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**Dynamic Selection: An Idea Flows Theory of Entry, Trade  
and Growth**

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## **Abstract**

This paper develops an idea flows theory of trade and growth with heterogeneous firms. New firms learn from incumbent firms, but the diffusion technology ensures entrants learn not only from frontier technologies, but from the entire technology distribution. By shifting the productivity distribution upwards, selection on productivity causes technology diffusion and this complementarity generates endogenous growth without scale effects. On the balanced growth path, the productivity distribution is a traveling wave with an increasing lower bound. Growth of the lower bound causes dynamic selection. Free entry mandates that trade liberalization increases the rates of technology diffusion and dynamic selection to offset the profits from new export opportunities. Consequently, trade integration raises long-run growth. The dynamic selection effect is a new source of gains from trade not found when firms are homogeneous. Calibrating the model implies that dynamic selection approximately triples the gains from trade relative to heterogeneous firm economies with static steady states.

Key words: International trade, firm heterogeneity, technology diffusion, endogenous growth  
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# 1 Introduction

How does trade affect welfare? The sources and size of the gains from trade inform any discussion of either international or intra-national integration and shape our understanding of the costs and benefits of globalization.<sup>1</sup> Motivated by the observation that only high performing firms participate in international trade (Bernard and Jensen 1995) recent work has studied the implications of firm heterogeneity for the gains from trade. The existence of substantial productivity differences between firms producing very similar products (Syverson 2011) introduces two channels for aggregate productivity gains that are absent when all firms produce on the technology frontier. First, cross-firm resource reallocation from less to more productive firms (Melitz 2003; Hsieh and Klenow 2009). Second, technology diffusion between firms (Luttmer 2007; Lucas and Moll 2013).

The literature on firm heterogeneity and trade has, with few exceptions, focused on the reallocation channel and studied economies with static steady states (Melitz 2003; Atkeson and Burstein 2010; Arkolakis, Costinot and Rodríguez-Clare 2012; Melitz and Redding 2013).<sup>2</sup> However, abstracting from technology diffusion overlooks a dynamic complementarity that exists between selection-induced reallocation and technology diffusion. Selection on productivity causes less productive firms to exit and shifts the productivity distribution of incumbent firms upwards. Whenever knowledge spillovers depend upon the entire distribution of technologies used in an economy, an upwards shift in the productivity distribution leads to technology diffusion. Thus, selection causes technology diffusion. Moreover, since technology diffusion raises average productivity it leads to low productivity firms becoming unprofitable and generates further selection. To understand the consequences of this complementarity, I move beyond static steady state economies and incorporate technology diffusion into a dynamic open economy with heterogeneous firms. I find that firm heterogeneity matters for understanding the long run effects of trade because the combination of selection and technology diffusion creates a new channel through which trade increases growth and generates a new source of dynamic gains from trade.

To allow for technology diffusion I develop a dynamic version of Melitz (2003) featuring knowledge spillovers from incumbent firms to entrants. In most endogenous growth theory the source of growth is either knowledge spillovers that reduce the relative cost of entry in an expanding varieties framework (Romer 1990) or productivity spillovers that allow entrants to improve the frontier technology in a quality ladders framework (Aghion and Howitt 1992; Grossman and Helpman 1991). However, Bollard, Klenow and Li (2013) find that entry costs do not fall relative to the cost of labor as economies grow. Moreover, the persistence of large within-industry productivity differences and the fact that most entrants do not use frontier technologies imply that not only innovation, but also the diffusion of existing technologies are important for aggregate productivity growth. Motivated by this observation recent work on idea flows has studied technology diffusion by assuming that agents can learn from meetings with other randomly chosen agents

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<sup>1</sup>For example, the European Commission's website on the proposed Transatlantic Trade and Investment Partnership (TTIP) between the United States and European Union highlights an estimate that TTIP will boost the world economy by €310 billion. Such estimates are central to evaluating the importance of trade negotiations.

<sup>2</sup>Throughout this paper I use the term "static steady state economies" to refer to both static models and papers such as Melitz (2003) and Atkeson and Burstein (2010) that incorporate dynamics, but do not allow for growth and, consequently, have a steady state that is constant over time.

in an economy (Alvarez, Buera and Lucas 2008; Lucas and Moll 2013; Perla and Tonetti 2014).

To model knowledge spillovers I build upon the idea flows literature by assuming that: (i) spillovers affect productivity, but not the cost of entry, and; (ii) spillovers depend not on the frontier technology, but upon the entire distribution of productivity. To be specific, each firm has both a product and a process technology. Product ownership gives a firm the monopoly right to produce a particular variety and is protected by an infinitely lived patent.<sup>3</sup> The firm's process technology determines its productivity and is non-rival and partially non-excludable. When a new product is created, the entering firm adopts a process technology by learning from incumbent firms. In this manner knowledge about how to organize, manage and implement production diffuses between firms. However, learning frictions such as information asymmetries and adoption capacity constraints mean that not all entrants learn from the most productive firms. Instead, knowledge spillovers depend on the productivity of all active firms and spillovers increase as the distribution of incumbent firm productivity improves. This formalization of knowledge spillovers is consistent with evidence that the productivity distributions of entrants and incumbent firms move together over time (Aw, Chen and Roberts 2001; Foster, Haltiwanger and Krizan 2001; Disney, Haskel and Heden 2003).

In the language of the Melitz model, the knowledge spillover process implies that instead of drawing from an exogenous distribution, entrants sample from a distribution that is endogenous to the productivity distribution of incumbent firms. Consequently, when selection increases the productivity cut-off below which firms exit, it also generates spillovers that shift upwards the productivity distribution of future entrants and lead to technology diffusion. Entry then causes further selection by raising industry competitiveness and making low productivity firms unprofitable. In equilibrium the positive feedback between selection and technology diffusion generates endogenous growth through dynamic selection. On the balanced growth path, the firm size distribution is stationary and the productivity distribution of incumbent firms is a traveling wave that shifts upwards over time as the exit cut-off grows.<sup>4</sup>

In the open economy firms face both fixed and variable trade costs. Only high productivity firms export and selection increases the exit cut-off and shifts the productivity distribution of incumbent firms upwards as in Melitz (2003). Consequently, trade liberalization generates technology diffusion and the expected productivity of future entrants rises. Unsurprisingly, this technology diffusion magnifies the rise in average productivity following trade liberalization. More importantly, it leads to a permanent increase in the long-run growth rate. To understand why, consider the free entry condition. In equilibrium, the cost of entry must equal an entrant's expected discounted lifetime profits. In the absence of technology diffusion, free entry mandates that an increase in expected profits from exporting is offset by a reduced probability of survival leading to the static selection effect found in Melitz (2003). However, with technology diffusion an increase in the level of the exit cut-off does not change the distribution of entrants' productivity relative to the exit cut-off. Instead, I show that free entry requires an increase in the growth rate of the exit cut-off which raises the rate at which a successful entrant's technology becomes obsolete and reduces entrants' expected

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<sup>3</sup>For a theory of product technology diffusion see product cycle models such as those considered by Grossman and Helpman (1991).

<sup>4</sup>Luttmer (2010) notes that the U.S. firm employment distribution appears to be stationary. König, Lorenz and Zilibotti (2012) show using European firm data that the observed firm productivity distribution behaves like a traveling wave with increasing mean.

discounted lifetime profits.<sup>5</sup> This dynamic selection effect of trade increases the growth rate of average productivity and, consequently, consumption per capita. Thus, the complementarity between selection and technology diffusion implies that trade liberalization raises growth.<sup>6</sup>

How does higher growth affect the gains from trade? In static steady state economies that follow Melitz (2003) the equilibrium exit cut-off and export threshold are efficient, implying that any adjustments in their levels following changes in trade costs generate welfare gains absent from homogeneous firm models (Melitz and Redding 2013). However, Atkeson and Burstein (2010) find that these welfare gains are small relative to increases in average firm productivity since in general equilibrium the gains from selection and reallocation are offset by reductions in entry and technology investment. Similarly, Arkolakis, Costinot and Rodríguez-Clare (2012) argue that firm heterogeneity is not important for quantifying the aggregate gains from trade. In particular, they show that in both Krugman (1980) and a version of Melitz (2003) with a Pareto productivity distribution, the gains from trade can be expressed as the same function of two observables: the import penetration ratio and the elasticity of trade with respect to variable trade costs (the trade elasticity). By raising the growth rate, the dynamic selection effect generates a new source of gains from trade that is not found in either static steady state economies with heterogeneous firms or dynamic economies with homogeneous firms. However, given the findings of Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012) it is natural to ask whether the benefits from an increase in the dynamic selection rate are offset by other general equilibrium effects.

To address this question, the paper shows that the welfare effects of trade can be decomposed into two terms. First, a static term that is identical to the gains from trade in Melitz (2003) (assuming a Pareto productivity distribution) and can be expressed as the same function of the import penetration ratio and trade elasticity that gives the gains from trade in Arkolakis, Costinot and Rodríguez-Clare (2012). Second, a dynamic term that depends on trade only through the growth rate of consumption per capita. The dynamic term is strictly increasing in the growth rate because dynamic selection generates a positive externality by raising the productivity of future entrants.<sup>7</sup> Since trade raises growth, the welfare decomposition implies that the gains from trade in this paper are strictly higher than in Melitz (2003). Conditional on the observed import penetration ratio and trade elasticity, the gains from trade are also strictly higher than in the class of static steady state economies studied by Arkolakis, Costinot and Rodríguez-Clare (2012).<sup>8</sup> It follows that the combination of firm heterogeneity and technology diffusion raises the gains from trade.

To assess the magnitude of the gains from trade-induced dynamic selection I calibrate the model using U.S. data. As in Arkolakis, Costinot and Rodríguez-Clare (2012) the import penetration ratio is a sufficient

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<sup>5</sup>Atkeson and Burstein (2010) also highlight the role played by the free entry condition in determining the general equilibrium gains from trade. However, while in a static steady state economy the free entry condition limits the gains from static selection, in this paper free entry is critical in ensuring dynamic gains from trade.

<sup>6</sup>The empirical literature on trade and growth faces the twin challenges of establishing causal identification and separating level and growth effects. However, the balance of evidence suggests a positive effect of trade on growth. See, for example, Frankel and Romer (1999) or Wacziarg and Welch (2008).

<sup>7</sup>Starting from the decentralized equilibrium a social planner can raise welfare by increasing the dynamic selection rate through subsidizing entry or, equivalently, taxing the fixed production cost.

<sup>8</sup>An important distinction to note is that in this paper the predicted import penetration ratio and trade elasticity are the same functions of underlying parameters as in Melitz (2003). However, they differ from the predictions made by other models considered by Arkolakis, Costinot and Rodríguez-Clare (2012). See Melitz and Redding (2013) for further discussion of this distinction.

statistic for the level of trade integration and the welfare effects of trade can be calculated in terms of a small number of observables and parameters. In addition to the import penetration ratio and trade elasticity, the calibration uses the rate at which new firms are created, the population growth rate, the intertemporal elasticity of substitution, the discount rate and the elasticity of substitution between goods. The baseline calibration implies that U.S. growth is 11 percent higher than it would be under autarky. More importantly, the increase in the dynamic selection rate triples the gains from trade relative to the static steady state economies considered by Arkolakis, Costinot and Rodríguez-Clare (2012). The finding that dynamic selection is quantitatively important for the gains from trade is extremely robust. For plausible parameter variations the dynamic selection effect always at least doubles the gains from trade.

As well as contributing to the debate over the gains from trade, this paper is closely related to the endogenous growth literature. Open economy endogenous growth theories with homogeneous firms find that the effects of trade on growth in a single sector economy depend on scale effects and international knowledge spillovers (Rivera-Batiz and Romer 1991; Grossman and Helpman 1991). By contrast, neither scale effects nor international knowledge spillovers are necessary for trade to raise growth through dynamic selection. To highlight the novelty of the dynamic selection mechanism I assume that there are no international knowledge spillovers and I show that the equilibrium growth rate does not depend on population size – there are no scale effects. Thus, this paper implies neither the counterfactual prediction that larger economies grow faster (Jones 1995a) nor the semi-endogenous growth prediction that population growth is the only source of long-run growth (Jones 1995b). Scale effects are absent from this paper because both the productivity distribution and the mass of varieties produced are endogenous. In equilibrium a larger population leads to a proportional increase in the mass of varieties produced (unlike in quality ladders growth models), but since the creation of new goods does not reduce the cost of future entry (unlike in expanding varieties growth models) the growth rate is unaffected.

Selection based growth in closed economies has been studied in recent work on idea flows by Luttmer (2007, 2012), Alvarez, Buera and Lucas (2008), Lucas and Moll (2013) and Perla and Tonetti (2014). The model developed in this paper extends the idea flows literature along a number of dimensions. First, it allows for the free entry of firms in an open economy. By contrast, Lucas and Moll (2013) and Perla and Tonetti (2014) assume a fixed mass of producers, while Alvarez, Buera and Lucas (2008) use an Eaton and Kortum (2002) framework that abstracts from firms and entry. Luttmer (2007) includes entry, but focuses on how post-entry productivity shocks shape the equilibrium productivity distribution and does not give a complete characterization of the balanced growth path or analyze the effects of trade. By abstracting from post-entry firm level productivity shocks this paper identifies the determinants of aggregate growth and shows that the free entry condition is central in determining the relationship between trade and growth. In addition, this paper introduces a new methodology for modeling knowledge spillovers that provides a more flexible and tractable way to represent technology diffusion. In Section 5 I show how this approach facilitates extending the technology diffusion model to allow for international knowledge spillovers, alternative productivity distributions, frontier technology growth and firm level productivity dynamics. The finding that trade raises growth by increasing the dynamic selection rate is robust to these extensions.

Most closely related to this paper is the work on trade, growth and selection by Baldwin and Robert-

Nicoud (2008), Alvarez, Buera and Lucas (2011) and Perla, Tonetti and Waugh (2014). Baldwin and Robert-Nicoud (2008) show that incorporating firm heterogeneity into an expanding variety growth model leads to an ambiguous effect of trade on growth that depends on the extent of international knowledge spillovers. However, since knowledge spillovers affect entry costs instead of entrants' productivity the model has three counter-factual implications. First, the equilibrium productivity distribution is time invariant. Second, entry costs decline relative to labor costs as the economy grows. Third, average firm size decreases as the economy grows. Alvarez, Buera and Lucas (2011) show that international knowledge spillovers increase growth in an Eaton and Kortum (2002) trade model, but assume that the rate of technology diffusion is independent of agents' optimization decisions and do not model firm level behavior. Perla, Tonetti and Waugh (2014) develop an open economy extension of Perla and Tonetti (2014) in which growth is driven by technology diffusion between incumbent firms, but the mass of firms is fixed. They find that trade can raise or lower growth depending on how the costs of searching for a better technology are specified, but since the mass of firms is exogenous they do not include the free entry condition which, as this paper shows, ensures a positive effect of trade on growth.

The remainder of the paper is organized as follows. Section 2 introduces the model, while Section 3 solves for the balanced growth path equilibrium and discusses the effects of trade on growth. In Section 4 I characterize household welfare on the balanced growth path and then calibrate the model and quantify the gains from trade. Finally, Section 5 demonstrates the robustness of the paper's results to relaxing some of the simplifying assumptions made in the baseline model, before Section 6 concludes.

## 2 Technology diffusion model

Consider a world comprised of  $J + 1$  symmetric economies. When  $J = 0$  there is a single autarkic economy, while for  $J > 0$  we have an open economy model. Time  $t$  is continuous and the preferences and production possibilities of each economy are as follows.

### 2.1 Preferences

Each economy consists of a set of identical households with dynastic preferences and discount rate  $\rho$ . The population  $L_t$  at time  $t$  grows at rate  $n \geq 0$  where  $n$  is constant and exogenous. Each household has constant intertemporal elasticity of substitution preferences and seeks to maximize:

$$U = \int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} dt, \quad (1)$$

where  $c_t$  denotes consumption per capita and  $\gamma > 0$  is the intertemporal elasticity of substitution. The numeraire is chosen so that the price of the consumption good is unity. Households can lend or borrow at interest rate  $r_t$  and  $a_t$  denotes assets per capita. Consequently, the household's budget constraint expressed in per capita terms is:

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t, \quad (2)$$

where  $w_t$  denotes the wage. Note that households do not face any uncertainty.

Under these assumptions and a no Ponzi game condition the household's utility maximization problem is standard<sup>9</sup> and solving gives the Euler equation:

$$\frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho), \quad (3)$$

together with the transversality condition:

$$\lim_{t \rightarrow \infty} \left\{ a_t \exp \left[ - \int_0^t (r_s - n) ds \right] \right\} = 0. \quad (4)$$

## 2.2 Production and trade

Output is produced by monopolistically competitive firms each of which produces a differentiated good. Labor is the only factor of production and all workers are homogeneous and supply one unit of labor per period. There is heterogeneity across firms in labor productivity  $\theta$ . A firm with productivity  $\theta$  at time  $t$  has marginal cost of production  $\frac{w_t}{\theta}$  and must also pay a fixed cost  $f$  per period in order to produce. The fixed cost is denominated in units of labor. The firm does not face an investment decision and firm productivity remains constant over time.<sup>10</sup> The final consumption good is produced under perfect competition as a constant elasticity of substitution aggregate of all available goods with elasticity of substitution  $\sigma > 1$  and is non-tradable.<sup>11</sup>

Firms can sell their output both at home and abroad. However, as in Melitz (2003) firms that select into exporting face both fixed and variable costs of trade. Exporters incur a fixed cost  $f_x$  denominated in units of domestic labor, per export market per period, while variable trade costs take the iceberg form. In order to deliver one unit of output to a foreign market a firm must ship  $\tau$  units. I assume  $\tau^{\sigma-1} f_x > f$  which is a necessary and sufficient condition to ensure that in equilibrium not all firms export. Since I consider a symmetric equilibrium, all parameters and endogenous variables are invariant across countries.

Conditional on the distribution of firm productivity, the structure of production and demand in this economy is equivalent to that in Melitz (2003) and solving firms' static profit maximization problems is straightforward. Firms face isoelastic demand and set factory gate prices as a constant mark-up over marginal costs. Firms only choose to produce if their total variable profits from domestic and foreign markets are sufficient to cover their fixed production costs and firms only export to a given market if their variable profits in that market are sufficient to cover the fixed export cost. Variable profits in each market are strictly increasing in productivity and since  $\tau^{\sigma-1} f_x > f$  the productivity above which firms export exceeds the minimum productivity for entering the domestic market. In particular, there is a cut-off productivity  $\theta_t^*$  such that firms choose to produce at time  $t$  if and only if their productivity is at least  $\theta_t^*$ . This exit cut-off is given by:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left( \frac{f w_t^\sigma}{c_t L_t} \right)^{\frac{1}{\sigma-1}}. \quad (5)$$

<sup>9</sup>See, for example, Chapter 2 of Barro and Sala-i-Martin (2004).

<sup>10</sup>Sections 5.2 and 5.3 analyze extensions of the model that include firm level productivity dynamics.

<sup>11</sup>This is equivalent to assuming households have constant elasticity of substitution preferences over differentiated goods.

In addition, there is a threshold  $\tilde{\theta}_t > \theta_t^*$  such that firms choose to export at time  $t$  if and only if their productivity is at least  $\tilde{\theta}_t$ . The export threshold is:

$$\tilde{\theta}_t = \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \tau \theta_t^*. \quad (6)$$

Firms can lend or borrow at interest rate  $r_t$  and the market value  $V_t(\theta)$  of a firm with productivity  $\theta$  is given by the present discounted value of future profits:

$$V_t(\theta) = \int_t^\infty \pi_v(\theta) \exp\left(-\int_t^v r_s ds\right) dv, \quad (7)$$

where  $\pi_v$  denotes the profit flow from both domestic and export sales at time  $v$  net of fixed costs and  $\pi_v(\theta) = 0$  if the firm does not produce.

In what follows, it will be convenient to use the change of variables  $\phi_t \equiv \frac{\theta}{\tilde{\theta}_t}$ , where  $\phi_t$  is firm productivity relative to the exit cut-off. I will refer to  $\phi_t$  as a firm's relative productivity. Let  $W_t(\phi_t)$  be the value of a firm with relative productivity  $\phi_t$  at time  $t$ . Obviously, only firms with  $\phi_t \geq 1$  will choose to produce and only firms with  $\phi_t \geq \tilde{\phi} \equiv \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \tau$  will choose to export. For these firms prices, employment and profits are given by:

$$p_t^d(\phi_t) = \frac{\sigma}{\sigma-1} \frac{w_t}{\phi_t \theta_t^*}, \quad p_t^x(\phi_t) = \tau p_t^d(\phi_t),$$

$$l^d(\phi_t) = f [(\sigma-1)\phi_t^{\sigma-1} + 1], \quad l^x(\phi_t) = f \tau^{1-\sigma} [(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}], \quad (8)$$

$$\pi_t^d(\phi_t) = f w_t (\phi_t^{\sigma-1} - 1), \quad \pi_t^x(\phi_t) = f \tau^{1-\sigma} w_t (\phi_t^{\sigma-1} - \tilde{\phi}^{\sigma-1}), \quad (9)$$

where I have used  $d$  and  $x$  superscripts to denote the domestic and export markets, respectively. Observe that employment is a stationary function of relative productivity and that, conditional on relative productivity  $\phi_t$ , both domestic and export profits are proportional to the fixed cost of production. Since there are  $J$  export markets, total firm employment is given by  $l(\phi_t) = l^d(\phi_t) + J l^x(\phi_t)$  and total firm profits are  $\pi_t(\phi_t) = \pi_t^d(\phi_t) + J \pi_t^x(\phi_t)$ .

### 2.3 Knowledge spillovers and entry

To invent new goods, entrants must employ workers to undertake research and development (R&D). Employing  $R_t f_e$  R&D workers produces a flow  $R_t$  of innovations where  $f_e > 0$  is an entry cost parameter. Each innovation generates both an idea for a new good (product innovation) and a production technology for producing the good (process innovation). Product ownership is protected by an infinitely lived patent, but knowledge spillovers occur because firms' process technologies are non-rival and partially non-excludable. Consequently, innovators can learn from the production techniques (technologies, managerial

methods, organizational forms, input choices, etc.) used by existing firms.<sup>12</sup> However, due to frictions that limit knowledge diffusion such as information asymmetries and absorption capacity constraints not all entrants learn from the most productive incumbent firms.<sup>13</sup> Instead, knowledge spillovers depend upon the entire distribution of technologies used by incumbent firms.

To formalize knowledge spillovers, I assume that the productivity of entrants is given by:

$$\theta = x_t \psi, \tag{10}$$

where  $x_t$  is a summary statistic of the productivity distribution of incumbent firms and  $\psi$  is a stochastic component drawn from a time invariant sampling distribution with cumulative distribution function  $F(\psi)$ . Knowledge spillovers are captured by variation in  $x_t$  and I assume  $x_t$  has the following three properties. First,  $x_t$  is a location statistic such that if  $G_t(\theta)$  is the cumulative productivity distribution function for firms that produce at time  $t$  and  $G_{t_1}(\theta) = G_{t_0}(\theta/\kappa)$  then  $x_{t_1} = \kappa x_{t_0}$ . Thus, if  $G_t$  shifts to the right by a proportional factor  $\kappa$  then  $x_t$  increases by the same factor  $\kappa$ . Second, holding  $G_t(\theta)$  constant,  $x_t$  is independent of the mass of incumbent firms. This ensures  $x_t$  is independent of the size of the economy. Third,  $x_t$  is independent of changes that vary the maximum of the incumbent firm productivity distribution, while leaving the remainder of the distribution unaffected. This implies knowledge spillovers are not driven by the frontier technology and only shifts in the entire productivity distribution cause spillovers. Summary statistics that satisfy these three properties include the minimum, median and mean, among many others, but not the maximum.

Modeling entrants' productivity draws using (10) implies that the cumulative distribution function of entrants' productivity  $\tilde{G}_t$  is given by:  $\tilde{G}_t(\theta) = F(\theta/x_t)$ . This implication is consistent with the observations that: (i) there is substantial productivity heterogeneity within an entering cohort, and; (ii) the productivity distributions of entrants and incumbents move together closely over time.<sup>14</sup>

The specification of knowledge spillovers introduced above differs in important ways from that used in either expanding variety (Romer 1990) or quality ladders (Aghion and Howitt 1992; Grossman and Helpman 1991) growth models. In expanding variety models knowledge accumulation lowers entry costs relative to labor costs and average firm employment falls as the economy grows. However, observed variation in firm sizes is inconsistent with these predictions. Bollard, Klenow and Li (2013) use cross-country, cross-industry data on the number and size of firms to infer that entry costs are approximately proportional to labor costs and do not fall with development. In addition, the U.S. firm employment distribution is roughly stable over time (Luttmer 2010). In quality ladders models entrants learn from frontier technologies and are more productive than incumbent firms. Yet empirical studies find that most entrants do not use frontier technologies (Foster,

<sup>12</sup>A large literature documents the importance of learning from other producers in agricultural technology diffusion (e.g. Foster and Rosenzweig 1995; Bandiera and Rasul 2006; Conley and Udry 2010). Robertson, Swan and Newell (1996) discuss the role of information networks in shaping the adoption of computer-aided production management (CAPM) in UK manufacturing firms. See Baptista (1999) for an overview of the literature on process technology diffusion.

<sup>13</sup>Conley and Udry (2010) find that pineapple farmers learn from other producers even when those producers use sub-optimal input levels.

<sup>14</sup>For evidence, see Foster, Haltiwanger and Krizan (2001) for the U.S.; Aw, Chen and Roberts (2001) for Taiwan, and; Disney, Haskel and Heden (2003) for the United Kingdom. For example, Aw, Chen and Roberts (2001), p.71, conclude that: "the productivity distributions of entering firms and incumbents shift over time in similar ways." Selection effects could rationalize this finding without requiring any knowledge spillovers, but selection alone is insufficient to generate endogenous long run growth.

Haltiwanger and Krizan 2001). In contrast to expanding variety models, the knowledge spillovers studied in this paper affect productivity not entry costs, while in contrast to quality ladders models the spillovers are a function of not only frontier technologies, but of all technologies used in the economy.

The structure of knowledge spillovers embodied in (10) builds upon epidemic models of technology diffusion, the search model of technological change developed by Kortum (1997) and recent work on idea flows (Luttmer 2007; Alvarez, Buera and Lucas 2008; Lucas and Moll 2013; Perla and Tonetti 2014). In epidemic models of technology diffusion the rate at which a new technology spreads depends upon the proportion of the population that uses the technology (Griliches 1957; Mansfield 1961). Epidemic models explain the lags in technology diffusion and why the rate at which a new technology is adopted is S-shaped over time (Stoneman 2002). However, I consider the case where there are a continuum of productivity levels rather than a binary technology use variable. Kortum (1997) analyzes a closed economy, quality ladders model where knowledge spillovers cause improvements in the productivity distribution from which new ideas are drawn and the strength of spillovers depends on the stock of R&D. By contrast, in this paper only R&D that causes shifts in the firm productivity distribution leads to knowledge spillovers.

The idea flows literature studies the evolution of the productivity distribution when agents learn from meeting other agents with higher knowledge. Since meetings result from random matching between agents, the technology diffusion process depends upon the distribution of knowledge in an economy. Applied to this paper, learning through random matching implies the productivity distribution of entrants is identical to the productivity distribution of incumbent firms. As in the idea flows literature I model knowledge spillovers as a function of the entire productivity distribution, but instead of assuming random matching equation (10) takes a reduced form approach in which the productivity of entrants depends upon the location of the incumbent firm productivity distribution and a random component. Consequently, the productivity distributions of entrants and incumbents may differ. For the baseline model considered in Sections 3 and 4 this difference is relatively unimportant. I show in Appendix B that if knowledge spillovers result from random matching between entrants and incumbent firms, the balanced growth path and the effects of trade integration obtained in the baseline model are unaffected. However, since equation (10) provides a more flexible representation of knowledge spillovers than random matching it makes general equilibrium analysis more tractable and Section 5 takes advantage of this tractability to extend the analysis by relaxing some of the simplifying assumptions made in the baseline model.

A final observation regarding equation (10) is that knowledge spillovers are intra-national not international in scope. Section 5.1 analyzes an extension of the model with international knowledge spillovers, but in the baseline model entrants only learn from domestic firms.

There is free entry into R&D, implying that in equilibrium the expected cost of innovating equals the expected value of creating a new firm:

$$f_e w_t = \int_{\theta} V_t(\theta) d\tilde{G}_t(\theta). \quad (11)$$

Entry is financed by a competitive and costless financial intermediation sector which owns the firms and, thereby, enables investors to pool the risk faced by innovators. Consequently, each household effectively

owns a balanced portfolio of all firms and R&D projects.<sup>15</sup>

How does the relative productivity distribution evolve over time? Let  $H_t$  and  $\tilde{H}_t$  be the cumulative distribution functions of relative productivity  $\phi$  for existing firms and entrants, respectively. Given the structure of productivity spillovers we must have  $\tilde{H}_t(\phi) = F\left(\frac{\phi\theta_t^*}{x_t}\right)$ . To characterize the intertemporal evolution of  $H_t$  I will first formulate a law of motion for  $H_t(\phi)$  between  $t$  and  $t + \Delta$  and then take the continuous time limit. Let  $M_t$  be the mass of producers in the economy at time  $t$  and assume the exit cut-off is strictly increasing over time.<sup>16</sup> Then the mass of firms with relative productivity less than  $\phi$  at time  $t + \Delta$  is:

$$M_{t+\Delta}H_{t+\Delta}(\phi) = M_t \left[ H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \phi \right) - H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \right) \right] + \Delta R_t \left[ F \left( \frac{\phi \theta_{t+\Delta}^*}{x_t} \right) - F \left( \frac{\theta_{t+\Delta}^*}{x_t} \right) \right]. \quad (12)$$

Since  $\phi_{t+\Delta} \leq \phi \Leftrightarrow \phi_t \leq \frac{\theta_{t+\Delta}^*}{\theta_t^*} \phi$  the first term on the right hand side is the mass of time  $t$  incumbents that have relative productivity less than  $\phi$ , but greater than one, at time  $t + \Delta$ .  $M_t H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \phi \right)$  gives the mass of time  $t$  producers with relative productivity less than  $\phi$  at time  $t + \Delta$ , while  $M_t H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \right)$  is the mass of time  $t$  incumbents that exit between  $t$  and  $t + \Delta$  because their productivity falls below the exit cut-off. The second term on the right hand side gives the mass of entrants between  $t$  and  $t + \Delta$  whose relative productivity falls between one and  $\phi$ .

Letting  $\phi \rightarrow \infty$  in (12) implies:

$$M_{t+\Delta} = M_t \left[ 1 - H_t \left( \frac{\theta_{t+\Delta}^*}{\theta_t^*} \right) \right] + \Delta R_t \left[ 1 - F \left( \frac{\theta_{t+\Delta}^*}{x_t} \right) \right], \quad (13)$$

and taking the limit as  $\Delta \rightarrow 0$  gives:<sup>17</sup>

$$\frac{\dot{M}_t}{M_t} = -H_t'(1) \frac{\dot{\theta}_t^*}{\theta_t^*} + \left[ 1 - F \left( \frac{\theta_t^*}{x_t} \right) \right] \frac{R_t}{M_t}. \quad (14)$$

This expression illustrates the two channels which affect the mass of incumbent firms. R&D generates a flow  $R_t$  of innovations, but a fraction  $F\left(\frac{\theta_t^*}{x_t}\right)$  of innovators receive a productivity draw below the exit cut-off and choose not to produce. In addition, as the exit cut-off increases firms' relative productivity levels decline and a firm exits when its relative productivity falls below one. The rate at which firms exit due to growth in the exit cut-off depends on the density of the relative productivity distribution at the exit cut-off  $H_t'(1)$ .

Now using (13) to substitute for  $M_{t+\Delta}$  in (12), rearranging and taking the limit as  $\Delta \rightarrow 0$  we obtain the following law of motion for  $H_t(\phi)$ :

<sup>15</sup>Since countries are symmetric it is irrelevant whether asset markets operate at the national or global level.

<sup>16</sup>When solving the model I will restrict attention to balanced growth paths on which  $\theta_t^*$  is strictly increasing in  $t$  meaning firms will never choose to temporarily cease production. In an economy with a declining exit cut-off, equilibrium would depend on whether exit from production was temporary or irreversible. I abstract from these issues in this paper.

<sup>17</sup>In obtaining both this expression and equation (15) I assume that  $\theta_t^*$  is differentiable with respect to  $t$  and  $H_t(\phi)$  is differentiable with respect to  $\phi$ . Both these conditions will hold on the balanced growth path considered below.

$$\begin{aligned}\dot{H}_t(\phi) &= \left\{ \phi H'_t(\phi) - H'_t(1) [1 - H_t(\phi)] \right\} \frac{\dot{\theta}_t^*}{\theta_t^*} \\ &+ \left\{ F\left(\frac{\phi \theta_t^*}{x_t}\right) - F\left(\frac{\theta_t^*}{x_t}\right) - H_t(\phi) \left[ 1 - F\left(\frac{\theta_t^*}{x_t}\right) \right] \right\} \frac{R_t}{M_t}.\end{aligned}\quad (15)$$

Thus, the evolution of the relative productivity distribution is driven by growth in the exit cut-off and the entry of new firms. When  $\dot{H}_t(\phi) = 0$  for all  $\phi \geq 1$  the relative productivity distribution is stationary.

## 2.4 Equilibrium

In addition to consumer and producer optimization, equilibrium requires the labor and asset markets to clear in each economy in all periods. Labor market clearing implies:

$$L_t = M_t \int_{\phi} l(\phi) dH_t(\phi) + R_t f_e, \quad (16)$$

while asset market clearing requires that aggregate household assets equal the combined worth of all firms:

$$a_t L_t = M_t \int_{\phi} W_t(\phi) dH_t(\phi). \quad (17)$$

Finally, as an initial condition I assume that at time zero there exists in each economy a mass  $\hat{M}_0$  of potential producers with productivity distribution  $\hat{G}_0(\theta)$ . We can now define the equilibrium.

An equilibrium of the world economy is defined by time paths for  $t \in [0, \infty)$  of consumption per capita  $c_t$ , assets per capita  $a_t$ , wages  $w_t$ , the interest rate  $r_t$ , the exit cut-off  $\theta_t^*$ , the export threshold  $\tilde{\theta}_t$ , firm values  $W_t(\phi)$ , the mass of firms in each economy  $M_t$ , the flow of innovations in each economy  $R_t$  and the relative productivity distribution  $H_t(\phi)$  such that: (i) households choose  $c_t$  to maximize utility subject to the budget constraint (2) implying the Euler equation (3) and the transversality condition (4); (ii) producers maximize profits implying the exit cut-off satisfies (5), the export threshold satisfies (6) and firm value is given by (7); (iii) free entry into R&D implies (11); (iv) the exit cut-off is strictly increasing over time and the evolution of  $M_t$  and  $H_t(\phi)$  are governed by (14) and (15); (v) labor and asset market clearing imply (16) and (17), respectively, and; (vi) at time zero there are  $\hat{M}_0$  potential producers in each economy with productivity distribution  $\hat{G}_0(\theta)$ .

## 3 Balanced growth path

I will solve for a balanced growth path equilibrium of the world economy. On a balanced growth path  $c_t, a_t, w_t, \theta_t^*, \tilde{\theta}_t, W_t(\phi), M_t$  and  $R_t$  grow at constant rates,  $r_t$  is constant and the distribution of relative productivity  $\phi$  is stationary, meaning  $\dot{H}_t(\phi) = 0 \forall t, \phi$ . When solving for the balanced growth path I impose the following assumption on the sampling distribution  $F$  from which the stochastic components of entrants' productivity levels are drawn.

**Assumption 1.** (i) The sampling productivity distribution  $F$  is Pareto:  $F(\psi) = 1 - \left(\frac{\psi}{\psi_{\min}}\right)^{-k}$  for  $\psi \geq \psi_{\min}$  with  $k > \max\{1, \sigma - 1\}$ .

(ii) Knowledge spillovers are such that:  $\frac{x_t}{\theta_t^*} \psi_{\min} \leq 1$ .

The first part of Assumption 1 simply states that  $F$  is a Pareto distribution with scale parameter  $\psi_{\min}$  and shape parameter  $k$ .<sup>18</sup> The second part of the assumption implies that not all entrants draw productivity levels above the exit cut-off and provided the inequality is strict some entrants receive productivity draws below the exit cut-off and choose not to produce. Let us define  $\lambda \equiv x_t \psi_{\min} / \theta_t^*$ .  $\lambda$  is a measure of the strength of knowledge spillovers. The fraction of entrants that draw productivity levels below the exit cut-off is  $F(\psi_{\min} / \lambda)$ .

Using Assumption 1 to substitute for  $F$  in (15), setting  $\dot{H}_t(\phi) = 0$  and solving the resulting first order differential equation for  $H(\phi)$  implies that the unique stationary relative productivity distribution is a Pareto distribution with scale parameter one and shape parameter  $k$ .

**Lemma 1.** Given Assumption 1 there exists a unique stationary relative productivity distribution:  $H(\phi) = 1 - \phi^{-k}$ .

Lemma 1 implies that on any balanced growth path the productivity distribution has a stable shape and looks like a traveling wave that shifts upwards as the exit cut-off grows. Aw, Chen and Roberts (2001) find that industry level productivity distributions tend to maintain stable shapes as they shift upwards in Taiwan, while König, Lorenz and Zilibotti (2012) show that the productivity distribution of western European firms behaves like a traveling wave. An immediate corollary of Lemma 1 is that the upper tails of the firm employment, revenue and profit distributions follow Pareto distributions and that the employment distribution is stationary.<sup>19</sup>

On the balanced growth path the relative productivity distribution of entrants is:

$$\tilde{H}(\phi) = F\left(\frac{\phi \psi_{\min}}{\lambda}\right) = H\left(\frac{\phi}{\lambda}\right).$$

Thus, entrants' relative productivity is drawn from a distribution that has the same functional form as the incumbents' relative productivity distribution, but is shifted inwards by a factor  $1/\lambda$ . If  $\lambda = 1$  then entrants and incumbents have identical productivity distributions.

Now let  $\frac{\dot{c}_t}{c_t} = q$  be the growth rate of consumption per capita. Then the household budget constraint (2) implies that assets per capita and wages grow at the same rate as consumption per capita:

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = q,$$

while the Euler equation (3) gives:

$$q = \gamma(r - \rho), \tag{18}$$

<sup>18</sup>Section 5.2 characterizes the balanced growth path when there are no functional form restrictions on  $F$ .

<sup>19</sup>It is well known that the upper tails of the distributions of firm sales and employment are well approximated by Pareto distributions (Luttmer 2007). Axtell (2001) argues that Pareto distributions provide a good fit to the entire sales and employment distributions in the U.S.

and the transversality condition (4) requires:

$$r > n + q \Leftrightarrow \frac{1 - \gamma}{\gamma} q + \rho - n > 0, \quad (19)$$

where the equivalence follows from (18). This inequality is also sufficient to ensure that household utility is well-defined. Since all output is consumed each period and economies are symmetric, output per capita is always equal to consumption per capita.

Next, differentiating equation (5) which defines the exit cut-off implies:

$$q = g + \frac{n}{\sigma - 1}. \quad (20)$$

where  $g = \frac{\dot{\theta}_t^*}{\theta_t^*}$  is the rate of growth of the exit cut-off and, therefore, the rate at which the productivity distribution shifts upwards. From equation (6) the export threshold is proportional to the exit cut-off meaning that  $g$  is also the growth rate of the export threshold and since each firm's productivity  $\theta$  remains constant over time  $g$  is the rate at which a firm's relative productivity  $\phi_t$  decreases.

Equation (20) makes clear that there are two sources of growth in this economy. First, productivity growth resulting from dynamic selection as the exit cut-off grows. Growth in the exit cut-off is driven by the dynamic complementarity between selection and technology diffusion. Selection causes knowledge spillovers and as new firms enter competition becomes tougher, which leads to further selection. As the exit cut-off grows, the least productive firms are forced to exit and this leads to a reallocation of resources to more productive firms raising average labor productivity and output per capita. This effect is the dynamic analogue of the static selection effect that results from changes in the level of the exit cut-off. Henceforth, I will refer to  $g$  as the dynamic selection rate. Understanding what determines the dynamic selection rate is the central concern of this paper.

The second source of growth is population growth. Using the employment function (8), the labor market clearing condition (16) simplifies to:

$$L_t = \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} M_t f \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] + R_t f_e. \quad (21)$$

Consequently, on a balanced growth path we must have that the mass of producers and the flow of innovations grow at the same rate as population:

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{M}_t}{M_t} = \frac{\dot{R}_t}{R_t} = n.$$

Thus, the link between population growth and consumption per capita growth arises because when the population increases the number of varieties produced grows and, since the final good production technology exhibits love of varieties, this raises consumption per capita.

To solve for the dynamic selection rate we can now substitute the profit function (9) and  $\phi_t = \frac{\theta}{\theta_t^*}$  into (7) and solve for the firm value function obtaining:

$$\begin{aligned}
V_t(\theta) &= W_t(\phi_t), \\
&= fw_t \left[ \frac{\phi_t^{\sigma-1}}{(\sigma-1)g+r-q} \left( 1 + I[\phi_t \geq \tilde{\phi}] \frac{Jf_x \tilde{\phi}^{1-\sigma}}{f} \right) \right. \\
&\quad + \frac{(\sigma-1)g}{r-q} \frac{\phi_t^{\frac{q-r}{g}}}{(\sigma-1)g+r-q} \left( 1 + I[\phi_t \geq \tilde{\phi}] \frac{Jf_x \tilde{\phi}^{\frac{r-q}{g}}}{f} \right) \\
&\quad \left. - \frac{1}{r-q} \left( 1 + I[\phi_t \geq \tilde{\phi}] \frac{Jf_x}{f} \right) \right]. \tag{22}
\end{aligned}$$

where  $I[\phi_t \geq \tilde{\phi}]$  is an indicator function that takes value one if a firm's relative productivity is greater than or equal to the export threshold and zero otherwise. Thus, the value of a firm with relative productivity  $\phi$  grows at rate  $q$ . Substituting (22) into the free entry condition (11), using  $\tilde{G}_t(\theta) = \tilde{H}(\phi) = H\left(\frac{\phi}{\lambda}\right)$  and integrating to obtain the expected value of an innovation implies:

$$q = kg + r - \frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k}{f_e} \left( f + Jf_x \tilde{\phi}^{-k} \right). \tag{23}$$

Together with (18) and (20), (23) gives us three equations for the three unknowns  $q$ ,  $g$  and  $r$ . Solving we obtain the growth rate of consumption per capita:

$$q = \frac{\gamma}{1+\gamma(k-1)} \left[ \frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + \frac{kn}{\sigma-1} - \rho \right]. \tag{24}$$

Given (24) we can use (18) to obtain  $r$  and (20) to obtain  $g$ .

Finally, recall that when characterizing the evolution of the relative productivity distribution in Section 2.3 I assumed  $g > 0$ . To ensure this condition is satisfied and the transversality condition (19) holds I impose the following parameter restrictions.

**Assumption 2.** *The parameters of the world economy satisfy:*

$$\begin{aligned}
\frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k f}{f_e} &> \rho + \frac{1-\gamma}{\gamma} \frac{n}{\sigma-1}, \\
\frac{(1-\gamma)(\sigma-1)}{k+1-\sigma} \frac{\lambda^k f}{f_e} \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] &> \gamma k(n-\rho) - (1-\gamma) \frac{k+1-\sigma}{\sigma-1} n.
\end{aligned}$$

The first inequality ensures that  $g > 0$  holds for any  $J \geq 0$ , while the second inequality is implied by the transversality condition.

This completes the proof that the world economy has a unique balanced growth path. Note that the proof holds for any non-negative value of  $J$  including the closed economy case where  $J = 0$ .

**Proposition 1.** *Given Assumptions 1 and 2 the world economy has a unique balanced growth path equilib-*

rium on which consumption per capita grows at rate:

$$q = \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + \frac{kn}{\sigma - 1} - \rho \right].$$

Remembering that Assumption 1 imposes  $k > \max\{1, \sigma - 1\}$ , we immediately obtain a corollary of Proposition 1 characterizing the determinants of the growth rate.

**Corollary 1.** *The growth rate of consumption per capita is strictly increasing in the fixed production cost  $f$ , the strength of knowledge spillovers  $\lambda$ , the intertemporal elasticity of substitution  $\gamma$ , the population growth rate  $n$  and the number of trading partners  $J$ , but is strictly decreasing in the entry cost  $f_e$ , the fixed export cost  $f_x$ , the variable trade cost  $\tau$  and the discount rate  $\rho$ .*

To understand Proposition 1 and Corollary 1 let us start by considering how trade integration affects growth. The equilibrium growth rate is higher in the open economy than in autarky. Moreover, either increasing the number of countries  $J$  in the world economy, reducing the variable trade cost  $\tau$  or reducing the fixed export cost  $f_x$  raises growth. To see why openness raises growth, consider the free entry condition (11). Using (7) and  $\tilde{G}_t(\theta) = H\left(\frac{\phi}{\lambda}\right)$  the free entry condition on the balanced growth path can be rewritten as:

$$f_e w_t = \int_{\phi} \left[ \int_t^{\infty} \pi_v(\phi_v) e^{-(v-t)r} dv \right] dH\left(\frac{\phi}{\lambda}\right).$$

The cost of entry on the left hand sides equals the expected present discounted value of entry on the right hand side. Conditional on a firm's relative productivity and the wage level, (9) shows that domestic profits are independent of trade integration, while trade increases the profits of exporters. Therefore, the new export opportunities that follow trade liberalization raise the value of entry, ceteris paribus. In addition, trade liberalization does not change entrants' relative productivity distribution  $\tilde{H}(\phi) = H\left(\frac{\phi}{\lambda}\right)$ . Consequently, trade liberalization causes an increase in the flow of entrants relative to the mass of incumbent firms  $\frac{R_t}{M_t}$ , which raises the dynamic selection rate  $g$ . To see this note that since  $M_t$  grows at rate  $n$ , the exit cut-off  $\theta_t^*$  grows at rate  $g$ ,  $H'_t(1) = k$  and  $F\left(\frac{\theta_t^*}{x_t}\right) = 1 - \lambda^k$ , equation (14) implies that on a balanced growth path:

$$\frac{R_t}{M_t} = \frac{n + gk}{\lambda^k}. \quad (25)$$

As the dynamic selection rate rises, firms' relative productivity levels decline at a faster rate and this reduces a firm's expected future profits and its expected lifespan. In equilibrium, the negative effect of increased dynamic selection on future profits exactly offsets the increase in expected profits from exporting. Thus, free entry mandates that trade liberalization raises growth because faster dynamic selection is required to offset the value of improved export opportunities.<sup>20</sup>

It is useful to compare Proposition 1 with the effects of trade liberalization when new entrants receive a productivity draw from an exogenously fixed distribution and there are no productivity spillovers as in

<sup>20</sup>Note that this analysis holds both for comparisons of the open economy with autarky and for the consequences of a partial trade liberalization resulting from an increase in  $J$  or a reduction in either  $\tau$  or  $f_x$ .

Melitz (2003). In the absence of knowledge spillovers trade liberalization still creates new export profit opportunities that increase the value of entry, *ceteris paribus*. However, in static steady state models such as Melitz (2003) the offsetting negative profit effect, which ensures the free entry condition is satisfied, comes from an increase in the level of the exit cut-off. A higher exit cut-off reduces both entrants' probability of obtaining a productivity draw above the exit cut-off and entrants' expected relative productivity conditional on successful entry. By contrast, in this paper knowledge spillovers imply that shifts in the level of the exit cut-off do not affect the relative productivity distribution of entrants. On the balanced growth path entrants draw relative productivity from a stationary distribution  $H\left(\frac{\phi}{\lambda}\right)$  that is unaffected by trade liberalization. Thus, although free entry implies that trade generates selection both with and without knowledge spillovers, when entrants learn from incumbents trade has a dynamic selection effect.

Two additional features of Proposition 1 are particularly noteworthy. First, growth is independent of population size meaning there are no scale effects. Second growth is increasing in the fixed production cost.<sup>21</sup> Let us consider each of these findings in turn. Scale effects are a ubiquitous feature of the first generation of endogenous growth models (Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) where growth depends on the size of the R&D sector which, on a balanced growth path, is proportional to population. However, Jones (1995a) documents that despite continuous growth in both population and the R&D labor force, growth rates in developed countries have been remarkably stable since the second world war.<sup>22</sup> This finding prompted Jones (1995b) to pioneer the development of semi-endogenous growth models in which the allocation of resources to R&D remains endogenous, but there are no scale effects because diminishing returns to knowledge creation mean that population growth is the only source of long-run growth. Semi-endogenous growth models have in turn been criticized for attributing long-run growth to a purely exogenous factor and understating the role of incentives to perform R&D in driving growth.<sup>23</sup>

There are three features of the technology diffusion model which imply the absence of scale effects. First, the mass of goods produced is endogenous. In quality ladders growth models the number of goods produced is constant and, consequently, the profit flow received by innovators is increasing in population, which generates a scale effect. In this paper population growth increases the mass of goods produced. Thus, in larger economies producers face more competitors and the incentive to innovate does not depend on market size. Second, unlike in expanding varieties growth models, the creation of new goods does not reduce the cost of R&D for future innovators implying that population growth does not generate horizontal knowledge spillovers. Third, and most important, knowledge spillovers depend upon the productivity distribution of all incumbent firms. In particular, I assumed in Section 2.3 that the variable  $x_t$  which captures knowledge spillovers is independent of both the mass of incumbent firms and the maximum of the incumbent productivity distribution. Consequently, when an increase in population raises the mass of goods produced it does not affect the incumbent firm productivity distribution and does not generate knowledge spillovers. As equation (25) makes clear, the dynamic selection rate depends not on the innovation rate, which is proportional to

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<sup>21</sup>Luttmer (2007) also finds that the consumption growth rate is increasing in  $\frac{f}{f_e}$  when there are productivity spillovers from incumbents to entrants.

<sup>22</sup>Although, see Kremer (1993) for evidence that scale effects may be present in the very long run.

<sup>23</sup>Jones (2005) draws a distinction between strong scale effects where the scale of an economy affects output growth and weak scale effects where scale affects the level of output. Using this terminology, the technology diffusion model features weak scale effects (see equations (31) and (32) below), but not strong scale effects.

population, but on the innovation rate relative to the mass of producers which is scale independent. Together these three features ensure that an increase in scale does not generate knowledge spillovers and, therefore, does not affect the equilibrium growth rate. A related model that features endogenous growth without scale effects is developed by Young (1998) who allows for R&D to raise both the quality and the number of goods produced, but assumes that knowledge spillovers only occur along the vertical dimension of production. However, in Young (1998) there is no selection on productivity, implying that the dynamic selection effect analyzed in this paper is missing.

Early work on the effects of trade in endogenous growth models found that global integration increases growth via the scale effect provided knowledge spillovers are sufficiently international in scope (Rivera-Batiz and Romer 1991; Grossman and Helpman 1991).<sup>24</sup> More recent papers have shown that if firm heterogeneity is included in standard expanding variety (Baldwin and Robert-Nicoud 2008) or quality ladders (Haruyama and Zhao 2008) models the relationship between trade and growth continues to depend on the extent of international knowledge spillovers. In models without scale effects such as Young (1998) and the semi-endogenous growth model of Dinopoulos and Segerstrom (1999) the long run growth rate is independent of an economy's trade status because trade is equivalent to an increase in scale. By contrast, in this paper growth is driven by selection, not scale and the dynamic selection mechanism through which trade increases growth does not require the existence of scale effects or international knowledge spillovers. Instead, it relies on the combination of firm heterogeneity and technology diffusion.

A higher fixed production cost increases growth through a similar mechanism to trade integration. From the profit function (9) we see that, for a given relative productivity  $\phi$  and wage  $w_t$ , profits are proportional to  $f$ . Since on the balanced growth path entrants' relative productivity distribution is independent of  $f$  it follows that the expected initial profit flow received by a new entrant (relative to the wage) is increasing in  $f$ . However, the free entry condition (11) implies that in equilibrium the expected value of innovating (relative to the wage) is independent of  $f$ . Therefore, to satisfy the free entry condition the increase in an entrant's expected initial profits generated by a rise in  $f$  must be offset by a fall in the entrant's expected future profits which requires that relative productivity  $\phi$  declines at a faster rate and the firm's expected lifespan falls. Thus, higher  $f$  increases the rate of dynamic selection  $g$  which raises the growth rate  $q$ . Substituting (25) back into the labor market clearing condition implies:

$$M_t = \left[ \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} f \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) + (n + gk) \frac{f_e}{\lambda^k} \right]^{-1} L_t. \quad (26)$$

It follows that raising  $f$  reduces the mass of goods produced. It is this reduction in competition among incumbents that leads to higher profits conditional on  $\phi$ .

The effects of the remaining parameters on the growth rate are unsurprising. Increasing the entry cost by raising  $f_e$  must, in equilibrium, lead to an increase in the expected value of innovating and this is achieved through lower growth which increases firms' expected lifespans. Similarly, growth is strictly increasing in the strength of knowledge spillovers  $\lambda$  because when spillovers are stronger an entrant's expected initial

<sup>24</sup>A complementary line of research examines how trade integration affects the incentives of asymmetric countries with multiple production sectors to undertake R&D (Grossman and Helpman 1991).

relative productivity is higher. Consequently, to ensure the free entry condition (11) holds the dynamic selection rate must increase to offset the rise in initial profits. A higher intertemporal elasticity of substitution or a lower discount rate raise growth by making households more willing to invest now and consume later, while, as discussed above, population growth raises consumption per capita growth through its impact on the growth rate of the mass of producers  $M_t$ . The elasticity of substitution  $\sigma$  and the Pareto shape parameter  $k$  have an ambiguous effect on growth.

### 3.1 Transition dynamics

Lemma 1 shows that there exists a unique stationary relative productivity distribution, but in equilibrium does  $H_t(\phi)$  converge to this distribution? The answer depends on the properties of the initial productivity distribution  $\hat{G}_0(\theta)$ . As the exit cut-off increases the functional form of the relative productivity distribution  $H_t(\phi)$  depends on the right tail properties of  $\hat{G}_0(\theta)$  and of the sampling distribution  $F$ . When productivity is sufficiently high, whichever distribution has the thicker right tail dominates and if  $F$  has the thicker right tail then as  $t$  becomes large  $H_t(\phi)$  inherits the functional form of  $F$  and converges to a Pareto distribution.

Formally, suppose  $\hat{G}_0(\theta)$  satisfies the following assumption.

**Assumption 3.** *The sampling distribution  $F$  has a weakly thicker right tail than the initial productivity distribution  $\hat{G}_0(\theta)$ :*

$$\lim_{\theta \rightarrow \infty} \frac{1 - \hat{G}_0(\theta)}{\theta^{-k}} = \kappa,$$

where  $\kappa \geq 0$ .

Note that any bounded initial productivity distribution satisfies Assumption 3 with  $\kappa = 0$ . Assumption 3 is a necessary and sufficient condition to ensure that the relative productivity distribution converges to the balanced growth path distribution whenever there is dynamic selection.

**Proposition 2.** *When Assumption 1 holds and the exit cut-off  $\theta_t^*$  is unbounded as  $t \rightarrow \infty$  then in equilibrium  $\lim_{t \rightarrow \infty} H_t(\phi) = 1 - \phi^{-k}$  if and only if Assumption 3 is satisfied.*

The proof of Proposition 2 is in Appendix A. The requirement that  $\theta_t^* \rightarrow \infty$  is necessary to ensure that for large  $t$  only the right tail properties of  $\hat{G}_0$  and  $F$  matter.

## 4 Gains from trade

Both static and dynamic selection create new sources of gains from trade that do not exist when firms are homogeneous. However, as shown by Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012), in general equilibrium the welfare gains generated by the static selection effect are offset by lower entry. Are the gains from dynamic selection offset by other general equilibrium effects? To answer this question we must move beyond simply considering the equilibrium growth rate and solve for the welfare effects of trade.

## 4.1 Balanced growth path welfare

The technology diffusion model has two state variables: the relative productivity distribution  $H_t(\phi)$  and the mass of incumbent firms  $M_t$ . Lemma 1 implies that the stationary relative productivity distribution is independent of trade. In addition, equation (26) shows that trade liberalization reduces the mass of incumbent firms. This decline in  $M_t$  occurs instantaneously following a trade liberalization as a consequence of an upwards jump in the exit cut-off  $\theta_t^*$ . It follows that provided the economy is on a balanced growth path when trade liberalization occurs, then the equilibrium jumps instantaneously to the new balanced growth path and there are no transition dynamics. Therefore, to characterize the welfare effects of trade liberalization it is sufficient to compare welfare on the pre-liberalization and post-liberalization balanced growth paths.<sup>25</sup>

Suppose that at time zero the productivity distribution of potential producers  $\hat{G}_0(\theta)$  is Pareto with shape parameter  $k$  and scale parameter  $\hat{\theta}_0^*$  and the mass of potential producers  $\hat{M}_0$  is such that in equilibrium some firms have productivity below the exit cut-off at time zero and choose to exit immediately. This refinement of the initial condition assumed in Section 2.4 ensures the economy is always on a balanced growth path.

Substituting  $c_t = c_0 e^{qt}$  into the household welfare function (1) and integrating implies:

$$U = \frac{\gamma}{\gamma - 1} \left[ \frac{\gamma c_0^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} - \frac{1}{\rho - n} \right]. \quad (27)$$

Therefore, household welfare depends on both the consumption growth rate  $q$  and the level of consumption  $c_0$ . From the household budget constraint (2), the Euler equation (3) and the transversality condition (19) we can write the initial level of consumption per capita  $c_0$  in terms of initial wages and assets as:<sup>26</sup>

$$c_0 = w_0 + \left( \frac{1-\gamma}{\gamma} q + \rho - n \right) a_0, \quad (28)$$

where  $\frac{1-\gamma}{\gamma} q + \rho - n$  is the marginal propensity to consume out of wealth, which is positive by the transversality condition.

Now using (22) to substitute for  $W_t(\phi)$  in the asset market clearing condition (17), integrating the right hand side to obtain average firm value and using (23) gives:

$$a_t L_t = \frac{f_e}{\lambda^k} w_t M_t, \quad (29)$$

which has the intuitive interpretation that the value of the economy's assets at any given time equals the expected R&D cost of replacing all incumbent firms.

Next, using the initial condition given above, the time zero exit cut-off  $\theta_0^*$  is given by:

$$\theta_0^* = \hat{\theta}_0^* \left( \frac{\hat{M}_0}{M_0} \right)^{\frac{1}{k}}. \quad (30)$$

<sup>25</sup>Transition dynamics may arise following a reduction in trade integration (a fall in  $J$ , an increase in  $\tau$  or an increase in  $f_x$ ) since in this case  $\frac{M_t}{L_t}$  increases by (26). The details of the adjustment process will depend on whether or not firm exit is assumed to be irreversible. In this paper I will abstract from these considerations and focus on balanced growth path welfare.

<sup>26</sup>This is a textbook derivation. See, for example, Barro and Sala-i-Martin (2004), pp.93-94.

We can now solve for initial consumption per capita by combining this expression with equations (5), (20), (24), (26), (28) and (29) to obtain:

$$c_0 = A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}} \left[ 1 + \frac{\sigma-1}{k\sigma+1-\sigma} \frac{n+gk}{n+gk + \frac{1-\gamma}{\gamma}q + \rho - n} \right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \quad (31)$$

where:

$$A_1 \equiv (\sigma-1) \left( \frac{k}{k+1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} \left( \frac{k+1-\sigma}{k\sigma+1-\sigma} \right)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}} > 0. \quad (32)$$

Remember that Assumption 2 ensures  $g > 0$  and  $\frac{1-\gamma}{\gamma}q + \rho - n > 0$ . Thus, both the numerator and the denominator of the final term in (31) are positive.

Armed with the equilibrium growth rate (24) and the initial consumption level (31) we can now analyze the welfare effects of trade integration. Observe that trade affects both growth and the consumption level only through the value of  $T \equiv J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}$ .  $T$  measures the extent of trade integration between countries.  $T$  is strictly increasing in the number of countries  $J$  in the world economy and the fixed production cost  $f$ , but strictly decreasing in the variable trade cost  $\tau$  and the fixed export cost  $f_x$ . When calibrating the model in Section 4.2 I show that the import penetration ratio is a sufficient statistic for  $T$  and that  $T$  is monotonically increasing in the import penetration ratio.

Trade affects welfare through two channels. First, trade raises welfare by increasing  $c_0$  for any given growth rate. These static gains from trade  $z^s$  are given by the term:

$$z^s = \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}} = (1+T)^{\frac{1}{k}}$$

in (31). The static gains from trade result from the net effect of increased access to imported goods, a reduction in the number of goods produced domestically and average productivity gains caused by an increase in the level of the exit cut-off. Interestingly, the static gains equal the total gains from trade in comparable economies with firm heterogeneity, but without knowledge spillovers. Thus, both in static steady state economies such as the variant of Melitz (2003) considered by Arkolakis, Costinot and Rodríguez-Clare (2012) where entrants draw productivity from a Pareto distribution and in a version of the model above where innovators draw productivity from a time invariant Pareto distribution (in this case the exit cut-off is constant on the balanced growth path and trade does not affect the consumption growth rate) the gains from trade equal  $z^s$ .

Second, trade affects welfare by raising the growth rate through the dynamic selection effect. I will refer to the change in welfare caused by trade-induced variation in the growth rate as the dynamic gains from trade. From (27) we see that increased growth has a direct positive effect on welfare, but (31) shows that it also affects the level of consumption. The level effect is made up of two components. First, there is the increase in  $n + gk$  which from (25) occurs because trade raises the innovation rate relative to the

mass of producers. This requires a reallocation of labor between production and R&D that decreases the consumption level. Second, variation in  $q$  changes households' marginal propensity to consume out of wealth  $\frac{1-\gamma}{\gamma}q + \rho - n$ . The sign of this effect on  $c_0$  depends on the intertemporal elasticity of substitution  $\gamma$ , but it is positive when  $\gamma < 1$ . In general, the net effect of higher growth on the consumption level can be either positive or negative and substituting  $g = q - \frac{n}{\sigma-1}$  into (31) and differentiating with respect to  $q$  shows that higher growth increases  $c_0$  if and only if:

$$n \left( 1 - \frac{1}{k} \frac{1-\gamma}{\gamma} \frac{k+1-\sigma}{\sigma-1} \right) > \rho.$$

However, regardless of the sign of the level effect, substituting for  $c_0$  using (31) and then differentiating (27) with respect to growth shows that the dynamic gains from trade are positive. Thus, the direct positive effect of growth on welfare always outweighs any possibly negative indirect effect resulting from variation in  $c_0$ . Proposition 3 summarizes the welfare effects of trade. The proposition is proved in Appendix A.

**Proposition 3.** *Trade integration resulting from either an increase in the number of trading partners  $J$ , a reduction in the fixed export cost  $f_x$ , or a reduction in the variable trade cost  $\tau$  increases welfare through two channels: (i) by raising the level of consumption for any given growth rate (static gains), and; (ii) by raising the growth rate of consumption per capita (dynamic gains). The static gains equal the total gains from trade in Melitz (2003) if productivity has a Pareto distribution.*

Two observations follow immediately from Proposition 3. First, since both the static and dynamic gains from trade are positive, trade is welfare improving. Second, since the dynamic gains are positive, the total gains from trade in this paper are strictly larger than in a static steady state economy such as Melitz (2003). This demonstrates that the combination of firm heterogeneity with knowledge spillovers that depend upon the entire distribution of incumbent firm productivity generates a new source of gains from trade that is not offset by other general equilibrium effects. In contrast to the findings of Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012), in this paper firm heterogeneity matters for the gains from trade.<sup>27</sup>

To understand why the higher growth resulting from trade liberalization is welfare improving consider the efficiency properties of the decentralized equilibrium. By equation (25) the dynamic selection rate is increasing in  $\frac{R_t}{M_t}$ . As the exit cut-off increases, knowledge spillovers cause the productivity distribution of entrants to shift upwards, but innovators do not internalize the social value of these spillovers. Thus, there is a positive externality from investment in R&D and the flow of entrants relative to the mass of incumbents  $\lambda^k \frac{R_t}{M_t}$  is inefficiently low in the decentralized equilibrium. I show in Appendix C that a benevolent government can raise welfare using either a R&D subsidy or a tax on the fixed production cost since both policies

<sup>27</sup>This result is related to the literature that studies the gains from trade in economies not covered by Arkolakis, Costinot and Rodríguez-Clare (2012). Ossa (2012) shows that cross-sectoral heterogeneity in trade elasticities increase the gains from trade relative to Arkolakis, Costinot and Rodríguez-Clare (2012)'s estimates, but his argument applies regardless of whether or not there is firm level heterogeneity. Edmond, Midrigan and Xu (2012) and Impullitti and Licandro (2012) find that when there are variable mark-ups pro-competitive effects can substantially increase the gains from trade, although Arkolakis et al. (2012) show that this will not always be the case. By contrast, this paper focuses on understanding whether firm heterogeneity matters for the gains from trade in a dynamic single sector economy with constant mark-ups.

incentivize entry relative to production and raise the dynamic selection rate.<sup>28</sup> Since trade raises growth by increasing  $\frac{R_t}{M_t}$ , the dynamic selection effect of trade exploits the knowledge spillovers externality and leads to dynamic welfare gains.

## 4.2 Quantifying the gains from trade

How large are the dynamic gains from trade? This section assesses the quantitative importance of the dynamic selection effect in determining the overall gains from trade. To quantify the gains from trade I will start by calibrating the model using U.S. data and then perform robustness checks against this baseline, but it should be remembered when interpreting the calibration results that the theory assumes symmetry across countries. The key to the calibration is showing that the gains from trade can be expressed in terms of a small number of observables and commonly used parameters. In particular, it is not necessary to specify values of  $J$ ,  $f$ ,  $f_x$ ,  $f_e$  or  $\lambda$ .

Define the gains from trade  $z$  in equivalent variation terms as the proportional increase in the autarky level of consumption required to obtain the open economy welfare level. Thus,  $z$  satisfies  $U(zc_0^A, q^A) = U(c_0, q)$  where  $U$ ,  $q$  and  $c_0$  are defined by (27), (24) and (31), respectively, and  $A$  superscripts denote autarky values.<sup>29</sup> From (27) we have:

$$z = \frac{c_0}{c_0^A} \left[ \frac{(1-\gamma)q^A + \gamma(\rho-n)}{(1-\gamma)q + \gamma(\rho-n)} \right]^{\frac{\gamma}{\gamma-1}}.$$

Observe that if  $q = q^A$  the gains from trade are given by the increase in the initial consumption level, which from (31) equals the static gains from trade  $z^s$ . The dynamic gains from trade  $z^d$  are defined by  $z^d = \frac{z}{z^s}$ .

The static gains from trade depend only on the import penetration ratio (IPR) and the trade elasticity (TE). To see this first calculate import expenditure in each country (IMP) which is given by:

$$IMP_t = \frac{k\sigma}{k+1-\sigma} M_t w_t f J \tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}. \quad (33)$$

Equation (33) shows that  $k$  equals the trade elasticity (the elasticity of imports with respect to variable trade costs). Now divide (33) by total domestic sales  $c_t L_t$  to obtain:

$$z^s = \left( \frac{1}{1-IPR} \right)^{\frac{1}{TE}}. \quad (34)$$

This expression is identical to the formula for calibrating the gains from trade obtained by Arkolakis, Costinot and Rodríguez-Clare (2012). It follows that the calibrated static gains from trade in the technology diffusion model developed in this paper equal the calibrated total gains from trade in the class of

<sup>28</sup>I assume that the policies are financed by lump sum transfers to households. Acemoglu et al. (2013) also find that it is welfare improving to tax fixed production costs, but for a different reason. In their model, exit induced by taxing the fixed cost of production reduces competition for skilled workers to perform R&D. By contrast, in this paper exit induced by the tax leads to knowledge spillovers and increases the dynamic selection rate.

<sup>29</sup>In this section I compare welfare at observed levels of trade with autarky welfare. However, the same methodology could be used to compare welfare in two equilibria with different levels of trade integration.

static steady state economies studied by Arkolakis, Costinot and Rodríguez-Clare (2012). Models covered by Arkolakis, Costinot and Rodríguez-Clare (2012) include Anderson (1979), Krugman (1980) and Eaton and Kortum (2002) in addition to the variant of Melitz (2003) with a Pareto productivity distribution.

The U.S. import penetration ratio for 2000, defined as imports of goods and services divided by gross output, was 0.081.<sup>30</sup> Anderson and Van Wincoop (2004) conclude based on available estimates that the trade elasticity is likely to lie between five to ten. I set  $k = 7.5$  for the baseline calibration, while in the robustness checks I allow  $k$  to vary between two and ten. This interval includes the trade elasticity of four estimated by Simonovska and Waugh (2011).

To calibrate the dynamic gains from trade we can express  $\frac{\lambda^k f}{f_e}$ , which enters the expression for the growth rate  $q$ , as a function of  $n, k, \sigma, \gamma, \rho, IPR$  and the entry rate of new firms relative to the mass of existing firms (NF). Since a fraction  $\lambda^k$  of innovations lead to the creation of new firms we have  $NF = \lambda^k \frac{R_t}{M_t}$  and using (20), (24) and (25) gives:

$$\frac{\lambda^k f}{f_e} = \frac{k + 1 - \sigma}{\gamma k (\sigma - 1)} (1 - IPR) \left\{ [1 + \gamma(k - 1)] (NF - n) + \frac{k(1 - \gamma)}{\sigma - 1} n + \gamma k \rho \right\}.$$

The U.S. Small Business Administration reports an entry rate of 11.6% per annum in 2002 (Luttmer 2007). Therefore, I set  $NF = 0.116$ . For the population growth rate I use  $n = 0.011$  based on average annual U.S. population growth from 1980-2000 as reported in the World Development Indicators.

Finally, there are three parameters to calibrate:  $\sigma, \gamma$  and  $\rho$ . To calibrate  $\sigma$  observe that the right tail of the firm employment distribution is a power function with index  $\frac{-k}{\sigma-1}$ . Luttmer (2007) shows that for U.S. firms in 2002 the right tail index of the employment distribution equals  $-1.06$ . Therefore, I let the elasticity of substitution  $\sigma = k/1.06 + 1$  implying  $\sigma = 8.1$ . Note that  $k > \max\{1, \sigma - 1\}$  as required by Assumption 1. Helpman, Melitz and Yeaple (2004) use European firm sales data to estimate  $k + 1 - \sigma$  at the industry level, obtaining estimates that mostly lie in the interval between 0.5 and 1 implying  $\sigma \in [k, k + 1/2]$ . In the robustness checks I allow  $\sigma$  to vary over a range that includes this interval.

Although controversy exists over the value of the intertemporal elasticity of substitution, estimates typically lie between 0.2 and 1.<sup>31</sup> Following García-Peñalosa and Turnovsky (2005) I let  $\gamma = 1/3$  in the baseline calibration. A low intertemporal elasticity of substitution will tend to reduce the dynamic gains from trade by making consumers less willing to substitute consumption over time. I also follow García-Peñalosa and Turnovsky (2005) in choosing the discount rate and set  $\rho = 0.04$ . In the robustness checks I allow  $\gamma$  to vary between 0.2 and 1 and  $\rho$  to vary between 0.01 and 0.15. Table 1 summarizes the data and parameter values used for the baseline calibration. Assumption 2 is satisfied both for the baseline calibration and in all the robustness checks.

Table 2 shows the calibration results. The model predicts that consumption per capita growth is 10.7% higher at observed U.S. trade levels than in a counterfactual autarkic economy. Due to the dynamic welfare gains resulting from higher growth, the total calibrated gains from trade are 3.2 times higher than the static gains. Thus, dynamic selection has a quantitatively important effect on the gains from trade.

<sup>30</sup>Imports of goods and services are from the World Development Indicators (Edition: April 2012) and gross output is from the OECD STAN Database for Structural Analysis (Vol. 2009).

<sup>31</sup>See, for example, Hall (1988), Vissing-Jorgensen (2002), Yogo (2004) and Guvenen (2006).

Next, I consider the robustness of these results. First, with respect to the import penetration ratio. Unsurprisingly, the gains from trade are higher when trade integration is greater (Figure 1). Increasing the import penetration ratio from 0.051 (Japan) to 0.36 (Belgium) raises welfare gains from 2.2% to 19.2%. More importantly, the ratio of the total gains to the static gains, which measures the proportional increase in the gains from trade due to dynamic selection, remains approximately constant as the import penetration ratio varies. Figure 2 plots the growth rate under trade relative to the autarky growth rate on the left hand axis and the total gains from trade relative to the static gains from trade on the right hand axis. The total gains are a little over three times larger than the static gains for all levels of the import penetration ratio between zero and 0.5.

Finally, Figure 3 shows the consequences of allowing for variation in the remaining calibration variables. I plot the growth effect of trade (left hand axis) and the ratio of the total gains from trade to the static gains (right hand axis) as a function of each variable in turn, while holding the other parameters fixed at their baseline values.<sup>32</sup> In all cases the dynamic gains from trade are quantitatively important and the results suggest that dynamic selection at least doubles the gains from trade. For example, either lowering the intertemporal elasticity of substitution or raising the discount rate reduces the dynamic gains from trade because it lowers the value of future consumption growth. However, even if the intertemporal elasticity of substitution is reduced to 0.2, the gains from trade are 2.4 times higher with dynamic selection, while the discount rate must exceed 14% before the total gains from trade are less than double the static gains.

## 5 Extensions of the technology diffusion model

The baseline model above establishes that incorporating technology diffusion into an otherwise standard open economy model with heterogeneous firms leads to a new source of dynamic gains from trade. In this section I analyze the implications of relaxing some of the simplifying assumptions made in the baseline model. I consider four extensions. First, I include international knowledge spillovers from foreign exporters to domestic firms. Second, I allow for firms to experience post-entry productivity growth and for entrants to draw productivity from distributions other than the Pareto distribution. Third, I consider knowledge spillovers that benefit both entrants and incumbent firms. Fourth, I introduce an alternative specification of the R&D technology which allows for decreasing returns to scale in R&D and congestion in the technology diffusion process. The finding that trade raises growth by increasing the dynamic selection rate is broadly robust across these alternative specifications. Moreover, the extensions identify additional channels through which trade affects technology diffusion and growth.

### 5.1 International knowledge spillovers

Suppose that entrants learn not only from domestic firms, but also from foreign firms that sell in the domestic market. To formalize this idea, let  $x_t$ , which captures knowledge spillovers, equal the average productivity of all firms that sell in a given market. All other assumptions are unchanged. This version of the model can be solved using the same reasoning applied above. There exists a unique balanced growth path on which the

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<sup>32</sup>The sole exception is Figure 3c, where I adjust  $\sigma$  to ensure  $\sigma = k/1.06 + 1$  always holds as the trade elasticity varies.

stationary relative productivity distribution is Pareto since Lemma 1 holds and the equilibrium growth rate is still given by (24). The only difference from the baseline model is the value of the knowledge spillovers parameter  $\lambda$  which, by definition, equals  $x_t \psi_{\min} / \theta_t^*$ . Calculating the average productivity of all firms selling in a market gives that on the balanced growth path:

$$x_t = \frac{k\theta_t^*}{k-1} \frac{1 + J\tilde{\phi}^{1-k}}{1 + J\tilde{\phi}^{-k}},$$

which implies:

$$\lambda = \frac{k\psi_{\min}}{k-1} \tilde{\lambda} \quad \text{where} \quad \tilde{\lambda} \equiv \frac{1 + J\tilde{\phi}^{1-k}}{1 + J\tilde{\phi}^{-k}}. \quad (35)$$

In the absence of international knowledge spillovers  $\tilde{\lambda} = 1$  and  $\lambda$  is independent of trade integration. With international knowledge spillovers  $\tilde{\lambda} > 1$  whenever not all firms are exporters (i.e. whenever  $\tilde{\phi} > 1$ ). Thus, international knowledge spillovers increase the strength of knowledge spillovers by raising  $\lambda$ . This increase in  $\lambda$  occurs because exporters are a select group of high productivity firms. Consequently, entrants learn more from the average foreign firm than from the average domestic firm.

Inspection of the equilibrium growth rate (24) shows that growth is increasing in  $\lambda$ . Therefore, trade has a larger effect on growth when there are international knowledge spillovers because in addition to the direct positive effect of trade on growth identified in Section 3, there is an indirect positive effect caused by the rise in  $\lambda$ . In addition, the consumption level  $c_0$  is independent of  $\lambda$  by (31). It follows immediately that international knowledge spillovers do not affect the static gains from trade, but increase the dynamic gains from trade. Proposition 4 summarizes these results.

**Proposition 4.** *Suppose Assumptions 1 and 2 hold and not all firms are exporters. Compared to autarky, trade raises growth and welfare by more when there are international knowledge spillovers than if there are only domestic knowledge spillovers.*

By exposing domestic entrants to the superior technologies used by foreign exporters, international knowledge spillovers open a second channel through which trade increase the dynamic selection rate. This channel operates if and only if the average foreign exporter is more productive than the average domestic firm. When all firms are exporters this condition is not satisfied and the effects of trade are identical to the baseline model. Similarly, if we assume an alternative specification for international knowledge spillovers in which entrants learn from all domestic and foreign firms and  $x_t$  equals the average productivity of all incumbent firms anywhere in the world, then symmetry across economies implies that  $\lambda$  and the effects of trade are unchanged from the baseline model.<sup>33</sup>

Proposition 4 compares the open economy equilibrium to autarky. The effects of marginal reductions in trade costs are more subtle. Differentiating (35) shows that  $\tilde{\lambda}$  is strictly increasing in  $J$ , an increase in the number of trading partners strengthens knowledge spillovers. However, the effect of reducing either  $\tau$  or

<sup>33</sup>Developing a technology diffusion model with asymmetric economies is beyond the scope of this paper, but it is reasonable to expect that international knowledge spillovers will play a particularly important role in shaping the gains from North-South trade where the productivity distribution differs across countries.

$f_x$  on  $\tilde{\lambda}$  is in general ambiguous. Lower variable or fixed trade costs reduce the export threshold  $\tilde{\phi}$  and this has two offsetting effects. First, the fraction of firms that are exporters increases, which raises  $\tilde{\lambda}$  because exporters are more productive than non-exporters. Second, the average productivity of exporters decreases which reduces  $\tilde{\lambda}$ . By differentiating the expression for  $\tilde{\lambda}$  we obtain:

$$\frac{d\tilde{\lambda}}{d\tilde{\phi}} \propto \frac{k}{\tilde{\phi}} + \frac{J}{\tilde{\phi}^k} - (k-1).$$

Therefore,  $\tilde{\lambda}$  is inverse U-shaped as a function of  $\tilde{\phi}$  as shown in Figure 4 and knowledge spillovers are strongest for some interior  $\tilde{\phi} \in (1, \infty)$ . The effect of marginal reductions in either  $\tau$  or  $f_x$  on growth and the dynamic gains from trade is also ambiguous since for sufficiently low  $\tilde{\phi}$  the negative effect of lower trade costs on  $\tilde{\lambda}$  can outweigh the direct positive effect of lower trade costs on the growth rate. Thus, although international knowledge spillovers imply higher gains from trade, they also imply that marginal reductions in trade costs may reduce growth when initial trade costs are low.

## 5.2 Frontier expansion and post-entry productivity growth

In the baseline model firms draw productivity from a Pareto distribution and each firm's productivity  $\theta$  does not change over time. By simplifying the model and ensuring the existence of a closed form solution for the balanced growth path, these assumptions facilitate a clear exposition of the dynamic selection mechanism through which trade raises growth. In this section I relax these assumptions and show that neither assumption is necessary to obtain the paper's finding that trade increases growth.

First, let us generalize  $F$  to allow for non-Pareto sampling distributions. Instead of Assumptions 1 and 2, I make the following assumption which allows for entrants to sample productivity from any differentiable distribution.

- Assumption 4.** (i)  $F$  is a differentiable cumulative distribution function with support  $[\psi_{\min}, \psi_{\max}]$ .  
(ii) Knowledge spillovers are given by:  $x_t = x\theta_t^*$  where  $x$  is a constant that satisfies  $x\psi_{\min} \leq 1$  and  $x\psi_{\max} > 1$ .  
(iii) The parameters of the world economy satisfy:  $\rho + \frac{1-\gamma}{\gamma} \frac{n}{\sigma-1} > 0$  and  $(\sigma-1) + \frac{1-\gamma}{\gamma} > 0$ .  
(iv) The transversality condition is satisfied and the dynamic selection rate  $g > 0$ .

Since  $\psi_{\max}$  can be infinite, Assumption 4 allows for the sampling distribution  $F$  to be either bounded or unbounded above.

On any balanced growth path the value of a firm with relative productivity  $\phi$  is given by (22). Note that  $\frac{W_t(\phi)}{w_t}$  is a stationary function of  $\phi$ . By differentiating this function we can show: (i)  $\frac{W_t(\phi)}{w_t}$  is strictly increasing in  $J$  and strictly decreasing in  $\tau$  or  $f_x$  for all  $\phi > \tilde{\phi}$ , but is independent of  $J$ ,  $\tau$  and  $f_x$  for all  $\phi \leq \tilde{\phi}$ , and; (ii)  $\frac{W_t(\phi)}{w_t}$  is strictly decreasing in  $q$  for all  $\phi > 1$  provided part (iii) of Assumption 4 holds.<sup>34</sup> Thus, holding relative productivity constant, trade integration increases the present discounted value (relative to the wage) of all exporters. In addition, the parameter restrictions in part (iii) of Assumption 4 are sufficient to ensure higher growth decreases the present discounted value (relative to the wage) of firms at all relative

<sup>34</sup>See the proof of Proposition 5 in Appendix A for details.

productivity levels. A sufficient condition for the parameter restrictions in part (iii) to hold is  $\gamma \leq 1$  and, as discussed in Section 4.2, empirical studies generally estimate  $\gamma < 1$ .

Using Assumption 4 the free entry condition (11) can be written as:

$$f_e = \int_{\phi} \frac{W_t(\phi)}{w_t} dF\left(\frac{\phi}{x}\right). \quad (36)$$

Part (ii) of Assumption 4 ensures that entrants' relative productivity distribution is stationary and independent of trade integration. Therefore, using the properties of  $\frac{W_t(\phi)}{w_t}$  derived above, the free entry condition implies that trade integration (a rise in  $J$ , a reduction in  $\tau$  or a reduction in  $f_x$ ) strictly increases the growth rate  $q$  by raising the dynamic selection rate  $g$ . Higher growth is required to offset the increase in the expected value of entry caused by higher expected export profits.

Neither parts (ii) or (iii) of Assumption 4 are necessary for trade to increase growth (for example, see Proposition 1), but they allow us to characterize the effects of trade integration on  $q$  without imposing any structure on the sampling distribution  $F$ . If part (ii) does not hold and  $F$  is not Pareto then entrants' relative productivity distribution  $\tilde{H}(\phi) = F\left(\frac{\phi\theta_t^*}{x_t}\right)$  is endogenous to trade integration through  $\frac{\theta_t^*}{x_t}$ . This endogeneity could either reinforce or weaken the positive effect of trade on growth depending on how trade affects the incumbent firm relative productivity distribution  $H(\phi)$  and how knowledge spillovers  $x_t$  are specified. Analyzing this effect would be an interesting topic for future research.

I have established that trade integration leads to higher growth on any balanced growth path. To show that a balanced growth path exists I also need to prove the existence of a stationary relative productivity distribution. Setting  $\dot{H}_t(\phi) = 0$  in (15) and using (14) to eliminate  $\frac{R_t}{M_t}$  gives:

$$\phi H'(\phi) = \frac{n}{g} \left[ H(\phi) - \frac{F\left(\frac{\phi}{x}\right) - F\left(\frac{1}{x}\right)}{1 - F\left(\frac{1}{x}\right)} \right] + H'(1) \frac{1 - F\left(\frac{\phi}{x}\right)}{1 - F\left(\frac{1}{x}\right)}. \quad (37)$$

The proof of Proposition 5 shows that this differential equation has a unique solution, implying the existence of a unique stationary relative productivity distribution  $H(\phi)$ . The solution  $H(\phi)$  depends on trade integration through the dynamic selection rate  $g$ . Whenever  $\psi_{\max} < \infty$ , relative productivity is bounded above with  $\phi_{\max} = x\psi_{\max}$ . In this case growth is driven both by the diffusion of existing technologies as in the baseline model and by the expansion of the technology frontier as  $\theta_{\max} = \theta_t^* \phi_{\max}$  increases. Thus, it is not necessary for the productivity distribution to have an unbounded right tail in order for trade to raise growth by increasing the dynamic selection rate. Combining the results above gives Proposition 5.

**Proposition 5.** *Given Assumption 4, the world economy has a unique balanced growth path equilibrium for any sampling distribution  $F$ . Trade integration raises the growth rate of consumption per capita on the balanced growth path.*

Without imposing any functional form restrictions on  $F$  it is not possible to characterize how trade affects welfare analytically, but the equilibrium conditions in Section 4 could be used to solve for balanced growth path welfare numerically for any given  $F$ .

Now, let us extend the baseline model to allow for firms' productivity levels to change over time. Developing a theory to explain the post-entry dynamics of firm productivity is not the purpose of this paper.

Instead, I show that the dynamic selection effect of trade is robust to allowing for general firm level productivity dynamics that are independent of trade integration.<sup>35</sup> Suppose that instead of Assumptions 1 and 2 the following assumption holds.

**Assumption 5.** (i) Entrants at time  $t$  draw both an initial relative productivity  $\phi$  and a set of productivity growth rates  $\zeta = \{\zeta_s\}_{s \geq t}$  where  $\zeta_s = \frac{\dot{\theta}_s}{\theta_s}$  from a stationary distribution  $\tilde{H}(\phi, \zeta)$  that is independent of trade integration.

(ii) The intertemporal elasticity of substitution satisfies:  $\gamma \leq 1$ .

(iii) The transversality condition is satisfied and the dynamic selection rate  $g > 0$ .

Part (i) of Assumption 5 implies that entrants draw not a productivity level, but a productivity path. The assumption allows for an arbitrary distribution of post-entry productivity dynamics at the firm level and firms that enter with the same productivity may have different post-entry growth rates. Implicit in part (i) is also the assumption that the structure of knowledge spillovers is such that trade integration does not change entrants' sampling distribution.

When Assumption 5 holds, analogous reasoning to that used for Proposition 5 shows that trade integration and higher growth have opposite effects on the expected value of entry. Consequently, the free entry condition gives Proposition 6. The proof is in Appendix A.

**Proposition 6.** *Given Assumption 5, on any balanced growth path equilibrium of the world economy with post-entry firm level productivity dynamics, trade integration raises the growth rate of consumption per capita.*

Part (ii) of Assumption 5 is sufficient to ensure Proposition 5 holds for an arbitrary sampling distribution  $\tilde{H}(\phi, \zeta)$ , but it is not necessary. Note also that Proposition 6 does not prove there exists a balanced growth path equilibrium. To prove existence requires showing there exists a stationary relative productivity distribution, which is not possible without imposing greater structure on  $\tilde{H}(\phi, \zeta)$ .

Propositions 5 and 6 show that the dynamic selection mechanism through which trade increases growth in the baseline model is also present when the productivity distribution is not Pareto and when firms' productivity levels vary over time. In both cases the result is driven by the same logic: trade integration raises export profits and free entry requires an offsetting increase in the dynamic selection rate.

### 5.3 Learning by incumbent firms

The focus of this paper is on knowledge spillovers from incumbent firms to entrants. However, existing firms may also benefit from knowledge spillovers. A simple way to incorporate learning by incumbents into the model is to assume firm productivity at time  $t$  is given by:

$$\theta_t = x_t \psi,$$

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<sup>35</sup>Lileeva and Trefler (2009), Atkeson and Burstein (2010) and Bustos (2011) analyze economies in which trade affects incumbent firms' incentives to undertake technology investment. To the extent that trade-induced technology upgrading raises export profits it is likely to magnify the positive effect of trade on growth identified in this paper. Section 5.3 considers a particular example in which trade leads to incumbent firm productivity growth.

where  $x_t$  has the same properties as in the baseline model and  $\psi$  continues to be a stochastic component drawn at entry from the sampling distribution  $F$ . The only difference between this specification and the baseline model is that each firm's productivity depends on the current value of  $x_t$ . Thus, upwards shifts in the productivity distribution generate spillovers that raise the productivity of both entrants and incumbents and the relative productivity of each firm remains constant over time. The remainder of the model is unchanged except that Assumption 2 is replaced by the following assumption.

**Assumption 6.** *The parameters of the world economy satisfy:*

$$1 > \gamma > \left( 1 + \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) - \rho \right] \frac{\sigma - 1}{n} \right)^{-1},$$

$$n < \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right).$$

The first inequality guarantees  $g > 0$ , while the second inequality ensures the transversality condition holds. Evidence supporting the assumption  $\gamma < 1$  was discussed in Section 4.2.

We can now solve for a balanced growth path equilibrium following the same steps used in Sections 3 and 4. The stationary relative productivity distribution is Pareto as in Lemma 1 and Proposition 7 shows that on the balanced growth path trade integration raises the growth rate leading to positive dynamic gains from trade. The proof is in Appendix A.

**Proposition 7.** *Given Assumptions 1 and 6, when knowledge spillovers raise the productivity of both entrants and incumbents, the world economy has a unique balanced growth path equilibrium on which consumption per capita grows at rate:*

$$q = \frac{\gamma}{1 - \gamma} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) - \rho \right].$$

*Trade integration increases the growth rate of consumption per capita. The positive effect of trade on growth increases the welfare gains from trade.*

When knowledge spillovers to incumbents are sufficiently strong that each firm's relative productivity remains constant over time, an increase in the dynamic selection rate does not affect firms' expected lifespans. Instead, under the maintained assumption that  $\gamma < 1$ , higher growth decreases the expected value of entry by raising  $r - q$  and reducing the present discounted value of future profits. Thus, the channel through which an increase in the dynamic selection rate lowers the value of innovating differs from the baseline model, but free entry continues to imply that trade integration raises growth. Moreover, the positive effect of trade on growth leads to dynamic welfare gains that are additional to the gains from trade in static steady state economies. Allowing for weaker knowledge spillovers to incumbent firms such that relative productivity is declining in  $g$  gives a hybrid between the baseline model and the variant considered in this section. Unsurprisingly, the effect of trade integration on growth remains positive.

## 5.4 R&D technology

The baseline model features constant returns to scale in R&D. In this section I generalize the R&D technology to allow for congestion in technology diffusion. Assume that when  $R_t f_e$  workers are employed in R&D the flow of new innovations is  $\Psi(R_t, M_t)$  where  $\Psi$  is homogeneous of degree one, strictly increasing in  $R_t$ , weakly increasing in  $M_t$  and  $\Psi(0, 0) = 0$ .<sup>36</sup>  $\Psi$  gives the mass of innovators who successfully learn from incumbents' production techniques. Allowing  $\Psi$  to depend on  $M_t$  introduces decreasing returns to scale in R&D investment and implies that R&D is more productive when there are more incumbent firms to learn from.

Given this R&D technology we can solve for a balanced growth path equilibrium following the same reasoning applied above. Modifying the R&D technology does not affect households' welfare maximization problem or firms' static profit maximization problem meaning that (18) and (20) continue to hold. In addition, the stationary relative productivity distribution is unchanged and Lemma 1 still holds. However, the free entry condition now implies:

$$q = kg + r - \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] \psi \left( \frac{M_t}{R_t} \right),$$

where  $\psi \left( \frac{M_t}{R_t} \right) \equiv \Psi \left( 1, \frac{M_t}{R_t} \right) = \frac{1}{R_t} \Psi(R_t, M_t)$ . Combining this expression with (18) and (20) gives:

$$q = \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) \psi \left( \frac{M_t}{R_t} \right) + \frac{kn}{\sigma - 1} - \rho \right]. \quad (38)$$

Comparing (38) with the baseline economy growth rate given by equation (24), the only difference is the inclusion of  $\psi \left( \frac{M_t}{R_t} \right)$ . To obtain the equilibrium value of  $\frac{R_t}{M_t}$  note that in this version of the model equation (14), which gives the rate at which new firms are created, becomes:

$$\frac{R_t}{M_t} \psi \left( \frac{M_t}{R_t} \right) = \frac{1}{\lambda^k} \left( kq - \frac{k + 1 - \sigma}{\sigma - 1} n \right). \quad (39)$$

Equations (38) and (39) define a system of two equations in the two unknowns  $q$  and  $\frac{R_t}{M_t}$ . It is not possible to solve for  $q$  explicitly. However, assuming the transversality condition holds and  $g > 0$ , the proof of Proposition 8 shows that  $q$  is higher under trade than in autarky and is strictly increasing in  $J$ , strictly decreasing in  $\tau$  and strictly decreasing in  $f_x$ . Thus, as in the baseline model, trade raises growth by increasing the rate of dynamic selection. Moreover, solving for the initial consumption level shows  $c_0$  is still given by (31). It follows that even with decreasing returns to scale in R&D there exist dynamic gains resulting from the pro-growth effects of trade and these dynamic gains increase the total gains from trade relative to static steady state versions of the model. Proposition 8 summarizes these results.

**Proposition 8.** *Given Assumption 1, when there is congestion in technology diffusion the world economy*

<sup>36</sup>The baseline model corresponds to the case  $\Psi(R_t, M_t) = R_t$ . Assuming  $\Psi$  is homogeneous of degree one ensures the existence of a balanced growth path equilibrium.

*has a unique balanced growth path equilibrium. Trade integration raises the growth rate of consumption per capita. The positive effect of trade on growth increases the welfare gains from trade.*

To calibrate the model with congestion in technology diffusion let  $\Psi(R_t, M_t) = R_t^\alpha M_t^{1-\alpha}$  where  $\alpha \in (0, 1]$  parameterizes the returns to scale in R&D. Figure 5 shows how trade affects growth and welfare as  $\alpha$  varies between zero and one with other observables and parameters held constant at their baseline values from Table 1.<sup>37</sup> Reducing the returns to scale in R&D lowers the dynamic gains from trade as reallocating labor from production to R&D has a smaller effect on growth. However, provided the returns to scale exceed one half, the dynamic selection effect of trade at least doubles the gains from trade.

## 6 Conclusions

A complete analysis of the welfare effects of trade integration must account for the relationship between trade and growth. Yet existing work on open economies with heterogeneous firms has mostly overlooked dynamic effects. By incorporating knowledge spillovers into a dynamic version of the Melitz model this paper shows that the combination of firm heterogeneity and technology diffusion has novel and important implications for understanding the dynamic consequences of trade.

Motivated by evidence that there is substantial productivity dispersion within entering cohorts and that the productivity distributions of entrants and incumbents move together over time, the paper assumes that entrants learn not only from frontier technologies, but from the entire distribution of technologies used in an economy. This creates a complementarity between selection and technology diffusion and generates endogenous growth through dynamic selection. Growth through dynamic selection is only possible when firms are heterogeneous. Trade liberalization raises growth by increasing the rate of dynamic selection. Faster dynamic selection is required to offset higher export profits and ensure the free entry condition is satisfied. The dynamic selection effect is a new channel for gains from trade and I prove that it strictly increases the gains from trade compared to static steady state economies with heterogeneous firms. Calibrating the model shows the dynamic gains are quantitatively important and at least double the overall gains from trade.

The baseline model makes a number of simplifying assumptions. In particular, I assume entrants sample from a Pareto productivity distribution and I do not model firms' life cycle dynamics. These assumptions make it possible to solve the model in closed form, but they are not responsible for the finding that trade raises growth. Extending the model shows that whenever the productivity distributions of entrants and incumbents move together over time, the free entry condition mandates that trade increases the dynamic selection rate. This result is robust to alternative distributional assumptions and theories of firm life cycles. I also show that the effects of trade on growth and welfare are qualitatively unchanged when both incumbents and entrants benefit from technology diffusion and that allowing for international knowledge spillovers further increases the dynamic gains from trade.

This paper's modeling of knowledge spillovers builds upon the idea flows literature, but introduces a

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<sup>37</sup>The balanced growth path equilibrium conditions do not imply the existence of an observable that can be used to calibrate  $\alpha$  directly and I am not aware of any empirical work that estimates the returns to scale in R&D when R&D is aimed at learning about existing technologies. Allowing for congestion in technology diffusion does not affect the calibration of the static gains from trade.

reduced form specification of learning which enables tractable, open economy, general equilibrium analysis. In contrast to expanding variety and quality ladders growth models, that paper finds that if spillovers depend upon the entire technology distribution in an economy then growth does not feature scale effects. Moreover, the balanced growth path equilibrium is consistent with empirical work showing that the firm size distribution is stable over time, while the firm productivity distribution shifts to the right as a traveling wave. In addition, the impact of knowledge spillovers on the firm productivity distribution generates a rich set of predictions concerning technology diffusion that can be tested using standard firm level data. For example, testing the impact of shocks to the incumbent firm productivity distribution on the productivity of entrants would shed further light on the knowledge spillover process.

This paper focuses primarily on within-country technology diffusion with symmetric economies. However, the framework it develops to model technology diffusion could also be used to study cross-country technology diffusion with asymmetric economies or to analyze geographic variation in spillovers within countries. In this way it will, hopefully, further contribute to improving our understanding of the dynamic consequences of globalization.

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## Appendix A – Proofs

### Proof of Proposition 2

A necessary and sufficient condition for Proposition 2 to hold is that  $\frac{1-G_t(\theta_1)}{\theta_1^{-k}} \frac{\theta_2^{-k}}{1-G_t(\theta_2)} \rightarrow 1$  as  $t \rightarrow \infty$  for all  $\theta_1, \theta_2 > \theta_t^*$  since this ensures that  $G_t(\theta)$  converges asymptotically to a Pareto distribution with shape parameter  $k$ .

Let  $Z_t(\theta)$  denote the mass of firms with productivity greater than or equal to  $\theta$  at time  $t$  where  $\theta > \theta_t^*$ . Since incumbent firm productivity remains constant and there is a flow  $R_t$  of entrants who draw productivity from distribution  $\tilde{G}_t(\theta) = F\left(\frac{\theta}{x_t}\right)$  we have:

$$Z_{t+\Delta}(\theta) = Z_t(\theta) + \Delta R_t \left[ 1 - F\left(\frac{\theta}{x_t}\right) \right].$$

Taking the limit as  $\Delta \rightarrow 0$  and using the functional form of  $F$  from Assumption 1 gives:

$$\dot{Z}_t(\theta) = R_t \left( \frac{\theta}{x_t \psi_{\min}} \right)^{-k},$$

and solving this differential equation implies:

$$Z_t(\theta) = Z_0(\theta) + \left( \frac{\theta}{\psi_{\min}} \right)^{-k} \int_0^t R_s x_s^k ds.$$

Now substituting  $Z_t(\theta) = M_t [1 - G_t(\theta)]$  into this equation implies that for all  $\theta_1, \theta_2 > \theta_t^*$  we have:

$$\frac{1 - G_t(\theta_1)}{\theta_1^{-k}} \frac{\theta_2^{-k}}{1 - G_t(\theta_2)} = \frac{M_0 \frac{1 - \hat{G}_0(\theta_1)}{\theta_1^{-k}} + \psi_{\min}^k \int_0^t R_s x_s^k ds}{M_0 \frac{1 - \hat{G}_0(\theta_2)}{\theta_2^{-k}} + \psi_{\min}^k \int_0^t R_s x_s^k ds}.$$

As  $t \rightarrow \infty$  we know  $\theta_t^* \rightarrow \infty$ . Therefore, a necessary and sufficient condition for the right hand side of the above equation to converge to one for all  $\theta_1, \theta_2 > \theta_t^*$  is that  $\frac{1 - \hat{G}_0(\theta)}{\theta^{-k}} \rightarrow \kappa$  as  $\theta \rightarrow \infty$  for some  $\kappa \geq 0$ , i.e. that Assumption 3 holds.

### Proof of Proposition 3

To show that the dynamic gains from trade are positive substitute (31) and (20) into (27) and differentiate with respect to  $q$  to obtain:

$$\begin{aligned} \frac{dU}{dq} &\propto -(k\sigma + 1 - \sigma)\gamma D_1 \left( kD_1 - \frac{1 - \gamma}{\gamma} D_2 \right) + k\gamma(D_1 + D_2) [k\sigma(D_1 + D_2) - (\sigma - 1)D_1], \\ &= k^2\gamma\sigma D_2^2 + D_1 D_2 [k^2\gamma\sigma + (k\sigma + 1 - \sigma)(1 + \gamma(k - 1))], \\ &> 0 \end{aligned}$$

where  $D_1 \equiv \frac{1 - \gamma}{\gamma} q + \rho - n$  and  $D_2 \equiv n + gk$ . In the first line of the above expression, the first term on

the right hand side captures the indirect effect of higher growth on welfare through changes in  $c_0$ , while the second term captures the direct effect. The final inequality comes from observing that Assumption 2 implies both  $D_1 > 0$  and  $D_2 > 0$ .

To obtain a version of the model developed in this paper without productivity spillovers assume that new entrants receive a productivity draw from a Pareto distribution with scale parameter one and shape parameter  $k$ . Thus,  $\tilde{G}(\theta) = 1 - \theta^{-k}$  is independent of  $t$ . Assuming the baseline model is otherwise unchanged, the same reasoning used in Section 2.3 above implies:

$$\frac{\dot{M}_t}{M_t} = -k \frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{R_t}{M_t} \theta_t^{*-k}.$$

It immediately follows that on a balanced growth path the exit cut-off must be constant implying  $g = 0$ . Consumer optimization and the solution for the exit cut-off (5) then give  $q = \frac{n}{\sigma-1}$  meaning that the growth rate is independent of trade integration. With this result in hand it is straightforward to solve the remainder of the model and show  $c_0 \propto z^s$ .

### Proof of Proposition 5

The proof has two parts. First, I show that trade integration (an increase in  $J$ , a reduction in  $\tau$  or a reduction in  $f_x$ ) raises the growth rate  $q$  on any balanced growth path. Second, I prove that there exists a unique balanced growth path by showing that there exists a unique stationary relative productivity distribution.

Define  $E(\phi) \equiv \frac{W_t(\phi)}{w_t}$ . On a balanced growth path  $E(\phi)$  is given by (22). Differentiating gives:

$$\begin{aligned} \frac{\partial E(\phi)}{\partial \tau} &\propto -(\sigma - 1) \left[ \left( \frac{\phi}{\tilde{\phi}} \right)^{\sigma-1} - \left( \frac{\phi}{\tilde{\phi}} \right)^{\frac{q-r}{g}} \right] \frac{d\tilde{\phi}}{d\tau} && \forall \phi > \tilde{\phi}, \\ &< 0, \end{aligned}$$

where the second line follows because  $\frac{d\tilde{\phi}}{d\tau} > 0$  and the transversality condition implies  $r > q$ . Similarly we have:

$$\begin{aligned} \frac{\partial E(\phi)}{\partial f_x} &\propto -1 + \left( \frac{\phi}{\tilde{\phi}} \right)^{\frac{q-r}{g}} && \forall \phi > \tilde{\phi}, \\ &< 0, \end{aligned}$$

where the second line again follows from the transversality condition. We also have  $\frac{\partial E(\phi)}{\partial J} > 0$  for all  $\phi > \tilde{\phi}$  and  $\frac{\partial E(\phi)}{\partial \tau} = \frac{\partial E(\phi)}{\partial f_x} = \frac{\partial E(\phi)}{\partial J} = 0$  for all  $1 \leq \phi \leq \tilde{\phi}$ .

Next, write  $E(\phi) = E^d(\phi) + E^x(\phi)$  where  $E^d$  denotes the present discounted value from domestic sales relative to the wage (i.e. set  $J = 0$  in (22) to obtain  $E^d$ ) and  $E^x$  denotes the value created by exporting. Using (18) and (20) to substitute for  $g$  and  $r$  in (22) and differentiating gives:

$$\frac{\partial^2 E^d(\phi)}{\partial \phi \partial q} \propto - \left( \rho + \frac{1-\gamma}{\gamma} \frac{n}{\sigma-1} \right) \frac{\log \phi}{g^2} \phi^{\frac{q-r}{g}-1} - \frac{(\sigma-1) + \frac{1-\gamma}{\gamma}}{(\sigma-1)g+r-q} \frac{1}{\phi} \left( \phi^{\sigma-1} - \phi^{\frac{q-r}{g}} \right),$$

which is negative for all  $\phi > 1$  by part (iii) of Assumption 4 and the transversality condition. Since  $\frac{\partial E^d(\phi)}{\partial q} = 0$  when  $\phi = 1$  it follows that  $\frac{\partial E^d(\phi)}{\partial q} < 0$  for all  $\phi > 1$ . Similar reasoning shows that  $\frac{\partial E^x(\phi)}{\partial q} < 0$  for all  $\phi > \tilde{\phi}$  and obviously  $\frac{\partial E^x(\phi)}{\partial q} = 0$  for all  $1 \leq \phi \leq \tilde{\phi}$ . Thus, higher growth reduces  $E(\phi)$  whenever  $\phi > 1$ . Combining this result with the effects of trade integration on  $E(\phi)$  obtained above implies that in order for the free entry condition (36) to hold we must have:

$$\frac{dq}{d\tau} < 0, \quad \frac{dq}{df_x} < 0, \quad \frac{dq}{dJ} > 0.$$

For the second part of the proof, showing that there exists a unique stationary relative productivity distribution is equivalent to proving that the differential equation (37) has a unique solution. Suppose  $H'(1) = \chi$ . Under the initial condition  $H(1) = 0$ , equation (37) is a first order ordinary differential equation for  $H(\phi)$ . Since (37) is continuous in  $\phi$  and Lipschitz continuous in  $H$ , the Picard-Lindelöf theorem implies that there exists a unique solution  $H(\phi; \chi)$  with  $H'(1; \chi) = \chi$ .

$H'(\phi; \chi)$  is strictly increasing in both  $H(\phi)$  and  $\chi$ . Consequently,  $\frac{\partial H(\phi; \chi)}{\partial \chi} > 0$  for all  $\phi > 1$ . Moreover,  $H'(1; 0) < 0$  and  $H''(1; 0) < 0$  meaning  $H(\phi; 0) < 0$  for all  $\phi > 1$ . In addition, for any  $\phi > 1$ ,  $H(\phi; \chi)$  can be made arbitrarily large by choosing a sufficiently high  $\chi$ . It follows that there exists a unique  $\chi^* > 0$  such that  $H(\phi_{\max}; \chi^*) = 1$ .<sup>38</sup> The unique solution to (37) is  $H(\phi) = H(\phi; \chi^*)$ .

## Proof of Proposition 6

Under Assumption 5, the free entry condition (11) is replaced on a balanced growth path by:

$$f_e = \int_{(\phi, \zeta)} E(\phi, \zeta) d\tilde{H}(\phi, \zeta),$$

where  $E(\phi, \zeta) \equiv \frac{W_t(\phi, \zeta)}{w_t}$ . Using (9) and observing that a firm with relative productivity  $\phi$  at time zero and future productivity growth  $\zeta$  has relative productivity  $\phi e^{\int_0^t \zeta_s ds} e^{-gt}$  at time  $t$ , we have that  $E(\phi, \zeta)$  is given by:

$$\begin{aligned} E(\phi, \zeta) &= I[\phi_t \geq 1] f \int_0^\infty \left[ \left( \phi e^{\int_0^t \zeta_s ds} \right)^{\sigma-1} e^{-(\sigma-1)gt} - 1 \right] e^{-(r-q)t} dt \\ &\quad + I[\phi_t \geq \tilde{\phi}] f J \tau^{1-\sigma} \int_0^\infty \left[ \left( \phi e^{\int_0^t \zeta_s ds} \right)^{\sigma-1} e^{-(\sigma-1)gt} - \tilde{\phi}^{\sigma-1} \right] e^{-(r-q)t} dt. \end{aligned}$$

We can now differentiate this expression and show: (i) holding  $q$  constant,  $E(\phi, \zeta)$  is strictly increasing in  $J$  and strictly decreasing in  $\tau$  and  $f_x$  for all  $\phi \geq \tilde{\phi}$  and independent of these variables for all  $1 \leq \phi \leq \tilde{\phi}$ ,

<sup>38</sup>If  $\phi_{\max} = \infty$  this should be interpreted as meaning there exists a unique  $\chi^*$  such that  $\lim_{\phi \rightarrow \infty} H(\phi; \chi^*) = 1$ .

and; (ii)  $E(\phi, \zeta)$  is strictly decreasing in  $q$  given that part (ii) of Assumption 5 holds and that  $g$  and  $r$  satisfy (18) and (20) on a balanced growth path. The proposition then follows from the free entry condition.

### Proof of Proposition 7

Let us solve for a balanced growth path taking Assumptions 1 and 6 as given. Since the relative productivity of incumbent firms is time invariant, equations (14) and (15), which govern the evolution of the mass of firms and the relative productivity distribution, respectively, are replaced by:

$$\frac{\dot{M}_t}{M_t} = \left[ 1 - F\left(\frac{\theta_t^*}{x_t}\right) \right] \frac{R_t}{M_t},$$

$$\dot{H}_t(\phi) = \left\{ F\left(\frac{\phi \theta_t^*}{x_t}\right) - F\left(\frac{\theta_t^*}{x_t}\right) - H_t(\phi) \left[ 1 - F\left(\frac{\theta_t^*}{x_t}\right) \right] \right\} \frac{R_t}{M_t}.$$

Setting  $\dot{H}_t(\phi) = 0$  immediately gives that the unique stationary relative productivity distribution is  $H(\phi) = 1 - \phi^{-k}$ . We also have  $\tilde{H}(\phi) = H\left(\frac{\phi}{\lambda}\right)$  as in the baseline model.

On a balanced growth path equations (18)-(21) continue to hold, but instead of (22) the firm value function is given by:

$$W_t(\phi) = \frac{f w_t}{r - q} \left\{ (\phi^{\sigma-1} - 1) + I[\phi \geq \tilde{\phi}] \frac{J f_x}{f} \left[ \left(\frac{\phi}{\tilde{\phi}}\right)^{\sigma-1} - 1 \right] \right\}.$$

Integrating to obtain the expected value of entry and using the free entry condition, (18) and (20) implies that there is a unique balanced growth path with growth rate:

$$q = \frac{\gamma}{1 - \gamma} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left( 1 + J \tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \right) - \rho \right].$$

Assumption 6 ensures the transversality condition is satisfied and  $g > 0$ . Note that  $\gamma < 1$  implies  $q$  is strictly increasing in  $J$  and strictly decreasing in  $\tau$  and  $f_x$ .

Welfare on the balanced growth path is given by (27) and solving for  $c_0$  we obtain:

$$c_0 = A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[ 1 + J \tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}} \left[ 1 + \frac{\sigma - 1}{k\sigma + 1 - \sigma} \frac{n}{\frac{1-\gamma}{\gamma} q + \rho} \right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \quad (40)$$

where  $A_1$  is given by (32). Observe that the static gains from trade are the same as in the baseline model. Since  $\gamma < 1$ , equation (40) implies  $\frac{\partial c_0}{\partial q} > 0$ . Therefore, higher growth is welfare increasing since it raises both  $q$  and  $c_0$ . It follows that the dynamic gains from trade are strictly positive.

### Proof of Proposition 8

To prove the proposition I need to show that  $q$  is strictly increasing in  $T = J \tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$ . The result can be proved by taking the total derivatives of (38) and (39) and rearranging to obtain  $\frac{dq}{dT}$ , but here is

a simpler argument. Suppose  $T$  increases, but  $q$  does not. Then (38) implies that  $\psi\left(\frac{M_t}{R_t}\right)$  must decrease which requires a fall in  $\frac{M_t}{R_t}$ . From the definition of  $\psi$  we have that  $\frac{R_t}{M_t}\psi\left(\frac{M_t}{R_t}\right) = \Psi\left(\frac{R_t}{M_t}, 1\right)$  which increases when  $\frac{M_t}{R_t}$  falls. Therefore, we must have that the left hand side of (39) increases, while the right hand side does not giving a contradiction. It follows that an increase in  $T$  must lead to an increase in  $q$ .

## Appendix B – Knowledge spillovers through random matching

Suppose that instead of equation (10), knowledge spillovers result from random matching between entrants and incumbents. In particular, I assume that each innovator searches for a process technology to use and is randomly matched with an incumbent firm whose technology she imperfectly imitates. Formally, this implies that the productivity distribution of innovators is a scaled version of the productivity distribution of incumbent firms where the scaling parameter  $\lambda$  measures the strength of spillovers. Thus,  $\tilde{G}_t(\theta) = G_t(\theta/\lambda)$  where  $\lambda \in (0, 1]$ .

With this specification of knowledge spillovers, equation (14) for the growth rate of the mass of firms becomes:

$$\frac{\dot{M}_t}{M_t} = -H'_t(1) \frac{\dot{\theta}_t^*}{\theta_t^*} + \left[ 1 - H_t\left(\frac{1}{\lambda}\right) \right] \frac{R_t}{M_t},$$

and equation (15) which characterizes the dynamics of the relative productivity distribution is replaced by:

$$\begin{aligned} \dot{H}_t(\phi) = & \left\{ \phi H'_t(\phi) - H'_t(1) [1 - H_t(\phi)] \right\} \frac{\dot{\theta}_t^*}{\theta_t^*} \\ & + \left\{ H_t\left(\frac{\phi}{\lambda}\right) - H_t\left(\frac{1}{\lambda}\right) - H_t(\phi) \left[ 1 - H_t\left(\frac{1}{\lambda}\right) \right] \right\} \frac{R_t}{M_t}. \end{aligned} \quad (41)$$

All other equations in Section 2 are unchanged.

Now, observe that if  $\phi$  has a Pareto distribution at time  $t$  then by (41)  $\dot{H}_t(\phi) = 0$ , implying that the Pareto distribution is a stationary relative productivity distribution.<sup>39</sup> Instead of Assumption 1 I impose the following initial condition.

**Assumption 7.** *The productivity distribution at time zero is Pareto:  $G_0(\theta) = 1 - \left(\frac{\theta}{\theta_t^*}\right)^{-k}$  for  $\theta \geq \theta_t^*$  with  $k > \max\{1, \sigma - 1\}$ .*

When Assumption 7 holds there is a unique stationary relative productivity distribution  $H(\phi) = 1 - \phi^{-k}$ . It follows that on a balanced growth path the relative productivity distribution of both entrants and incumbents is the same as in Section 3. Having established this result, the same reasoning used in Sections 3 and 4 shows that, under Assumptions 2 and 7, Propositions 1 and 3 continue to hold. Therefore, on the balanced growth path, the equilibrium growth rate and the effects of trade on growth and welfare are the same with random matching as when knowledge spillovers are given by (10).<sup>40</sup>

<sup>39</sup>More generally, solving (41) with  $\dot{H}_t(\phi) = 0$  implies:

$$H(\phi) = 1 - \phi^{-k} + \phi^{-k} \int_1^\phi B(s) s^{k-1} ds,$$

where  $k > 0$  and  $B(\phi)$  satisfies:

$$B'(\phi) \phi \frac{\dot{\theta}_t^*}{\theta_t^*} = B(\phi) \frac{\dot{M}_t}{M_t} - B\left(\frac{\phi}{\lambda}\right) \frac{R_t}{M_t},$$

with  $B(1) = 0$ . Obviously,  $B(\phi) = 0$  solves this equation and implies  $\phi$  has a Pareto distribution, but it is not known whether other solutions exist.

<sup>40</sup>Note that the parameter  $\lambda$ , which captures the strength of knowledge spillovers, has slightly different interpretations in each

## Appendix C – Tax policy

A complete optimal policy analysis of the dynamic selection model lies beyond the scope of this paper. However, to better understand its efficiency properties we can analyze the welfare effects of linear taxes on fixed costs and R&D. Consider a single autarkic economy in which the government taxes the fixed cost of production at rate  $v$  and subsidizes R&D at rate  $v_e$ .<sup>41</sup> Thus, each firm must pay  $w_t f(1 + v)$  per period in order to produce and employing an R&D worker costs  $w_t(1 - v_e)$ . Also, assume that the government balances its budget through lump sum transfers to households and that entry is governed by the general form of the R&D technology introduced in Section 5.4 which allows for the possibility of congestion in technology diffusion.

Under these assumptions it is straightforward to solve for the balanced growth path equilibrium using reasoning analogous to that applied in Section 3 above. Provided  $g > 0$  and the transversality condition holds there exists a unique balanced growth path equilibrium on which:

$$q = \frac{\gamma}{1 + \gamma(k - 1)} \left[ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \psi \left( \frac{M_t}{R_t} \right) \frac{1 + v}{1 - v_e} + \frac{kn}{\sigma - 1} - \rho \right], \quad (42)$$

$$c_0 = A_1 (k\sigma + 1 - \sigma)^{\frac{k\sigma + 1 - \sigma}{k(\sigma - 1)}} f^{-\frac{k + 1 - \sigma}{k(\sigma - 1)}} (1 + v) * \left[ (k + 1 - \sigma) + k(\sigma - 1)(1 + v) + (\sigma - 1) \frac{1 + v}{1 - v_e} \frac{n + gk}{n + gk + \frac{1 - \gamma}{\gamma} q + \rho - n} \right]^{-\frac{k\sigma + 1 - \sigma}{k(\sigma - 1)}}, \quad (43)$$

where  $\frac{R_t}{M_t}$  satisfies (39) as before. Observe that either taxing the fixed production cost or subsidizing R&D leads to higher growth by increasing the ratio of fixed costs to R&D costs and raising the dynamic selection rate. Also, by comparing (42) with (24) and (43) with (31) we see that while tax policy can mimic the growth effect of trade integration it cannot simultaneously replicate the effect of trade on the level of consumption.

Household welfare on the balanced growth path still depends on  $q$  and  $c_0$  through equation (27). Therefore, to analyze the welfare effects of tax policy we can substitute (42) and (43) into (27) and then differentiate with respect to  $v$  and  $v_e$  while using (39) to account for the endogeneity of  $\frac{R_t}{M_t}$ . For the sake of brevity the resulting algebra is omitted, but there are two main findings.

First, when  $v = v_e = 0$  welfare is strictly increasing in  $v$ . Moreover, provided  $\Psi(R_t, M_t) = R_t$  welfare is also strictly increasing in  $v_e$ . This means that in the baseline model with constant returns to scale in R&D either taxing the fixed cost of production or subsidizing R&D raises welfare relative to an economy without taxes. In each case the policy is welfare improving because it increases the firm creation rate  $\lambda^k \frac{R_t}{M_t}$  which is inefficiently low in the decentralized equilibrium since innovators do not internalize the knowledge spillovers that entry generates.

Second, if the government chooses  $v$  and  $v_e$  simultaneously to maximize welfare the optimal tax rates

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case. In the baseline model  $\lambda = x_t \psi_{\min} / \theta_t^*$ , while with random matching  $\lambda$  measures the effectiveness with which entrants learn incumbents' technologies.

<sup>41</sup>The results in this appendix generalize immediately to the open economy model, but only if we abstract from strategic policy interactions across countries by imposing symmetric taxes in all economies.

satisfy:<sup>42</sup>

$$v = \frac{f_e}{\lambda^k f} \frac{n + gk}{\psi\left(\frac{M_t}{R_t}\right)},$$

$$v_e = 1 - \frac{\psi\left(\frac{M_t}{R_t}\right)}{\psi\left(\frac{M_t}{R_t}\right) - \frac{M_t}{R_t} \psi'\left(\frac{M_t}{R_t}\right)} \left[ 1 + \frac{k + 1 - \sigma}{\sigma - 1} \frac{v}{1 + v} \frac{\psi\left(\frac{M_t}{R_t}\right)}{\psi\left(\frac{M_t}{R_t}\right) - \frac{M_t}{R_t} \psi'\left(\frac{M_t}{R_t}\right)} \right]^{-1}.$$

It immediately follows that the government sets  $v > 0$  implying a tax on the fixed costs of production. In addition, in the baseline model with constant returns to scale in R&D we have  $v_e > 0$  meaning entry is subsidized. However, when there is congestion in technology diffusion R&D may either be subsidized or taxed depending on the shape of  $\Psi$ .

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<sup>42</sup>These results hold assuming the government's maximization problem is concave. In general concavity is not guaranteed, but a sufficient condition that ensures concavity is  $\Psi(R_t, M_t) = R_t^\alpha M_t^{1-\alpha}$  with  $\alpha \in (0, 1]$  and  $(1 - \gamma)(k + 1 - \sigma) > \alpha\gamma k(\sigma - 1)$ .

**Table 1: Calibration observables and parameters**

Observable/parameter		Value	Source
Import penetration ratio	IPR	0.081	U.S. import penetration ratio in 2000
Firm creation rate	NF	0.116	U.S. Small Business Administration 2002
Population growth rate	n	0.011	U.S. average 1980-2000
Trade elasticity	k	7.5	Anderson and Van Wincoop (2004)
Elasticity of substitution across goods	$\sigma$	8.1	$\sigma = k/1.06 + 1$ to match right tail index of employment distribution
Intertemporal elasticity of substitution	$\gamma$	0.33	García-Peñalosa and Turnovsky (2005)
Discount rate	$\rho$	0.04	García-Peñalosa and Turnovsky (2005)

**Table 2: Calibration results**

Outcome		Value
Growth rate - trade	q	0.0156
Growth rate - autarky	$q^A$	0.0141
<b>Growth (trade vs. autarky)</b>	<b><math>q/q^A</math></b>	<b>1.107</b>
Consumption level (trade vs. autarky)	$c_0/c_0^A$	1.010
Static gains from trade	$z^s$	1.011
Dynamic gains from trade	$z^d$	1.025
Total gains from trade	z	1.036
<b>Gains from trade (total vs. static)</b>	<b><math>(z-1)/(z^s-1)</math></b>	<b>3.2</b>

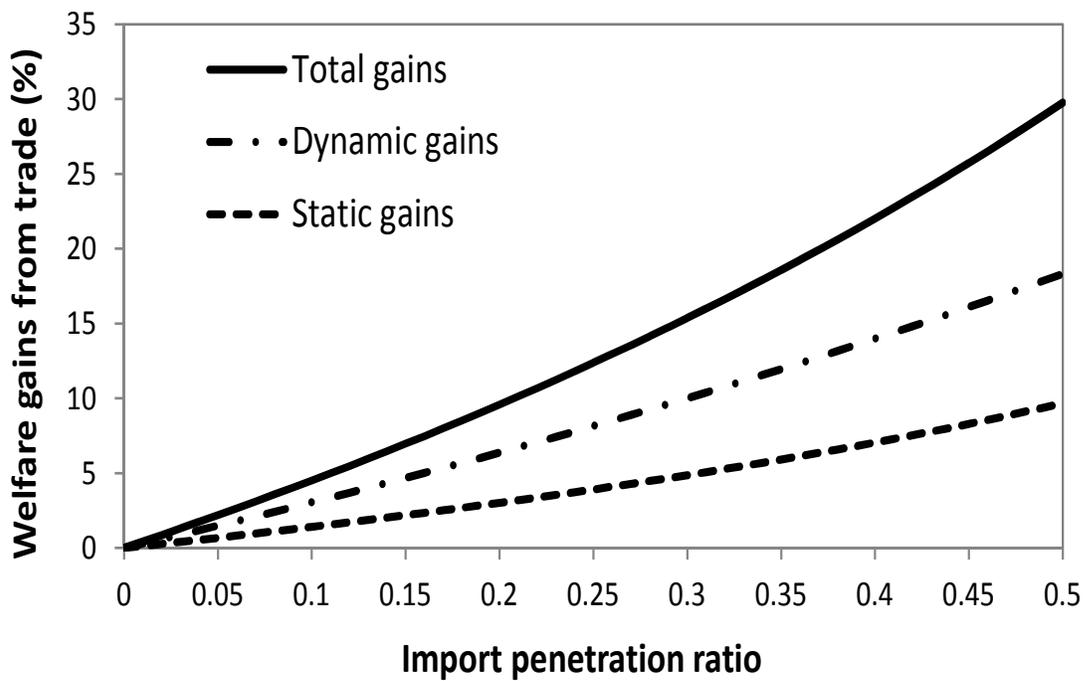


Figure 1: Import Penetration Ratio and the Gains from Trade

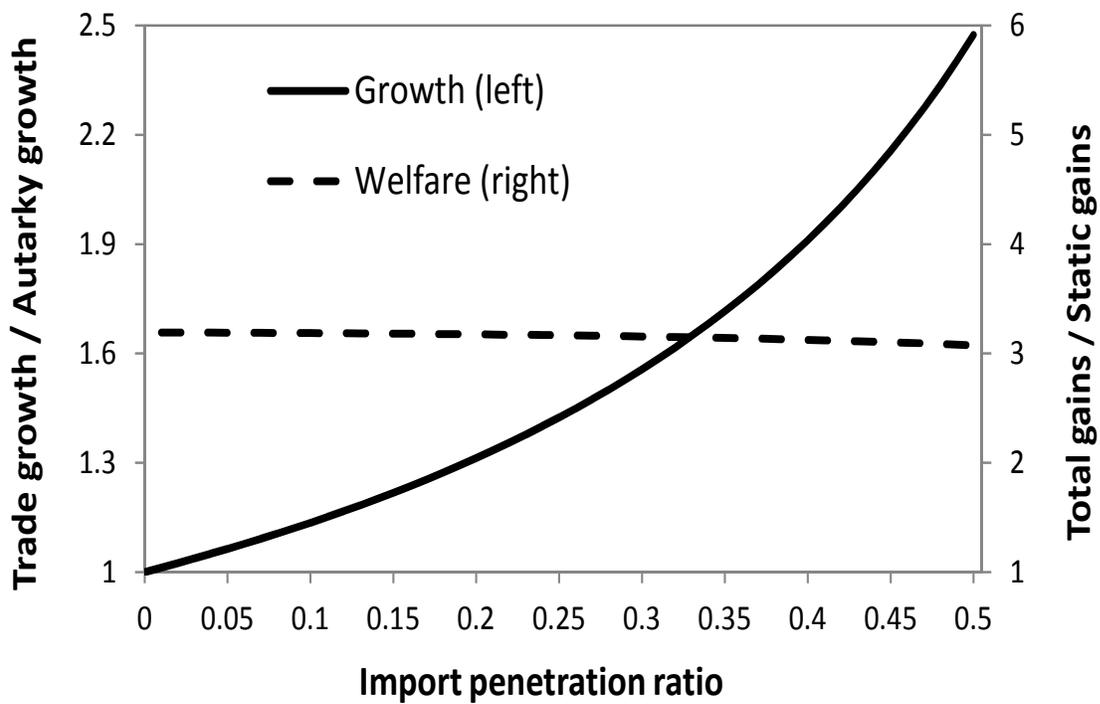


Figure 2: Import Penetration Ratio and the Dynamic Gains from Trade

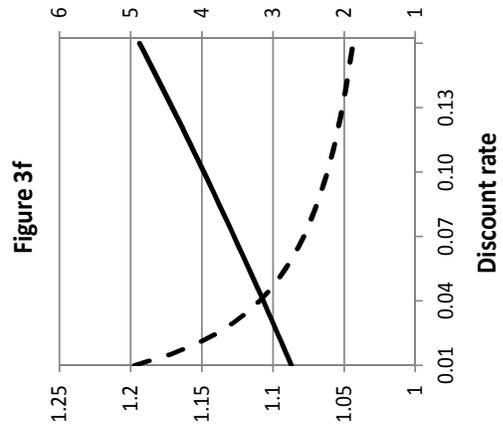
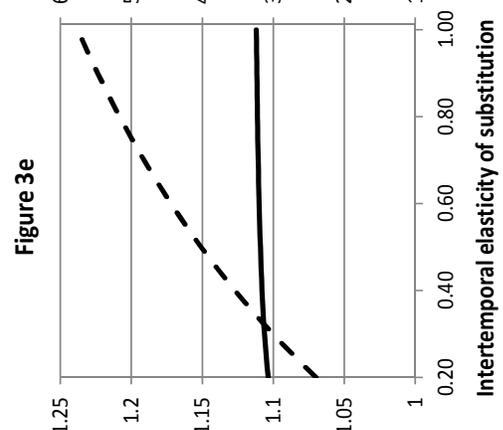
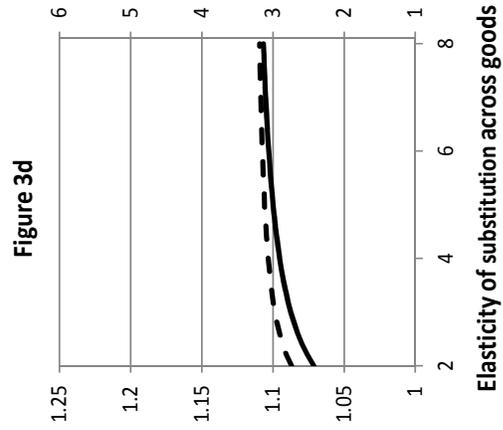
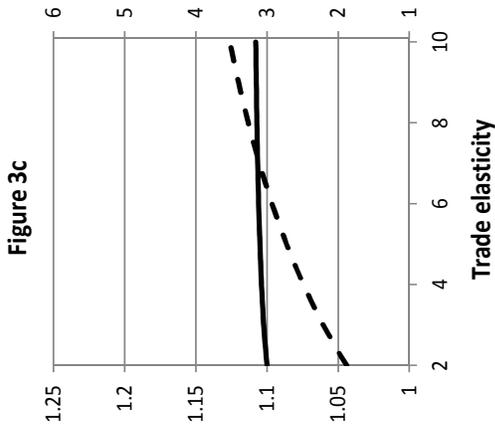
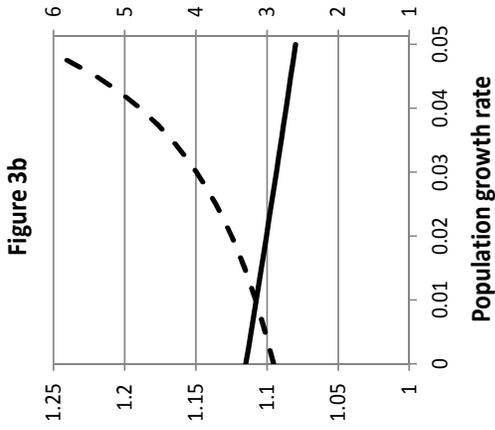
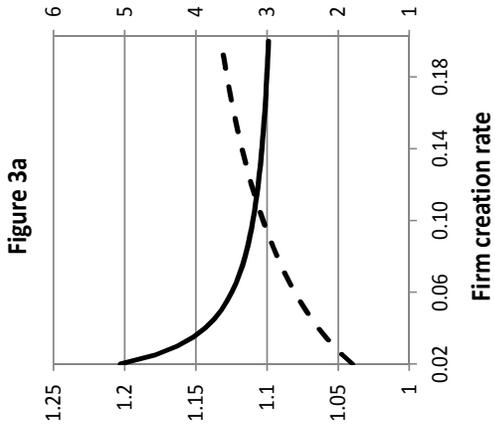


Figure 3: Dynamic Gains from Trade

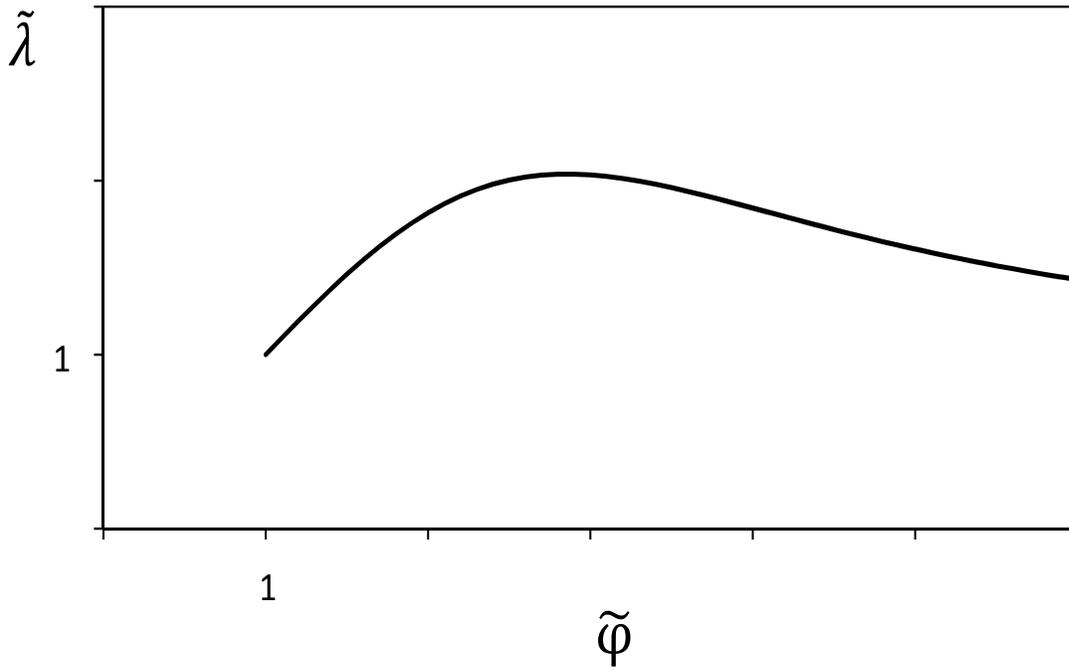


Figure 4: Trade Integration and International Knowledge Spillovers

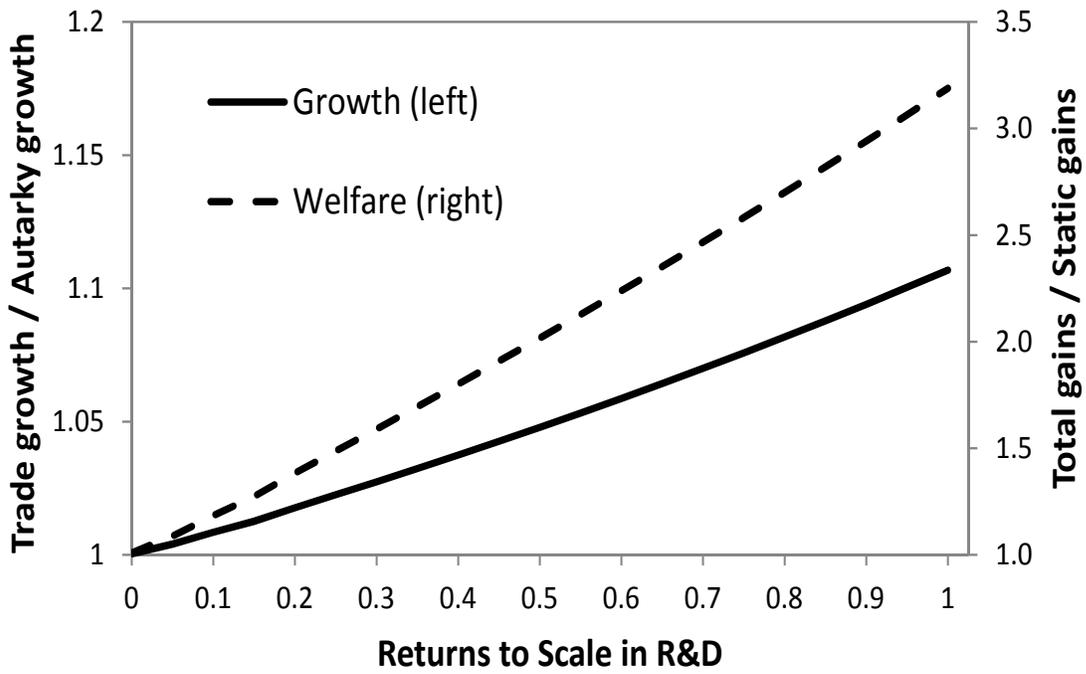


Figure 5: Returns to Scale in R&D and the Dynamic Gains from Trade

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