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# Distorted Monopolistic Competition

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**Abstract**
We characterize the equilibrium and optimal resource allocations in a general equilibrium model of monopolistic competition with multiple asymmetric sectors and heterogeneous firms. We first derive general results for additively separable preferences and general productivity distributions, and then analyze specific examples that allow for closed-form solutions and a simple quantification procedure. Using data for France and the United Kingdom, we find that the aggregate welfare distortion -- due to inefficient labour allocation and firm entry between sectors and inefficient selection and output within sectors -- is equivalent to the contribution of 6-8% of the total labour input.

Keywords: monopolistic competition, welfare distortion, intersectoral distortions, intrasectoral distortions
JEL codes: D43; D50; L13

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# 1 Introduction

The first welfare theorem states that market equilibria are efficient under perfect competition in the absence of externalities and other market failures. A laissez-faire market allocation corresponds, in that case, to an *optimal* allocation that a benevolent social planner would choose. When firms operate in a monopolistically competitive environment, however, the market economy does typically *not* lead to an efficient outcome.

This insight has a long tradition in the literature, dating back at least to Spence (1976) and Dixit and Stiglitz (1977). More recently, the welfare distortions under monopolistic competition have been revisited by Dhingra and Morrow (2014), Nocco, Ottaviano, and Salto (2014), and Parenti, Ushchev, and Thisse (2016) who argue that the market delivers, in general, the wrong selection of firms and the wrong firm-level outputs from a social point of view. Those analyses have been limited to models with a single monopolistically competitive industry that consists of heterogeneous firms.

Such settings ignore the salient heterogeneity across different sectors that we observe in the data. In France in 2008, for example, there are 4,889 textile and footwear producers, which vastly differ in size and compete for an aggregate expenditure share of 2% by the French consumers. Those firms operate, arguably, in a different market and face different demands than the 4,607 manufacturers of wood products or the 124,202 health and personal service providers, on which French consumers spend less than 0.1% and almost 20% of aggregate income, respectively.[1]

When the economy is represented by heterogeneous sectors consisting of heterogeneous firms, a new margin for misallocations arises: the market may not only allocate resources inefficiently *within*, but also *between* sectors in general equilibrium. For example, the textile industry may not only have some firms that produce too little, and others that produce too much from a social perspective. It may also have the wrong overall size, i.e., employ too many (or too few) workers in equilibrium, which in turn means that some other industries may have fewer (or more) workers than is socially optimal. Characterizing those distortions theoretically and quantifying their implied welfare losses are the two objectives of this paper.

To achieve our first goal, we develop a general equilibrium model of monopolistic competition with multiple asymmetric sectors and heterogeneous firms. We build on Zhelobodko, Kokovin, Parenti, and Thisse (2012) and Dhingra and Morrow (2014), who study the pos-

[1]See Section 4 below, in particular Table 1 for more details about the data. Notice the large number of competitors in each sector, which suggests that monopolistic competition may be a reasonable approximation of the market structure.

itive and normative aspects of a single monopolistically competitive industry, respectively. We extend their approach to a multi-sector model and allow the sectors to differ in various dimensions. Imposing standard assumptions on the upper-tier utility function, we establish existence and uniqueness of the equilibrium and the optimal allocations and, by comparing the two, we characterize the distortions that arise in our economy. The latter include inefficient firm selection and output distortions within sectors — as in existing models — and inefficiencies in the labor allocation and the masses of entrants between sectors. These intersectoral distortions are the novel feature of our framework.

We derive general results that the *revenue-to-utility ratio* and the *elasticity of the upper-tier utility* are crucial for characterizing labor and entry distortions between sectors, and we explain the intuition for those two sectoral statistics in detail below. Contrary to the conventional approach in industrial organization, which has studied single industries in partial equilibrium while ignoring interdependencies between them, we analyze those inefficiencies in a framework that fully recognizes the general equilibrium nature of the problem: there cannot be too many (or too few) workers and entrants simultaneously in *all* sectors, and the distortions in one sector depend on the characteristics of all sectors in the economy.

Our second goal is to explore the magnitude of entry and selection distortions at the sectoral level, and to assess *how large* the aggregate welfare loss is from a quantitative point of view. For this purpose we develop two specific parametrized examples of our general model. Those examples allow for closed-form solutions and lend themselves naturally to a simple quantification exercise which only requires data that is easily accessible for many countries.

The first example uses Cobb-Douglas upper-tier and constant elasticity of substitution (CES) subutility functions. This ubiquitous CES model has dominated much of the literature on monopolistic competition in various fields, and it exhibits some very special properties (Zhelobodko et al., 2012; Dhingra and Morrow, 2014). In particular, from the one-sector model by Dhingra and Morrow (2014) we know that selection and firm-level outputs are efficient if the subutility function is of the CES form. However, in this multi-sector example, distortions in entry and the sectoral labor allocation disappear if and only if the revenue-to-utility ratio happens to be identical in all sectors. Otherwise, the allocation is efficient *within* but not *between* sectors.[2]

Our second example is a fully tractable model with variable elasticity of substitution

[2]This insight is consistent with Epifani and Gancia (2011), who compare equilibrium and optimal allocations in a multi-sector CES model with homogeneous firms. Our paper, by contrast, develops a model with general consumer preferences and heterogeneous firms where CES subutilities are considered as one example.

(VES), where demands exhibit smaller price elasticity at higher consumption levels of a variety. Unlike the CES model, this VES model can account for the empirically well-documented facts of variable markups and incomplete pass-through across firms within industries (e.g., Hottman, Redding, and Weinstein, 2016; Yilmazkuday, 2016). It features all distortions in the allocation of resources within and between sectors that were highlighted in the general framework. In particular, we show that productive firms always produce too little and unproductive firms too much from a social perspective, and that the market delivers too little selection compared to the social optimum. Entry and the labor allocation are also inefficient, and the market allocates too many firms and workers to sectors with a higher concentration of low-productivity firms.

Quantifying the CES and VES models using data from France and the United Kingdom, we obtain four key results. First, there is a substantial aggregate welfare distortion both in France and in the UK. In the multi-sector VES model, it is equivalent to 6–8% of the total labor input in either country.[3] Second, intersectoral misallocations are crucial for this aggregate distortion. When we constrain the economy to consist of a single sector, thereby shutting down inefficiencies in entry and the labor allocation, the aggregate distortion can be 30% lower than the one predicted in the multi-sector economy. Put differently, a single-sector model yields downward-biased predictions for the total welfare loss. Third, we find that the multi-sector CES model predicts a much smaller aggregate distortion (of less than 1% in both France and the UK) than the VES model. The intuition is that the CES model displays efficient selection and firm-level outputs by construction. It therefore misses the cutoff and output distortions, which according to our results from the VES model account for an important part of the aggregate welfare loss. Last, at the sectoral level, we find similar patterns of inefficient entry and selection between the two countries. Insufficient entry arises almost exclusively for services, while manufacturing sectors tend to exhibit excessive entry. Manufacturing sectors are, however, more efficient when it comes to selecting the right set of surviving firms from the pools of entrants, i.e., they tend to exhibit smaller cutoff distortions than most service sectors.

Our paper is most closely related to the recent literature that investigates the efficiency of market allocations in models with a single monopolistically competitive sector, most notably Dhingra and Morrow (2014), Nocco, Ottaviano, and Salto (2014), and Parenti, Ushchev and Thisse (2016).[4] We make two contributions: first, our model considers multiple monopolis-

[3]This result relies on the concept of the *Allais surplus* (see Allais 1943, 1977) which determines the resource-cost minimizing allocation to achieve the equilibrium utility level.

[4]Our equilibrium analysis is related to Zhelobodko et al. (2012) and Mrazova and Neary (2014). It is more broadly related to the work by Fabinger and Weyl (2013) on pass-through under general demand structures.

tically competitive sectors while maintaining general additively separable preferences and productivity distributions. Second, while those papers focus exclusively on theory, we also explore the quantitative importance of various distortions.

Our work is also related to the classic literature in industrial organization that studies welfare implications of market power and inefficient entry for single industries in partial equilibrium. Harberger (1954) is a seminal reference for the former, and Mankiw and Whinston (1986) for the latter aspect. Our monopolistic competition model is complementary to this line of research, and recognizes general equilibrium interdependencies between sectors.

The rest of the paper is organized as follows. Section 2 presents our general model, while Section 3 turns to the specific solvable examples. The quantification procedure and results are discussed in Section 4. Section 5 concludes.

# 2 General model

Consider an economy with a mass $L$ of agents. Each agent is both a consumer and a worker, and supplies inelastically one unit of labor, which is the only factor of production. There are $j = 1, 2, \ldots, J$ sectors producing final consumption goods. Each good is supplied as a continuum of differentiated varieties, and each variety is produced by a single firm under monopolistic competition. Firms can differ by productivity, both within and between sectors. We denote by $G_j$ the continuously differentiable cumulative distribution function, from which firms draw their marginal labor requirement, $m$, after entering sector $j$. An entrant need not operate and only firms with high productivity $1/m$ survive. Let $N_j^E$ and $m_j^d$ be the mass of entrants and the marginal labor requirement of the least productive firm in sector $j$, respectively. Given $N_j^E$, a mass $N_j^E G_j(m_j^d)$ of varieties are then supplied by firms with $m \leq m_j^d$.

## 2.1 Equilibrium allocation

The utility maximization problem of a representative consumer is given by:

$$\max_{\{q_j(m),\ \forall j,m\}} \quad U \equiv U\big(U_1, U_2, \ldots, U_J\big)$$

$$U_j \equiv N_j^E \int_0^{m_j^d} u_j\big(q_j(m)\big) \mathrm{d}G_j(m)$$

$$\text{s.t.} \quad \sum_{j=1}^{J} N_j^E \int_0^{m_j^d} p_j(m) q_j(m) \mathrm{d}G_j(m) = w, \tag{1}$$

where $U$ is a strictly increasing and strictly concave *upper-tier utility* function that is twice continuously differentiable in all its arguments; $u_j$ is a strictly increasing, strictly concave, and thrice continuously differentiable sector-specific *subutility* function satisfying $u_j(0) = 0$; $p_j(m)$ and $q_j(m)$ are the price and consumption of a sector-$j$ variety produced with marginal labor requirement $m$; and $w$ denotes a consumer's income. We assume that $\lim_{U_j \to 0}(\partial U/\partial U_j) = \infty$ for all sectors to be active.

Let $\lambda$ denote the Lagrange multiplier associated with (1). The utility-maximizing consumptions satisfy the following first-order conditions:

$$u_j'\big(q_j(m)\big) = \lambda_j p_j(m), \quad \text{where} \quad \lambda_j \equiv \frac{\lambda}{\partial U/\partial U_j}. \tag{2}$$

To alleviate notation, let $p_j^d \equiv p_j(m_j^d)$ and $q_j^d \equiv q_j(m_j^d)$ denote the price set and quantity sold by the least productive firm operating in sector $j$, respectively. From the first-order conditions (2), which hold for any sector $j$ and any firm with $m \leq m_j^d$, we then have

$$\frac{u_j'(q_j^d)}{u_j'\big(q_j(m)\big)} = \frac{p_j^d}{p_j(m)} \quad \text{and} \quad \frac{u_j'(q_j^d)}{u_\ell'(q_\ell^d)} = \frac{\lambda_j}{\lambda_\ell}\frac{p_j^d}{p_\ell^d}, \tag{3}$$

which determine the equilibrium intra- and intersectoral consumption patterns, respectively.

We assume that the labor market is competitive, and that workers are mobile across sectors. All firms hence take the common wage $w$ as given. Turning to technology, entry into each sector $j$ requires to hire a sunk amount $F_j$ of labor paid at the market wage. After paying the sunk cost, $F_j w > 0$, each firm draws its marginal labor requirement from $G_j$, which is known to all firms. Conditional on survival, production takes place with constant marginal cost, $mw$, and sector-specific fixed cost, $f_j w \geq 0$.

Let $\pi_j(m)$ denote the operating profit of a firm with productivity $1/m$, divided by the wage rate $w$. Making use of condition (2), and of the equivalence between price and quantity as the firm's choice variable under monopolistic competition with a continuum of firms (Vives, 1999), the firm maximizes its operating profit

$$\pi_j(m) = L\left[\frac{u_j'\big(q_j(m)\big)}{\lambda_j w} - m\right] q_j(m) - f_j \tag{4}$$

with respect to quantity $q_j(m)$. Although $\lambda_j w$ contains the information of all the other sectors by (2), each firm takes this market aggregate as given because there is a continuum of firms. From (4), the profit-maximizing price satisfies

$$p_j(m) = \frac{mw}{1 - r_{u_j}\left(q_j(m)\right)}, \tag{5}$$

where $r_{u_j}(x) \equiv -xu_j''(x)/u_j'(x)$ denotes the 'relative risk aversion' or the 'relative love for variety' (Behrens and Murata, 2007; Zhelobodko et al., 2012).[5] In what follows, we refer to $1/[1 - r_{u_j}(q_j(m))]$ as the *private markup* charged by a firm that produces output $q_j(m)$.

To establish the existence and uniqueness of an equilibrium cutoff, $(m_j^d)^{\text{eqm}}$, and equilibrium quantities, $q_j^{\text{eqm}}(m)$ for all $m \in [0, m_j^d]$, we consider the *zero cutoff profit* (ZCP) condition, given by $\pi_j(m_j^d) = 0$, and the *zero expected profit* (ZEP) condition, defined as $\int_0^{m_j^d} \pi_j(m)\mathrm{d}G_j(m) = F_j$. Using (2), (4), and (5), the ZCP and ZEP conditions can be expressed respectively as follows:

$$\left[\frac{1}{1 - r_{u_j}\left(q_j^d\right)} - 1\right] m_j^d q_j^d = \frac{f_j}{L}, \tag{6}$$

$$L \int_0^{m_j^d} \left[\frac{1}{1 - r_{u_j}(q_j(m))} - 1\right] m q_j(m)\mathrm{d}G_j(m) = f_j G_j(m_j^d) + F_j, \tag{7}$$

which – even in our multi-sector economy – allow us to prove the existence and uniqueness of the sectoral cutoff and quantities as in the single-sector analysis by Zhelobodko et al. (2012).

**Proposition 1 (Equilibrium cutoff and quantities)** *Assume that the fixed costs, $f_j$, and sunk costs, $F_j$, are not too large. Then, the equilibrium cutoff and quantities $\{(m_j^d)^{\text{eqm}},\ q_j^{\text{eqm}}(m),\ \forall m \in [0, (m_j^d)^{\text{eqm}}]\}$ in each sector $j$ are uniquely determined.*

**Proof** *See Appendix A.1.* □

Turning to the equilibrium labor allocation, $L_j$, and the equilibrium mass of entrants, $N_j^E$, in each sector $j$, we first provide two important expressions that must hold in equilibrium.[6] We then establish the existence and uniqueness of the equilibrium labor allocation and entry.

**Lemma 1 (Labor allocation and entry)** *Any equilibrium labor allocation in sector $j = 1, 2, \ldots, J$ satisfies*

$$L_j = e_j L = \frac{\frac{R_j}{U_j}\mathcal{E}_{U,U_j}}{\sum_{\ell=1}^J \frac{R_\ell}{U_\ell}\mathcal{E}_{U,U_\ell}} L, \tag{8}$$

[5] We assume that the second-order conditions for profit maximization, $r_{u_j'}(x) \equiv -xu_j'''(x)/u_j''(x) < 2$ for all $j = 1, 2, \ldots, J$, hold (Zhelobodko et al., 2012, p.2771).

[6] To alleviate notation, we henceforth suppress the 'eqm' superscript when there is no possible confusion.

*where* $e_j \equiv N_j^E \int_0^{m_j^d} p_j(m)q_j(m)\mathrm{d}G_j(m)/w$ *is the expenditure share for sector-*$j$ *varieties;* $R_j/U_j$ *is the real revenue-to-utility ratio, where* $R_j \equiv N_j^E \int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)$ *is the sectoral real revenue; and* $\mathcal{E}_{U,U_j} \equiv (\partial U/\partial U_j)(U_j/U)$ *is the elasticity of the upper-tier utility function with respect to the lower-tier utility in sector* $j$*. Furthermore, any equilibrium mass of entrants satisfies*

$$N_j^E = e_j L \left\{ \frac{1 - \int_0^{m_j^d}[1 - r_{u_j}(q_j(m))]\nu_j(q_j(m))\mathrm{d}G_j(m)}{f_j G_j(m_j^d) + F_j} \right\}, \tag{9}$$

*where* $\nu_j(q_j(m)) = u_j'(q_j(m))q_j(m)/\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)$ *is the revenue share of a variety produced with marginal labor requirement* $m$ *in sector* $j$*.*

**Proof** *See Appendix B.1.* □

Lemma 1 shows that, in any equilibrium, the labor share $L_j/L$ must be the same as the expenditure share $e_j$ for all sectors. More importantly, the latter can be expressed by the real revenue-to-utility ratios $R_j/U_j$ and the elasticities $\mathcal{E}_{U,U_j}$ of the upper-tier utility function. We will discuss the intuition of those terms in Section 2.3. The mass of entrants is more complicated since it is affected not only by $e_j$, but also by effective entry cost $f_j G_j(m_j^d) + F_j$, the distribution of the markup terms $1 - r_{u_j}(q_j(m))$, and the revenue shares $\nu_j(q_j(m))$. It is worth emphasizing that we have not specified functional forms for either utility or productivity distributions to derive those results.

Note that Lemma 1 does not yet imply existence and uniqueness of the equilibrium labor allocation and the equilibrium mass of entrants. The reason is that, while the expression in the braces in (9) is uniquely determined by Proposition 1, the expenditure share $e_j$ can depend on $\{N_j^E\}_{j=1,2,\dots,J}$ via $\mathcal{E}_{U,U_j}$. Thus, to establish those properties, we impose some separability on the upper-tier utility function. More specifically, assume that the derivative of the upper-tier utility function with respect to the lower-tier utility in each sector can be divided into an own-sector and an economy-wide component as follows:

$$\frac{\partial U}{\partial U_j} = \gamma_j U_j^{\xi_j} U^{\xi}, \tag{10}$$

where $\gamma_j > 0$, $\xi_j < 0$, and $\xi > 0$ are parameters.[7] Specification (10) includes, for example,

[7]The crucial points are that, under condition (10), the ratio of the derivatives in (2) with respect to $j$ and $\ell$ depends on $N_j^E$ and $N_\ell^E$ only, and that it satisfies some monotonicity properties. Should the ratio of the derivatives in (2) depend on all $N_i^E$ terms, the system of equations becomes generally intractable.

the cases where the upper-tier utility function is of either the Cobb-Douglas or the CES form. When condition (10) holds, we can prove the following result:

**Proposition 2 (Equilibrium labor allocation and entry)** *Assume that* (10) *holds. Then, the equilibrium labor allocation and masses of entrants* $\{L_j^{\text{eqm}}, (N_j^E)^{\text{eqm}}\}_{j=1,2,\ldots,J}$ *are uniquely determined by* (8) *and* (9).

**Proof** *See Appendix A.2.* □

## 2.2 Optimal allocation

Having analyzed the equilibrium allocation, we turn to the optimal allocation.[8] Assume that the planner chooses the quantities, cutoffs, and masses of entrants to maximize welfare subject to the resource constraint of the economy as follows:

$$\max_{\{q_j(m), m_j^d, N_j^E, \forall j, m\}} \quad L \cdot U\big(U_1, U_2, \ldots, U_J\big)$$

$$U_j \equiv N_j^E \int_0^{m_j^d} u_j(q_j(m)) \mathrm{d}G_j(m)$$

$$\text{s.t.} \quad \sum_{j=1}^{J} N_j^E \left\{ \int_0^{m_j^d} \left[ Lmq_j(m) + f_j \right] \mathrm{d}G_j(m) + F_j \right\} = L. \tag{11}$$

The planner has no control over the uncertainty of the draws of $m$, but knows the underlying distributions $G_j$. Let $\delta$ denote the Lagrange multiplier associated with (11). The first-order conditions with respect to quantities, cutoffs, and the masses of entrants are given by:

$$u_j'(q_j(m)) = \delta_j m, \quad \delta_j \equiv \frac{\delta}{\partial U / \partial U_j} \tag{12}$$

$$L \frac{u_j(q_j^d)}{\delta_j} = Lm_j^d q_j^d + f_j \tag{13}$$

$$L \int_0^{m_j^d} \frac{u_j(q_j(m))}{\delta_j} \mathrm{d}G_j(m) = \int_0^{m_j^d} \left[ Lmq_j(m) + f_j \right] \mathrm{d}G_j(m) + F_j. \tag{14}$$

[8]In the main text, we consider the 'primal' first-best problem where the planner maximizes utility subject to the economy's resource constraint. When quantifying the gap between the equilibrium and the optimum in Section 4, we will analyze a 'dual' problem where the planner minimizes the resource cost subject to a utility level. The latter allows us to derive a welfare measure – the so-called Allais surplus (Allais, 1943, 1977) – that can be used in contexts where equivalent or compensating variations (or related criteria to compare different equilibria) cannot be readily applied. More details are relegated to Appendix D and the supplementary Appendix F.

From the first-order conditions (12), which hold for any sector $j$ and any firm with $m \leq m_j^d$, we then have

$$\frac{u_j'(q_j^d)}{u_j'\big(q_j(m)\big)} = \frac{m_j^d}{m} \quad \text{and} \quad \frac{u_j'(q_j^d)}{u_\ell'(q_\ell^d)} = \frac{\delta_j}{\delta_\ell}\frac{m_j^d}{m_\ell^d}, \tag{15}$$

which determine the optimal intra- and intersectoral consumption patterns, respectively.

We start again with the cutoff and quantities. Noting that $\delta_j = u_j'(q_j(m))/m$ for any value of $m$ from (12), we can rewrite condition (14) as follows:

$$L\int_0^{m_j^d}\left[\frac{1}{\mathcal{E}_{u_j,q_j(m)}} - 1\right] mq_j(m)\mathrm{d}G_j(m) = f_jG_j(m_j^d) + F_j, \tag{16}$$

where $\mathcal{E}_{u_j,q_j(m)} \equiv q_j(m)u_j'(q_j(m))/u_j(q_j(m))$ is the elasticity of the subutility $u_j$. We refer to $1/\mathcal{E}_{u_j,q_j(m)}$ as the *social markup* that a firm with marginal labor requirement $m$ should optimally charge, and to $m/\mathcal{E}_{u_j,q_j(m)}$ as the *shadow price* of a variety produced by a firm with $m$ in sector $j$.[9] Condition (16) may then be understood as the *zero expected social profit* (ZESP) condition, which is analogous to the ZEP condition (7). Furthermore, evaluating (12) at $m_j^d$ and plugging the resulting expression into (13), we obtain an expression similar to the ZCP condition (6) as follows:

$$\left(\frac{1}{\mathcal{E}_{u_j,q_j^d}} - 1\right) m_j^d q_j^d = \frac{f_j}{L}, \tag{17}$$

which we call the *zero cutoff social profit* (ZCSP) condition. Using (16) and (17), we can establish the existence and uniqueness of the sectoral cutoff and quantities.

**Proposition 3 (Optimal cutoff and quantities)** *Assume that the fixed costs, $f_j$, and the sunk costs, $F_j$, are not too large. Then, the optimal cutoff and quantities $\{(m_j^d)^{\text{opt}}, q_j^{\text{opt}}(m),\ \forall m \in [0, (m_j^d)^{\text{opt}}]\}$ in each sector $j$ are uniquely determined.*

**Proof** *See Appendix A.3.* □

Turning next to the optimal labor allocation, $L_j$, and the optimal masses of entrants, $N_j^E$, we proceed in the same way as for the equilibrium case, and provide the following two expressions.

[9] Dhingra and Morrow (2014) refer to $1 - \mathcal{E}_{u_j,q_j(m)} = [u_j(q_j(m)) - \delta_j mq_j(m)]/u_j(q_j(m))$ as the social markup, which captures the utility from consumption of a variety net of its resource costs. Moreover, they label $[p_j(m) - mw]/p_j(m) = r_{u_j}(q_j(m))$ as the private markup. We adopt their terminology but redefine the two markups in a slightly different way.

**Lemma 2 (Labor allocation and entry)** *Any optimal labor allocation in sector $j = 1, 2, \ldots, J$ satisfies*

$$L_j = e_j L = \frac{\mathcal{E}_{U,U_j}}{\sum_{\ell=1}^{J} \mathcal{E}_{U,U_\ell}} L, \tag{18}$$

*where $e_j \equiv N_j^E \int_0^{m_j^d} m q_j(m) / \mathcal{E}_{u_j,q_j(m)} \mathrm{d}G_j(m)$ is the social expenditure share for sector-$j$ varieties constructed by using their shadow prices and optimal quantities. Furthermore, any optimal mass of entrants satisfies*

$$N_j^E = e_j L \left\{ \frac{1 - \int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)} \zeta_j(q_j(m)) \mathrm{d}G_j(m)}{f_j G_j(m_j^d) + F_j} \right\}, \tag{19}$$

*where $\zeta_j(q_j(m)) \equiv u_j(q_j(m)) / \int_0^{m_j^d} u_j(q_j(m)) \mathrm{d}G_j(m)$ captures the relative contribution of a variety produced with marginal labor requirement $m$ to utility in sector $j$.*

**Proof** *See Appendix B.2.* □

Lemma 2 shows that, in any optimal allocation, the sectoral labor share must be the same as the sectoral expenditure share. Observe that this expression is analogous to that in equilibrium, with the private expenditure share being replaced by the social expenditure share. The latter can be expressed solely in terms of the elasticities $\mathcal{E}_{U,U_j}$ of the upper-tier utility function, and does not involve the real revenue-to-utility ratio. The optimal mass of entrants is again more complicated since it also includes effective entry costs, the distribution of social markup terms $\mathcal{E}_{u_j,q_j(m)}$, and the shares $\zeta_j(q_j(m))$ that capture the relative contribution of a variety produced with marginal labor requirement $m$ to utility in sector $j$.

Finally, similarly as in the equilibrium analysis, Lemma 2 does not yet imply the existence and uniqueness of the optimal labor allocation and the optimal masses of entrants. We thus impose again the separability condition (10) to establish those properties as follows:

**Proposition 4 (Optimal labor allocation and entry)** *Assume that* (10) *holds. Then, the optimal labor allocation and masses of entrants $\{L_j^{\text{opt}}, (N_j^E)^{\text{opt}}\}_{j=1,2,\ldots,J}$ are uniquely determined by* (18) *and* (19).

**Proof** *See Appendix A.4.* □

## 2.3 Equilibrium versus optimum

Having established existence and uniqueness of the equilibrium and optimal allocations in Propositions 1–4, we are now ready to investigate how equilibrium and optimum generally differ.

First, there are cutoff and output distortions within sectors. By the proofs of Propositions 1 and 3, we know that $\lambda_j w$ and $\delta_j$ are uniquely determined without any information on the other sectors. Hence, we can study the equilibrium and optimal cutoffs and quantities on a sector-by-sector basis. The analysis of cutoff and quantity distortions in each sector $j$ then works as in the single-sector model by Dhingra and Morrow (2014) who characterize those inefficiencies solely by the properties of $u_j$ and $G_j$. We shall not repeat their general analysis here, but we illustrate it in the next section using specific examples.

The novel feature of our model lies in labor and entry distortions between sectors. It is important to notice that, unlike the cutoff and quantity distortions, characterizing labor and entry distortions for one sector requires information on all sectors. Put differently, the labor allocation and, thus, entry are interdependent when there are multiple sectors. Hence, entry distortions in our multi-sector model generally differ from those in models with a single imperfectly competitive sector such as Mankiw and Whinston (1986) and Dhingra and Morrow (2014).

To characterize the labor and entry distortions, compare expressions (8) and (9) from Lemma 1 with (18) and (19) from Lemma 2. We then obtain the following expressions:

$$\frac{L_j^{\mathrm{eqm}}}{L_j^{\mathrm{opt}}} = \frac{e_j^{\mathrm{eqm}}}{e_j^{\mathrm{opt}}} \tag{20}$$

$$\frac{(N_j^E)^{\mathrm{eqm}}}{(N_j^E)^{\mathrm{opt}}} = \frac{e_j^{\mathrm{eqm}}}{e_j^{\mathrm{opt}}} \cdot \frac{f_j G_j((m_j^d)^{\mathrm{opt}}) + F_j}{f_j G_j((m_j^d)^{\mathrm{eqm}}) + F_j} \cdot \frac{1 - \int_0^{(m_j^d)^{\mathrm{eqm}}} [1 - r_{u_j}(q_j(m))]\nu_j(q_j(m))\mathrm{d}G_j(m)}{1 - \int_0^{(m_j^d)^{\mathrm{opt}}} \mathcal{E}_{u_j, q_j(m)} \zeta_j(q_j(m))\mathrm{d}G_j(m)}. \tag{21}$$

Starting with the labor allocation, expression (20) implies that the equilibrium labor allocation is efficient – i.e., $L_j^{\mathrm{eqm}} = L_j^{\mathrm{opt}}$ – if and only if equilibrium expenditure shares coincide with their optimal counterparts — i.e., $e_j^{\mathrm{eqm}} = e_j^{\mathrm{opt}}$. Otherwise, the labor share is excessive (insufficient) in sectors with a suboptimally large (small) budget share.

Turning to the entry distortions, expression (21) shows that $(N_j^E)^{\mathrm{eqm}}/(N_j^E)^{\mathrm{opt}}$ depends on three terms. The first term $e_j^{\mathrm{eqm}}/e_j^{\mathrm{opt}}$ is equivalent to $L_j^{\mathrm{eqm}}/L_j^{\mathrm{opt}}$ by (20). In a single-sector model, this term vanishes because $L_j^{\mathrm{eqm}} = L_j^{\mathrm{opt}} = L$. In a multi-sector model, however, the gap between $e_j^{\mathrm{eqm}}$ and $e_j^{\mathrm{opt}}$ plays a crucial role. We will come to this term below. The second and third terms capture two additional margins, namely 'effective fixed costs' and 'private and social markups', which we explain in turn.[10]

[10]Note that the cutoff and quantity distortions can influence the entry distortions, although the former inefficiencies do not depend on the latter ones.

**Effective fixed costs.** The second term in (21) shows that if the market delivers too little selection, $(m_j^d)^{\text{eqm}} > (m_j^d)^{\text{opt}}$, entry tends to be insufficient. The reason is that the higher survival probability in equilibrium, as compared to the optimum, increases the expected fixed costs that entrants have to pay. This reduces expected profitability and discourages entry more in equilibrium than in optimum. In contrast, other things equal, too much equilibrium selection, $(m_j^d)^{\text{eqm}} < (m_j^d)^{\text{opt}}$, leads to excessive entry.

**Private and social markups.** The last term in (21) shows that the gap between equilibrium and optimal entry depends on the private and social markup terms, which may exacerbate or attenuate excess entry (Mankiw and Whinston, 1986; Dhingra and Morrow, 2014). The numerator can be related to the *business stealing effect*: the higher the private markups $1/[1 - r_{u_j}(q_j(m))]$, the more excessive the entry. The denominator, in turn, captures the *limited appropriability effect*: the greater the social markups $1/\mathcal{E}_{u_j,q_j(m)}$, the more insufficient the entry. Thus, the last term in (21) depends on the relative strength of these two effects, as well as on the weighting schemes $\nu_j(q_j(m))$ and $\zeta_j(q_j(m))$ that are determined by the properties the subutility function $u_j$ and the productivity distribution function $G_j$.

The following Proposition summarizes the general result for distortions in the labor allocation (20) and thus for the first term of entry distortions (21).

**Proposition 5 (Distortions in the labor allocation)** *The equilibrium and optimal labor allocations satisfy* $L_j^{\text{eqm}} \gtreqless L_j^{\text{opt}}$ *if and only if the equilibrium and optimal expenditure shares satisfy* $e_j^{\text{eqm}} \gtreqless e_j^{\text{opt}}$*, which is equivalent to*

$$\frac{\frac{R_j}{U_j}\mathcal{E}_{U,U_j}^{\text{eqm}}}{\sum_{\ell=1}^{J}\frac{R_\ell}{U_\ell}\mathcal{E}_{U,U_\ell}^{\text{eqm}}} \gtreqless \frac{\mathcal{E}_{U,U_j}^{\text{opt}}}{\sum_{\ell=1}^{J}\mathcal{E}_{U,U_\ell}^{\text{opt}}}, \tag{22}$$

*where the left- and right-hand sides are evaluated at the equilibrium and at the optimum, respectively. Assume, without loss of generality, that sectors are ordered such that* $e_j^{\text{eqm}}/e_j^{\text{opt}}$ *is non-decreasing in* $j$*. If there are at least two different* $e_j^{\text{eqm}}/e_j^{\text{opt}}$*'s, then there exists a threshold* $j^* \in \{1, 2, \dots, J-1\}$ *such that the equilibrium labor allocation is insufficient for sectors* $j \le j^*$*, whereas it is excessive for sectors* $j > j^*$*. The equilibrium labor allocation is optimal if and only if all* $e_j^{\text{eqm}}/e_j^{\text{opt}}$ *terms are the same.*

**Proof** *See Appendix A.5.* □

As can be seen from (22), the interdependence of heterogeneous sectors is important for distortions in the labor allocation. Which sectors display excess labor allocation depends on two types of heterogeneity: the sectoral real revenue-to-utility ratios, $R_j/U_j$ evaluated at the equilibrium; and the elasticities of the upper-tier utility function, $\mathcal{E}_{U,U_j}$, evaluated at the equilibrium and optimum. We now shed more light on the importance of those two components to build the intuition for this result.

**Real revenue-to-utility ratios.** In our setting with heterogeneous firms, the real revenue-to-utility ratio in sector $j$ can be expressed as follows:

$$\frac{R_j}{U_j} = \frac{\int_0^{m_j^d} u_j'(q_j(m))q_j(m)\mathrm{d}G_j(m)}{\int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)} = \int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)}\zeta_j(q_j(m))\mathrm{d}G_j(m) < 1, \tag{23}$$

where $\zeta_j(q_j(m))$ is the relative contribution of a variety to utility in sector $j$; and where the inverse of the social markup, $\mathcal{E}_{u_j,q_j(m)}$, captures the appropriability of $\zeta_j(q_j(m))$ by a firm with productivity $1/m$. The inequality in (23) holds because $\mathcal{E}_{u_j,q_j(m)} < 1$ for all $m \in [0, m_j^d]$ by concavity of $u_j$, and because $\int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)}\zeta_j(q_j(m))\mathrm{d}G_j(m) < \int_0^{m_j^d} \zeta_j(q_j(m))\mathrm{d}G_j(m) = 1$ by the definition of $\zeta_j(q_j(m))$ in Lemma 2.

Expression (23) shows that the revenue-to-utility ratio $R_j/U_j$ tends to increase with a greater appropriability, especially for varieties associated with a greater utility weight $\zeta_j(q_j(m))$. By construction, $R_j/U_j$ depends on the properties of $u_j$ and $G_j$. Using the definition of the covariance and $\int_0^{m_j^d} \zeta_j(q_j(m))\mathrm{d}G_j(m) = 1$, we can also rewrite (23) as:

$$\begin{aligned}\frac{R_j}{U_j} &= \left[\int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)}\mathrm{d}G_j(m)\right]\left[\int_0^{m_j^d} \zeta_j(q_j(m))\mathrm{d}G_j(m)\right] + \mathrm{cov}\left(\mathcal{E}_{u_j,q_j(m)}, \zeta_j(q_j(m))\right) \\ &= \overline{\mathcal{E}}_{u_j,q_j(m)} + \mathrm{cov}\left(\mathcal{E}_{u_j,q_j(m)}, \zeta_j(q_j(m))\right),\end{aligned}$$

where $\overline{\mathcal{E}}_{u_j,q_j(m)} \equiv \int_0^{m_j^d} \mathcal{E}_{u_j,q_j(m)}\mathrm{d}G_j(m)$. Hence, the labor allocation tends to be more excessive in sectors with higher average appropriability $\overline{\mathcal{E}}_{u_j,q_j(m)}$ and with larger covariance, i.e., when varieties with higher approporiability tend to contribute more to the consumers' utility.

**Elasticities of the upper-tier utility function.** The second key ingredient of the labor distortions in (22) are the equilibrium and optimal elasticities of the upper-tier utility function, $\mathcal{E}_{U,U_j}^{\mathrm{eqm}}$ and $\mathcal{E}_{U,U_j}^{\mathrm{opt}}$. Consumers allocate more expenditure to varieties produced in sectors with higher elasticity. Other things equal, the higher the equilibrium upper-tier elasticities relative to the optimal counterpart in one sector, the higher the equilibrium expenditure share relative to the optimal counterpart in that sector, thereby leading to more excessive entry.

It is worth emphasizing that even when the equilibrium and optimal elasticities of upper-tier utility in each sector are the same, i.e., $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j}$, their sectoral heterogeneity plays a crucial role in the labor and entry distortions. Indeed, although $\mathcal{E}^{\text{eqm}}_{U,U_j}$ and $\mathcal{E}^{\text{opt}}_{U,U_j}$ in the numerator of (22) cancel each other out when they are identical, the elasticities in the denominator remain. We will elaborate on this point in the next section where we illustrate some examples.

To sum up, the difference between market equilibrium and social optimum in terms of the labor allocation and firm entry across heterogeneous sectors depends, in general, on four key ingredients: effective fixed costs; private and social markups; real revenue-to-utility ratios; and the elasticities of the upper-tier utility. While distortions in a single-sector model are characterized solely by $u_j$ and $G_j$ *for that sector* (Dhingra and Morrow, 2014), in a multi-sector setting characterizing distortions *for one sector* requires additional information on the revenue-to-utility ratios $R_j/U_j$ and the elasticities of the upper-tier utility $\mathcal{E}_{U,U_j}$ *for all sectors*. Hence, when assessing distortions we need to take into account the interdependence between heterogeneous sectors.

# 3 Examples

We have so far made only few assumptions on functional forms. To derive sharper results, and ultimately to take our model to the data, we now consider specific functional forms for both the subutilities and the upper-tier utility that allow for simple closed-form solutions.

Starting with the subutility function $u_j$, we first analyze in Section 3.1 the ubiquitous CES case that has dominated much of the literature on monopolistic competition. We then turn to a tractable 'variable elasticity of substitution' (VES) model in Section 3.2

In doing so, notice that the lower-tier utility $U_j$ in specification (1) does not nest the standard homothetic CES aggregator. To nest it, we consider a simple monotonic transformation of the lower-tier utility in (1) as $\widetilde{U}_j(U_j)$. In Section 3.1 we assume that $\widetilde{U}_j(U_j) = U_j^{1/\rho_j} = [N_j^E \int_0^{m_j^d} q_j(m)^{\rho_j} \mathrm{d}G_j(m)]^{1/\rho_j}$, whereas we retain $\widetilde{U}_j(U_j) = U_j$ in Section 3.2.

Even with the transformation $\widetilde{U}_j$ of the lower-tier utility, we can re-establish the general results shown in Section 2, as long as we let $\widetilde{U}_j(0) = 0$, $\widetilde{U}_j' > 0$, and $\lim_{U_j \to \infty} \widetilde{U}_j(U_j) = \infty$, while replacing the condition in (10) with[11]

$$\frac{\partial U}{\partial \widetilde{U}_j}\frac{\partial \widetilde{U}_j}{\partial U_j} = \gamma_j \widetilde{U}_j^{\xi_j} U^{\xi}, \tag{24}$$

where $\gamma_j > 0$, $\xi_j < 0$, and $\xi > 0$ are parameters.

Turning to the upper-tier utility function, we consider in the remainder of this paper the standard CES form: $U = \{\sum_{j=1}^J \beta_j [\widetilde{U}_j(U_j)]^{(\sigma-1)/\sigma}\}^{\sigma/(\sigma-1)}$, where $\sigma \geq 1$, $\beta_j > 0$ for all $j$, and $\sum_{j=1}^J \beta_j = 1$. Thus, the elasticity of the upper-tier utility function is given by $\mathcal{E}_{U,U_j} \equiv (\partial U/\partial \widetilde{U}_j)(\partial \widetilde{U}_j/\partial U_j)(U_j/U) = \beta_j(\partial \widetilde{U}_j/\partial U_j)(U_j/\widetilde{U}_j)(\widetilde{U}_j/U)^{(\sigma-1)/\sigma}$. When $\sigma \to 1$, the upper-tier utility reduces to the Cobb-Douglas form, $U = \prod_{j=1}^J [\widetilde{U}_j(U_j)]^{\beta_j}$, so that

$$\mathcal{E}_{U,U_j} = \beta_j \left(\frac{\partial \widetilde{U}_j}{\partial U_j}\frac{U_j}{\widetilde{U}_j}\right). \tag{25}$$

The Cobb-Douglas upper-tier utility function always satisfies condition (24) that guarantees the existence and uniqueness of the equilibrium and optimal allocations. When the upper-tier utility function is of the CES form, whereas the lower-tier utility is of the homothetic CES form with $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$, we have $(\partial U/\partial \widetilde{U}_j)(\partial \widetilde{U}_j/\partial U_j) = (\beta_j/\rho_j)\widetilde{U}_j^{\frac{\sigma-1}{\sigma}-\rho_j} U^{1/\sigma}$. Hence, in that case, it is required that $(\sigma-1)/\sigma < \rho_j$ for condition (24) to hold with $\xi_j < 0$.[12]

Those specifications for the upper-tier utility function significantly simplify the analysis of entry and labor distortions. Retaining $\sigma \to 1$ for now, we consider two specific forms for the subutility functions for which the real revenue-to-utility ratios display a simple behavior. We will return to the case with $\sigma > 1$ in Section 4 where we quantify the model.

### 3.1 CES subutility

We first discuss the case of the CES subutility that has been widely used in the literature. Assume that $u_j(q_j(m)) = q_j(m)^{\rho_j}$ and $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$, where $\rho_j \in (0,1)$ for all sectors $j$. Using (25), the elasticity of the upper-tier utility function can be rewritten as $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j} = \beta_j/\rho_j$, whereas the elasticity of substitution between any pair of varieties in sector $j$ is given

[11] The proofs are virtually identical to the ones in Appendices A and B, except that $\partial U/\partial U_j$ needs to be replaced with $(\partial U/\partial \widetilde{U}_j)(\partial \widetilde{U}_j/\partial U_j)$. Observe that in a single-sector model, the choice of $\widetilde{U}_j$ does not affect distortions because it is a monotonic transformation of the overall utility in that case. In a multi-sector model, however, sectoral allocations and thus aggregate distortions are affected by $\widetilde{U}_j$.

[12] Should $(\sigma-1)/\sigma > \rho_j$ hold, goods are Hicks-Allen complements (see, e.g., Matsuyama, 1995), so that multiple equilibria with some inactive sectors may arise, a case that we exclude from our analysis in what follows.

by $1/(1-\rho_j)$. Notice that we allow $\rho_j$ to differ across sectors.

Since the cutoff and quantity distortions can be separated from the labor and entry distortions, we can use the single-sector result by Dhingra and Morrow (2014), i.e., in the CES case $(m_j^d)^{\text{eqm}} = (m_j^d)^{\text{opt}}$ and $q_j^{\text{eqm}}(m) = q_j^{\text{opt}}(m)$ for all $m$ irrespective of the underlying productivity distribution $G_j$. However, the equilibrium labor allocation and entry need not be optimal. By Proposition 5, the real revenue-to-utility ratios $R_j/U_j$ evaluated at the equilibrium, together with the elasticities of upper-tier utility $\mathcal{E}_{U,U_j}^{\text{eqm}} = \mathcal{E}_{U,U_j}^{\text{opt}} = \beta_j/\rho_j$, are crucial for those inefficiencies. When the subutility function is of the CES form, we know that $\mathcal{E}_{u_j,q_j(m)} = \rho_j$, so that $R_j/U_j = \rho_j$ by (23). Furthermore, since $(m_j^d)^{\text{eqm}} = (m_j^d)^{\text{opt}}$, $q_j^{\text{eqm}}(m) = q_j^{\text{opt}}(m)$, $\mathcal{E}_{u_j,q_j(m)} = 1 - r_j(q_j(m))$, and $\nu_j(q_j(m)) = \zeta_j(q_j(m))$ for all $m$ holds in the CES case, the second and the third terms in (21) vanish, so that $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} = e_j^{\text{eqm}}/e_j^{\text{opt}} = L_j^{\text{eqm}}/L_j^{\text{opt}}$. Hence, we can restate Proposition 5 for this specific example as follows:

**Corollary 1 (Distortions in the labor allocation and entry with CES subutility)** *Assume that the subutility function in each sector is of the CES form, $u_j(q_j(m)) = q_j(m)^{\rho_j}$. Then, the labor allocations and the masses of entrants satisfy $L_j^{\text{eqm}} \gtreqless L_j^{\text{opt}}$ and $(N_j^E)^{\text{eqm}} \gtreqless (N_j^E)^{\text{opt}}$, respectively, if and only if*

$$\sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell} \gtreqless \frac{1}{\rho_j}. \tag{26}$$

*Assume, without loss of generality, that sectors are ordered such that $\rho_j$ is non-decreasing in $j$. If there are at least two different $\rho_j$'s, there exists a threshold $j^* \in \{1, 2, \ldots, J-1\}$ such that sectors $j \leq j^*$ display insufficient entry and insufficient labor allocation, whereas sectors $j > j^*$ display excess entry and excess labor allocation. The equilibrium allocation in the CES case is optimal if and only if all $\rho_j$'s are the same across sectors.*

**Proof** *See above.* □

Several comments are in order. First, since there are no cutoff and quantity distortions in the case of CES subutility functions, the market equilibrium is fully efficient if and only if the $\rho_j$'s are the same across all sectors. However, there are distortions in the labor allocation and in the masses of entrants when the $\rho_j$'s vary across sectors.[13]

Second, $\rho_j$ in the CES model can be related not only to the inverse of the markup, but also to the elasticity of the upper-tier utility $\mathcal{E}_{U,U_j}$ and to the elasticity of the subutility $\mathcal{E}_{u_j,q_j(m)}$. It is the latter two elasticities that matter for the labor and entry distortions. The reason

[13]Hsieh and Klenow (2009) consider a heterogeneous firms model where $\rho_j$'s are the same across all sectors. In contrast, Epifani and Gancia (2011) allow for heterogeneity in $\rho_j$ across sectors yet consider homogeneous firms within sectors.

is that the difference between the equilibrium and optimal expenditure shares comes from $R_j/U_j = \rho_j$ and $\mathcal{E}_{U,U_j} = \beta_j/\rho_j$, which are determined by the first derivatives of $u_j$ and $\widetilde{U}_j$ as seen from (23) and (25). In contrast, the markup depends on $r_{u_j}$, which involves the second derivative of $u_j$. Thus, in the case of the Cobb-Douglas upper-tier utility and CES subutility functions, markup heterogeneity is not a determinant of labor and entry distortions.

Third, Corollary 1 holds irrespective of the functional form for $G_j$. Hence, productivity distributions play no role in the optimality of the market outcome for the standard case with the Cobb-Douglas upper-tier utility and CES subutility functions.

Last, since Corollary 1 only pertains to the class of CES subutility functions, it must not be read as a general 'if and only if' result for any subutility function. Indeed, as we show in the next subsection, the labor allocation and entry can be efficient even when the subutility function is *not* of the CES form.

## 3.2 VES subutility

We have so far examined the case of CES subutility functions without cutoff and quantity distortions. We now turn to our VES example where all types of distortions – cutoff, quantity, labor, and entry distortions – can operate. Specifically, we consider the 'constant absolute risk aversion' (CARA) subutility as in Behrens and Murata (2007), $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, where $\alpha_j$ is a strictly positive parameter.

This specification can be viewed as an example of VES preferences analyzed in the seminal paper by Krugman (1979). It is analytically tractable, and generates demand functions exhibiting smaller price elasticity at higher consumption levels. Unlike the CES model, this VES case can therefore account for the empirically well-documented facts of incomplete pass-through and higher markups charged by more productive firms within each sector.

To derive closed-form solutions, assume that $G_j$ follows a Pareto distribution $G_j(m) = \left(m/m_j^{\max}\right)^{k_j}$, where both the upper bounds $m_j^{\max} > 0$ and the shape parameters $k_j \geq 1$ may differ across sectors. In what follows, we assume that $\widetilde{U}_j(U_j) = U_j$, so that $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j} = \beta_j$ by (25). We relegate most analytical details for the case with CARA subutilities and Pareto productivity distributions to the supplementary Appendix E. We show there that the equilibrium and optimal cutoffs for this case are given as follows:

$$(m_j^d)^{\text{eqm}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j}}{\kappa_j L}\right]^{\frac{1}{k_j+1}} \quad \text{and} \quad (m_j^d)^{\text{opt}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j} (k_j+1)^2}{L}\right]^{\frac{1}{k_j+1}}, \qquad (27)$$

where $\kappa_j \equiv k_j \mathrm{e}^{-(k_j+1)} \int_0^1 (1+z)\left(z^{-1}+z-2\right)(z\mathrm{e}^z)^{k_j} \mathrm{e}^z \mathrm{d}z > 0$ is a function of the shape parameter $k_j$ only. Using expressions (27), we can establish the following result:

**Proposition 6 (Distortions in the cutoff and quantities with CARA subutility)** *Assume that the subutility function in each sector is of the CARA form $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, and that the productivity distribution follows a Pareto distribution, $G_j(m) = (m/m_j^{\max})^{k_j}$. Then, the equilibrium cutoff exceeds the optimal cutoff in each sector, i.e., $(m_j^d)^{\text{eqm}} > (m_j^d)^{\text{opt}}$. Furthermore, there exists a unique threshold $m_j^* \in (0, (m_j^d)^{\text{opt}})$ such that $q_j^{\text{eqm}}(m) < q_j^{\text{opt}}(m)$ for all $m \in [0, m_j^*)$ and $q_j^{\text{eqm}}(m) > q_j^{\text{opt}}(m)$ for all $m \in (m_j^*, (m_j^d)^{\text{eqm}})$.*

**Proof** *See Appendix A.6.* □

Three comments are in order. First, in this model, more productive firms with $m < m_j^*$ underproduce, whereas less productive firms with $m > m_j^*$ overproduce in equilibrium as compared to the optimum in each sector $j$. Notice that both types of firms coexist in equilibrium since the threshold $m_j^*$ satisfies the inequalities $0 < m_j^* < (m_j^d)^{\text{opt}} < (m_j^d)^{\text{eqm}}$.[14]

Second, using (27), the gap between the equilibrium and optimal selection can be expressed as a simple function of the sectoral shape parameter only: $(m_j^d)^{\text{opt}}/(m_j^d)^{\text{eqm}} = \left[\kappa_j(k_j+1)^2\right]^{1/(k_j+1)} < 1$. Since this expression increases with $k_j$, the larger the value of $k_j$ (i.e., a larger mass of the productivity distribution is concentrated on low-productivity firms) the smaller is the magnitude of insufficient selection in sector $j$.

Finally, Proposition 6 holds on a sector-by-sector basis, regardless of the labor allocation and the masses of entrants. Thus, our results on cutoff and quantity distortions would also apply to a single-sector version of the CARA model.

Turning to the labor and entry distortions, the combination of CARA subutility functions and Pareto productivity distributions yields the equilibrium and optimal masses of entrants as follows (see expressions (E-17) and (E-30) in the supplementary Appendix E):

$$(N_j^E)^{\text{eqm}} = \frac{e_j^{\text{eqm}} L}{(k_j+1)F_j} = \frac{L_j^{\text{eqm}}}{(k_j+1)F_j} \quad \text{and} \quad (N_j^E)^{\text{opt}} = \frac{e_j^{\text{opt}} L}{(k_j+1)F_j} = \frac{L_j^{\text{opt}}}{(k_j+1)F_j}. \tag{28}$$

Thus, as in the CES case, we have $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} = e_j^{\text{eqm}}/e_j^{\text{opt}} = L_j^{\text{eqm}}/L_j^{\text{opt}}$. From Proposition 5 we know that distortions in the labor allocation are determined by the real

[14]This need not always be the case, however. For example, Dhingra and Morrow (2014) derive general conditions for cutoff and quantity distortions in a single-sector framework. In their model with an arbitrary subutility function and an arbitrary productivity distribution, it is possible that $m_j^*$ exceeds $(m_j^d)^{\text{eqm}}$. In that case, all firms (even the least productive ones) would underproduce, whereas in our model some firms (the least productive ones) always overproduce from a social point of view.

revenue-to-utility ratios $R_j/U_j$ evaluated at the equilibrium, together with the elasticities of the upper-tier utility function $\mathcal{E}^{\text{eqm}}_{U,U_j} = \mathcal{E}^{\text{opt}}_{U,U_j} = \beta_j$. When the subutility function is of the CARA form and the productivity distribution follows a Pareto distribution, we can show that $R_j/U_j$ depends solely on the sectoral shape parameter $k_j$ as follows:

**Lemma 3** *With a CARA subutility function and a Pareto productivity distribution, we have:*

$$\frac{R_j}{U_j} = \frac{\int_0^1 (1-z)\mathrm{e}^{z-1}(1+z)\mathrm{e}^{z-1}\left(z\mathrm{e}^{z-1}\right)^{k_j-1}\mathrm{d}z}{\int_0^1 (1-\mathrm{e}^{z-1})(1+z)\mathrm{e}^{z-1}\left(z\mathrm{e}^{z-1}\right)^{k_j-1}\mathrm{d}z} \equiv \theta_j. \tag{29}$$

**Proof** *See Appendix B.3.* □

To characterize the labor and entry distortions, we rank sectors by their real revenue-to-utility ratios such that $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_J$. Since $\theta_j$ is increasing in $k_j$, ranking sectors by $\theta_j$ is equivalent to ranking them by $k_j$. Plugging (29) into (22), using $\mathcal{E}_{U,U_j} = \beta_j$ from the upper-tier Cobb-Douglas specification, and noting that $(N_j^E)^{\text{eqm}}/(N_j^E)^{\text{opt}} = L_j^{\text{eqm}}/L_j^{\text{opt}}$ by (28), we can restate Proposition 5 for this example as follows:

**Corollary 2 (Distortions in the labor allocation and entry with CARA subutility)** *Assume that the subutility function in each sector is of the CARA form, $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, and that the productivity distribution follows a Pareto distribution, $G_j(m) = (m/m_j^{\max})^{k_j}$. Then, the labor allocation and the masses of entrants satisfy $L_j^{\text{eqm}} \gtreqless L_j^{\text{opt}}$ and $(N_j^E)^{\text{eqm}} \gtreqless (N_j^E)^{\text{opt}}$, respectively, if and only if*

$$\theta_j \gtreqless \sum_{\ell=1}^{J} \beta_\ell \theta_\ell. \tag{30}$$

*Assume, without loss of generality, that sectors are ordered such that $\theta_j$ is non-decreasing in $j$. If there are at least two different $\theta_j$'s, there exists a threshold $j^* \in \{1, 2, \ldots, J-1\}$ such that sectors $j \leq j^*$ display insufficient entry and insufficient labor allocation, whereas sectors $j > j^*$ display excess entry and excess labor allocation. The equilibrium labor allocation and entry in the CARA case are optimal if and only if all $\theta_j$'s, and thus all $k_j$'s, are the same across sectors.*

**Proof** *See above.* □

Corollary 2 states that sectors with larger values of $k_j$ (i.e., sectors where a larger mass of the productivity distribution is concentrated on low-productivity firms) are more likely to display excess entry and excess labor allocation in equilibrium. As mentioned after Proposition 6, sectors with larger values of $k_j$ also display smaller cutoff distortions. Thus, more excessive entry comes with more efficient selection. Furthermore, Corollary 2 shows that all sectors with $\theta_j$ above the weighted average $\sum_{\ell=1}^{J} \beta_\ell \theta_\ell$ display excess entry and labor allocation, whereas the opposite is true for all sectors with $\theta_j$ below that threshold. Hence, interdependence of heterogeneous sectors matters for those distortions: If there is no heterogeneity in $k_j$, then the labor allocation and entry are efficient although the cutoffs and quantities are inefficient in all sectors.

# 4 Quantification

In this section, we take our model to the data in order to quantify the gap between the equilibrium and optimal allocations.[15] Our approach is based on the two examples in the previous section, and only requires data that is accessible for many countries. In particular, we need the expenditure shares across sectors, and some aggregate statistics of the firm-size distribution within sectors. We make use of firm-level data from France in 2008 and from the United Kingdom (UK) in 2005. Using two different countries enables us to assess the robustness of our quantification approach, and to compare the distortions in those two different cases.

We first focus on the VES model from Section 3.2 that captures all types of distortions. We then quantify the CES model from Section 3.1, where cutoff and output distortions are absent. Finally, we put the quantitative predictions of the two models into perspective.

## 4.1 Data

Our quantification procedure requires firm-level employment data, as well as expenditure shares and R&D outlays at the sectoral level.[16] For France, the firm-level employment data comes from the 'Élaboration des Statistiques Annuelles d'Entreprises' (ESANE) database, which combines administrative and survey data to produce structural business statistics. We use the administrative part of the dataset that contains employment figures for almost all business organizations in France. It is compiled from annual tax returns that companies file to the tax authorities and from annual social security data that supply information on the employees. We focus on the year 2008, for which there are 1,100,220 firms with positive

[15] Our paper differs from a different strand of literature that quantifies the aggregate welfare impacts of public policies. Hsieh and Klenow (2009), for example, compare observed equilibria in China and India with counterfactual equilibria in which those countries would attain the "U.S. efficiency" level. Unlike this literature, we compare the observed market equilibrium and the optimal allocation that the social planner would choose.

[16] Further details concerning the datasets can be found in Appendix C.1.

employment records.[17] For each firm, we also have information about its sectoral affiliation. The French input-output tables contain information on 35 sectors, the public sector plus 34 private sectors, roughly corresponding to 2-digit NACE (revision 1.1) codes. This dictates the level of aggregation in our analysis. We discard the public sector (12.12% of expenditure) and focus on the remaining 34 private sectors. For those sectors, we obtain expenditure shares, $\widehat{e}_j$, by re-scaling total expenditure such that the shares sum up to one. These observed expenditure shares are reported in Table 1.

The data for the UK have the same structure. We use the 'Business Structure Database' (BSD), which contains a small number of variables, including employment and sectoral affiliation, for almost all business organizations in the UK. The BSD is derived primarily from the 'Inter-Departmental Business Register' (IDBR), which is a live register of data collected by 'Her Majesty's Revenue and Customs' (HMRC) via VAT and 'Pay As You Earn' (PAYE) records. We focus on the year 2005 for which there are 1,704,543 firms with positive employment records (excluding the firm owners). We can distinguish the exact same 34 sectors as for France for the sectoral affiliation of those firms, for which we obtain expenditure shares from the British input-output tables. These observed expenditure shares, $\widehat{e}_j$, re-scaled again to sum to one, are reported in Table 2.

## 4.2 Quantifying distortions: the CARA subutility case

To quantify the VES model, we first match a theory-based moment of the sector-specific firm-size distribution to its empirical counterpart. To this end, we derive an analytical expression for the standard deviation of (log) firm-level employment in sector $j$, excluding the labor input $F_j$ that all firms have to bear as a sunk entry cost. The resulting expression depends only on the shape parameter $k_j$ of the sector-specific Pareto productivity distribution (see equation (C-1) in Appendix C.2). To construct its empirical counterpart, we compute for each sector $j$ the ratio of R&D expenditure (our proxy for sunk entry costs) to gross output and then multiply the ratio by total employment in that sector. Dividing this by the number of firms gives us a measure for $F_j$, which we then subtract from the total employment of each firm in the respective sector (see Appendix C.1 for more details). Finally, we calculate for each sector $j$ the standard deviation of the resulting (log) number of employees. This data moment and the number of firms in each sector are reported in Table 1 for France, and in Table 2 for the UK.

[17]The dataset contains 3 employment variables. We use employment on December 31st from the French Business Register (OCSANE) source.

With the standard deviation of the (log) number of employees at hand, we can then uniquely back out $\widehat{k}_j$ for each sector and compute $\widehat{\theta}_j$ and $\widehat{\kappa}_j$, which depend solely on $\widehat{k}_j$. Using $\widehat{\theta}_j$ and the observed expenditure shares $\widehat{e}_j$, we obtain $\widehat{\beta}_j$ by solving $\sum_{\ell=1}^{J} \widehat{\beta}_\ell = 1$ and $\widehat{e}_j = \widehat{\beta}_j \widehat{\theta}_j / \sum_{\ell=1}^{J} \widehat{\beta}_\ell \widehat{\theta}_\ell$, which corresponds to (8) in the case of Cobb-Douglas upper-tier utility and CARA subutility functions. We can proceed in a similar way in the case of CES upper-tier utility, and the details are provided in the supplementary Appendix F.

We summarize the structural parameters that we obtain for the two countries in Tables 1 and 2. Observe the substantial heterogeneity across French sectors: the shape parameters $\widehat{k}_j$ of the sectoral Pareto distributions range from 2.0 to 24.3, with an (unweighted) average of 5.7. In the UK, the differences are even larger, as the values of $\widehat{k}_j$ range from 1.5 to 41.3, with an (unweighted) average of 7.4.

**Cutoff distortions.** Given the values of $\widehat{k}_j$, $\widehat{\theta}_j$, $\widehat{\kappa}_j$, and $\widehat{\beta}_j$, we are now in a position to quantify the distortions in France and in the UK. We first compare the equilibrium and optimal cutoffs in each sector. Using the expressions in (27), we compute for each sector $j$ the following measure of cutoff distortions:

$$\frac{(m_j^d)^{\text{eqm}} - (m_j^d)^{\text{opt}}}{(m_j^d)^{\text{opt}}} \times 100 = \left\{ \left[ \kappa_j (k_j + 1)^2 \right]^{-\frac{1}{k_j+1}} - 1 \right\} \times 100, \tag{31}$$

which depends only on $k_j$ as $\kappa_j$ is a function of $k_j$ only. Since there is too little selection by Proposition 6, $(m_j^d)^{\text{eqm}} > (m_j^d)^{\text{opt}}$ holds, so that expression (31) is always positive. The gap between the equilibrium and optimal cutoffs is smaller the larger is the sectoral shape parameter $k_j$, i.e., a larger mass of the productivity distribution is concentrated on low-productivity firms.

Tables 1 and 2 report the magnitudes of cutoff distortions for all sectors in France and the UK, which we illustrate in Figures 1 and 2 for those two countries. We find substantial distortions due to insufficient selection. For France, the simple average across sectors is 15.9%, but with huge sectoral variation from only 2.8% to almost 30%. In the UK, the average is 16.7% and the range goes from 1.7% to 37.8%. The correlation of those distortions between the two countries is 0.356, while the Spearman rank correlation is 0.328. Thus, the model makes roughly similar predictions on which sectors in France and the UK exhibit greater cutoff distortions. We discuss this point in more detail below.

**Entry distortions.** Turning to the gap between the equilibrium and optimal entry, or equivalently the gap between the equilibrium and optimal labor allocations in our examples, we

Table 1: Sectoral data, parameter values, and distortions for France in 2008.

| | | | | | CARA + Cobb-Douglas & Pareto | | | | | | CES + Cobb-Douglas & Pareto | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Std. dev. | | | | | Cutoff | Entry | | | Cutoff | Entry |
| Sector | Description | Firms | $\widehat{e}_j$ | log emp | $\widehat{k}_j$ | $\widehat{\theta}_j$ | $\widehat{\kappa}_j$ | $\widehat{\beta}_j$ | distortions | | $\widehat{\rho}_j$ | $\widehat{\beta}_j$ | distortions | |
| 1 | Agriculture | 5551 | 0.0188 | 1.0038 | 2.8670 | 0.8721 | 0.0312 | 0.0188 | 21.8406 | -0.1886 | 0.7421 | 0.0188 | 0 | 0.3470 |
| 2 | Mining and quarrying | 1132 | 0.0002 | 1.0523 | 3.5570 | 0.8911 | 0.0227 | 0.0002 | 17.9533 | 1.9848 | 0.7892 | 0.0002 | 0 | 6.7070 |
| 3 | Food products, beverages, tobacco | 38582 | 0.0697 | 0.9858 | 2.6642 | 0.8653 | 0.0346 | 0.0704 | 23.3225 | -0.9765 | 0.7242 | 0.0697 | 0 | -2.0711 |
| 4 | Textiles, leather and footwear | 4889 | 0.0205 | 1.0354 | 3.2891 | 0.8845 | 0.0255 | 0.0203 | 19.2867 | 1.2213 | 0.7730 | 0.0205 | 0 | 4.5251 |
| 5 | Wood products | 4607 | 0.0008 | 1.1811 | 8.4447 | 0.9471 | 0.0055 | 0.0007 | 7.9290 | 8.3958 | 0.9089 | 0.0008 | 0 | 22.8950 |
| 6 | Pulp, paper, printing and publishing | 12136 | 0.0086 | 1.1805 | 8.3928 | 0.9469 | 0.0055 | 0.0079 | 7.9764 | 8.3625 | 0.9083 | 0.0086 | 0 | 22.8198 |
| 7 | Coke, refined petroleum, nuclear fuel | 27 | 0.0168 | 1.1447 | 6.1501 | 0.9303 | 0.0094 | 0.0158 | 10.7480 | 6.4650 | 0.8756 | 0.0168 | 0 | 18.3985 |
| 8 | Chemicals and chemical products | 1194 | 0.0285 | 1.1688 | 7.5071 | 0.9413 | 0.0067 | 0.0264 | 8.8810 | 7.7318 | 0.8977 | 0.0285 | 0 | 21.3827 |
| 9 | Rubber and plastics products | 2760 | 0.0037 | 1.0332 | 3.2565 | 0.8836 | 0.0259 | 0.0037 | 19.4626 | 1.1220 | 0.7709 | 0.0037 | 0 | 4.2374 |
| 10 | Other non-metallic mineral products | 3426 | 0.0020 | 1.0428 | 3.4013 | 0.8873 | 0.0243 | 0.0019 | 18.7050 | 1.5521 | 0.7801 | 0.0020 | 0 | 5.4774 |
| 11 | Basic metals | 602 | 0.0001 | 1.2166 | 13.1203 | 0.9646 | 0.0025 | 0.0001 | 5.1666 | 10.3951 | 0.9410 | 0.0001 | 0 | 27.2453 |
| 12 | Fabricated metal products | 17249 | 0.0021 | 1.1442 | 6.1290 | 0.9301 | 0.0095 | 0.0020 | 10.7833 | 6.4415 | 0.8752 | 0.0021 | 0 | 18.3419 |
| 13 | Machinery and equipment | 8227 | 0.0053 | 1.1003 | 4.5835 | 0.9109 | 0.0153 | 0.0050 | 14.1902 | 4.2470 | 0.8345 | 0.0053 | 0 | 12.8416 |
| 14 | Office, accounting, computing mach. | 160 | 0.0033 | 1.0684 | 3.8519 | 0.8976 | 0.0201 | 0.0032 | 16.6828 | 2.7305 | 0.8045 | 0.0033 | 0 | 8.7831 |
| 15 | Electrical machinery and apparatus | 1656 | 0.0034 | 1.2466 | 24.2501 | 0.9802 | 0.0008 | 0.0030 | 2.8241 | 12.1791 | 0.9680 | 0.0034 | 0 | 30.8871 |
| 16 | Radio, TV, communication equip. | 786 | 0.0042 | 1.1439 | 6.1119 | 0.9299 | 0.0095 | 0.0040 | 10.8121 | 6.4223 | 0.8749 | 0.0042 | 0 | 18.2957 |
| 17 | Medical, precision, optical instr. | 3753 | 0.0050 | 1.0383 | 3.3327 | 0.8856 | 0.0250 | 0.0049 | 19.0565 | 1.3517 | 0.7758 | 0.0050 | 0 | 4.9020 |
| 18 | Motor vehicles and (semi-)trailers | 835 | 0.0326 | 1.1046 | 4.7020 | 0.9127 | 0.0147 | 0.0312 | 13.8546 | 4.4568 | 0.8386 | 0.0326 | 0 | 13.3862 |
| 19 | Other transport equipment | 452 | 0.0028 | 1.1128 | 4.9432 | 0.9162 | 0.0135 | 0.0026 | 13.2186 | 4.8581 | 0.8462 | 0.0028 | 0 | 14.4165 |
| 20 | Manufacturing n.e.c; recycling | 9802 | 0.0130 | 1.1760 | 8.0324 | 0.9447 | 0.0060 | 0.0120 | 8.3212 | 8.1207 | 0.9043 | 0.0130 | 0 | 22.2727 |
| 21 | Electricity, gas and water supply | 1279 | 0.0225 | 0.9745 | 2.5480 | 0.8610 | 0.0368 | 0.0228 | 24.2650 | -1.4664 | 0.7129 | 0.0225 | 0 | -3.6039 |
| 22 | Construction | 188513 | 0.0082 | 0.9992 | 2.8127 | 0.8704 | 0.0320 | 0.0083 | 22.2182 | -0.3915 | 0.7376 | 0.0082 | 0 | -0.2700 |
| 23 | Wholesale and retail trade; repairs | 274437 | 0.1377 | 1.0151 | 3.0067 | 0.8765 | 0.0291 | 0.1373 | 20.9236 | 0.3099 | 0.7532 | 0.1377 | 0 | 1.8463 |
| 24 | Hotels and restaurants | 113317 | 0.0489 | 0.9489 | 2.3083 | 0.8512 | 0.0420 | 0.0502 | 26.4702 | -2.5803 | 0.6866 | 0.0489 | 0 | -7.1669 |
| 25 | Transport and storage | 26847 | 0.0291 | 0.9962 | 2.7783 | 0.8692 | 0.0326 | 0.0292 | 22.4649 | -0.5232 | 0.7346 | 0.0291 | 0 | -0.6727 |
| 26 | Post and telecommunications | 1144 | 0.0191 | 1.0374 | 3.3186 | 0.8852 | 0.0252 | 0.0188 | 19.1303 | 1.3099 | 0.7749 | 0.0191 | 0 | 4.7813 |
| 27 | Finance and insurance | 12383 | 0.0376 | 0.9141 | 2.0264 | 0.8379 | 0.0498 | 0.0393 | 29.6331 | -4.1024 | 0.6494 | 0.0376 | 0 | -12.1881 |
| 28 | Real estate activities | 36902 | 0.1649 | 0.9517 | 2.3334 | 0.8523 | 0.0414 | 0.1691 | 26.2215 | -2.4570 | 0.6895 | 0.1649 | 0 | -6.7672 |
| 29 | Renting of machinery and equipment | 4815 | 0.0022 | 1.1101 | 4.8613 | 0.9151 | 0.0139 | 0.0021 | 13.4279 | 4.7255 | 0.8437 | 0.0022 | 0 | 14.0777 |
| 30 | Computer and related activities | 16355 | 0.0010 | 1.1944 | 9.7504 | 0.9535 | 0.0042 | 0.0010 | 6.8991 | 9.1285 | 0.9209 | 0.0010 | 0 | 24.5238 |
| 31 | Research and development | 1562 | 0.0074 | 1.2375 | 19.2934 | 0.9754 | 0.0012 | 0.0067 | 3.5386 | 11.6260 | 0.9598 | 0.0074 | 0 | 29.7810 |
| 32 | Other Business Activities | 132159 | 0.0073 | 1.0964 | 4.4803 | 0.9092 | 0.0159 | 0.0070 | 14.4958 | 4.0571 | 0.8309 | 0.0073 | 0 | 12.3453 |
| 33 | Education | 11401 | 0.0799 | 1.0726 | 3.9371 | 0.8994 | 0.0194 | 0.0776 | 16.3484 | 2.9297 | 0.8085 | 0.0799 | 0 | 9.3287 |
| 34 | Health, social work, personal services | 124202 | 0.1930 | 0.9659 | 2.4642 | 0.8577 | 0.0385 | 0.1966 | 24.9935 | -1.8394 | 0.7042 | 0.1930 | 0 | -4.7851 |

*Notes:* Column 1 reports the number of firms in each sector in the ESANE database for France in 2008, column 2 the observed (rescaled) expenditure shares from the French input-output table, and column 3 the observed standard deviation of the log number of employees across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-1) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (31) and (32), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-2) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (35).

Table 2: Sectoral data, parameter values, and distortions for the United Kingdom in 2005.

| | | | | | **CARA + Cobb-Douglas & Pareto** | | | | | | **CES + Cobb-Douglas & Pareto** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sector | Description | Firms | $\widehat{e}_j$ | Std. dev. log emp | $\widehat{k}_j$ | $\widehat{\theta}_j$ | $\widehat{\kappa}_j$ | $\widehat{\beta}_j$ | Cutoff distortions | Entry distortions | $\widehat{\rho}_j$ | $\widehat{\beta}_j$ | Cutoff distortions | Entry distortions |
| 1 | Agriculture | 57969 | 0.0127 | 0.8424 | 1.5152 | 0.8069 | 0.0706 | 0.0138 | 37.7850 | -8.1349 | 0.5607 | 0.0127 | 0 | -24.3233 |
| 2 | Mining and quarrying | 1124 | 0.0008 | 1.2580 | 35.5036 | 0.9863 | 0.0004 | 0.0007 | 1.9363 | 12.2922 | 0.9781 | 0.0008 | 0 | 32.0111 |
| 3 | Food products, beverages. tobacco | 4606 | 0.0442 | 1.1260 | 5.3830 | 0.9220 | 0.0118 | 0.0421 | 12.1970 | 4.9662 | 0.8584 | 0.0442 | 0 | 15.8529 |
| 4 | Textiles, leather and footwear | 9041 | 0.0213 | 1.1829 | 8.6063 | 0.9480 | 0.0053 | 0.0198 | 7.7852 | 7.9348 | 0.9106 | 0.0213 | 0 | 22.8954 |
| 5 | Wood products | 7301 | 0.0014 | 1.1079 | 4.7949 | 0.9141 | 0.0142 | 0.0013 | 13.6024 | 4.0730 | 0.8416 | 0.0014 | 0 | 13.5846 |
| 6 | Pulp, paper, printing and publishing | 24882 | 0.0112 | 1.1142 | 4.9862 | 0.9168 | 0.0133 | 0.0108 | 13.1112 | 4.3825 | 0.8475 | 0.0112 | 0 | 14.3787 |
| 7 | Coke, refined petroleum, nuclear fuel | 122 | 0.0104 | 1.1442 | 6.1295 | 0.9301 | 0.0095 | 0.0098 | 10.7826 | 5.8902 | 0.8752 | 0.0104 | 0 | 18.1245 |
| 8 | Chemicals and chemical products | 1989 | 0.0088 | 1.2614 | 41.2898 | 0.9882 | 0.0003 | 0.0079 | 1.6669 | 12.5055 | 0.9812 | 0.0088 | 0 | 32.4242 |
| 9 | Rubber and plastics products | 5152 | 0.0035 | 1.1077 | 4.7899 | 0.9140 | 0.0142 | 0.0034 | 13.6159 | 4.0646 | 0.8414 | 0.0035 | 0 | 13.5630 |
| 10 | Other non-metallic mineral products | 3412 | 0.0017 | 1.0171 | 3.0332 | 0.8773 | 0.0287 | 0.0017 | 20.7588 | -0.1201 | 0.7552 | 0.0017 | 0 | 1.9273 |
| 11 | Basic metals | 1203 | 0.0003 | 1.1800 | 8.3555 | 0.9466 | 0.0056 | 0.0002 | 8.0108 | 7.7767 | 0.9079 | 0.0003 | 0 | 22.5386 |
| 12 | Fabricated metal products | 24116 | 0.0019 | 1.2025 | 10.7654 | 0.9575 | 0.0035 | 0.0017 | 6.2663 | 9.0180 | 0.9283 | 0.0019 | 0 | 25.2887 |
| 13 | Machinery and equipment | 8719 | 0.0064 | 1.1206 | 5.1953 | 0.9196 | 0.0125 | 0.0061 | 12.6131 | 4.6993 | 0.8534 | 0.0064 | 0 | 15.1825 |
| 14 | Office, accounting, computing mach. | 898 | 0.0006 | 1.0715 | 3.9145 | 0.8989 | 0.0196 | 0.0006 | 16.4360 | 2.3441 | 0.8075 | 0.0006 | 0 | 8.9841 |
| 15 | Electrical machinery and apparatus | 2694 | 0.0015 | 1.0675 | 3.8347 | 0.8973 | 0.0202 | 0.0014 | 16.7521 | 2.1569 | 0.8037 | 0.0015 | 0 | 8.4690 |
| 16 | Radio, TV, communication equip. | 1004 | 0.0057 | 1.2070 | 11.4206 | 0.9598 | 0.0032 | 0.0052 | 5.9160 | 9.2724 | 0.9324 | 0.0057 | 0 | 25.8380 |
| 17 | Medical, precision, optical instr. | 2443 | 0.0016 | 1.0956 | 4.4595 | 0.9089 | 0.0160 | 0.0016 | 14.5590 | 3.4788 | 0.8301 | 0.0016 | 0 | 12.0353 |
| 18 | Motor vehicles and (semi-)trailers | 2059 | 0.0272 | 1.1459 | 6.2088 | 0.9308 | 0.0093 | 0.0256 | 10.6513 | 5.9773 | 0.8768 | 0.0272 | 0 | 18.3347 |
| 19 | Other transport equipment | 1012 | 0.0036 | 1.2551 | 31.7979 | 0.9848 | 0.0005 | 0.0032 | 2.1599 | 12.1161 | 0.9756 | 0.0036 | 0 | 31.6677 |
| 20 | Manufacturing n.e.c; recycling | 16028 | 0.0109 | 1.0735 | 3.9535 | 0.8997 | 0.0193 | 0.0107 | 16.2857 | 2.4335 | 0.8093 | 0.0109 | 0 | 9.2289 |
| 21 | Electricity, gas and water supply | 428 | 0.0261 | 1.1854 | 8.8336 | 0.9492 | 0.0050 | 0.0241 | 7.5915 | 8.0711 | 0.9128 | 0.0261 | 0 | 23.2015 |
| 22 | Construction | 156266 | 0.0085 | 0.9638 | 2.4443 | 0.8569 | 0.0389 | 0.0087 | 25.1733 | -2.4391 | 0.7020 | 0.0085 | 0 | -5.2515 |
| 23 | Wholesale and retail trade; repairs | 306437 | 0.1850 | 0.9788 | 2.5911 | 0.8626 | 0.0359 | 0.1884 | 23.9071 | -1.7932 | 0.7172 | 0.1850 | 0 | -3.2016 |
| 24 | Hotels and restaurants | 130213 | 0.0781 | 0.9975 | 2.7940 | 0.8697 | 0.0323 | 0.0789 | 22.3519 | -0.9789 | 0.7359 | 0.0781 | 0 | -0.6720 |
| 25 | Transport and storage | 31912 | 0.0392 | 0.9289 | 2.1417 | 0.8436 | 0.0464 | 0.0408 | 28.2533 | -3.9495 | 0.6655 | 0.0392 | 0 | -10.1799 |
| 26 | Post and telecommunications | 4654 | 0.0181 | 0.9526 | 2.3417 | 0.8527 | 0.0412 | 0.0186 | 26.1401 | -2.9224 | 0.6905 | 0.0181 | 0 | -6.8087 |
| 27 | Finance and insurance | 15890 | 0.0807 | 0.9190 | 2.0638 | 0.8398 | 0.0486 | 0.0844 | 29.1713 | -4.3838 | 0.6548 | 0.0807 | 0 | -11.6270 |
| 28 | Real estate activities | 80146 | 0.1104 | 0.8570 | 1.6199 | 0.8141 | 0.0654 | 0.1192 | 35.7739 | -7.3083 | 0.5813 | 0.1104 | 0 | -21.5440 |
| 29 | Renting of machinery and equipment | 13615 | 0.0061 | 1.0636 | 3.7599 | 0.8957 | 0.0209 | 0.0059 | 17.0596 | 1.9760 | 0.8000 | 0.0061 | 0 | 7.9678 |
| 30 | Computer and related activities | 102580 | 0.0010 | 0.8645 | 1.6720 | 0.8176 | 0.0630 | 0.0010 | 34.8511 | -6.9194 | 0.5911 | 0.0010 | 0 | -20.2240 |
| 31 | Research and development | 1603 | 0.0001 | 1.0575 | 3.6486 | 0.8932 | 0.0218 | 0.0001 | 17.5386 | 1.6963 | 0.7942 | 0.0001 | 0 | 7.1867 |
| 32 | Other Business Activities | 371014 | 0.0041 | 0.9100 | 1.9952 | 0.8363 | 0.0508 | 0.0043 | 30.0287 | -4.7829 | 0.6448 | 0.0041 | 0 | -12.9669 |
| 34 | Education | 23494 | 0.0625 | 1.1440 | 6.1179 | 0.9300 | 0.0095 | 0.0591 | 10.8019 | 5.8774 | 0.8750 | 0.0625 | 0 | 18.0934 |
| 35 | Health, social work, personal services | 215336 | 0.2044 | 1.0816 | 4.1275 | 0.9031 | 0.0180 | 0.1988 | 15.6477 | 2.8157 | 0.8170 | 0.2044 | 0 | 10.2671 |

*Notes:* Column 1 reports the number of firms in each sector in the BSD database for the UK in 2005, column 2 the observed (rescaled) expenditure shares from the UK input-output table, and column 3 the observed standard deviation of the log number of employees across firms, where data are constructed as described in Appendix C.1. Column 4 reports the values of $\widehat{k}_j$ that we obtain by matching the numbers from column 3 to expression (C-1) in Appendix C.2. Columns 5 and 6 report the values of $\widehat{\theta}_j$ and $\widehat{\kappa}_j$ which are transformations of $\widehat{k}_j$. Column 7 reports the value $\widehat{\beta}_j$ obtained as described in Section 4.1. In columns 8 and 9 we report the magnitudes of cutoff and entry distortions at the sectoral level obtained from (31) and (32), respectively. Column 10 reports the value of $\widehat{\rho}_j$ obtained by matching the numbers from column 3 to expression (C-2) in Appendix C.2 while using $\widehat{k}_j$ from column 4. Column 11 reports the values $\widehat{\beta}_j$ which correspond to the expenditure shares from column 2. Finally, column 12 reports only zeroes as the CES model does not exhibit cutoff distortions, and column 13 reports the magnitudes of entry distortions as computed in (35).

use expressions (28) and Proposition 5, together with (29), to compute the following measure of intersectoral distortions for each sector $j$:

$$\frac{(N_j^E)^{\text{eqm}} - (N_j^E)^{\text{opt}}}{(N_j^E)^{\text{opt}}} \times 100 = \frac{(L_j)^{\text{eqm}} - (L_j)^{\text{opt}}}{(L_j)^{\text{opt}}} \times 100 = \left( \frac{\theta_j}{\sum_{\ell=1}^J \beta_\ell \theta_\ell} - 1 \right) \times 100. \qquad (32)$$

Based on (32), our model predicts that 25 sectors in the French economy exhibit excess entry by up to 12.2%. The remaining 9 sectors display insufficient entry by up to -4.1%. In the UK, excess entry arises in 23 sectors, whereas insufficient entry occurs in 11 sectors, with a range of entry distortions from -8.1% to 12.5%. See Tables 1 and 2 for the detailed numbers, and Figures 1 and 2 for a graphical illustration of those distortions.

Digging deeper into these patterns, we find some similarities between France and the UK. In both countries, excess entry typically (though not always) occurs in manufacturing. See, for example, [11] 'Basic metals' and [15] 'Electrical machinery and apparatus' in France, or [8] 'Chemical products' and [19] 'Transport equipment' in the UK, where it is particularly strong. By contrast, insufficient entry is almost exclusively a phenomenon of service sectors.[18] See, for example, [24] 'Hotels and restaurants' and [27] 'Finance and insurance' in France, or [28] 'Real estate' and [32] 'Other business services' in the UK, where we find strongly negative values. Overall, the correlation of entry distortions across sectors in the two countries is 0.330 and the Spearman rank correlation is 0.328. Furthermore, the direction or 'sign' of inefficient entry is the same in 26 out of 34 sectors, i.e., in more than three-quarter of the sectors. Put differently, the model makes similar predictions as to which sectors in the two countries tend to display excessive or insufficient entry.

Recall that in the CARA model the larger the value of $k_j$, the more excessive is the firm entry (and the labor allocation) but the smaller is the magnitude of insufficient selection. In other words, manufacturing sectors in both countries not only tend to exhibit excess entry, but also display relatively smaller cutoff distortions, i.e., equilibrium firm selection relatively closer to the optimum. By contrast, there are too few entrants in many service sectors, and firm selection is far less severe than it should be from a social point of view. It is worth emphasizing that those predictions are based on a full-fledged general equilibrium model that recognizes all interdependencies across sectors in the economy. Thus, our analysis is in contrast to the conventional approach in industrial organization that has typically studied

[18]The sector [1] 'Agriculture' also exhibits insufficient entry in both countries, and particularly so in the UK, but hardly any manufacturing sector in either country has too few entrants. Notice that these findings do, of course, *not* imply that the mass of entrants in manufacturing is larger than that in services in equilibrium, since they refer to a sector-by-sector comparison of the equilibrium and the optimal entry.

Figure 1: Cutoff and entry disortions, CARA model for France in 2008.

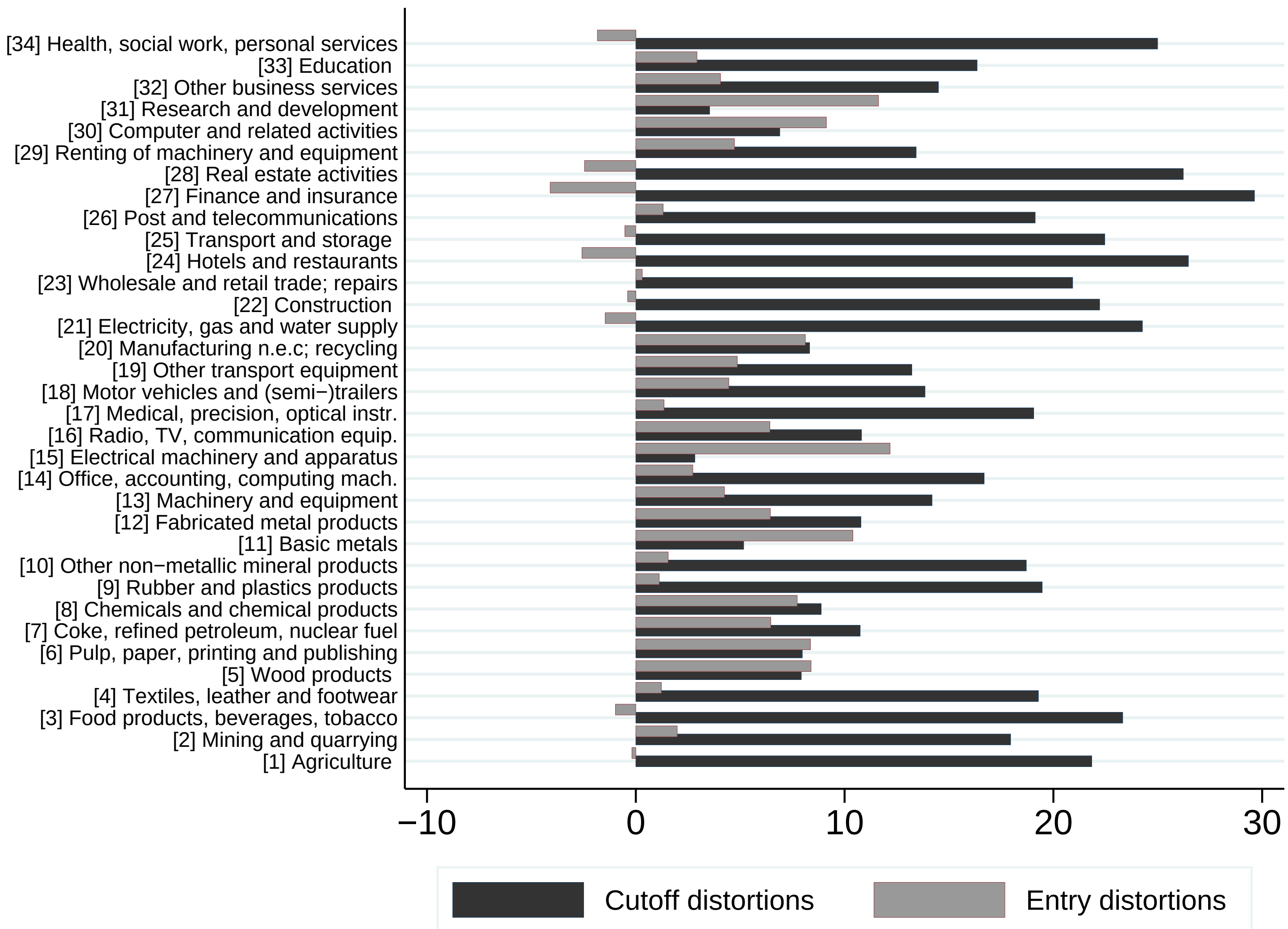
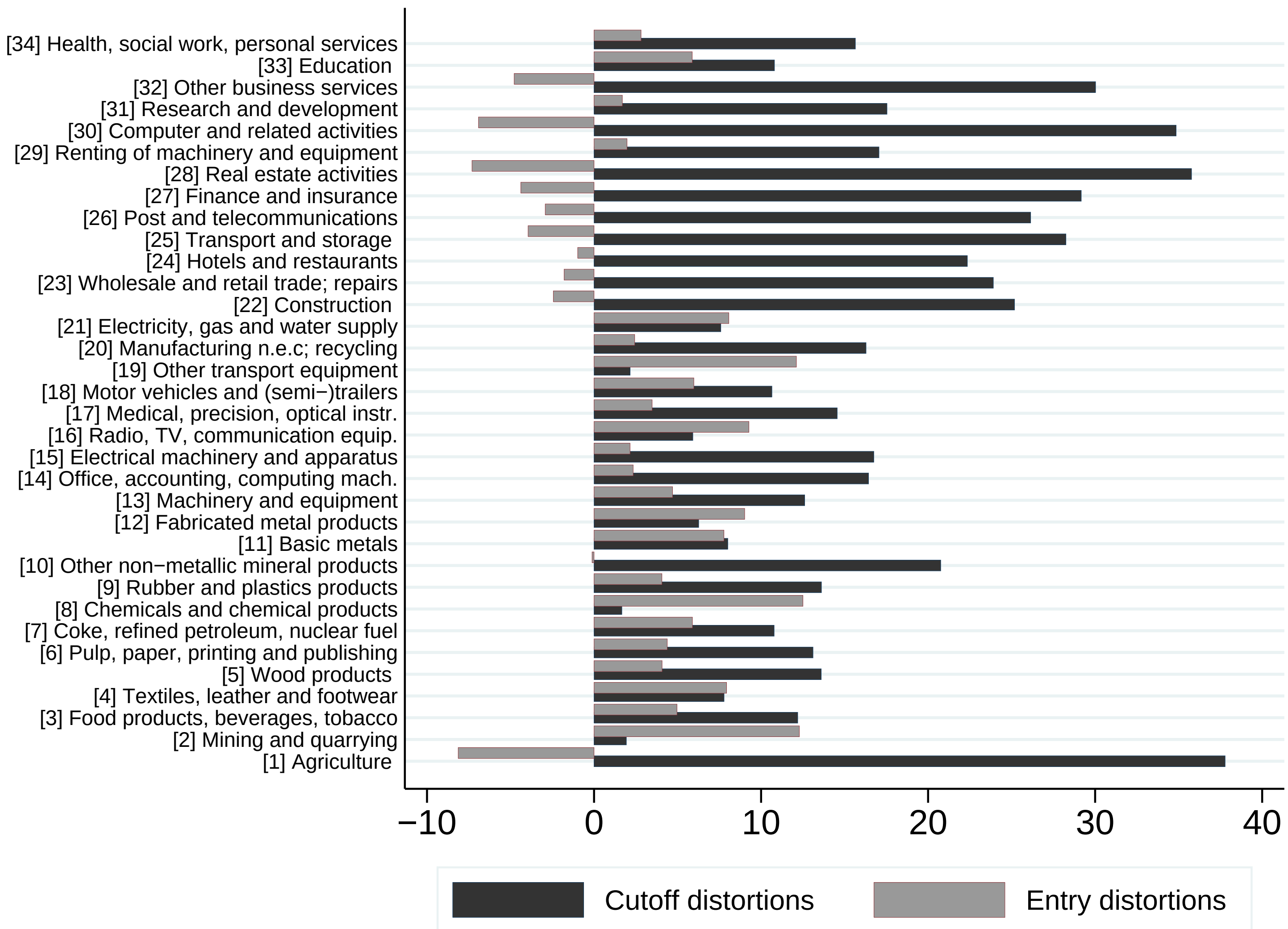


Figure 2: Cutoff and entry disortions, CARA model for the United Kingdom in 2005.



entry and selection for a single industry in partial equilibrium.

**Aggregate welfare distortion.** Having analyzed cutoff and entry distortions in each sector, we now consider the aggregate welfare distortion in the economy. To this end, we use the concept of the *Allais surplus* (Allais, 1943, 1977) since compensating and equivalent variations, which are used to analyze the welfare change *between two equilibria*, are not readily applicable to measuring the welfare distortion, i.e., the welfare gap *between the equilibrium and optimum*. Intuitively, we measure the amount of labor – which is taken as the numeraire – that can be saved when the planner minimizes the resource cost of attaining the equilibrium utility level.

Let $L^A(U^{\text{eqm}})$ denote the minimum amount of labor that the social planner requires to attain the equilibrium utility level. By construction, $L^A(U^{\text{eqm}})$ is not greater than the amount of labor $L$ that the market economy requires to reach the equilibrium utility level because the labor market clears in equilibrium and because their may be distortions. As shown in Appendix D, we can define a measure of the aggregate welfare distortion based on the Allais surplus as follows:

$$-\frac{L^A(U^{\text{eqm}}) - L}{L} \times 100 = \left\{ 1 - \frac{\prod_{j=1}^{J} \left[ (k_j+1)^2 \kappa_j \right]^{\frac{\beta_j}{k_j+1}}}{\sum_{\ell=1}^{J} \beta_\ell \theta_\ell} \right\} \times 100. \tag{33}$$

Plugging the values of $\widehat{k}_j$, $\widehat{\theta}_j$, $\widehat{\kappa}_j$, and $\widehat{\beta}_j$ from Tables 1 and 2 into (33), we can compute the magnitude of the aggregate welfare distortion in France and in the UK, respectively.

Table 3: Aggregate welfare distortions as measured by the Allais surplus.

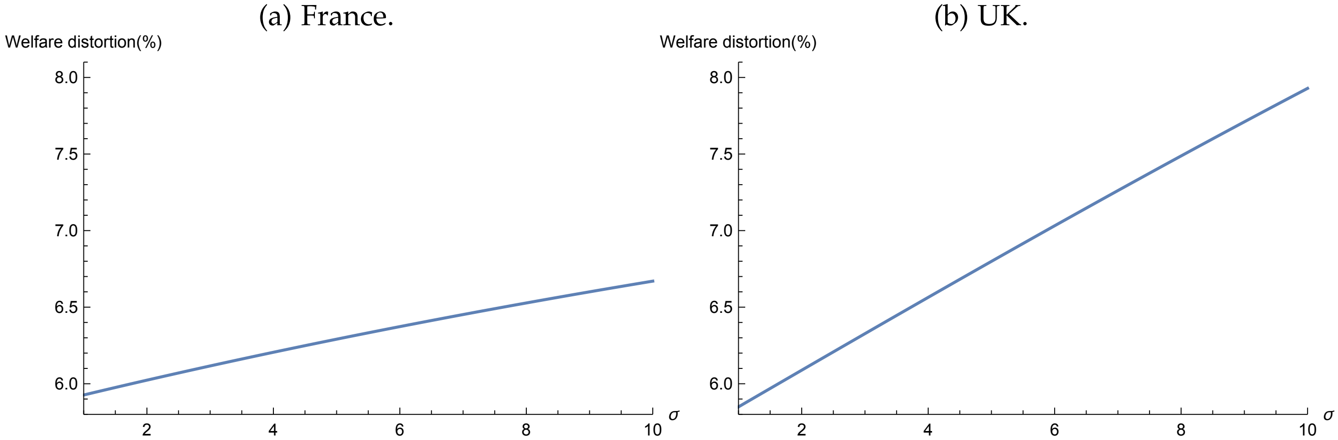
| | France | | UK | |
|---|---|---|---|---|
| | VES | CES | VES | CES |
| Aggregate distortion<br>(% of aggregate labor input saved) | 5.93 | 0.34 | 5.85 | 0.99 |
| Cutoff and quantity distortion | 81.81 | 0 | 95.11 | 0 |
| Entry and labor distortion<br>(as % of aggregate distortion) | 18.19 | 100 | 4.89 | 100 |

Table 3 summarizes our results. For France, the aggregate welfare distortion is 5.93%, and for the UK it is 5.85%. In words, to achieve the equilibrium utility level in each of the two countries, the social planner requires almost 6% less aggregate labor input when compared to the case with utility maximizing consumers and profit maximizing firms.

Disentangling the relative contribution of the cutoff and the entry distortion is difficult, since it is generally not possible to shut down one without affecting the other.[19] To gauge the potential importance of within and between sector distortions, we hence proceed as follows. We pool our data across all sectors and proceed *as if* there were only a single sector. Distortions in the labor allocation cannot arise in this single-sector case – since by definition $L_j^{\text{eqm}} = L_j^{\text{opt}} = L$ – and entry is efficient by (28). Therefore, the welfare gap between the equilibrium and optimum depends only on cutoff and output distortions. We then estimate the value of $k$ for that single sector in the same way as before, by matching the standard deviation of the (log) employment distribution across all firms. This yields $\widehat{k} = 3.5687$ for France and $\widehat{k} = 3.0598$ for the UK. Plugging that common value into (33), we compute the associated Allais surplus for the single-sector economy and compare it with the Allais surplus in the multi-sector case. The results are summarized in the bottom part of Table 3. As can be seen, the distortions in the single-sector case are 18.19% smaller for France, and 4.89% smaller for the UK. Put differently, disregarding entry and labor distortions would lead to an underestimation of the aggregate welfare distortion by 5%–18% in our CARA example with a Cobb-Douglas upper-tier utility function.

Figure 3: Aggregate welfare distortions in the CES-CARA model as a function of $\sigma$.

(a) France. (b) UK.




**Robustness check.** We have also conducted a robustness check with respect to the choice of the upper-tier utility function. In particular, we have replaced the Cobb-Douglas upper-tier function with the CES function $U = \{\sum_{j=1}^{J} \beta_j [\widetilde{U}_j(U_j)]^{(\sigma-1)/\sigma}\}^{\sigma/(\sigma-1)}$, and the Allais

[19] We know from the results in Corollary 2 that entry in the CARA case is efficient if and only if all $k_j$'s are the same. Hence, one could think of setting all $k_j$'s to same common value to shut down entry distortions. However, the common value of $k$ that is chosen has an effect on the magnitude of cutoff distortions.

surplus for that case with CES upper-tier utility and CARA subutility is given by (see the supplementary Appendix F for details):

$$-\frac{L^A(U^{\text{eqm}}) - L}{L} \times 100 = \left[1 - \frac{1}{\sum_{\ell=1}^J \beta_\ell \theta_\ell} \cdot \left\{ \sum_{j=1}^J \beta_j \left[(k_j+1)^2 \kappa_j\right]^{\frac{1-\sigma}{k_j+1}} \right\}^{\frac{1}{1-\sigma}} \right] \times 100. \qquad (34)$$

Notice that, once we choose a value of $\sigma$, this expression for the aggregate welfare distortion can be computed using $\widehat{k}_j$, $\widehat{\theta}_j$, $\widehat{\kappa}_j$, and $\widehat{\beta}_j$ from Tables 1 and 2, respectively.

Figure 3 illustrates the magnitude of the aggregate welfare distortion given by (34) as a function of $\sigma$ for France (panel (a)) and the UK (panel (b)). We find that the higher is the elasticity of substitution between sectors, the stronger is the aggregate welfare distortion in both countries. It ranges between 6% and 7% in France, and between 6% and 8% in the UK. Treating the economy *as if* it consisted of a single sector, as before, we re-quantify the magnitude of entry and labor distortions. For $\sigma \in (1, 10)$, it ranges between 18% and 27% in France, and between 5% and 29% in the UK. Thus, the higher the elasticity of substitution for the upper-tier utility function, the stronger the underestimation of the aggregate welfare distortion due to inefficient entry and labor allocation, and it can reach almost 30% for reasonable parameter values.

## 4.3 Quantifying distortions: the CES subutility case

Finally, we quantify the workhorse model with Cobb-Douglas upper-tier and CES subutility functions. Recall that there are no cutoff distortions with CES subutility functions. However, by Corollary 1, there are still labor and entry distortions due to heterogeneity in the real revenue-to-utility ratio $R_j/U_j$ and in the elasticity of upper-tier utility $\mathcal{E}_{U,U_j}$ when the $\rho_j$ terms differ across sectors. How large are the welfare distortions for France and the UK predicted by the CES model?

To quantify this model, we use the same sector-specific statistics as before: the standard deviation of (log) firm-level employment, not including the labor input for R&D which we use as a proxy for sunk entry and fixed costs. To match this observed data moment, we also assume sector-specific Pareto distributions for productivity draws, and then derive the corresponding theoretical expression for the CES case. As can be seen from equation (C-2) in Appendix C.2, this expression now depends on two parameters: $\rho_j$ and $k_j$. Since the $k_j$'s are technology parameters that do not depend on consumer preferences, we keep the same values of $\widehat{k}_j$ from the VES model above. We can then uniquely back out the corresponding values for $\widehat{\rho}_j$. Since by (22) the equilibrium expenditure share is $\beta_j$ for this case, the value of $\widehat{\beta}_j$ for each sector can be obtained by setting $\widehat{\beta}_j = \widehat{e}_j$, where $\sum_{j=1}^{J} \widehat{\beta}_j = \sum_{j=1}^{J} \widehat{e}_j = 1$ by definition of the observed expenditure share.

The parameter values thus obtained for France and the UK are reported in Tables 1 and 2. Equipped with those numbers, we can quantify the magnitude of entry distortions for each sector $j$ as follows:

$$\frac{(N_j^E)^{\text{eqm}} - (N_j^E)^{\text{opt}}}{(N_j^E)^{\text{opt}}} \times 100 = \frac{L_j^{\text{eqm}} - L_j^{\text{opt}}}{L_j^{\text{opt}}} \times 100 = \left( \rho_j \sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell} - 1 \right) \times 100. \tag{35}$$

As can be seen from Tables 1 and 2, in both countries the CES and VES models make very similar predictions as to which sectors display excess or insufficient entry. Yet, the CES model implies larger magnitudes than the VES model. In France, the range of inefficient entry and labor allocation goes from -12.2% to 30.9%, and in the UK from -24.3% to 32.4%.

To quantify the aggregate welfare distortion, we again rely on the Allais surplus and compute the following expression (see Appendix D for details):

$$-\frac{L^A(U^{\text{eqm}}) - L}{L} \times 100 = \left[ 1 - \prod_{j=1}^{J} \left( \rho_j \sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell} \right)^{\frac{\beta_j/\rho_j}{\sum_{\ell=1}^{J} (\beta_\ell/\rho_\ell)}} \right] \times 100. \tag{36}$$

The results are 0.34% for France, and 0.99% for the UK, as summarized in Table 3. In other words, less than 1% of the aggregate labor input could be saved if the social planner minimized the resource cost to attain the equilibrium utility level. Compared to the VES model, where the corresponding number is roughly 6%, it appears that the aggregate welfare distortion in the CES model is much smaller than that in the VES model. However, correcting the inefficiencies between sectors would still lead to substantial changes in entry patterns and sectoral employment shares.

# 5 Conclusions

We have developed a general equilibrium model of monopolistic competition with multiple sectors and heterogeneous firms. Comparing the equilibrium and optimal allocations, we have characterized the various distortions that operate in our economy. Concrete specifications of our general model allow for closed-form solutions that can be readily taken to the data. Applying this approach to French data for 2008 and UK data for 2005, we have quantified the aggregate welfare distortions while uncovering substantial sectoral heterogeneity and assessing contribution of each type of distortions to the overall welfare losses.

Our preferred specification implies substantial aggregate welfare distortions for France and for the UK, each of which amounts to almost 6% of the respective economy's aggregate labor input. Our results suggest that inefficiencies within and between sectors both matter in practice. Removing those distortions would presumably require rather different interventions: industrial policy tools to address the latter problem, combined with policies targeted at specific firms to address the former. A general lesson that one can deduce from our analysis is that interdependencies are important for the design of such programs: the optimal policy for one sector is not only influenced by conditions of that particular sector, but it depends on the characteristics of all sectors in the economy. We leave it to future work to explore the details of feasible policy schemes that alleviate misallocations. In this paper we have taken a first step, and provided a novel approach to derive quantitative predictions for the welfare distortion. More work is needed in the future to derive robust lessons for policy.

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# Appendix

## A. Proofs of the propositions

This appendix provides all the proofs of the propositions. To alleviate notation, we suppress indices for sectors and arguments wherever possible.

**A.1. Proof of Proposition 1.** This result can be established using a similar method as in Zhelobodko et al. (2012). However, we provide an alternative proof that can be readily applied to the optimal cutoff and quantities (see Appendix A.3). Using the profit-maximizing price (5) for the marginal variety, we can rewrite the ZCP condition (6) as

$$\frac{r_{u_j}(q_j^d)}{1-r_{u_j}(q_j^d)} m_j^d q_j^d = r_{u_j}(q_j^d)\frac{p_j^d}{w} q_j^d = \frac{f_j}{L},$$

which, together with the first-order condition (2) for the marginal variety, $u_j'(q_j^d) = \lambda_j p_j^d$, yields

$$r_{u_j}(q_j^d) u_j'(q_j^d) q_j^d = -(q_j^d)^2 u_j''(q_j^d) = \frac{f_j}{L}\lambda_j w.$$

The left-hand side is increasing in $q_j^d$ since

$$\frac{\partial}{\partial q_j^d}\left(-(q_j^d)^2 u_j''(q_j^d)\right) = -q_j^d u_j''(q_j^d)\left[2 - \left(-\frac{q_j^d u_j'''(q_j^d)}{u_j''(q_j^d)}\right)\right] = -q_j^d u_j''(q_j^d)\left[2 - r_{u_j'}(q_j^d)\right] > 0,$$

where we use the second-order condition $r_{u_j'}(q_j(m)) < 2$. Thus, we know that $q_j^d$ is increasing in the market aggregate $\lambda_j w$.

Furthermore, using the first-order condition (2) and the profit-maximizing price (5) for the marginal variety, we have

$$\left[1 - r_{u_j}(q_j^d)\right] u_j'(q_j^d) = (\lambda_j w) m_j^d. \tag{A-1}$$

The left-hand side is decreasing in $q_j^d$ since

$$\frac{\partial}{\partial q_j^d}\left\{\left[1 - r_{u_j}(q_j^d)\right] u_j'(q_j^d)\right\} = u_j''(q_j^d)\left[2 - r_{u_j'}(q_j^d)\right] < 0.$$

Hence, since we have shown above that $\partial q_j^d/\partial(\lambda_j w) > 0$, the left-hand side in (A-1) decreases as $\lambda_j w$ on the right-hand side of (A-1) increases. It then follows that $m_j^d$ is decreasing in $\lambda_j w$.

Similarly, using the first-order conditions (2) and the profit-maximizing prices (5) for other varieties, we have

$$\left[1 - r_{u_j}(q_j(m))\right] u_j'(q_j(m)) = (\lambda_j w)m.$$

Since the left-hand side is decreasing in $q_j(m)$, we know that $q_j(m)$ is decreasing in $\lambda_j w$.

Next, we rewrite the ZEP condition (7) as

$$L \int_0^{m_j^d} \left\{ \left[ \frac{1}{1 - r_{u_j}(q_j(m))} - 1 \right] m q_j(m) - \frac{f_j}{L} \right\} \mathrm{d}G_j(m) = F_j. \tag{A-2}$$

Given that $m_j^d$ and $q_j(m)$ are decreasing in $\lambda_j w$, we differentiate the left-hand side of this expression with respect to $\lambda_j w$ as follows:

$$\begin{aligned} &L \left\{ \left[ \frac{1}{1 - r_{u_j}(q_j^d)} - 1 \right] m_j^d q_j(m_j^d) - \frac{f_j}{L} \right\} g_j(m_j^d) \frac{\partial m_j^d}{\partial(\lambda_j w)} \\ &\qquad + L \int_0^{m_j^d} \left\{ \frac{r_{u_j}'(q_j(m))}{[1 - r_{u_j}(q_j(m))]^2} q_j(m) + \frac{r_{u_j}(q_j(m))}{1 - r_{u_j}(q_j(m))} \right\} m \frac{\partial q_j(m)}{\partial(\lambda_j w)} \mathrm{d}G_j(m). \end{aligned}$$

The first-term is zero by the ZCP condition (6). Noting that

$$\begin{aligned} r_{u_j}(q_j(m)) &= -\frac{q_j(m) u_j''(q_j(m))}{u_j'(q_j(m))} \\ r_{u_j}'(q_j(m)) &= -\frac{[u_j''(q_j(m)) + u_j'''(q_j(m)) q_j(m)] u_j'(q_j(m)) - q_j(m)[u_j''(q_j(m))]^2}{[u_j'(q_j(m))]^2}, \end{aligned}$$

the second term can be expressed as:

$$L \int_0^{m_j^d} \left\{ \frac{[2 - r_{u_j'}(q_j(m))] r_{u_j}(q_j(m))}{[1 - r_{u_j}(q_j(m))]^2} \right\} m \frac{\partial q_j(m)}{\partial(\lambda_j w)} \mathrm{d}G_j(m) < 0,$$

where we use the second-order condition $r_{u_j'}(q_j(m)) < 2$. Hence, the left-hand side of the ZEP condition (A-2) is decreasing in $\lambda_j w$.

Assume that fixed costs, $f_j$, and sunk costs, $F_j$, are not too large. The former ensures that profits are non-negative (see the ZCP condition in (6)). The latter ensures existence. The left-hand side of the ZEP condition is strictly decreasing in $\lambda_j w$, whereas the right-hand side is constant. Hence, if fixed costs, $f_j$, and sunk costs, $F_j$, are not too large, then there exists a unique solution for $\lambda_j w$. Using the unique $\lambda_j w$ thus obtained, we can establish the existence and uniqueness of $m_j^d$ and $q_j(m)$ since both are decreasing in $\lambda_j w$. □

**A.2. Proof of Proposition 2.** The first-order conditions (2) and (3), when combined with equation (10), imply that

$$\frac{\left[N_j^E \int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)\right]^{\xi_j}}{\left[N_\ell^E \int_0^{m_\ell^d} u_\ell(q_\ell(m))\mathrm{d}G_\ell(m)\right]^{\xi_\ell}} = \frac{p_j^d}{p_\ell^d}\frac{\gamma_\ell}{\gamma_j}\frac{u_\ell'(q_\ell^d)}{u_j'(q_j^d)}. \tag{A-3}$$

When $f_j$ and $F_j$ are not too large, the market aggregate $\lambda_j w$ is uniquely determined by the ZEP condition and so are sector-specific cutoffs $m_j^d$ and the associated prices $p_j^d$ and quantities $q_j^d$ and $q_j(m)$ (see Appendix A.1). Since the ZEP condition does not include $N_j^E$, those variables are independent of $N_j^E$. Thus, the integrals in (A-3) are independent of $N_j^E$ and $N_\ell^E$. The right-hand side of equation (A-3) is strictly positive and finite. By monotonicity, there clearly exists a unique $N_j^E(N_\ell^E)$. This relationship satisfies $(N_j^E)' > 0$, $N_j^E(0) = 0$ and $\lim_{N_\ell^E \to \infty} N_j^E(N_\ell^E) = \infty$.

In each sector $j$, labor supply $L_j$ equals labor demand $N_j^E\{\int_0^{m_j^d}[Lmq_j(m) + f_j]\,\mathrm{d}G_j(m) + F_j\}$, so that

$$\frac{L_j}{N_j^E} - L\int_0^{m_j^d} mq_j(m)\mathrm{d}G_j(m) = f_j G_j(m_j^d) + F_j. \tag{A-4}$$

Plugging expression (A-4) into (7) yields

$$N_j^E \int_0^{m_j^d} \frac{mq_j(m)}{1 - r_{u_j}\big(q_j(m)\big)}\mathrm{d}G_j(m) = \frac{L_j}{L}. \tag{A-5}$$

Summing over $j$ and using the overall labor market clearing condition $L = \sum_{j=1}^J L_j$, we then have the following equilibrium condition:

$$\sum_{j=1}^J N_j^E(N_\ell^E)\int_0^{m_j^d} \frac{mq_j(m)}{1 - r_{u_j}(q_j(m))}\mathrm{d}G_j(m) = 1. \tag{A-6}$$

Observe that all integral terms on the left-hand side of (A-6) are positive and independent of the masses of entrants, whereas the right-hand side equals one. Since the limit of the left-hand side is zero when $N_\ell^E$ goes to zero, and infinity when $N_\ell^E$ goes to infinity, the existence and uniqueness of a solution for $N_\ell^E$ follows directly by the properties of $N_j^E(\cdot)$. Since the terms in braces of the right-hand side of (9) are uniquely determined by Proposition 1, the existence and uniqueness of $N_j^E$ implies those of $e_j L$ and thus those of $L_j$ in (8), which proves Proposition 2. □

**A.3. Proof of Proposition 3.** Plugging the first-order condition for the marginal variety $m_j^d = u_j'(q_j^d)/\delta_j$ into (17), we have

$$u_j(q_j^d) - u_j'(q_j^d)q_j^d = \frac{f_j}{L}\delta_j. \tag{A-7}$$

The left-hand side is increasing in $q_j^d$ (since $u_j'' < 0$), which establishes that $q_j^d$ is increasing in $\delta_j$. Thus, $u'(q_j^d)$ is decreasing in $\delta_j$. Then, from the first-order condition for the marginal variety, we see that when $\delta_j$ increases, $m_j^d$ must decrease because $u_j'(q_j^d)/\delta_j$ decreases. Hence, $m_j^d$ is a decreasing function of $\delta_j$. From the first-order conditions for the other varieties, $u'(q_j(m)) = \delta_j m$, we know that $q_j(m)$ is decreasing in $\delta_j$.

Next, we rewrite the ZESP condition (16) as

$$L\int_0^{m_j^d}\left[\left(\frac{1}{\mathcal{E}_{u_j,q_j(m)}} - 1\right) m q_j(m) - \frac{f_j}{L}\right] g_j(m)\mathrm{d}m = F_j.$$

Given that $m_j^d$ and $q_j(m)$ are decreasing in $\delta_j$, we differentiate the left-hand side of this expression with respect to $\delta_j$ as follows:

$$L\left[\left(\frac{1}{\mathcal{E}_{u_j,q_j^d}} - 1\right) m_j^d q_j^d - \frac{f_j}{L}\right] g_j(m_j^d)\frac{\partial m_j^d}{\partial \delta_j}$$
$$+L\int_0^{m_j^d}\left[-\frac{1}{\mathcal{E}_{u_j,q_j(m)}}\frac{\partial \mathcal{E}_{u_j,q_j(m)}}{\partial q_j(m)}\frac{q_j(m)}{\mathcal{E}_{u_j,q_j(m)}} + \frac{1-\mathcal{E}_{u_j,q_j(m)}}{\mathcal{E}_{u_j,q_j(m)}}\right] m\frac{\partial q_j(m)}{\partial \delta_j} g_j(m)\mathrm{d}m,$$

where the first term is zero by (17). Using

$$\frac{\partial \mathcal{E}_{u_j,q_j(m)}}{\partial q_j(m)}\frac{q_j(m)}{\mathcal{E}_{u_j,q_j(m)}} = 1 - r_{u_j}(q_j(m)) - \mathcal{E}_{u_j,q_j(m)},$$

we finally have

$$L\int_0^{m_j^d}\frac{r_{u_j}(q_j(m))}{\mathcal{E}_{u_j,q_j(m)}} m\frac{\partial q_j(m)}{\partial \delta_j} g_j(m)\mathrm{d}m < 0,$$

where the inequality comes from $\partial q_j(m)/\partial\delta_j < 0$.

Assume that fixed costs, $f_j$, and sunk costs, $F_j$, are not too large. The former ensures that social profits are non-negative (see the ZCSP condition (17)), and the latter ensures existence. The left-hand side of the ZESP condition is strictly decreasing in $\delta_j$, whereas the right-hand side is constant. Hence, if fixed costs, $f_j$, and sunk costs, $F_j$, are not too large, then there exists a unique solution for $\delta_j$. Using the unique $\delta_j$ thus obtained, we can establish the existence and uniqueness of $m_j^d$ and $q_j(m)$ since both are decreasing in $\delta_j$. $\square$

**A.4. Proof of Proposition 4.** The first-order conditions (12) and (15), when combined with equation (10), imply that

$$\frac{\left[N_j^E \int_0^{m_j^d} u_j(q_j(m))\mathrm{d}G_j(m)\right]^{\xi_j}}{\left[N_\ell^E \int_0^{m_\ell^d} u_\ell(q_\ell(m))\mathrm{d}G_\ell(m)\right]^{\xi_\ell}} = \frac{m_j^d}{m_\ell^d}\frac{\gamma_\ell}{\gamma_j}\frac{u_\ell'(q_\ell^d)}{u_j'(q_j^d)}. \tag{A-8}$$

When $f_j$ and $F_j$ are not too large, $\delta_j$ is uniquely determined by the ZESP condition, and so are the sector-specific cutoffs $m_j^d$ and the associated quantities $q_j^d$ and $q_j(m)$. Since the ZESP condition does not include $N_j^E$, those variables are independent of $N_j^E$. Thus, the integrals in (A-8) are independent of $N_j^E$ and $N_\ell^E$. The right-hand side of equation (A-8) is strictly positive and finite. By monotonicity, there clearly exists a unique $N_j^E(N_\ell^E)$. This relationship satisfies $(N_j^E)' > 0$, $N_j^E(0) = 0$ and $\lim_{N_\ell^E \to \infty} N_j^E(N_\ell^E) = \infty$.

Plugging expression (A-4) for the optimal allocation into (16) yields

$$N_j^E \int_0^{m_j^d} \frac{mq_j(m)}{\mathcal{E}_{u_j,q_j(m)}}\mathrm{d}G_j(m) = \frac{L_j}{L}. \tag{A-9}$$

Substituting $N_j^E(N_\ell^E)$ obtained from (A-8) into (A-9), making use of $L_j = e_j L$ and summing over $j$, we then have the following equilibrium condition:

$$\sum_{j=1}^{J} N_j^E(N_\ell^E) \int_0^{m_j^d} \frac{mq_j(m)}{\mathcal{E}_{u_j,q_j(m)}}\mathrm{d}G_j(m) = 1. \tag{A-10}$$

Observe that all integral terms on the left-hand side of (A-10) are positive and independent of the masses of entrants, whereas the right-hand side equals one. Since the limit of the left-hand side is zero when $N_\ell^E$ goes to zero, and infinity when $N_\ell^E$ goes to infinity, the existence and uniqueness of a solution for $N_\ell^E$ follows directly by the properties of $N_j^E(\cdot)$. Since the terms in braces of the right-hand side of (19) are uniquely determined by Proposition 3, the existence and uniqueness of $N_j^E$ implies those of $e_j L$ and thus those of $L_j$ in (18), which proves Proposition 4. $\square$

**A.5. Proof of Proposition 5.** The former claim can readily be obtained from (8) and (18). The latter claim can be shown as follows. Without loss of generality, we order sectors by non-decreasing private-to-social expenditure ratios:

$$\frac{e_1^{\text{eqm}}}{e_1^{\text{opt}}} \leq \frac{e_2^{\text{eqm}}}{e_2^{\text{opt}}} \leq \ldots \leq \frac{e_J^{\text{eqm}}}{e_J^{\text{opt}}}.$$

Then, by definition of the expenditure shares, we must have

$$\frac{e_1^{\text{eqm}}}{e_1^{\text{opt}}} \leq \frac{e_j^{\text{eqm}}}{e_j^{\text{opt}}}, \quad \forall j \quad \Rightarrow \quad e_1^{\text{eqm}} = e_1^{\text{eqm}} \sum_{j=1}^{J} e_j^{\text{opt}} \leq e_1^{\text{opt}} \sum_{j=1}^{J} e_j^{\text{eqm}} = e_1^{\text{opt}},$$

which implies $e_1^{\text{eqm}}/e_1^{\text{opt}} \leq 1$. Conversely,

$$\frac{e_j^{\text{eqm}}}{e_j^{\text{opt}}} \leq \frac{e_J^{\text{eqm}}}{e_J^{\text{opt}}}, \quad \forall j \quad \Rightarrow \quad e_J^{\text{opt}} = e_J^{\text{opt}} \sum_{j=1}^{J} e_j^{\text{eqm}} \leq e_J^{\text{eqm}} \sum_{j=1}^{J} e_j^{\text{opt}} = e_J^{\text{eqm}},$$

which implies $1 \leq e_J^{\text{eqm}}/e_J^{\text{opt}}$. Since $e_j^{\text{eqm}}/e_j^{\text{opt}}$ is non-decreasing in $j$, we have $e_1^{\text{eqm}}/e_1^{\text{opt}} < e_J^{\text{eqm}}/e_J^{\text{opt}}$ if there are at least two different $e_j^{\text{eqm}}/e_j^{\text{opt}}$'s. In that case, there exists a threshold $j^* \in \{1, 2, \ldots, J-1\}$ such that all sectors with $j \leq j^*$ attract too little expenditure, whereas all sectors with $j > j^*$ attract too much expenditure. Using (8) and (18) then yields the result in terms of the intersectoral labor allocation.

To see that the intersectoral allocation is optimal if and only if all expenditure ratios are constant, we proceed as follows. First, assume that $e_j^{\text{eqm}}/e_j^{\text{opt}} = c$ for all $j$, where $c$ is a constant. Then, $\sum_{j=1}^{J} e_j^{\text{eqm}} = c \sum_{j=1}^{J} e_j^{\text{opt}} = 1$ must hold, which yields $c = 1$ and $e_j^{\text{eqm}} = e_j^{\text{opt}}$ for all $j$. Since $L_j^{\text{eqm}} = e_j^{\text{eqm}} L$ and $L_j^{\text{opt}} = e_j^{\text{opt}} L$, this proves the if part. To see the only if part, assume that $L_j^{\text{eqm}} = L_j^{\text{opt}}$ for all $j$. Clearly, this is only possible if $e_j^{\text{eqm}}/e_j^{\text{opt}} = 1$ for all $j$. This completes the proof of Proposition 5. □

**A.6. Proof of Proposition 6.** Taking the ratio of $(m_j^d)^{\text{opt}}$ and $(m_j^d)^{\text{eqm}}$ from (27) yields

$$\left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{k_j+1} = \kappa_j (k_j + 1)^2. \tag{A-11}$$

Since $\kappa_j (k_j + 1)^2 < 1$ (see the discussion below (E-15) in Appendix E.1), this immediately implies that $(m_j^d)^{\text{opt}}/(m_j^d)^{\text{eqm}} < 1$.

Next, taking the difference between the optimal quantity (E-22) and the equilibrium quantity (E-2), evaluated at the equilibrium price (E-3), we have the following two cases.

First, when $0 \leq m \leq (m_j^d)^{\text{opt}}$, we obtain

$$q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m) = \frac{1}{\alpha_j} \ln \left[ \frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}} \right] - \frac{1}{\alpha_j} \ln W \left( \mathrm{e} \frac{m}{(m_j^d)^{\text{eqm}}} \right). \tag{A-12}$$

Recalling that $W(0) = 0$ by the property of the Lambert $W$ function, we know that $\lim_{m \to +0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] > 0$. Second, when $(m_j^d)^{\text{opt}} < m < (m_j^d)^{\text{eqm}}$, we know that $q_j^{\text{opt}}(m) = 0$, and that $q_j^{\text{eqm}}(m) > 0$, so that

$$q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m) = \frac{1}{\alpha_j} \ln \left[ \frac{m}{(m_j^d)^{\text{eqm}}} \right] - \frac{1}{\alpha_j} \ln W \left( \mathrm{e} \frac{m}{(m_j^d)^{\text{eqm}}} \right) < 0. \tag{A-13}$$

Recalling that $W(\mathrm{e}) = 1$ by the property of the Lambert $W$ function, we know that $\lim_{m \to (m_j^d)^{\text{eqm}} - 0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] = 0$. Noting that (A-13) is strictly increasing in $m$,[20] and that $(m_j^d)^{\text{opt}} < (m_j^d)^{\text{eqm}}$, it is verified that $\lim_{m \to (m_j^d)^{\text{opt}} + 0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] < 0$.

Finally, since $q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)$ is continuous at $(m_j^d)^{\text{opt}}$ by expressions (A-12) and (A-13), $\lim_{m \to (m_j^d)^{\text{opt}} - 0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] < 0$ must hold in (A-12). Noting that expression (A-12) is strictly decreasing in $m$, and that $\lim_{m \to +0}[q_j^{\text{opt}}(m) - q_j^{\text{eqm}}(m)] > 0$, we know that there exists a unique $m_j^* \in (0, (m_j^d)^{\text{opt}})$ such that $q_j^{\text{opt}}(m) > q_j^{\text{eqm}}(m)$ for $m \in (0, m_j^*)$ and $q_j^{\text{opt}}(m) < q_j^{\text{eqm}}(m)$ for $m \in (m_j^*, (m_j^d)^{\text{opt}}]$. This, together with the inequality in (A-13) for $m \in ((m_j^d)^{\text{opt}}, (m_j^d)^{\text{eqm}})$ proves our claim. □

## B. Proofs of the lemmas

**B.1. Proof of Lemma 1.** By definition, the consumer's expenditure share for sector-$j$ varieties is given by

$$e_j \equiv \frac{N_j^E}{w} \int_0^{m_j^d} p_j(m) q_j(m) \mathrm{d}G_j(m). \tag{B-1}$$

Combining (5) and (A-5) and using (B-1) yield $L_j/L = e_j$. Let

$$R_j \equiv N_j^E \int_0^{m_j^d} u_j'\big(q_j(m)\big) q_j(m) \mathrm{d}G_j(m) = \lambda_j e_j w, \tag{B-2}$$

be a measure of real revenue in sector $j$, where we have made use of the first-order conditions (2) and of (B-1). Let further $\mathcal{E}_{U,U_j} \equiv (\partial U / \partial U_j)(U_j/U)$ be the elasticity of the upper-tier utility function. The expenditure share can then be expressed as a function of $R_j$ and $\mathcal{E}_{U,U_j}$.

[20]To derive this property, we use $W'(x) = W(x)/\{x[1 + W(x)]\}$.

To see this, we use (B-2) to get

$$\frac{e_j}{e_\ell} = \frac{R_j \lambda_\ell}{R_\ell \lambda_j} = \frac{R_j \mathcal{E}_{U,U_j} \frac{U}{U_j}}{R_\ell \mathcal{E}_{U,U_\ell} \frac{U}{U_\ell}} \quad \Rightarrow \quad e_j \sum_{\ell=1}^{J} \frac{R_\ell}{U_\ell} \mathcal{E}_{U,U_\ell} = \frac{R_j}{U_j} \mathcal{E}_{U,U_j},$$

where we use (2) and the property that the expenditure shares sum to one. We thus have

$$e_j = \frac{\frac{R_j}{U_j} \mathcal{E}_{U,U_j}}{\sum_{\ell=1}^{J} \frac{R_\ell}{U_\ell} \mathcal{E}_{U,U_\ell}}, \tag{B-3}$$

as stated in (8). Finally, turning to the mass of entrants, from (A-4) and (8) we obtain

$$N_j^E = \frac{e_j L}{f_j G_j(m_j^d) + F_j + L \int_0^{m_j^d} m q_j(m) \mathrm{d}G_j(m)}. \tag{B-4}$$

Since $mq_j(m) = q_j(m)p_j(m)[1 - r_{u_j}(q_j(m))]/w = q_j(m)[1 - r_{u_j}(q_j(m))]u_j'(q_j(m))/(\lambda_j w)$ from profit maximization and the consumer's first-order conditions, and using $\lambda_j w = R_j/e_j$ from (B-2), we have $mq_j(m) = e_j q_j(m)[1 - r_{u_j}(q_j(m))]u_j'(q_j(m))/R_j$. Plugging this into equation (B-4), and noticing that $R_j$ depends on $N_j^E$, we can solve the resulting equation for $N_j^E$, which yields (9). This completes the proof of Lemma 1. □

**B.2. Proof of Lemma 2.** The proof is similar to that for Lemma 1. Using the shadow price $m/\mathcal{E}_{u_j,q_j(m)}$, the social expenditure share for sector-$j$ varieties is defined as

$$e_j \equiv N_j^E \int_0^{m_j^d} \frac{m q_j(m)}{\mathcal{E}_{u_j,q_j(m)}} \mathrm{d}G_j(m). \tag{B-5}$$

Combining (B-5) and (A-9) yield $L_j = e_j L$. The social expenditure share $e_j$ can be expressed in terms of the elasticities of the upper-tier utility function. To see this, we use the definition of $\mathcal{E}_{u_j,q_j(m)}$ and the first-order condition (12) to obtain

$$e_j = N_j^E \int_0^{m_j^d} \frac{m u_j(q_j(m))}{u_j'(q_j(m))} \mathrm{d}G_j(m) = \frac{U_j}{\delta_j}. \tag{B-6}$$

Taking the ratio of sectors $j$ and $\ell$ yields

$$\frac{e_j}{e_\ell} = \frac{U_j \delta_\ell}{U_\ell \delta_j} = \frac{\mathcal{E}_{U,U_j}}{\mathcal{E}_{U,U_\ell}} \quad \Rightarrow \quad e_j \sum_{\ell=1}^{J} \mathcal{E}_{U,U_\ell} = \mathcal{E}_{U,U_j},$$

where we have made use of condition (12) and of the property that the expenditure shares sum to one. We thus obtain

$$e_j = \frac{\mathcal{E}_{U,U_j}}{\sum_{\ell=1}^{J} \mathcal{E}_{U,U_\ell}}$$

as in equation (18). Finally, turning to the mass of entrants, from (A-4) and (18) we obtain (B-4). We know that $mq_j(m) = q_j(m)u_j'(q_j(m))/\delta_j = e_j q_j(m)u_j'(q_j(m))/U_j = e_j \mathcal{E}_{u_j,q_j(m)}$ $u_j(q_j(m))/U_j$ holds for the optimal allocation. Plugging this into (B-4), and noting that $U_j$ depends on $N_j^E$, we can solve the resulting equation for $N_j^E$, which yields (19). This completes the proof of Lemma 2. □

**B.3. Proof of Lemma 3.** We derive the ratio $R_j/U_j$ for the CARA case with Pareto productivity distributions. By definition of the CARA subutility,

$$R_j = N_j^E \int_0^{m_j^d} q_j(m)\alpha_j \mathrm{e}^{-\alpha_j q_j(m)} \mathrm{d}G_j(m) \;\text{ and }\; U_j = N_j^E \int_0^{m_j^d} \left[1 - \mathrm{e}^{-\alpha_j q_j(m)}\right] \mathrm{d}G_j(m). \quad \text{(B-7)}$$

As shown in the supplementary Appendix E, the equilibrium quantities are given by $q_j(m) = (1/\alpha_j)[1 - W(\mathrm{e}\, m/m_j^d)]$, where $W$ is the Lambert $W$ function defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$. To integrate the foregoing expressions, we use the change in variables suggested by Corless et al. (1996, p.341). Let

$$z \equiv W\left(\mathrm{e}\,\frac{m}{m_j^d}\right), \quad \text{so that} \quad \mathrm{e}\,\frac{m}{m_j^d} = z\mathrm{e}^z.$$

This change in variables then yields $\mathrm{d}m = (1+z)\mathrm{e}^{z-1} m_j^d \mathrm{d}z$, with the new integration bounds given by 0 and 1. Substituting the expressions for quantities into $R_j$ in (B-7), using the definition of $W$, and making the above change in variables, we have:

$$\begin{aligned} R_j &= N_j^E \int_0^{m_j^d} [1 - W(\mathrm{e}\, m/m_j^d)] \mathrm{e}^{W(\mathrm{e}\, m/m_j^d) - 1} g_j(m) \mathrm{d}m \\ &= N_j^E m_j^d \int_0^1 (1-z)\mathrm{e}^{z-1}(1+z)\mathrm{e}^{z-1} g_j(z\mathrm{e}^{z-1} m_j^d) \mathrm{d}z. \end{aligned}$$

Applying the same technique to the lower-tier utility $U_j$ in (B-7) we obtain

$$\begin{aligned} U_j &= N_j^E \int_0^{m_j^d} [1 - \mathrm{e}^{W(\mathrm{e}\, m/m_j^d) - 1}] g_j(m) \mathrm{d}m \\ &= N_j^E m_j^d \int_0^1 (1 - \mathrm{e}^{z-1})(1+z)\mathrm{e}^{z-1} g_j(z\mathrm{e}^{z-1} m_j^d) \mathrm{d}z. \end{aligned}$$

Taking the ratio $R_j/U_j$ then yields:

$$\frac{R_j}{U_j} = \frac{\int_0^1 (1-z)\mathrm{e}^{z-1}(1+z)\mathrm{e}^{z-1} g_j\left(z\mathrm{e}^{z-1} m_j^d\right) \mathrm{d}z}{\int_0^1 (1-\mathrm{e}^{z-1})(1+z)\mathrm{e}^{z-1} g_j\left(z\mathrm{e}^{z-1} m_j^d\right) \mathrm{d}z}, \tag{B-8}$$

where $(1-z)\mathrm{e}^{z-1} < 1 - \mathrm{e}^{z-1}$ for all $z \in [0,1)$. With a Pareto distribution, we have $g_j(z\mathrm{e}^{z-1}m_j^d) = k_j(z\mathrm{e}^{z-1}m_j^d)^{k_j-1}(m_j^{\max})^{-k_j}$, so that expression (B-8) can be written as (29). □

## C. Additional details for the quantification procedure

This appendix provides details on the data that we use and derives additional expressions required for the quantification procedure.

**C.1. Data.** Besides the firm-level ESANE dataset for France and the BSD dataset for the UK, we build on industry-level information from the OECD STAN database for both countries. More specifically, we obtain sectoral expenditure shares and R&D expenditure data by ISIC Rev. 3 from the French and UK input-output tables. These input-output tables contain information on 35 sectors and dictate the level of aggregation in our analysis. We discard the 'Public Administration and Defense' aggregate (12.12% of expenditure for France and 11.29% for the UK). Expenditure for each sector is computed as the sum of 'Households Final Consumption' (code C39) and 'General Government Final Consumption' (code C41). We use the ratio of R&D expenditure to gross output at basic prices to proxy for sunk entry costs and fixed costs, and trim the data by getting rid of the top and bottom 1.5% of the firm-level employment distribution across all sectors.[21]

**C.2. Additional expressions.** We derive the expressions needed to back out the structural parameters of the model in our quantification procedure.

**CARA subutility.** In the CARA case, firm variable employment used for production in the market equilibrium with Pareto productivity distribution is given by:

$$\text{varemp}^{\text{CARA}_j}(m) = \frac{m}{\alpha_j}\left(1 - W_j\right),$$

[21]We first match the R&D expenditure data with our 34 sectors and compute, for each sector, the ratio of R&D expenditure to gross output at basic prices (code R49) with the latter information coming from input-output tables. We then multiply the ratio by total employment in that sector, divide it by the number of firms to get a proxy measure of $F_j$ and $f_j$, and subtract it from the employment of each firm. We ignore those firms ending up with a non-positive employment.

where $W_j \equiv W(\mathrm{e}\, m/m_j^d)$ denotes the Lambert $W$ function. Using $z \equiv W(\mathrm{e}m/m_j^d)$, $\mathrm{e}m/m_j^d = z\mathrm{e}^z$ and $\mathrm{d}m = (1+z)\mathrm{e}^{z-1}m_j^d \mathrm{d}z$, the conditional mean of $\ln[\mathrm{varemp}_j^{\mathrm{CARA}}(m)]$ is given by:

$$\mathrm{mean}_j^{\mathrm{CARA}} = \frac{1}{G(m_j^d)} \int_0^{m_j^d} \ln\left[\frac{m}{\alpha_j}(1-W_j)\right] \mathrm{d}G(m) = M_j + \ln m_j^d - \ln \alpha_j,$$

where $M_j \equiv -1/k_j + k_j \int_0^1 (z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\ln(1-z)\mathrm{d}z$ is a function of $k_j$ only. In turn, the conditonal variance of $\ln[\mathrm{varemp}_j^{\mathrm{CARA}}(m)]$ becomes:

$$\begin{aligned}\left(\mathrm{sd}_j^{\mathrm{CARA}}\right)^2 &= \frac{1}{G(m_j^d)} \int_0^{m_j^d} \left\{\ln\left[\frac{m}{\alpha_j}(1-W_j)\right] - \mu_j^{\mathrm{CARA}}\right\}^2 \mathrm{d}G(m) \\ &= \frac{2}{k_j^2} - M_j^2 + k_j \int_0^1 \ln\left[(z\mathrm{e}^{z-1})^2(1-z)\right] (z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\ln(1-z)\mathrm{d}z,\end{aligned}$$

which yields the following expression:

$$\mathrm{sd}_j^{\mathrm{CARA}} = \sqrt{\frac{2}{k_j^2} - M_j^2 + k_j \int_0^1 \ln\left[(z\mathrm{e}^{z-1})^2(1-z)\right] (z\mathrm{e}^{z-1})^{k_j-1}(1+z)\mathrm{e}^{z-1}\ln(1-z)\mathrm{d}z}. \tag{C-1}$$

**CES subutility.** Turning to the CES case, firm variable employment used for production in the market equilibrium with Pareto productivity distribution is given by:

$$\mathrm{varemp}_j^{\mathrm{CES}}(m) = \frac{f_j\rho_j}{1-\rho_j}\left(\frac{m_j^d}{m}\right)^{\frac{\rho_j}{1-\rho_j}}.$$

The conditional mean of $\ln[\mathrm{varemp}_j^{\mathrm{CES}}(m)]$ is given by:

$$\mathrm{mean}_j^{\mathrm{CES}} = \frac{1}{G(m_j^d)} \int_0^{m_j^d} \ln\left[\frac{f_j\rho_j}{1-\rho_j}\left(\frac{m_j^d}{m}\right)^{\frac{\rho_j}{1-\rho_j}}\right] \mathrm{d}G(m) = \ln\left(\frac{f_j\rho_j}{1-\rho_j}\right) + \frac{\rho_j}{k_j(1-\rho_j)}.$$

Using the same approach than in the CARA case, one can obtain the standard deviation of $\ln[\mathrm{varemp}_j^{\mathrm{CES}}(m)]$, which depends on $k_j$ and $\rho_j$, as follows:

$$\mathrm{sd}_j^{\mathrm{CES}} = \frac{\rho_j}{k_j(1-\rho_j)}. \tag{C-2}$$

## D. Allais surplus

This appendix derives the *Allais surplus* (Allais, 1943, 1977), which is the welfare measure we use when quantifying aggregate welfare distortions. In our context, the Allais surplus is defined as the maximum amount of the numeraire that can be saved when the social planner minimizes the resource cost of providing the agents with the equilibrium utility. We thus consider the following optimization problem:

$$\begin{aligned} \min_{\{N_j^E, m_j^d, q_j(m)\}} \quad & L^A \equiv \sum_{j=1}^{J} N_j^E \left\{ \int_0^{m_j^d} \left[ Lmq_j(m) + f_j \right] \mathrm{d}G_j(m) + F_j \right\} \\ \text{s.t.} \quad & U\big(\widetilde{U}_1(U_1), \widetilde{U}_2(U_2), \ldots, \widetilde{U}_J(U_J)\big) \geq \overline{U}, \end{aligned} \tag{D-1}$$

where $\overline{U}$ is a fixed target utility level that needs to be provided to each agent. The solution to this problem yields the minimum resource cost, $L^A(\overline{U})$, required to achieve the target utility level. Setting $\overline{U} = U^{\text{eqm}}$, the Allais surplus is formally defined as:

$$A \equiv L - L^A(U^{\text{eqm}}), \tag{D-2}$$

where the first term $L$ is the amount of labor needed for the market economy to attain the equilibrium utility since the labor market clears in equilibrium. If there are distortions, the planner requires, by definition, less labor to attain the equilibrium utility than the market economy does. Thus, the minimum resource cost must satisfy $L^A(U^{\text{eqm}}) \leq L$, so that $A \geq 0$.

Let $\mu$ denote the Lagrange multiplier associated with the utility constraint. From (D-1), the first-order conditions with respect to $q_j(m)$, $m_j^d$, and $N_j^E$ are given by

$$u_j'(q_j(m)) = \frac{L}{\mu_j} m, \qquad \mu_j \equiv \mu \frac{\partial U}{\partial \widetilde{U}_j} \frac{\partial \widetilde{U}_j}{\partial U_j} \tag{D-3}$$

$$\mu_j u_j(q_j^d) = L m_j^d q_j^d + f_j \tag{D-4}$$

$$\mu_j \int_0^{m_j^d} u_j(q_j(m)) \mathrm{d}G_j(m) = \int_0^{m_j^d} [Lmq_j(m) + f_j] \mathrm{d}G_j(m) + F_j \tag{D-5}$$

as well as the constraint $\overline{U} = U\big(\widetilde{U}_1(U_1), \widetilde{U}_2(U_2), \ldots, \widetilde{U}_J(U_J)\big) \geq \overline{U}$. Comparing (D-3)–(D-5) with (12)–(14) reveals that the first-order conditions are isomorphic. Thus, we can conclude that the optimal cutoffs and quantities are the same in the Allais surplus problem and the 'primal' optimal problem in Section 2.2. In what follows, we focus on the optimal labor allocation and entry.

**D.1. CARA subutility.** Assume that the subutility function is of the CARA form $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, that the upper-tier utility function $U$ is of the CES form as in (E-1), that $\widetilde{U}_j(U_j) = U_j$, and that $G_j$ follows a Pareto distribution. We also assume that $f_j = 0$ in the CARA subutility case.

To derive the optimal masses of entrants, we use the multipliers $\mu_j \equiv \mu \mathcal{E}_{U,U_j} \frac{U}{U_j}$. Given the CES upper-tier utility, the ratio of multipliers in sectors $j$ and $\ell$ is

$$\frac{\mu_j}{\mu_\ell} = \frac{\beta_j}{\beta_\ell} \left( \frac{U_\ell}{U_j} \right)^{\frac{1}{\sigma}} = \frac{\alpha_\ell}{\alpha_j} \frac{m_j^d}{m_\ell^d}, \tag{D-6}$$

where we have used (D-3) evaluated at $m = m_j^d$ to get the last equality. It follows from (D-6) that

$$U_\ell = \left( \frac{\alpha_\ell \beta_\ell}{\alpha_j \beta_j} \frac{m_j^d}{m_\ell^d} \right)^\sigma U_j,$$

which, together with the utility constraint $\overline{U} = [\sum_{\ell=1}^J \beta_\ell U_\ell^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$, yields

$$\overline{U} = U_j \cdot \left[ \beta_j^{1-\sigma} \left( \frac{m_j^d}{\alpha_j} \right)^{\sigma-1} \sum_{\ell=1}^J \beta_\ell^\sigma \left( \frac{m_\ell^d}{\alpha_\ell} \right)^{1-\sigma} \right]^{\frac{\sigma}{\sigma-1}}. \tag{D-7}$$

Since the optimal quantities and cutoffs are the same in the 'primal' and 'dual' problems, we can plug (E-23) into (D-7) to eliminate $U_j$. We can then use $G_j(m_j^d) = \alpha_j F_j (k_j+1)^2/(L m_j^d)$ from the expression of the optimal cutoff (E-28) to solve for $N_j^E$ as follows

$$N_j^E = \frac{\alpha_j^{\sigma-1} \beta_j^\sigma [(m_j^d)^{\mathrm{opt}}]^{1-\sigma}}{\left[ \sum_{\ell=1}^J \alpha_\ell^{\sigma-1} \beta_\ell^\sigma [(m_\ell^d)^{\mathrm{opt}}]^{1-\sigma} \right]^{\frac{\sigma}{\sigma-1}}} \frac{L\overline{U}}{F_j(k_j+1)} = (N_j^E)^{\mathrm{opt}} \frac{\overline{U}}{U^{\mathrm{opt}}}, \tag{D-8}$$

where $U^{\mathrm{opt}} = \{\sum_{\ell=1}^J \alpha_\ell^{\sigma-1} \beta_\ell^\sigma [(m_\ell^d)^{\mathrm{opt}}]^{1-\sigma}\}^{1/(\sigma-1)}$ as given by (E-33) and where $(N_j^E)^{\mathrm{opt}}$ is given by (E-30). As can be seen from (D-8), the mass of entrants in sector $j$ needed to achieve $\overline{U}$ is proportional to this target utility level.

Summing up, to achieve the target utility $\overline{U}$ in the resource minimization problem, the planner imposes the socially optimal cutoffs $(m_j^d)^{\mathrm{opt}}$ and firm-specific quantities $q_j^{\mathrm{opt}}(m) = (1/\alpha_j) \ln(m_j^{\mathrm{opt}}/m)$, and chooses the mass of entrants (D-8) that is proportional to $\overline{U}$. Thus, to achieve a higher $\overline{U}$ the planner would allow more entrants, but always choose the same level of selection. The associated resource cost $L^A(\overline{U})$ can be obtained by plugging this solution back into the objective function as follows

$$L^A(\overline{U}) = \sum_{j=1}^{J} (N_j^E)^{\text{opt}} \frac{\overline{U}}{U^{\text{opt}}} \left[ \int_0^{(m_j^d)^{\text{opt}}} L m q_j^{\text{opt}}(m) \mathrm{d}G_j(m) + F_j \right] = \frac{\overline{U}}{U^{\text{opt}}} L.$$

The last equality holds because the optimal allocation in Appendix E, by definition, clears the labor market. Setting $\overline{U} = U^{\text{eqm}}$ yields

$$\frac{L^A(U^{\text{eqm}})}{L} = \frac{U^{\text{eqm}}}{U^{\text{opt}}} < 1, \quad \text{i.e.,} \quad \frac{L - L^A(U^{\text{eqm}})}{L} = \frac{U^{\text{opt}} - U^{\text{eqm}}}{U^{\text{opt}}}, \tag{D-9}$$

where the numerator of the left-hand side is the Allais surplus. This expression provides a measure of the aggregate welfare distortion in the economy. Note that we may use the welfare measure based on utility and the measure based on the Allais surplus interchangeably.

**D.2. CES subutility.** Assume that the subutility function is of the CES form $u_j(q_j(m)) = q_j(m)^{\rho_j}$, that the upper-tier utility function $U$ is of the CES form as in (E-1), that $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$, and that $G_j$ follows a Pareto distribution. We also assume that $f_j > 0$.

To derive the optimal masses of entrants, we use the multipliers $\mu_j \equiv \mu \frac{\partial U}{\partial \widetilde{U}_j} \frac{\partial \widetilde{U}_j}{\partial U_j}$. Given the CES upper-tier utility, the ratio of multipliers in sectors $j$ and $\ell$ is

$$\frac{\mu_j}{\mu_\ell} = \frac{\beta_j/\rho_j}{\beta_\ell/\rho_\ell} \frac{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}}{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}} = \frac{\rho_\ell (q_j^d)^{1-\rho_j} m_j^d}{\rho_j (q_\ell^d)^{1-\rho_\ell} m_\ell^d}, \tag{D-10}$$

where we have used (D-3) evaluated at $m = m_j^d$ in the second equality. Since the optimal cutoffs and quantities are as in Appendix E, using (E-35) allows us to rewrite expression (D-10) as follows:

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \left(\frac{\beta_j}{\beta_\ell}\right) \left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j - 1} \left[\frac{f_\ell\rho_\ell}{L(1-\rho_\ell)}\right]^{1-\rho_\ell} \frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}. \tag{D-11}$$

Since the right-hand side of (D-11) is the same as that of (E-40), we obtain

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \frac{(U_j^{\text{opt}})^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{(U_\ell^{\text{opt}})^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}}.$$

As in Appendix E, we now consider that $\sigma \to 1$ in order to derive closed-form solutions. We then have $U_j/U_\ell = U_j^{\text{opt}}/U_\ell^{\text{opt}}$ and from the definition of $\overline{U}$ we obtain:

$$\begin{aligned} \overline{U} &= \prod_{\ell=1}^{J} U_j^{\frac{\beta_\ell}{\rho_\ell}} \left(\frac{U_\ell}{U_j}\right)^{\frac{\beta_\ell}{\rho_\ell}} = \prod_{\ell=1}^{J} U_j^{\frac{\beta_\ell}{\rho_\ell}} \left(\frac{U_\ell^{\text{opt}}}{U_j^{\text{opt}}}\right)^{\frac{\beta_\ell}{\rho_\ell}} \\ &= \prod_{\ell=1}^{J} (U_\ell^{\text{opt}})^{\frac{\beta_\ell}{\rho_\ell}} \prod_{\ell=1}^{J} \left(\frac{U_j}{U_j^{\text{opt}}}\right)^{\frac{\beta_\ell}{\rho_\ell}} = U^{\text{opt}} \cdot \left(\frac{U_j}{U_j^{\text{opt}}}\right)^{\sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell}}. \end{aligned} \tag{D-12}$$

Using (E-38), and because $m_j^d = (m_j^d)^{\text{opt}}$, we know that $U_j/U_j^{\text{opt}} = N_j^E/(N_j^E)^{\text{opt}}$. Plugging this expression into (D-12), we obtain

$$N_j^E = (N_j^E)^{\text{opt}} \left(\frac{\overline{U}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell}}}.$$

Thus, we have

$$\begin{aligned} L^A(\overline{U}) &= \sum_{j=1}^{J} (N_j^E)^{\text{opt}} \left(\frac{\overline{U}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell}}} \left\{ \int_0^{(m_j^d)^{\text{opt}}} \left[ Lmq_j^{\text{opt}}(m) + f_j \right] \mathrm{d}G_j(m) + F_j \right\} \\ &= \left(\frac{\overline{U}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell}}} L, \end{aligned}$$

where the last equality holds because the optimal allocation clears the labor market. Hence, evaluating $\overline{U}$ at $U^{\text{eqm}}$, we obtain

$$\frac{L - L^A(U^{\text{eqm}})}{L} = 1 - \left(\frac{U^{\text{eqm}}}{U^{\text{opt}}}\right)^{\frac{1}{\sum_{\ell=1}^{J} \frac{\beta_\ell}{\rho_\ell}}}. \tag{D-13}$$

This expression provides a measure of the aggregate welfare distortion in the economy. Note that we may not use the welfare measure based on utility and the measure based on the Allais surplus interchangeably in this case, as we could in the CARA case in Appendix D.1. The reason is the presence of $\widetilde{U}_j$, which is a transformation of the lower-tier utility. Without that transformation, which in the CES case would amount to setting all $\rho_\ell$'s that appear in the power of (D-13) equal to one, the foregoing result that utility and the Allais surplus can be used interchangeably would still hold.

# Supplementary Appendix – for online publication

In this supplementary Appendix, we first provide details on the derivations of the equilibrium and optimal allocations for CARA subutility functions. We then provide a brief summary of the expressions for CES subutility functions. These expressions are required for the quantitative analysis. Last, we provide details on how we quantify the case with CARA subutility functions and CES upper-tier utility.

## E. Analytical expressions

We assume that the upper-tier utility is of the CES form:

$$U = \left\{ \sum_{j=1}^{J} \beta_j \left[ \widetilde{U}_j(U_j) \right]^{(\sigma-1)/\sigma} \right\}^{\sigma/(\sigma-1)}, \tag{E-1}$$

where $\sigma > 1$ is the intersectoral elasticity of substitution, and where the $\beta_j$ are strictly positive parameters that sum to one. The lower-tier utility is $U_j \equiv N_j^E \int_0^{m_j^d} u_j\big(q_j(m)\big) \mathrm{d}G_j(m)$. In what follows, we focus on cases in which the CES form in (E-1) satisfies condition (24), so that there exist unique intersectoral equilibrium and optimal allocations. As explained in the main text, this is always the case for CARA subutility functions and $\widetilde{U}_j(U_j) = U_j$, and it is the case for homothetic lower-tier CES utility functions with $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$ when the lower-tier elasticity of substitution exceeds the upper-tier elasticity of substitution. Observe that (E-1) includes the Cobb-Douglas form as a limit case. All results based on the Cobb-Douglas specification, as given in the main text, can be retrieved from the following expressions by letting $\sigma \to 1$.

**E.1. CARA subutility.** We provide detailed derivations of the equilibrium and optimal allocations in the CARA case.

**Equilibrium allocation.** We first derive the equilibrium cutoffs and quantities.[22] Assume that $\widetilde{U}_j(U_j) = U_j$, and that $u_j(q_j(m)) = 1 - \mathrm{e}^{-\alpha_j q_j(m)}$, so that $u_j'(q_j(m)) = \alpha_j \mathrm{e}^{-\alpha_j q_j(m)}$, $u_j''(q_j(m)) = -\alpha_j^2 \mathrm{e}^{-\alpha_j q_j(m)}$, and $r_u(q_j(m)) = \alpha_j q_j(m)$. We assume in what follows that there are no fixed costs for production, i.e., $f_j = 0$ for all sectors $j$. We can do so since, as in Melitz and Ottaviano (2008) but contrary to Melitz (2003), the marginal utility of each

[22]Additional information on the equilibrium cutoffs and quantities can be found in Behrens and Murata (2007) and in Behrens et al. (2014).

variety is bounded at zero consumption so that demand for a variety drops to zero when its price exceeds some threshold. Since for the least productive firm, which is indifferent between producing and not producing, we have $q_j^d \equiv q_j(m_j^d) = 0$, the first-order conditions (2) evaluated for any $m$ and at the cutoff $m_j^d$ imply the following demand functions:

$$q_j(m) = \frac{1}{\alpha_j} \ln \left[ \frac{p_j^d}{p_j(m)} \right] \quad \text{for} \quad 0 \leq m \leq m_j^d, \tag{E-2}$$

where $p_j^d \equiv p_j(m_j^d)$. Making use of the profit maximizing prices (5), $r_u(q_j(m)) = \alpha_j q_j(m)$, and $q_j^d = 0$, we have

$$q_j(m) = \frac{1}{\alpha_j} \ln \left[ \frac{m_j^d}{1 - r_{u_j}(q_j^d)} \frac{1 - r_u(q_j(m))}{m} \right] = \frac{1}{\alpha_j} \ln \left\{ \frac{m_j^d}{m} [1 - \alpha_j q_j(m)] \right\}.$$

This implicit equation can be solved for $q_j(m) = (1 - W_j)/\alpha_j$, where $W_j \equiv W(\mathrm{e}\, m/m_j^d)$ denotes the Lambert $W$ function, defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ (see Corless et al., 1996). We suppress its argument to alleviate notation whenever there is no possible confusion. Since $r_{u_j} = 1 - W_j$, we then also have the following profit maximizing prices, quantities, and operating profits:

$$p_j(m) = \frac{mw}{W_j}, \quad q_j(m) = \frac{1}{\alpha_j}(1 - W_j), \quad \pi_j(m) = \frac{Lmw}{\alpha_j}(W_j^{-1} + W_j - 2). \tag{E-3}$$

By definition of the Lambert $W$ function, we have $W(\varphi) \geq 0$ for all $\varphi \geq 0$. Taking logarithms on both sides of $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ and differentiating yields

$$W'(\varphi) = \frac{W(\varphi)}{\varphi[W(\varphi) + 1]} > 0$$

for all $\varphi > 0$. Finally, we have: $0 = W(0)\mathrm{e}^{W(0)}$, which implies $W(0) = 0$; and $\mathrm{e} = W(\mathrm{e})\mathrm{e}^{W(\mathrm{e})}$, which implies $W(\mathrm{e}) = 1$. Hence, we have $0 \leq W_j \leq 1$ if $0 \leq m \leq m_j^d$. The expressions in (E-3) show that a firm with a draw $m_j^d$ charges a price equal to marginal cost, faces zero demand, and earns zero operating profits. Furthermore, using the properties of $W'$, we readily obtain $\partial p_j(m)/\partial m > 0$, $\partial q_j(m)/\partial m < 0$, and $\partial \pi_j(m)/\partial m < 0$. In words, firms with higher productivity $1/m$ charge lower prices, produce larger quantities, and earn higher operating profits. Our specification with variable demand elasticity also features higher markups for more productive firms. Indeed, the markup

$$\Lambda_j(m) \equiv \frac{p_j(m)}{mw} = \frac{1}{W_j} \tag{E-4}$$

is such that $\partial \Lambda_j(m)/\partial m < 0$.

Using (E-3) and $r_{u_j} = 1 - W_j$, and recalling that $f_j = 0$, the zero expected profit condition (7) can be expressed as

$$\int_0^{m_j^d} m\left(W_j^{-1} + W_j - 2\right) \mathrm{d}G_j(m) = \frac{\alpha_j F_j}{L}. \tag{E-5}$$

To derive closed-form solutions for various expressions with CARA subutility functions, we need to compute integrals involving the Lambert $W$ function. This can be done by using the change in variables suggested by Corless et al. (1996, p.341). Let

$$z \equiv W\left(\mathrm{e}\,\frac{m}{m_j^d}\right), \quad \text{so that} \quad \mathrm{e}\,\frac{m}{m_j^d} = z\mathrm{e}^z.$$

The change in variables then yields $\mathrm{d}m = (1+z)\mathrm{e}^{z-1} m_j^d \mathrm{d}z$, with the new integration bounds given by 0 and 1. Using the change in variables, the LHS of (E-5) can be expressed as follows:

$$\int_0^{m_j^d} m\left(W_j^{-1} + W_j - 2\right) \mathrm{d}G_j(m) = (m_j^d)^2 \int_0^1 z(1+z)\mathrm{e}^{2(z-1)}(z^{-1} + z - 2) g_j\left(z\mathrm{e}^{z-1} m_j^d\right) \mathrm{d}z$$

for an arbitrary distribution $g_j(\cdot)$ of draws.

We consider the Pareto distribution $G_j(m) = (m/m_j^{\max})^{k_j}$ with upper bound $m_j^{\max} > 0$ and shape parameter $k_j \geq 1$. The associated density $g_j$ is 'multiplicatively quasi-separable' in the sense that $g_j(xy) \equiv g_j(x) \times h_j(y)$ for some function $h_j$ (see Behrens and Murata, 2007, Theorem 1, p.779). In that case, we have

$$\int_0^{m_j^d} m\left(W_j^{-1} + W_j - 2\right) \mathrm{d}G_j(m) = (m_j^d)^2 h_j(m_j^d) \int_0^1 z(1+z)\mathrm{e}^{2(z-1)}(z^{-1} + z - 2) g_j\left(z\mathrm{e}^{z-1}\right) \mathrm{d}z,$$

where the integral term is independent of the cutoff $m_j^d$. This property simplifies substantially the analysis. Indeed, the integral reduces to

$$\int_0^{m_j^d} m\left(W_j^{-1} + W_j - 2\right) \mathrm{d}G_j(m) = \kappa_j \left(m_j^{\max}\right)^{-k_j} (m_j^d)^{k_j+1}, \tag{E-6}$$

where $\kappa_j \equiv k_j \mathrm{e}^{-(k_j+1)} \int_0^1 (1+z)\left(z^{-1}+z-2\right)(z\mathrm{e}^z)^{k_j}\,\mathrm{e}^z \mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter $k_j$. Plugging (E-6) into (E-5), we obtain the equilibrium cutoffs

$$(m_j^d)^{\mathrm{eqm}} = \left[\frac{\alpha_j F_j (m_j^{\max})^{k_j}}{\kappa_j L}\right]^{\frac{1}{k_j+1}} \tag{E-7}$$

and quantities $q_j^{\mathrm{eqm}}(m) = [1 - W_j(\mathrm{e}\, m/(m_j^d)^{\mathrm{eqm}})]/\alpha_j$. Note that (E-7) implies that

$$\left[\frac{(m_j^d)^{\mathrm{eqm}}}{m_j^{\max}}\right]^{k_j} = G_j((m_j^d)^{\mathrm{eqm}}) = \frac{\alpha_j F_j}{\kappa_j L} \frac{1}{(m_j^d)^{\mathrm{eqm}}}, \tag{E-8}$$

a relationship that we will use in what follows.

We now turn to the equilibrium labor allocation and masses of entrants. Using (E-3), labor market clearing in sector $j$ can be written as

$$N_j^E \left[L \int_0^{m_j^d} m q_j(m) \mathrm{d}G_j(m) + F_j\right] = N_j^E \left[\frac{L}{\alpha_j} \int_0^{m_j^d} m\,(1-W_j)\,\mathrm{d}G_j(m) + F_j\right] = L_j. \tag{E-9}$$

Making use of the same change in variables for integration as before, and imposing the Pareto distribution, we have

$$\int_0^{m_j^d} m\,(1-W_j)\,\mathrm{d}G_j(m) = \kappa_{1j} \left(m_j^{\max}\right)^{-k_j} (m_j^d)^{k_j+1}, \tag{E-10}$$

where $\kappa_{1j} \equiv k_j \mathrm{e}^{-(k_j+1)} \int_0^1 (1-z^2)(z\mathrm{e}^z)^{k_j}\,\mathrm{e}^z \mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter $k_j$. It can be verified that $\kappa_{1j}/\kappa_j = k_j$, so that

$$\kappa_j(k_j+1) = \kappa_{1j} + \kappa_j. \tag{E-11}$$

Using (E-7)–(E-11), and $\sum_{j=1}^J L_j = L$, the labor market clearing condition thus reduces to

$$\sum_{j=1}^J L_j = \sum_{j=1}^J \frac{\kappa_{1j} + \kappa_j}{\kappa_j} N_j^E F_j = \sum_{j=1}^J (k_j+1) N_j^E F_j = L. \tag{E-12}$$

Computing $\partial U/\partial U_j$ from (E-1), inserting the definition of $\lambda_j$ into (3), and recalling that

$q_j^d = 0$ and $p_j^d = m_j^d w$ for all $j$, we obtain

$$\frac{\alpha_j}{\alpha_\ell} = \frac{m_j^d}{m_\ell^d}\frac{\lambda_j}{\lambda_\ell} \quad \Rightarrow \quad \frac{U_j}{U_\ell} = \left(\frac{\alpha_j}{\alpha_\ell}\right)^\sigma \left(\frac{\beta_j}{\beta_\ell}\right)^\sigma \left[\frac{(m_j^d)^{\text{eqm}}}{(m_\ell^d)^{\text{eqm}}}\right]^{-\sigma}. \tag{E-13}$$

To solve them for the masses of entrants, we first compute the expression for $U_j$ in (E-13). Using the demand functions (E-2) and the profit-maximizing prices in (E-3), the lower-tier utility is given by

$$U_j = N_j^E \left[G_j(m_j^d) - \frac{1}{m_j^d}\int_0^{m_j^d} m W_j^{-1} \mathrm{d}G_j(m)\right], \tag{E-14}$$

which can be integrated (using again the same change in variables as before) to obtain:

$$\int_0^{m_j^d} m W_j^{-1} \mathrm{d}G_j(m) = \kappa_{2j} \left(m_j^{\max}\right)^{-k_j} (m_j^d)^{k_j+1},$$

where $\kappa_{2j} \equiv k_j \mathrm{e}^{-(k_j+1)} \int_0^1 (z^{-1}+1)\left(z\mathrm{e}^z\right)^{k_j} \mathrm{e}^z \mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter $k_j$. One can verify that $1 - \kappa_{2j} = \frac{1}{k_j+1} - (\kappa_{1j} + \kappa_j)$, so that we can rewrite the $\kappa_{2j}$ term in terms of $\kappa_{1j}$ and $\kappa_j$ only. Thus, the lower-tier utility (E-14) becomes

$$U_j = \left[\frac{1}{k_j+1} - (\kappa_{1j} + \kappa_j)\right] N_j^E G_j(m_j^d). \tag{E-15}$$

Since $U_j > 0$ by construction of the lower-tier utility, we have $(\kappa_{1j} + \kappa_j)(k_j + 1) < 1$, which is equivalent to $\kappa_j (k_j+1)^2 < 1$ by (E-11).

We next insert (E-15) into (E-13) to obtain

$$\frac{N_j^E}{N_\ell^E} = \left(\frac{\alpha_j}{\alpha_\ell}\right)^\sigma \left(\frac{\beta_j}{\beta_\ell}\right)^\sigma \left[\frac{\frac{1}{k_\ell+1} - (\kappa_{1\ell}+\kappa_\ell)}{\frac{1}{k_j+1} - (\kappa_{1j}+\kappa_j)}\right] \left[\frac{(m_j^d)^{\text{eqm}}}{(m_\ell^d)^{\text{eqm}}}\right]^{-\sigma} \left[\frac{G_\ell((m_\ell^d)^{\text{eqm}})}{G_j((m_j^d)^{\text{eqm}})}\right], \tag{E-16}$$

which allows us to express the mass of entrants in sector $j$ as a function of the mass of entrants in sector $\ell$. Inserting $N_j^E = N_j^E(N_\ell^E)$ into the labor market clearing condition (E-12), and using (E-8), we can solve for the mass of entrants in sector $j$ as follows:

$$(N_j^E)^{\text{eqm}} = \frac{\alpha_j^{\sigma-1}\beta_j^\sigma \theta_j [(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^J \alpha_\ell^{\sigma-1}\beta_\ell^\sigma \theta_\ell [(m_\ell^d)^{\text{eqm}}]^{1-\sigma}} \frac{L}{(k_j+1)F_j}, \tag{E-17}$$

where $\theta_j \equiv \frac{\kappa_j(k_j+1)}{1/(k_j+1) - (\kappa_{1j}+\kappa_j)}$ is the ratio of real revenue-to-utility, which depends only on $k_j$.

Combining (E-17) and (E-12) yields the following equilibrium labor allocation to sector $j$:

$$L_j^{\mathrm{eqm}} = (k_j + 1)(N_j^E)^{\mathrm{eqm}} F_j = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}\theta_j[(m_j^d)^{\mathrm{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\theta_\ell[(m_\ell^d)^{\mathrm{eqm}}]^{1-\sigma}} L. \tag{E-18}$$

As shown in Lemma 1, the sectoral labor allocation satisfies $L_j = e_j L$, where $e_j$ is the sectoral expenditure share given by $e_j \equiv N_j^E \int_0^{m_j^d} p_j(m) q_j(m) \mathrm{d}G_j(m)/w$. From (E-18), we thus directly have

$$e_j^{\mathrm{eqm}} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}\theta_j[(m_j^d)^{\mathrm{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\theta_\ell[(m_\ell^d)^{\mathrm{eqm}}]^{1-\sigma}}. \tag{E-19}$$

Finally, inserting (E-8) and (E-17) into (E-15), and noting (E-19) and the definition of $\theta_j$, we can express the lower-tier utility from sector $j$ in the market equilibrium in a compact form as follows:

$$U_j^{\mathrm{eqm}} = \frac{\alpha_j e_j^{\mathrm{eqm}}}{\theta_j} \frac{1}{(m_j^d)^{\mathrm{eqm}}}. \tag{E-20}$$

Making use of the upper-tier utility (E-1) and of (E-20), the utility $U$ across all sectors is then

$$U^{\mathrm{eqm}} = \left\{ \frac{\sum_{j=1}^{J}\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\mathrm{eqm}}]^{1-\sigma}}{\left[\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\theta_\ell[(m_\ell^d)^{\mathrm{eqm}}]^{1-\sigma}\right]^{\frac{\sigma-1}{\sigma}}} \right\}^{\frac{\sigma}{\sigma-1}} = \frac{\left\{\sum_{j=1}^{J}\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\mathrm{eqm}}]^{1-\sigma}\right\}^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\theta_\ell[(m_\ell^d)^{\mathrm{eqm}}]^{1-\sigma}}. \tag{E-21}$$

When the upper-tier utility function is of the Cobb-Douglas form, $\sigma = 1$, so that (E-19) reduces to $e_j^{\mathrm{eqm}} = \beta_j\theta_j / \sum_{\ell=1}^{J}(\beta_\ell\theta_\ell)$. Expression (E-20) can then be rewritten as $U_j^{\mathrm{eqm}} = [\alpha_j\beta_j / \sum_{\ell=1}^{J}(\beta_\ell\theta_\ell)][1/(m_j^d)^{\mathrm{eqm}}]$. Hence, (E-21) reduces to

$$U^{\mathrm{eqm}} = \prod_{j=1}^{J}\left[\frac{\alpha_j\beta_j}{\sum_{\ell=1}^{J}(\beta_\ell\theta_\ell)} \frac{1}{(m_j^d)^{\mathrm{eqm}}}\right]^{\beta_j}.$$

**Optimal allocation.** We next derive the expressions for the optimal cutoffs and quantities in the CARA case. From the first-order conditions (12), the optimal consumptions must satisfy

$$\frac{\alpha_j \mathrm{e}^{-\alpha_j q_j(m_j^d)}}{\alpha_j \mathrm{e}^{-\alpha_j q_j(m)}} = \frac{m_j^d}{m} \quad \text{and} \quad \frac{\alpha_j \mathrm{e}^{-\alpha_j q_j(m_j^d)}}{\alpha_\ell \mathrm{e}^{-\alpha_\ell q_\ell(m_\ell^d)}} = \frac{\delta_j}{\delta_\ell}\frac{m_j^d}{m_\ell^d}.$$

The first conditions, together with $q_j(m_j^d) = 0$, can be solved to yield:

$$q_j(m) = \frac{1}{\alpha_j} \ln\left(\frac{m_j^d}{m}\right) \quad \text{for} \quad 0 \le m \le m_j^d. \tag{E-22}$$

Plugging (E-22) into $U_j$ and letting $\overline{m}_j \equiv (1/G_j(m_j^d)] \int_0^{m_j^d} m \mathrm{d}G_j(m)$ denote the average value of $m$, we obtain:

$$U_j = \left(1 - \frac{\overline{m}_j}{m_j^d}\right) N_j^E G_j(m_j^d) = \frac{N_j^E G_j(m_j^d)}{k_j + 1}, \tag{E-23}$$

where we have used the property of the Pareto distribution that $\overline{m}_j = [k_j/(k_j+1)]m_j^d$ to obtain the second equality. Plugging (E-22) into (11) and integrating, the resource constraint becomes

$$\sum_{j=1}^{J} N_j^E \left[\frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} m_j^d G_j(m_j^d) + F_j\right] = L. \tag{E-24}$$

Assuming that the upper-tier utility function is given by (E-1), the planner's problem can be redefined using (E-23) and (E-24) as follows:

$$\max_{\{N_j^E, m_j^d\}} \quad \widehat{V} \equiv L \cdot \left\{ \sum_{j=1}^{J} \beta_j \left[\frac{N_j^E G_j(m_j^d)}{k_j+1}\right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \tag{E-25}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} N_j^E \left[\frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} m_j^d G_j(m_j^d) + F_j\right] = L. \tag{E-26}$$

Denoting by $\widehat{\delta}$ the Lagrange multiplier of this redefined problem, the first-order conditions with respect to $N_j^E$ and $m_j^d$ are given by

$$\frac{\beta_j \widehat{V}}{N_j^E} \frac{\left[\frac{N_j^E G_j(m_j^d)}{k_j+1}\right]^{\frac{\sigma-1}{\sigma}}}{\sum_{\ell=1}^{J} \beta_\ell \left[\frac{N_\ell^E G_\ell(m_\ell^d)}{k_\ell+1}\right]^{\frac{\sigma-1}{\sigma}}} = \widehat{\delta} \left[\frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} m_j^d G_j(m_j^d) + F_j\right] \tag{E-27}$$

$$\frac{\beta_j \widehat{V}}{N_j^E} \frac{\left[\frac{N_j^E G_j(m_j^d)}{k_j+1}\right]^{\frac{\sigma-1}{\sigma}}}{\sum_{\ell=1}^{J} \beta_\ell \left[\frac{N_\ell^E G_\ell(m_\ell^d)}{k_\ell+1}\right]^{\frac{\sigma-1}{\sigma}}} = \widehat{\delta} \frac{L}{\alpha_j} \frac{k_j}{(k_j+1)^2} \frac{G_j(m_j^d)}{G_j'(m_j^d)} \left[G_j(m_j^d) + m_j^d G_j'(m_j^d)\right].$$

Because the left-hand side is common, we obtain the optimal cutoffs

$$(m_j^d)^{\text{opt}} = \left[ \frac{\alpha_j F_j (m_j^{\max})^{k_j} (k_j+1)^2}{L} \right]^{\frac{1}{k_j+1}} \tag{E-28}$$

and quantities $q_j^{\text{opt}}(m) = (1/\alpha_j) \ln[(m_j^d)^{\text{opt}}/m]$. Note that (E-28) implies that

$$\left[ \frac{(m_j^d)^{\text{opt}}}{m_j^{\max}} \right]^{k_j} = G_j((m_j^d)^{\text{opt}}) = \frac{\alpha_j F_j (k_j+1)^2}{L} \frac{1}{(m_j^d)^{\text{opt}}}, \tag{E-29}$$

a relationship that we will use repeatedly in what follows.

Using (E-29), the right-hand side of (E-27) becomes $\widehat{\delta} F_j (k_j+1)$. Moreover, taking the ratio of (E-27) for sectors $j$ and $\ell$, we have

$$\frac{N_j^E}{N_\ell^E} = \left( \frac{\beta_j}{\beta_\ell} \right)^{\sigma} \left[ \frac{G_j(m_j^d)}{k_j+1} \right]^{\sigma-1} \left[ \frac{G_\ell(m_\ell^d)}{k_\ell+1} \right]^{1-\sigma} \left[ \frac{(k_j+1)F_j}{(k_\ell+1)F_\ell} \right]^{-\sigma}$$

for all $j = 1, 2, \ldots, J$. Plugging this relationship into the resource constraint (E-26), and using (E-29), we readily obtain the optimal mass of entrants in sector $j$:

$$(N_j^E)^{\text{opt}} = \frac{\alpha_j^{\sigma-1} \beta_j^{\sigma} [(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J} \alpha_\ell^{\sigma-1} \beta_\ell^{\sigma} [(m_\ell^d)^{\text{opt}}]^{1-\sigma}} \frac{L}{(k_j+1)F_j}, \tag{E-30}$$

which implies the optimal labor allocation as follows:

$$L_j^{\text{opt}} = (k_j+1)(N_j^E)^{\text{opt}} F_j = \frac{\alpha_j^{\sigma-1} \beta_j^{\sigma} [(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J} \alpha_\ell^{\sigma-1} \beta_\ell^{\sigma} [(m_\ell^d)^{\text{opt}}]^{1-\sigma}} L.$$

Since the optimal labor allocation satisfies $L_j^{\text{opt}} = e_j^{\text{opt}} L$ by Lemma 2, the social expenditure share on good $j$ is therefore given by

$$e_j^{\text{opt}} = \frac{\alpha_j^{\sigma-1} \beta_j^{\sigma} [(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J} \alpha_\ell^{\sigma-1} \beta_\ell^{\sigma} [(m_\ell^d)^{\text{opt}}]^{1-\sigma}}. \tag{E-31}$$

Finally, plugging (E-29) and (E-30) into (E-23), the lower-tier utility from sector $j$ at the optimal allocation can be expressed as

$$U_j^{\text{opt}} = \alpha_j e_j^{\text{opt}} \frac{1}{(m_j^d)^{\text{opt}}}, \tag{E-32}$$

so that

$$U^{\text{opt}} = \left\{ \sum_{j=1}^{J} \alpha_j^{\sigma-1} \beta_j^{\sigma} [(m_j^d)^{\text{opt}}]^{1-\sigma} \right\}^{\frac{1}{\sigma-1}}. \tag{E-33}$$

When the upper-tier utility function is of the Cobb-Douglas form, $\sigma = 1$, so that (E-31) reduces to $e_j^{\text{opt}} = \beta_j$. Expression (E-32) can then be rewritten as $U_j^{\text{opt}} = \alpha_j \beta_j / (m_j^d)^{\text{opt}}$. Hence, (E-33) reduces to

$$U^{\text{opt}} = \prod_{j=1}^{J} \left[ \frac{\alpha_j \beta_j}{(m_j^d)^{\text{opt}}} \right]^{\beta_j}.$$

**E.2. CES subutility.** We briefly summarize the equilibrium and optimal allocations in the case with CES subutility functions, $u_j(q_j(m)) = q_j(m)^{\rho_j}$, where $0 < \rho_j < 1$, and Pareto distribution functions, $G_j(m) = (m/m_j^{\max})^{k_j}$. As in the existing literature, we also assume that $\widetilde{U}_j(U_j) = U_j^{1/\rho_j}$ and that $f_j > 0$.

First, with CES subutility functions, $1 - r_{u_j}(q_j(m)) = \mathcal{E}_{u_j, q_j(m)} = \rho_j$ holds for all $m$, and $q_j(m) = (m_j^d/m)^{1/(1-\rho_j)} q_j^d$ holds for both the equilibrium and optimal allocations. Thus, the ZEP and ZCP conditions, (7) and (6), are equivalent to the ZESP and ZCSP conditions, (16) and (17). The resulting equilibrium and optimum cutoffs are therefore the same and given by

$$(m_j^d)^{\text{eqm}} = (m_j^d)^{\text{opt}} = m_j^{\max} \left[ \frac{F_j}{f_j} \frac{k_j(1-\rho_j) - \rho_j}{\rho_j} \right]^{\frac{1}{k_j}}, \tag{E-34}$$

which implies that the demand functions $q_j(m)$ are common between the equilibrium and the optimum for all $m \leq m_j^d$. In particular $q_j^d$ can be obtained from (6) or (17) as follows:

$$q_j^d = \frac{f_j}{L} \frac{\rho_j}{1-\rho_j} \frac{1}{m_j^d}. \tag{E-35}$$

Second, given the foregoing results, $\nu_j(q_j(m)) = \zeta_j(q_j(m))$ holds for all $m \leq m_j^d$, so that the expressions in the braces of (9) and those of (19) are the same. Thus, the equilibrium and optimal masses of entrants satisfy

$$(N_j^E)^{\text{eqm}} = e_j^{\text{eqm}} \frac{L\rho_j}{k_j F_j} \quad \text{and} \quad (N_j^E)^{\text{opt}} = e_j^{\text{opt}} \frac{L\rho_j}{k_j F_j}. \tag{E-36}$$

Third, the conditions (3) for equilibrium intersectoral consumption can be rewritten as

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \left(\frac{\beta_j\rho_j}{\beta_\ell\rho_\ell}\right)\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[\frac{f_\ell\rho_\ell}{L(1-\rho_\ell)}\right]^{1-\rho_\ell}\frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}.$$

To obtain closed-form solutions, we assume that $\sigma = 1$, so that the above expression reduces to the Cobb-Douglas case:

$$\frac{U_j}{U_\ell} = \left(\frac{\beta_j\rho_j}{\beta_\ell\rho_\ell}\right)\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[\frac{f_\ell\rho_\ell}{L(1-\rho_\ell)}\right]^{1-\rho_\ell}\frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}. \tag{E-37}$$

Using (E-34) and (E-35), together with $q_j(m) = (m_j^d/m)^{1/(1-\rho_j)}q_j^d$ and the Pareto distribution, the lower-tier utility is given by

$$U_j = \frac{N_j^E k_j F_j}{L}\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}(m_j^d)^{-\rho_j}. \tag{E-38}$$

Plugging (E-38) into (E-37) and using (E-36), we then obtain

$$\frac{(N_j^E)^{\text{eqm}}}{(N_\ell^E)^{\text{eqm}}} = \frac{\beta_j\rho_j}{\beta_\ell\rho_\ell}\frac{k_\ell F_\ell}{k_j F_j} = \frac{e_j^{\text{eqm}}}{e_\ell^{\text{eqm}}}\frac{\rho_j}{\rho_\ell}\frac{k_\ell F_\ell}{k_j F_j} \quad \Rightarrow \quad e_\ell^{\text{eqm}} = \frac{\beta_\ell}{\beta_j}e_j^{\text{eqm}}.$$

Since $\sum_{\ell=1}^J e_\ell = 1$, we finally obtain

$$e_j^{\text{eqm}} = \frac{\beta_j}{\sum_{\ell=1}^J \beta_\ell} = \beta_j. \tag{E-39}$$

Using (E-36) and (E-39), expression (E-38) can be rewritten as

$$U_j^{\text{eqm}} = \beta_j\rho_j\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[(m_j^d)^{\text{eqm}}\right]^{-\rho_j},$$

which yields

$$U^{\text{eqm}} = \prod_{j=1}^J\left\{\beta_j\rho_j\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[(m_j^d)^{\text{eqm}}\right]^{-\rho_j}\right\}^{\frac{\beta_j}{\rho_j}}.$$

Turning to the optimal allocation, the conditions (15) for optimal intersectoral consumption can be rewritten as

$$\frac{U_j^{\frac{1-\sigma(1-\rho_j)}{\sigma\rho_j}}}{U_\ell^{\frac{1-\sigma(1-\rho_\ell)}{\sigma\rho_\ell}}} = \left(\frac{\beta_j}{\beta_\ell}\right)\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[\frac{f_\ell\rho_\ell}{L(1-\rho_\ell)}\right]^{1-\rho_\ell}\frac{(m_j^d)^{-\rho_j}}{(m_\ell^d)^{-\rho_\ell}}. \tag{E-40}$$

Assume again that the upper-tier utility is Cobb-Douglas, i.e., $\sigma \to 1$. In that case, we can use the same procedure as above to obtain

$$\frac{(N_j^E)^{\text{opt}}}{(N_\ell^E)^{\text{opt}}} = \frac{\beta_j}{\beta_\ell}\frac{k_\ell F_\ell}{k_j F_j} = \frac{e_j^{\text{opt}}}{e_\ell^{\text{opt}}}\frac{\rho_j}{\rho_\ell}\frac{k_\ell F_\ell}{k_j F_j} \quad \Rightarrow \quad e_\ell^{\text{opt}} = \frac{\beta_\ell/\rho_\ell}{\beta_j/\rho_j}e_j^{\text{opt}}$$

so that

$$e_j^{\text{opt}} = \frac{\beta_j/\rho_j}{\sum_{\ell=1}^J(\beta_\ell/\rho_\ell)}.$$

Using (E-36), expression (E-38) can be rewritten as

$$U_j^{\text{opt}} = \frac{\beta_j}{\sum_{\ell=1}^J(\beta_\ell/\rho_\ell)}\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[(m_j^d)^{\text{opt}}\right]^{-\rho_j},$$

which yields

$$U^{\text{opt}} = \prod_{j=1}^J\left\{\frac{\beta_j}{\sum_{\ell=1}^J(\beta_\ell/\rho_\ell)}\left[\frac{f_j\rho_j}{L(1-\rho_j)}\right]^{\rho_j-1}\left[(m_j^d)^{\text{opt}}\right]^{-\rho_j}\right\}^{\frac{\beta_j}{\rho_j}}.$$

## F. Expressions for quantifying the CES-CARA case.

Quantifying the Cobb-Douglas-CARA case is relatively easy because when $\sigma \to 1$ the equilibrium and optimal expenditure shares are independent of the $\alpha_j$ parameters and the cutoffs $m_j^d$ (which subsume other parameters such as the sunk entry costs $F_j$). This no longer holds in the CES-CARA case, which makes the quantification more involved. However, we can proceed as follows.

Let $\{\widehat{e}_j^{\text{eqm}}\}_{j=1}^J$ be the equilibrium expenditure shares from the data, and let $\{\widehat{\theta}_j\}_{j=1}^J$ be the revenue-to-utility ratios obtained from the standard deviation formula in Appendix C.2. Recall that in the Cobb-Douglas case those two pieces of information allows us to back out $\{\widehat{\beta}_j^{\text{eqm}}\}_{j=1}^J$ by solving

$$\widehat{e}_j^{\text{eqm}} = \frac{\widehat{\beta}_j^{\text{eqm}}\widehat{\theta}_j}{\sum_{\ell=1}^J\widehat{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell}, \quad \sum_j\widehat{\beta}_j^{\text{eqm}} = 1.$$

In the CES case, using (E-19), the equilibrium expenditure share can be rewritten as

$$\widehat{e}_j^{\text{eqm}} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}\widehat{\theta}_j[(m_j^d)^{\text{eqm}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\widehat{\theta}_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}} = \frac{\left[\frac{\alpha_j\beta_j}{(m_j^d)^{\text{eqm}}}\right]^{\sigma-1}\beta_j\widehat{\theta}_j}{\sum_{\ell=1}^{J}\left[\frac{\alpha_\ell\beta_\ell}{(m_\ell^d)^{\text{eqm}}}\right]^{\sigma-1}\beta_\ell\widehat{\theta}_\ell} = \frac{\widetilde{\beta}_j^{\text{eqm}}\widehat{\theta}_j}{\sum_{\ell=1}^{J}\widetilde{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell},$$

where $\widetilde{\beta}_j^{\text{eqm}} \equiv \left[(\alpha_j\beta_j)/(m_j^d)^{\text{eqm}}\right]^{\sigma-1}\beta_j$, and where $\widehat{e}_j^{\text{eqm}}$ and $\widehat{\theta}_j$ come from the data. Clearly, $\widetilde{\beta}_j^{\text{eqm}} = \text{const.} \times \widehat{\beta}_j^{\text{eqm}}$ is a solution to the foregoing equation, i.e., the CES $\beta$ parameters are proportional to the Cobb-Douglas $\beta$ parameters. The constant term is shown to disappear in the end.

Using the same transformation for the $\beta$ terms as above, the equilibrium utility (E-21) can be rewritten as

$$U^{\text{eqm}} = \frac{\left\{\sum_{j=1}^{J}\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{eqm}}]^{1-\sigma}\right\}^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}\widehat{\theta}_\ell[(m_\ell^d)^{\text{eqm}}]^{1-\sigma}} = \frac{\left(\sum_{j=1}^{J}\widetilde{\beta}_j^{\text{eqm}}\right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J}\widetilde{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell},$$

which, using $\widetilde{\beta}_j^{\text{eqm}} = \text{const.} \times \widehat{\beta}_j^{\text{eqm}}$, can be rewritten as

$$U^{\text{eqm}} = (\text{const.})^{\frac{\sigma}{\sigma-1}-1}\frac{\left(\sum_{j=1}^{J}\widehat{\beta}_j^{\text{eqm}}\right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\ell=1}^{J}\widehat{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell} = (\text{const.})^{\frac{\sigma}{\sigma-1}-1}\frac{1}{\sum_{\ell=1}^{J}\widehat{\beta}_\ell^{\text{eqm}}\widehat{\theta}_\ell}.$$

Turning to the optimal expenditure share, we have

$$e_j^{\text{opt}} = \frac{\alpha_j^{\sigma-1}\beta_j^{\sigma}[(m_j^d)^{\text{opt}}]^{1-\sigma}}{\sum_{\ell=1}^{J}\alpha_\ell^{\sigma-1}\beta_\ell^{\sigma}[(m_\ell^d)^{\text{opt}}]^{1-\sigma}} = \frac{\left[\frac{\alpha_j\beta_j}{(m_j^d)^{\text{opt}}}\right]^{\sigma-1}\beta_j}{\sum_{\ell=1}^{J}\left[\frac{\alpha_\ell\beta_\ell}{(m_\ell^d)^{\text{opt}}}\right]^{\sigma-1}\beta_\ell} = \frac{\widetilde{\beta}_j^{\text{opt}}}{\sum_{\ell=1}^{J}\widetilde{\beta}_\ell^{\text{opt}}},$$

where $\widetilde{\beta}_j^{\text{opt}} \equiv \left[(\alpha_j\beta_j)/(m_j^d)^{\text{opt}}\right]^{\sigma-1}\beta_j$. We know that

$$\frac{\widetilde{\beta}_j^{\text{eqm}}}{\widetilde{\beta}_j^{\text{opt}}} = \left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{\sigma-1} \quad \Rightarrow \quad \widetilde{\beta}_j^{\text{opt}} = \left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{1-\sigma}\widetilde{\beta}_j^{\text{eqm}} = \text{const.} \times \left[\frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}}\right]^{1-\sigma}\widehat{\beta}_j^{\text{eqm}},$$

The optimal utility can be rewritten as

$$U^{\text{opt}} = \left\{ \sum_{j=1}^{J} \alpha_j^{\sigma-1} \beta_j^{\sigma} [(m_j^d)^{\text{opt}}]^{1-\sigma} \right\}^{\frac{1}{\sigma-1}} = \left\{ \sum_{j=1}^{J} \left[ \frac{\alpha_j \beta_j}{(m_j^d)^{\text{opt}}} \right]^{\sigma-1} \beta_j \right\}^{\frac{1}{\sigma-1}} = \left( \sum_{j=1}^{J} \widetilde{\beta}_j^{\text{opt}} \right)^{\frac{1}{\sigma-1}}.$$

which can be rewritten as

$$U^{\text{opt}} = (\text{const.})^{\frac{1}{\sigma-1}} \left( \sum_{j=1}^{J} \widehat{\beta}_j^{\text{opt}} \right)^{\frac{1}{\sigma-1}},$$

where $\widehat{\beta}_j^{\text{opt}} = \left[ (m_j^d)^{\text{opt}} / (m_j^d)^{\text{eqm}} \right]^{1-\sigma} \widehat{\beta}_j^{\text{eqm}}$. Finally, taking the ratio of $U^{\text{eqm}}$ and $U^{\text{opt}}$, we obtain

$$\frac{U^{\text{eqm}}}{U^{\text{opt}}} = \frac{(\text{const.})^{\frac{\sigma}{\sigma-1}-1} \frac{1}{\sum_{\ell=1}^{J} \widehat{\beta}_\ell^{\text{eqm}} \widehat{\theta}_\ell}}{(\text{const.})^{\frac{1}{\sigma-1}} \left( \sum_{j=1}^{J} \widehat{\beta}_j^{\text{opt}} \right)^{\frac{1}{\sigma-1}}} = \frac{\frac{1}{\sum_{\ell=1}^{J} \widehat{\beta}_\ell^{\text{eqm}} \widehat{\theta}_\ell}}{\left\{ \sum_{j=1}^{J} \left[ \frac{(m_j^d)^{\text{opt}}}{(m_j^d)^{\text{eqm}}} \right]^{1-\sigma} \widehat{\beta}_j^{\text{eqm}} \right\}^{\frac{1}{\sigma-1}}}.$$

We already know $\widehat{\beta}_j^{\text{eqm}}$ and $\widehat{\theta}_j$. Since the cutoff ratio is a function of $k_j$ only, the above expression can be quantified for any given value of $\sigma$. Then, using (D-9), we can compute the associated Allais surplus required to quantify the distortions.

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