally, is there a set of parameters for which we observe an increase in $R_S$, but a proportionately larger increase in $R_N$?

To answer these questions, write the general form of the FOCs as:

$$\frac{\partial R^S}{\partial h^S} = \frac{\Pi^I - R^I + b_i(R^I) \Pi^H + b_j(R^I) \Pi^H}{\rho + b_i(R^I) + b_j(R^I)},$$

where $\Pi^H$ is pre invention pay-off, $\Pi^I$ is post invention payoff, and $\Pi^H$ is the post invention payoff for the loser. As we cannot find a closed form solution to this problem, make use of the implicit function rule to determine the signs of the derivatives. Start by defining a function $g$:
increase its R&D expenditure. Mexican

hold of a patent that allows them to continue producing legally. Unfortunately

northern

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can now receive a larger payo

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effects of a change in

N

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Π

and will hence be zero at the optimum. Thus we get:


\frac{\partial R_i}{\partial R_i} = \frac{\partial g}{\partial R_i} \quad \text{for} \quad \frac{\partial g}{\partial R_i} > 0

because of the concavity assumption. So we can find the derivatives of \( R_i \)

with respect to the variables of interest:

\frac{\partial R_i}{\partial \Pi^i} = - \frac{\partial g}{\partial \Pi^i} = \frac{\partial h_i(R_i)}{\partial \Pi^i} > 0

\frac{\partial R_i}{\partial \Pi^h} = \frac{\partial g}{\partial \Pi^h} = - \frac{\partial h_i(R_i)}{\partial \Pi^h} < 0

\frac{\partial R_i}{\partial \Pi^w} = \frac{\partial g}{\partial \Pi^w} = - \frac{\partial h_i(R_i)}{\partial \Pi^w} < 0

\frac{\partial R_i}{\partial \Pi^l} = - \frac{\partial g}{\partial \Pi^l} = \frac{\partial h_i(R_i)}{\partial \Pi^l} \leq 0

when \( \Pi^l = 0 \), the last expression will be greater than zero.

We are now able to answer the aforementioned questions. The effect on \( R_N \)

is ambiguous as the change in \( \Pi^i \), and \( \Pi^h \) work in different directions. As the

level of pre invention profits \( \Pi^h \) fall (to zero) \( R_S \) increases. Also, the decrease

of \( \Pi^l \) (to zero) means an increase in \( R_S \). Remains the effect of a change in

\( R_N \) on \( R_S \) that cannot clearly be determined. Hence the effect on \( R_S \) is also

ambiguous. Therefore, we cannot exclude the possibility that there exists a set

of parameters for which the regime change implies an increase in \( R_N \), and \( R_S \),

such that the relative probability of \( N \) making the invention increases. The

intuition is as follows: as imitation of northern products becomes illegal, \( S \)

will now remain empty handed if \( N \) is first to innovate. So it has an incentive to

increase its R&D expenditure. Mexican firms face closure, unless they can get

hold of a patent that allows them to continue producing legally. Unfortunately

the increase in R&D expenditure by Mexican firms, doesn’t guarantee in itself

that they will be more successful in the international market for inventions. As

northern firms can now receive a larger payoff for their inventions, they may

increase R&D spending themselves. Recall that increasing \( R \) means that the

probability of making the invention at any point in time increases. Since both

firms spend more on R&D the expected time of invention approaches. And it

may just be the case that the relative increase in \( R_N \) is such that the probability

of \( N \) making the invention first increases. So northern firms are still one step

ahead of southern firms.
2.3 Simulation

To explore the possible implications run a few simulations. For that we will need to assume a particular functional form for \( h(R) \). Assume \( h = R^{\alpha}, \) where \( \alpha < 1 \). We also need to specify the profits in the two markets. For that assume a simple linear demand curve \( p = a_i - b_i Q, \) \( i = N, S \). The profits in each market are a standard result. In case of a monopoly the profit level is \( \Pi_j = (a_i - c_j)^2 / (4b_i), \) \( j = l, h, \) and for Cournot duopoly we have \( \Pi_j = (a_i - c_j)^2 / (9b_i). \) Then the Nash equilibrium under the imitation scenario will be defined by the following two equations:

\[
\frac{\partial R_S}{\partial h_S} = \frac{\Pi_l S - h_S}{\rho} - \frac{n_l S_n + n_l S_h + h_N (R_N) \frac{n_l S}{\rho}}{\rho + h_N (R_N) + h_S (R_S)}
\]

\[
\frac{\partial R_N}{\partial h_N} = \frac{n_l N_n + n_l N_h - R_N + h_N}{\rho + h_N + h_S}
\]

Given the parameters of the model \((\alpha, a_N, a_S, b_N, b_S, c_l, c_h, \rho)\) these will be two equations in two unknowns.

{present typical results}

Under the no imitation scenario, the two FOCs were

\[
\frac{\partial R_S}{\partial h_S} = \frac{n_l S_n + n_l S_h - R_S}{\rho} - \frac{h_S}{\rho + h_N (R_N) + h_S (R_S)}
\]

\[
\frac{\partial R_N}{\partial h_N} = \frac{n_l N_n + n_l N_h - R_N + h_N}{\rho + h_N + h_S}
\]

{present typical results}

{can my hypothesis be confirmed? for what values, or what range can it be confirmed? intuition}

3 Model 2: Learning by imitating

- the role of learning by imitating, or learning by doing: a lot of firms have their origin in time periods when they were not protected by any IPR. they started off imitating/copying products that were patented in other countries, from there they developed into what they are today (eg. japan, Schiff (1971): industrialisation without patents: Ciba (then Novartis, now syngenta); jurgens and van den bergh (now Unilever))

- (West) Germany for instance introduced patent protection for pharmaceauticals in 1968. By that time it had become the largest exporter of pharmaceaultical products, and the third largest producer (cf. the French paper, p. 13, refering to Frischtak (1989))

- The intuitive point should be clear: it is very hard to come up with a great invention from scratch. Doing a task over and over, helps one to familiarise with the technique and the issues, and subsequently come up with ways to improve them. Lax patent protection may then act as a sort of infant industry protection. Allowing domestic firms to copy products, or techniques patented elsewhere, they can establish themselves until they are able to compete with the big international firms on an equal footing.
Introducing the TRIPs agreements in countries that were so far unable
to establish a domestic industry, may then preclude there ever arising one.

- Similar to the model presented in the first part, but now define the probability of any one firm to make an innovation in a time period \( dt \) as \( h_i(R_i, q_i) \) \( dt \), \( i = N, S \). \( R_i \) is as before, and \( q_i \) is the current level of production. We will assume again that \( h_i(.) \) is concave, and \( h_i(R_i, 0) > 0 \), as long as \( R_i > 0 \).

### 3.1 Autarky

Now firms will want to maximise:

\[
V = \int_0^\infty e^{-\rho t} e^{-h(R, q)} t \left( \Pi(c_h, q_h) - R + h(R, q_h) \frac{\Pi(c_i, q_i)}{\rho} \right) dt = \frac{\Pi(c_s, q_s) - R + h(R, q_s) \Pi(c_i, q_i)}{\rho + h(R, q_s)} \]  

The firm will pick its optimal level of RD expenditure by setting:

\[
\frac{dV}{dR} = 0
\]

This will give:

\[
\frac{\partial R}{\partial h} = \frac{\Pi(c_s, q_s) - R + h(R, q_s) \Pi(c_i, q_i)}{(\rho + h(R, q_s))} \frac{\Pi(c_s, q_s) - R + h(R, q_s) \Pi(c_i, q_i)}{\rho + h(R, q_s)}
\]

\[
\frac{\partial h}{\partial R} = \frac{\Pi(c_s, q_s) - R + h(R, q_s) \Pi(c_i, q_i)}{\rho + h(R, q_s)}
\]

Which very much resembles what we found before, except for the \( q \)'s. So this equation is relating the RD expenditure of the firm to the capital gain it can make.

The firm will choose the optimum output level by setting:

\[
\frac{dV}{dq} = 0
\]

That gives

\[
0 = \Pi_q (\rho + h(R, q_i)) + h_q (R + \Pi (c_i, q) - \Pi (c_s, q_s))
\]

Note that when \( h_q = 0 \) (as is the case under standard assumptions), this expression requires \( \Pi_q = 0 \), i.e. that output be chosen to maximise profits.

Now, since \( h_q > 0 \), it must be the case that \( \Pi_q < 0 \). So reducing the pre-invention output would increase pre-invention profits. In other words, the firm is producing more than the short term profit maximising level. This effect is not surprising. Since producing more increases the probability of invention, it pays off to produce more than what would be the profit maximising level in the short run. Consumers gain, less DWL.

### 3.2 Trade Equilibrium

What happens once we open the economy to international trade and competition? As before, I will make the distinction between a strict and a lax patent regime.

#### 3.2.1 No Imitation

Since the firm in \( N \) is the current patent holder, it is the only producer in the world. It makes a profit \( \Pi_{NN} \), and \( \Pi_{NS} \) in region \( N \), and \( S \) respectively. The
firm in $S$ is a competitor for the innovation, if its expected discounted profit is positive. For firm $S$ we have:

$$V_S = \begin{cases} \int_0^\infty e^{-\rho t} e^{-[h_S(R_S,0)+h_S(R_S,0)]t} \left( -R_S + h_S (R_S,0) \right) \frac{\Pi_{S,N}(c_i,q_i)+\Pi_{S,S}(c_j,q_j)}{\rho} \, dt, & \text{if } -R_S + h_S (R_S,0) \frac{\Pi_{S,N}(c_i,q_i)+\Pi_{S,S}(c_j,q_j)}{\rho} > 0 \\ 0, & \text{if } -R_S + h_S (R_S,0) \frac{\Pi_{S,N}(c_i,q_i)+\Pi_{S,S}(c_j,q_j)}{\rho} < 0 \end{cases},$$

So if $h_S$ were low enough, firm $S$ would not enter the market at all, and $N$ will be a monopolist before and after innovation. The model is then the same as in the previous autarky section, except that the total profit levels are now higher, since the firm can make an additional profit in the $S$ market. If $S$ decides to enter we have:

$$V_S = \frac{-R_S + h_S (R_S,0) \left( \frac{\Pi_{S,N}(c_i,q_i)+\Pi_{S,S}(c_j,q_j)}{\rho} \right)}{\rho + h_N (R_N,0)+h_S (R_S,0)}$$

the FOC gives

$$\frac{\partial R_S}{\partial h_S} = \frac{\Pi_{S,N}(c_i,q_i)+\Pi_{S,S}(c_j,q_j)}{\rho} - \frac{-R_S + h_S (R_S,0) \left( \frac{\Pi_{S,N}(c_i,q_i)+\Pi_{S,S}(c_j,q_j)}{\rho} \right)}{\rho + h_N (R_N,0)+h_S (R_S,0)} - V_S$$

Similarly to the autarky model, this equation can be interpreted as giving a relation between the research effort of a firm (the LHS) and the total gain (the RHS).

For a firm in $N$ we have

$$V_N = \int_0^\infty e^{-\rho t} e^{-[h_N(R_N,0)+h_S(R_S,0)]t} \times \left( \left( \Pi_{N,N}(c_h,q_h) + \Pi_{N,S}(c_h,q_h) - R_N + h_N (R_N,0) \right) \frac{\Pi_{N,N}(c_i,q_i)+\Pi_{N,S}(c_j,q_j)}{\rho} \right) \, dt$$

\[= \frac{\Pi_{N,N}(c_h,q_h) + \Pi_{N,S}(c_h,q_h) - R_N + h_N (R_N,0) \frac{\Pi_{N,N}(c_i,q_i)+\Pi_{N,S}(c_j,q_j)}{\rho}}{\rho + h_N (R_N,0)+h_S (R_S,0)} \]

The FOCs are:

$$\frac{\partial R_N}{\partial h_N} = 0$$

\[\frac{\partial h_N}{\partial q_h} = \frac{\Pi_{N,N} + \Pi_{N,S} - \Pi_{N,N} + \Pi_{N,S}}{\rho} \]

\[\frac{\partial h_N}{\partial q_N} = \frac{\Pi_{N,N} + \Pi_{N,S}}{\rho} - V_N \]

That expression lends itself to the usual interpretation of giving a relationship between RD and total gain.

The optimal level of output $q_h$ must satisfy:

$$0 = \frac{\partial \Pi_N}{\partial q_h} (\rho + h_N (R_N,0) + h_S (R_S,0)) + \partial h_N / \partial q_h \left( \Pi_N - \Pi_N + R_N + h_S (R_S,0) \frac{\Pi_S}{\rho} \right)$$

So once again we see that the firm will produce more than will maximise its pre-invention output (since stuff in brackets is positive and $\partial h_N / \partial q_h > 0$, so that it must be $\partial \Pi_N / \partial q_h < 0$).

### 3.2.2 Imitation

Assume as in the first section that firms engage in Cournot competition in $S$. Firm $N$ will then maximise:

$$V_N = \int_0^\infty e^{-\rho t} e^{-[h_N(R_N,0)+h_S(R_S,0)]t} \times$$
\( \times \left( \Pi_{NN}(c_h, q_h) + \Pi_{NS}(c_h, q_h) - R_N + h_N(R_N, q_h) \frac{\Pi_{NN}(c_l, q_l) + \Pi_{NS}(c_l, q_l)}{\rho} \right) \, dt \)

\[
= \frac{\Pi_{NN}(c_h, q_h) + \Pi_{NS}(c_h, q_h) - R_N + h_N(R_N, q_h) \frac{\Pi_{NN}(c_l, q_l) + \Pi_{NS}(c_l, q_l)}{\rho}}{\rho + h_N(R_N, q_h) + h_S(R_S, q_S)}
\]

This looks the same as under the no imitation scenario, except for the \( \Pi \) and the denominator is larger (\( h_S(R_S, q_S) > h_S(R_S, 0) \)). For firm \( S \) we get:

\[
V_S = \int_0^\infty e^{-\rho t} e^{-[h_N(R_N, q_h) + h_S(R_S, q_S)]t} \times \left( \Pi_{SS}(c_h, q_h) - R_S + h_S(R_S, q_S) \frac{\Pi_{SN}(c_l, q_l) + \Pi_{SS}(c_l, q_l)}{\rho} + h_N(R_N, q_N) \frac{\Pi_{SS}(c_l, q_l)}{\rho} \right) \, dt
\]

\[
= \frac{\Pi_{SS}(c_h, q_h) - R_S + h_S(R_S, q_S) \frac{\Pi_{SN}(c_l, q_l) + \Pi_{SS}(c_l, q_l)}{\rho} + h_N(R_N, q_N) \frac{\Pi_{SS}(c_l, q_l)}{\rho}}{\rho + h_N(R_N, q_N) + h_S(R_S, q_S)}
\]

Note the difference between \( \Pi_{SS} \) and \( \Pi_{SS} \). If firm \( S \) is the first to innovate it becomes a monopolist in \( N \) and \( S \). However, if firm \( N \) is the first to make the invention, \( S \) can copy the production technology, so that once again firms \( N \) and \( S \) form a duopoly in market \( S \). (Note that firm \( N \) can not choose to become a copier if \( S \) is first to innovate, since it is based in \( N \) and it cannot legally copy there, even if the goods are to be exported and sold in \( S \).)

### 4 The Path ahead:

- obviously finishing the model
- Try to find more and longer data to back up my story
- literature
- drastic and non-drastic innovation
Figure 1: Patent Applications (Total and from Residents)
Source: WIPO

Figure 2: Mexican Utility Patent Applications in US (Total and Share)
Source: USPTO

Figure 1:

Figure 2: