

Input Quality, Ownership Structure and International Trade

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Abstract

Ownership structure varies across input quality and countries through market thickness concept. We have a thick market for high quality input when demand for high quality input is high and most of the firms demanding high quality inputs are unintegrated. Under this situation, suppliers of high quality input have higher outside option and face less hold-up problem. Consequently, there will be more arm's length trade for high quality inputs. When trade costs for inputs are high, firms in the rich countries where market for high quality inputs is thick are likely to have an arm's length relationship with their suppliers of high quality inputs and vertical relationship with their suppliers of low quality inputs. The opposite happens in poor countries. The reduction in trade costs increases market thickness for both high and low quality industries in both countries. With low enough trade costs, both low quality industry in the North and high quality industry in the south will have more arms' length relationship.

1 Introduction

- Many papers have proposed several determinants in making ownership decision. Relevant papers are Antras 2003, Antras and Helpman 2004, Antras and Helpman 2008, and Grossman & Helpman 2002. These papers suggest that main determinants of ownership structure are
 1. how important final good producers' relationship-specific investment is relative to suppliers' (relevant proxies are capital intensity, R&D intensity, skill intensity),
 2. Firms' productivity,
 3. Judicial quality in the sourcing countries,
 4. Matching costs.

- This paper will complement these existing papers by showing how desired input quality level affects firms' decision on their ownership structure.
- Literature review on product quality;
 - Hallak 2005; rich countries import relatively more from countries that produce high-quality goods.
 - Schott 2003; rich countries (K and S abundant) use their endowment advantage to increase specialization within products.
 - Hummels&Klenow 2005; rich tends to export higher quantities at higher prices.
 - Baldwin Harrigan 2009; higher unit values with longer distance, also suggest that high productivity firms produce high quality goods.
 - Fajgelbaum, Grossman & Helpman 2009; the model has nested logit aggregate demand system which yields an increase in fraction of consumers buying high quality products when income rises. Under trade, rich country will be the net exporter of high quality varieties and poor country will be the net exporter of low quality varieties.
 - Verhoogen 2008, high productivity firms produce high quality goods.
 - Kugler&Verhoogen 2008 2009; high productivity firms use high quality inputs to produce high quality goods.
- "Pepsi grows potatoes in China" case study from Harvard Business Publishing is a good example showing how desired input quality level affects firms' ownership decision through market thickness concept. Pepsi's snack food business is conducted through Frito Lay Inc. and its well known products are Doritos corn chips and Lay's potato chips. The production of Lay's potato chips requires high quality potatoes that must be large, round, low in sugar and water content, and high in solids. In the US, Pepsi lets Black Gold company supplies potatoes to its US plant. On the other hand, Pepsi decided to vertically integrate potato growing business in order to supply its Lay's chip factory in China. Before the integration decision was made, Pepsi tried hard to persuade Chinese state farms, cooperatives and individual farmers to grow high quality potatoes for Pepsi. The firm even taught local farmers to grow potatoes commercially, set up a demonstration farm, gave out loans so that farmers can buy relevant set of equipment, and even offered them a buying price which is roughly twice of the market price in China. Chinese farmers were still reluctant to invest and become Pepsi's supplier. The ones who joined the scheme did not invest fully up to Pepsi's standard. This forces Pepsi to grows potatoes itself.
- I believe that the main reason why Chinese farmers were not willing to invest is simply because hold-up risk is high due to thin market for high quality potatoes in China. In the US, there is a thick market for high

quality potatoes and Black Gold faces relatively less hold-up risk. This is because if it is held up by Pepsi, it can still sell these high quality potatoes to other food manufacturers whereas Chinese farmers do not have this option.

- The model in this paper will have final goods market setting similar to the one in Fajgelbaum, Grossman & Helpman 2009 where richer countries demand more high quality varieties relative to poor. There is an input market where final good producers and input suppliers trade. The input market setting is similar to the one in McLaren 2000 (AER). Autarky case will be explored first then the model will be extended to analyze the international trade case (only trade in inputs is allowed). Then this model can be improved further by allowing the trade in final goods.

2 Model

2.1 Final goods market

2.1.1 Demand system

- There are 2 industries; Manufacturing and Agricultural sectors.
- Agricultural sector produces homogeneous goods under perfect competition. Assume that 1 worker can produce 1 unit of this homogeneous product. Hence $p^A = w$. We normalize $p^A = 1$, so we have $w = 1$.
- Manufacturing products are divided into 2 industries; High quality and Low quality industries.
- There are many varieties in each product category.
- Firms can enter manufacturing sector freely so manufacturing firms will make zero profit in equilibrium.
- Individual h has the following demand function

$$u_{qi}^h = zg_q + \varepsilon_{qi}^h \tag{1}$$

where

- $q \in \{H, L\}$ be the quality index and g_q is quality constant ($g_H > g_L$),
- z = consumption of homogeneous good,
- i = variety index and the set of varieties with quality q is Q_q .
- ε_{qi}^h represents individual h 's idiosyncratic evaluation of variety i with quality q .

- There are N individuals in the economy and they are endowed with different amounts of effective labor. Income distribution is captured by cumulative density function I_y . $I_y(y)$ represents the fraction of N individuals who are endowed with effective labor less than or equal to y . Different countries have different income distribution.
- Each individual is randomly assigned his preference schedule ε^h which is a vector consisting of ε_{qi}^h for all varieties. The distribution of ε^h follows Generalized Extreme Value (GEV) distribution which has the following cumulative density function

$$F_\varepsilon(\varepsilon) = e^{-\sum_{q \in \{H,L\}} [\sum_{i \in Q_q} e^{-\varepsilon_{q,i}/\theta_q}]^{\theta_q}} \quad (2)$$

where $0 < \theta_q < 1$.

- Each consumer consumes one unit of manufacturing good and spend the rest of his income on homogeneous good. Hence each consumer will choose quality level(q) and variety(i) in order to maximize $(y^h - p_{qi}) g_q + \varepsilon_{qi}^h$.
- This fraction of people with income y that decides to buy variety i with quality level q is

$$\rho_{q,i}(y) = \rho_{i|q} \rho_q(y),$$

where

$$\rho_{i|q} = \frac{e^{-p_{q,i} g_q / \theta_q}}{\sum_{l \in Q_q} e^{-p_{q,l} g_q / \theta_q}},$$

and

$$\rho_q(y) = \frac{(\sum_{l \in Q_q} e^{(y-p_{q,i}) g_q / \theta_q})^{\theta_q}}{\sum_{\omega \in \{H,L\}} (\sum_{l \in Q_\omega} e^{(y-p_{\omega,i}) g_\omega / \theta_\omega})^{\theta_\omega}}.$$

$\rho_{i|q}$ represents the probability of an individual buying variety i with quality q once that person has already decided to buy a variety with quality q and $\rho_q(y)$ is the fraction of people with income y that decide to buy manufacturing good with quality q .

- θ_q measures the degree of dissimilarity across all varieties in the set Q_q ¹. Here it is assume that $\theta_H > \theta_L$ and this implies that low quality varieties are better substitutes for one another than different varieties of high quality².
- Under this demand system, fraction of individuals who purchased variety i with quality q rises with income only when $g_q >$ average quality consumed by these individual $[\sum_{q \in \{H,L\}} g_q \rho_q(y)]$. As we have only 2 quality level and $g_H > g_L$, we know for sure that high-quality varieties are demanded more when there is an increase in income.

¹Larger θ_q -> varieties in Q_q are considered to be more dissimilar -> $\varepsilon_{q,i}$ and $\varepsilon_{q,i'}$ are less correlated.

²For example, low quality coffee like Tesco value or Sainsbury value coffee is very similar while Starbucks and Illy coffee has different unique taste.

- Then we can derive per capita demand for variety i with quality q which has the following expression

$$\begin{aligned}
x_{q,i}(y) &= \int_{y_{\min}}^{\infty} \rho_{q,i}(y) dI_y(y) \\
&= \frac{e^{-p_{q,i}g_q/\theta_q}}{\sum_{l \in Q_q} e^{-p_{q,l}g_q/\theta_q}} \int_{y_{\min}}^{\infty} \frac{(\sum_{l \in Q_q} e^{(y-p_{q,l})g_q/\theta_q})^{\theta_q}}{\sum_{\omega \in \{H,L\}} (\sum_{l \in Q_\omega} e^{(y-p_{\omega,l})g_\omega/\theta_\omega})^{\theta_\omega}} dI_y(y)
\end{aligned} \tag{3}$$

2.1.2 Production of final goods

- Each final good producer (F) must acquire an input from an input supplier (S).
- Like final goods, there are also 2 levels for input quality; high and low. It is assumed that in order to produce high quality output, an input supplier requires 1 unit of high quality input. Similarly, 1 unit of low quality input is needed when a unit of low quality output is produced. As input quality is the same as output quality, I can use output quality index (q) as the quality index for input quality.
- Each F is assigned with an input supplier S . Before the assignment, all F 's must have made their decision on the quality level of their varieties. Let's denote $F_{q,i}$ to be the final good producer who produce variety i with quality q and $S_{q,i}$ to be the input supplier who is paired with $F_{q,i}$.
- Let $f_{q,i}$ be the price that $F_{q,i}$ has to pay a supplier for an input and this does not vary with the quantity of variety i produced³. Hence, when variety i producer set its price in order to maximize his profit, he treats this cost as fixed. This fixed cost is determined in input market which will be described in the next section.
- Even though $f_{q,i}$ does not vary with the quality of the final good produced, $F_{q,i}$ still has to pay marginal cost of c_q . It is assumed that the marginal cost of producing high quality variety is higher than the marginal cost of producing low quality variety ($c_H > c_L$).
- Given per capita demand for variety i in equation 3 and this cost structure, $F_{q,i}$ will set price to maximize his profit and the pricing rule is

$$p_{q,i} = c_q + \frac{\theta_q}{g_q}. \tag{4}$$

³A good example for this assumption is accounting software. A firm pays a fixed amount of money to a software developer. Once the software is ready, the firm can ask for as many copies as it wants. Another example is some specialized chemical. A firm pays a fixed amount of money to a supplier to develop a special chemical formulae which costs a lot. Once the formulae is found, the marginal cost of producing more of this chemical is negligible comparing to the initial payment.

All the terms in the right hand side are not variety specific. This implies that all the varieties with the same quality level must have the same price. Also, the optimal price does not vary with the number of firms selling goods with quality q . Therefore varieties with the same quality level must received the same total demand.

- Now I will discuss about input market so that we will understand more about the fixed payment ($f_{q,i}$) to an input supplier. Then we will be ready to analyzed the autarky equilibrium.

2.2 Input market

- Each $F_{q,i}$ has already decided whether to produce high or low quality variety before they are paired with $S_{q,i}$.
- Let n_q denote the number of final good producers demanding inputs with quality q . Each $F_{q,i}$ is paired with a supplier $S_{q,i}$, hence there must be exactly n_q pairs in quality q input market.
- An input from $S_{q,i}$ can not be used to produce any variety with quality q' where $q' \neq q$. None the less, it can be used by any $F_{q,j}$ where $j \in Q_q$ and its productivity depends on how specialized the input is and whom it will be used by.
- Within each quality level, there are 2 types of inputs; specialized and general which are generated from maximal specialization and flexibility production strategy respectively. Specialized inputs give higher probability of getting a compatible input. Hence, these production strategies have different outcome matrices. Each outcome is represented by "success rate" or " $R_{q,ij}$ " which represents the probability that an input from $S_{q,i}$ is compatible with $F_{q,j}$'s production where i can be equal to j . Both parties only realize whether the input is workable or not after the exchange has taken place. Success rates under different production strategies are shown in the matrices below.
- Under maximal specialization, $S_{q,i}$ produces exactly the type of input that will maximize $F_{q,i}$'s success rate. In this case, I will assume that $F_{q,i}$'s success rate is one or $F_{q,i}$ will get a workable input for sure. None the less, this input is so specialized such that it is worthless outside their relationship and $S_{q,i}$ can never produce a maximally specialized input for $F_{q,j}$ ⁴. This is summarized in the table below.

	$R_{q,ii}$	$R_{q,ij}$
$q_i = H;$	1	0
$q_i = L;$	1	0

where $i \neq j$.

⁴ $S_{q,i}$ only know the exact type of input demanded by the final good producer that it is paired with.

- Under flexibility production strategy, $S_{q,i}$ produces an input which is less specific to $F_{q,i}$ in order to create positive value for its input outside this relationship. However, this strategy is subject to some extra risk. In other words, this input can either be effective or defective with probability $\rho_{q,i}$ and $(1 - \rho_{q,i})$ respectively. Under quality q , the relevant payoff matrix is

$$\begin{array}{cc}
 & R_{q,ii} & R_{q,ij} \\
 \text{Effective;} & e_q & e'_q \\
 \text{Defective;} & d_q & d'_q
 \end{array}$$

where $1 > e_q > e'_q > d_q > d'_q > 0$. Even though, F and S only realize whether their input is workable or not after the exchange has taken place, the effectiveness of each input is observed after its production and before the exchange occurs. This implies 2 things. Firstly, each input from maximal specialization strategy gives higher payoff to its intended user comparing to flexibly produced input. Secondly, an effective input has an absolute advantage over a defective input.

- These 2 outcome matrices imply that an input produced by $S_{q,i}$ will always give higher success rate to $F_{q,i}$ comparing to what $F_{q,j}$ will get, irrespective of whether the production strategy is maximal specialization or flexibility. In other words,

$$R_{q,ii} > R_{q,ij} \text{ with probability } 1, i \neq j \quad (5)$$

Hence, there will be only one situation where $F_{q,i}$ would want to use input designed for $F_{q,j}$ which is when $S_{q,i}$ chooses flexibility strategy and the input turns out to be defective ($e'_q > d_q$ hence it is worthwhile for $F_{q,i}$ to buy an input from another supplier).

2.3 Ownership structure

- There are 2 types of ownership structure: Merger (or vertical integration) or arm's length relationship. Under merger, $F_{q,i}$ and $S_{q,i}$ decide which production strategy to use and what to do with their input together whereas $S_{q,i}$ makes these decisions freely under arm's length relationship. $F_{q,i}$ has to pay "governance costs" of L_q effective units of labor if it integrates with $S_{q,i}$. Under both ownership structures, $S_{q,i}$ pays irreversible investment cost of K_q in production stage I.
- Relevant time line is shown in Figure 1.
- Entry stage has already been explained in final goods market section.
- Under merger stage, $F_{q,i}$ can give $S_{q,i}$ a take-it-or-leave-it merger offer. Consequently, we have a merger only when both $F_{q,i}$ and $S_{q,i}$ want to integrate. All pairs decide their ownership structure and the number of unintegrated pairs is publicly known at the end of this stage.

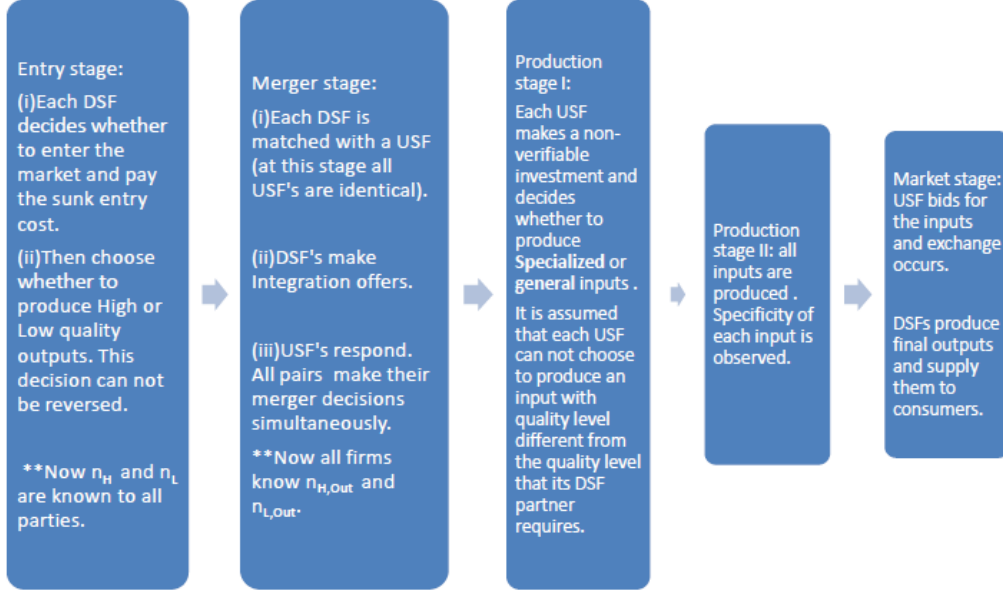


Figure 1: Timeline for manufacturing sector.

- Under production stage, $S_{q,i}$ can be held up by $F_{q,i}$ because $S_{q,i}$ makes a non-verifiable investment of K_q in production stage I (K_H or K_L) while the input is produced in production stage II (only observe the success rate in production stage II). This investment cost is borne by $S_{q,i}$ alone. Therefore, $S_{q,i}$ can increase its outside option and, hence, reduce hold-up risk by producing general input instead of specialized input which has no use outside its relationship with $F_{q,i}$.
- Under market stage, both merged and unintegrated pairs have an option to sell their inputs in the market through auction. Let b_{ij} denote $F_{q,i}$'s bid on the input produced by $S_{q,j}$. Each $F_{q,i}$ must place a bid on each available input. Obviously, if $F_{q,i}$ does not want an input from $S_{q,j}$ then the bid for this input must be zero ($b_{ij} = 0$).

3 Analysis of the model

- Let input i be the input that is intended to be used by $F_{q,i}$. Clearly, the price of input i ($P_{q,i}$) in the input market must be equal to the highest bid. In other words, $P_{q,i} = \max_j \{b_{ji}\}$.
- It is natural in this type of bidding game to have a tie in the equilibrium where the winning bid is equal to the bid from the runner-up bidder.

Otherwise, the winner is not bidding optimally because he can reduce his bid and still win this auction. Therefore, the winning bid and the price of input i is determined by the bid from the runner-up bidder. The tie-breaking rule is simply let the one that benefits from this input more wins this bidding game. Hence, $F_{q,i}$ will always win the bidding game for input i when there is a tie.

Proposition 1 *Integrated pairs will never sell their inputs and unintegrated $S_{q,j}$ will always sell its input to its intended user $F_{q,i}$.*

- The setting that I have introduced so far creates the situation where $R_{q,ii} > R_{q,ij}$ always which is the same as in McLaren 2000 paper. This is the crucial condition required for the derivation of this proposition and we have it in this model too. Consequently, the prove is exactly the same as in the appendix in McLaren 2000 paper. I'll type it up later.
- From pricing rule in equation 4, we know that all varieties with quality level q must have the same price. Given the demand structure in this model, these varieties must also have the same demand. Therefore, if all $F_{q,i}$'s that get a workable input will receive the same revenue and it is denoted by G_q .
- From this proposition, we know for sure that expected net return from selling your input to its intended user will always be higher than expected return from selling it to someone else. Mathematically, we have

$$R_{q,jj}G_q - P_{q,j} \geq R_{q,ij}G_q - P_{q,i} \text{ for } i \neq j. \quad (6)$$

This is also true when $F_{q,j}$ is integrated because, by keeping input j , he gets $R_{q,jj}G_q$ while he gets $P_{q,j} + (R_{q,ij}G_q - P_{q,i})$ from selling input j . From proposition 1, we must have $R_{q,jj}G_q \geq P_{q,j} + (R_{q,ij}G_q - P_{q,i})$ and this is the same as the condition in inequality 6. From this we can find the price of the input designed for $F_{q,i}$ to be

$$P_{q,i} \geq \max_{j \neq i} \{ \max [R_{q,ij}G_q - (R_{q,jj}G_q - P_{q,j}), 0] \}. \quad (7)$$

This implies that input i will have a positive price when at least one final good producer j ($i \neq j$) receives higher expected revenue comparing to what he will get if he uses input j .

Proposition 2 *All integrated pairs will choose maximal specialization strategy.*

- This is clear from Proposition 1. Integrated pair will always use the input that they produce. The maximal specialization strategy will always give the highest return with certainty.
- This seems like vertical integration will always be chosen but don't forget that merger option is associated with a fixed governance cost of L_q .

Proposition 3 *The equilibrium price of any maximally specialized input is always equal to zero.*

- This is because under maximal specialization strategy, $R_{q,ij}$ is 0. No one will place a bid which is higher than the value of the input to the bidder. Here the value is zero outside this relationship, hence the others will place bid exactly equal to zero, $b_{q,ij} = 0$. The price of this input must be zero ($P_{q,i} = 0$) because everyone will place zero bid for this maximally specialized input.
- Thus, it is clear that unintegrated $S_{q,i}$ will never choose maximal specialization strategy which will yield him a return of zero.
- With proposition 2, we know that all inputs produced by integrated pairs will always have prices equal to zero.
- There are many equilibria but let's focus on the lowest-price equilibrium.⁵

Proposition 4 *The lowest-price equilibrium is well defined and is as followed. If all inputs are all effective or all defective, the price of these inputs must be zero. If we have at least one defective input and one effective input, then the price of an effective input is $(e'_q - d_q) G_q$ while the price of a defective input must be strictly zero.*

- There is only one situation where you want to make a positive bid on the others' inputs. This happens when your partner has produced a defective input and at least one $S_{q,i}$'s has produced an effective input. This is because, if you manage to win the bid and get someone else's effective input, you have just increased your expected revenue by $(e'_q - d_q) G_q$. Every $F_{q,i}$ with input i being defective will place a bid of $(e'_q - d_q) G_q$ on all effective inputs. This is also equal to the bid from the runner-up bidder. Therefore, $(e'_q - d_q) G_q$ must also be the price of all effective inputs. No one will ever make a positive bid for someone else's defective input, so the price of a defective input must always be zero.
- If all inputs are effective, everyone uses its own input and no one places positive bid for the others' inputs.

4 Industry equilibrium under symmetric case

- The symmetric case is where all $\rho_{q,i} = \rho_q$ for all i . In other words, all $F - S$ pairs have the same probability of getting an effective input under flexibility strategy.

⁵An example of many equilibria case: let's assume that all inputs are general and turn out to be effective. If all $F_{q,i}$'s place the same bid of a , we have an equilibrium for any values of a within the range $[0, e'_q]$. But lowest-price equilibrium occurs when $a = 0$.

- Let's assume that S 's expected payoff from choosing flexibility strategy can cover its investment cost K_q or

$$\rho_q (e'_q - d_q) G_q > K_q$$

, otherwise none of S 's will choose flexibility strategy.

- Let U_q be the set of $F_{q,i}$'s or $S_{q,i}$'s who are unintegrated.
- Let $N(U_q) =$ number of unintegrated pairs.
- Now we can calculate the expected price of an input under flexibility strategy which is;

$$\mu_q(\rho_q, N(U_q)) = \rho_q \left[1 - \rho_q^{N(U_q)-1} \right] (e'_q - d_q) G_q$$

Note that $\rho_q \left[1 - \rho_q^{N(U_q)-1} \right]$ is the probability of an input supplier getting an effective input while at least one of the other suppliers get a defective input. When the market for inputs with quality q is thick ($N(U_q)$ is high), the expected price of an input produced under arm's length relationship is higher.

Proposition 5 $\mu_q(\rho_q, N(U_q))$ is an increasing function in $N(U_q)$.

- $S_{q,i}$ is willing to supply an input to $F_{q,i}$ under arm's length relationship only when expected input price is higher than the investment cost. In other words,

$$\mu_q(\rho_q, N(U_q)) > K_q.$$

As we know that $\mu_q(\rho_q, N(U_q))$ is an increasing function of $N(U_q)$, there must be a threshold value

$$\bar{n}_q = \min \{ m \mid \mu_q(\rho_q, N(m)) > K_q \} \quad (8)$$

such that $S_{q,i}$ is willing to supply an input to $F_{q,i}$ under arm's length relationship.

- Net expected return from merger is $R_{q,ii}G_q - L_q - K_q = G_q - L_q - K_q$.
- Combined expected return from arm's length relationship is $\rho_q e_q G_q + (1 - \rho_q) d_q G_q - K_q$ ⁶.
- Surely if net expected return from merger is larger than the combined expected return from arm's length relationship, integration will always be chosen. So we assume that

$$\rho_q e_q G_q + (1 - \rho_q) d_q G_q - K_q > G_q - L_q - K_q. \quad (9)$$

⁶From the propositions mentioned earlier, we know that each unintegrated $F_{q,i}$ will end up using the input produced by $S_{q,i}$ irrespective of the types of its input (ie. even though it is a defective input, $F_{q,i}$ still has to use it).

This implies that combined expected return from arm's length relationship is assumed to be larger than the expected return from merger and arm's length relationship will always be chosen when $S_{q,i}$ is willing to supply an input to $F_{q,i}$ under arm's length relationship. In other words, we have arm's length relationship when at least $(\bar{n}_q - 1)$ pairs are expected to have arm's length relationship.

- It is clear that there are only 2 Nash equilibria; complete vertical integration ($N(U_q) = 0$) or universal use of independent suppliers ($N(U_q) = n_q$). We will get the latter equilibrium if each firm expects that at least $(\bar{n}_q - 1)$ pairs will have arm's length relationship, otherwise equilibrium with pervasive integration will take place.

Proposition 6 *Assuming that firms are symmetric, a closed economy with small enough quality-q industry ($n_q < \bar{n}_q$) will end up with complete vertical integration. When this quality-q industry ($n_q \geq \bar{n}_q$) is large enough, we can have either complete vertical integration or universal use of independent suppliers.*

- Rich countries have income distribution that first-order stochastically dominates poor countries' income distribution.
- According to demand structure and final goods market setting, poor countries should have higher n_L and lower n_H comparing to rich countries in autarky. Therefore, poor countries are likely to have complete vertical integration in high-quality industry and pervasive arm's length relationship in low-quality industry. The opposite occurs in rich countries.
- If I can solve for equilibrium n_L and n_H as functions of other parameters (i.e., θ_q , income distribution, number of population etc.) like in Fajgelbaum, Grossman & Helpman 2009, then we can talk about the effect of a change in these parameters on ownership structure in autarky. I can not solve for the general equilibrium because
 - Different ownership structures result in different input costs and, hence, different fixed costs for final goods production. Hence it is important to solve for $n_{L,ver}$, $n_{H,ver}$, $n_{L,out}$ and $n_{H,out}$.
 - Some $F_{q,i}$'s will end up with incompatible inputs and produce zero unit of their varieties.
 - Hard to model the expectation of number of unintegrated pairs.
 - None the less, I think I can solve for $n_{L,ver}$, $n_{H,ver}$, $n_{L,out}$ and $n_{H,out}$ in each equilibrium, but I won't be able to determined which equilibrium will take place.

5 Open economy (only input trade is allowed)

- There are 2 countries; North and South. Superscript k is the country index (i.e. $k \in \{N, S\}$).
- These countries are identical except their income distributions. Let's assume that N's income distribution first-order stochastically dominates S's income distribution. Hence, p_q is identical across countries but x_q^k (overall demand for a variety with quality q in country k) varies across both q and k . From income distribution assumption, we should have $x_H^N \geq x_H^S$ and $x_L^N \leq x_L^S$. Hence the expected revenue from getting a workable input (G_q^k) varies across both q and k (i.e. $G_H^N > G_H^S$ and $G_L^N < G_L^S$).
- n_q^N and n_q^S will change after opening to trade if there is a change in ownership structure from autarky equilibrium to equilibrium under trade. Because different ownership structures are associated with different fixed costs. The zero profit conditions that pins down n 's are affected by the change in fixed costs, therefore n_q^N and n_q^S will change. Which direction?
- Now the global number of firms producing quality- q varieties is $n_q^N + n_q^S$.
- When $S_{q,i}^N$ is unintegrated, both $F_{q,i}^N$ and $F_{q,i}^S$ can bid for $S_{q,i}^N$'s input.
- Each export of an input is associated with a fixed transportation cost of t which is born by the supplier. Hence if unintegrated $S_{q,i}^N$ sells its input to a final good producer in the south, it will get $(e'_q - d_q) G_q^S - t$.
- Hence the expected price of an input with quality q producing under arm's length relationship in country k becomes

$$\mu_q^k(\rho_q, N(U_q^k), N(U_q^{k'})) = \rho_q \left[1 - \rho_q^{N(U_q^k)-1} \right] (e'_q - d_q) G_q^k + \rho_q \left[1 - \rho_q^{N(U_q^{k'})-1} \right] \max \left\{ \left[(e'_q - d_q) G_q^{k'} - t \right], 0 \right\}$$

- From equation 8, it is clear that \bar{n}_q^k varies across countries.
- Let's assume that under autarky we have $n_H^N > \bar{n}_H^N$ (universal use of independent suppliers for high quality industry in the North), $n_L^N < \bar{n}_L^N$ (pervasive vertical integration for low quality industry in the North), $n_H^S < \bar{n}_H^S$ (pervasive vertical integration for high quality industry in the South) and $n_L^S > \bar{n}_L^S$ (universal use of independent suppliers for low quality industry in the South).
- I will investigate each industry in different countries separately. So there are 4 industry-country pairs to be investigated which are (N,H), (S,H), (N,L) and (S,L).
- Clearly if $(e'_q - d_q) G_q^{k'} \leq t$, all 4 cases must have the same equilibrium as in autarky.

- There must be some values of t that give $(e'_q - d_q) G_H^N > t$, $(e'_q - d_q) G_L^S > t$, $(e'_q - d_q) G_L^N \leq t$ and $(e'_q - d_q) G_H^S \leq t$. It can be proved that there will be not change in ownership structure from autarky.
- When t is sufficiently low (i.e. $(e'_q - d_q) G_q^{k'} > t$), \bar{n}_q^k will be lower than its autarky value.
 - Case 1 (N,H); we have universal use of independent suppliers for high quality industry in the North in autarky. \bar{n}_H^N will be lower and the ownership structure will not change. Also, n_H^N will not change.
 - Case 2 (S,L); Similar to case 1. Nothing changes.
 - Case 3 (S,H); high quality input suppliers in the South now have an option of exporting their inputs to the North. \bar{n}_H^S is now lower. It is more likely now that $n_H^S \geq \bar{n}_H^S$ and we should have pervasive arm's length relationship in the South under some low enough t . (But n_H^S may falls due to the higher fixed costs)
 - Case 4 (N,L); low quality input suppliers in the North now have an option of exporting their inputs to the South. \bar{n}_L^N is now lower. It is more likely now that $n_L^S \geq \bar{n}_L^N$ and we should have pervasive arm's length relationship in the North under some low enough t . (But n_L^N may falls due to the higher fixed costs)
- As t decreases (an increase in globalization), markets become thicker and it is more likely to see pervasive arm's length relationship in both industries and counties. This is welfare improving due to the inequality condition 9.
- This simple model does not give insights about the trade flow of intermediate goods.

6 Extensions

- We can have heterogenous firms case by assigning firms with different $\rho_{q,i}$. The one with higher ρ are more productive as it tends to make effective input under flexibility strategy.
- Introduce how agents form expectation. (i.e. the way agents calculate \bar{n})
- Try to solve for equilibrium n_L and n_H as functions of some parameters (i.e. θ_q , income distribution, number of population etc.)
- Allow final goods to be tradable.