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Political Competition and the Limits of Political Compromise

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Abstract
We consider an economy where competing political parties alternate in office. Due to rent-seeking motives, incumbents have an incentive to set public expenditures above the socially optimum level. Parties cannot commit to future policies, but they can forge a political compromise where each party curbs excessive spending when in office if they expect future governments to do the same. We find that, if the government cannot manipulate state variables, more intense political competition fosters a compromise that yields better outcomes, potentially even the first best. By contrast, if the government can issue debt, vigorous political competition can render a compromise unsustainable and drive the economy to a low-welfare, high-debt, long-run trap. Our analysis thus suggests a legislative trade-off between restricting political competition and constraining the ability of governments to issue debt.

Key words: Political turnover, efficient policies, public debt
JEL: E61, E62, H30, H63

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1 Introduction

There is a broad agreement among economists that competition generally improves economic outcomes. Does that conclusion extend to political competition? Market and political competition are fundamentally different in that the latter affects economic outcomes indirectly, through policymaking. Still, a common presumption—as articulated for example by Wittman (1989)—is that it acts like market competition when it comes to fostering social welfare. In a nutshell, the idea is that elections allow voters to discipline bad governments and that competition among politicians let voters find alternatives to unskilled/selfish incumbents. Yet while such discipline and selection effects can be stronger when political competition is fiercer, they may still be insufficient to deliver efficient policies. For example, politicians may be inherently rent-seeking (so selection is a non-issue) and elections may be fought on multiple dimensions that include non-economic issues (so voting may fail to discipline incumbents’ economic policies).

We show that political competition matters for economic outcomes even when all political parties are identical and economic policies have no effect on electoral prospects, and whether it helps or hinders economic performance hinges on the ability of incumbents to influence the policy space available to future governments. If incumbents cannot affect state variables like the public debt, intense political competition facilitates the implementation of efficient policies. But if governments can use the debt to finance current government expenditures, then too much political competition leaves the economy trapped in a bad equilibrium. In particular, in a bipartisan society where political economy frictions are serious, the efficient policy is most likely to be sustainable if government access to the public debt is left unrestricted. More generally, our mechanism uncovers a legislative tradeoff for the sustainability of “good” economic policies. If the laws regulating the formation of political parties are loose, then constraints on the public debt must be tight. But if the restrictions on formal political participation are stringent, then the government should be left free to issue debt.

The key mechanism rests on the possibility that the political parties may cooperate intertemporally and establish a “political compromise” to neutralize the policy inefficiencies that stem from political frictions. The parties have an incentive to do so because policies affect their payoffs also when they are out of office. In that case they do not enjoy the perks and rents created by the policies, but bear the consequences of the inefficiencies they introduce in the economy. Thus, a political compromise puts a brake on the gains of the current incumbent but can improve its future payoff. Whether it is sustainable depends on the degree of political competition as well as on the constraints.

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1In the literature, the terms political competition, political instability, political turnover and political fragmentation are often used interchangeably to denote phenomena related to situations when the identity of those who hold power changes over time. For our purposes, we believe that ‘political competition’ is more adequate, but occasionally we also use the near-synonyms mentioned above.
on the public debt.

We embed the analysis in a simple, standard, neoclassical economic structure. In each period households decide how much to work and consume, while competitive firms decide how much to produce under a constant returns to scale technology that uses labor as input. The government provides a public good that is financed through taxes. The political structure is perhaps the simplest that allows us to study our main question. There is an exogenous number of competing parties that are unable to commit to policies. The political friction stems from incumbents and opposition parties having different preferences. Specifically, the period payoff of opposition parties is proportional to the representative household period utility, whereas incumbents enjoy some extra utility from government consumption. This results in incumbents having quasi-hyperbolic preferences, as defined by Laibson (1997), with the implication that the party in power has an incentive to spend more than is socially optimal.\textsuperscript{2} Political turnover is determined by a random process in which the probability that a given party will hold power in each period is inversely related to the degree of political competition, proxied by the number of active political parties.

Consider first the case in which an incumbent is unable to manipulate the action space of future governments. The efficient policy (i.e. the one that maximizes society’s welfare) is unachievable if politicians are too profligate, as in that case the short-run temptation to spend is too large. Otherwise, a political compromise where all parties implement the efficient policy when in power is sustainable through trigger strategies, provided that there is enough political competition. The intuition is simple. With strong competition, the probability that the incumbent will return to power and enjoy office rents in the future is low, while the probability that it will suffer the economic consequences of government rent-seeking when out of power is high. Hence, it pays to forge a compromise that limits rents (and improves the economy’s performance) when competition is fierce. This is not advantageous, however, if political competition is weak so that each party expects to hold office frequently.

In reality, incumbents will prefer a compromise that implements not the first best, but a policy that maximizes their own present value payoffs. A political compromise that yields this “politically optimal” policy is always achievable when public debt is ruled out, but its nature depends on the intensity of party competition. In particular, the politically optimal policy yields higher welfare, and generates greater gains for the parties, the more intense political competition is. Hence, in this restricted setting there is a clear sense in which more political competition is conducive of economic efficiency.

Suppose now that the government can issue public debt and, as a consequence,

\textsuperscript{2}Such preferences imply that, in period $t$, the marginal rate of substitution between $t$ and $t+1$ is lower than the marginal rate of substitution between $t+s$ and $t+s+1$, $s \geq 1$. Government preferences with this property are common in recent political economy models (e.g. Aguiar and Amador 2011, Halac and Yared 2013).
shape the action space of future administrations. The intuitive results just described are then largely overturned. We concentrate on the more interesting case where politicians’ prodigality is high enough so that there is a bad equilibrium in which the first incumbent increases government expenditures so much that the public debt reaches its maximum sustainable value. This would lock the society in a permanent state of low consumption and high debt. Under the shadow of this bad equilibrium, the incentive for the political parties to cooperate is greater. Considering trigger strategies to support better equilibrium outcomes, we find that the efficient policy can be sustained as an equilibrium outcome if political competition is not too intense. Similarly, the politically optimal policy is achievable if the degree of political competition is sufficiently low. Moreover, the incumbent’s gain from implementing the politically optimal policy decreases with the intensity of political competition. Therefore, curbing politicians’ profligacy requires weak, not strong political competition.

The intuition for these results is as follows. Without cooperation, an incumbent would enjoy extraordinarily high rents in office, but would leave the economy stuck in such a bad equilibrium that future governments would have little benefit from holding office. If instead a political compromise were forged, the incumbent would enjoy fewer rents today but more rents in the future, if it returned to power. A political compromise therefore not only secures a healthier state for the economy; it also preserves some rents for future governments. But those gains from future incumbency are more relevant to political parties when they are more likely to hold power in the future, which happens when political competition is less intense. Therefore, contrary to conventional wisdom, in this case political competition hampers the viability of efficient policies.

These polar cases of no debt and unconstrained debt provide the insights that help us understand the general case. A constraint on the debt lowers the incumbent’s short-run gain in the absence of cooperation and affects the future payoff gain for households from a political compromise, but neither of these effects depends on the degree of political competition. Conversely, a constraint on the debt also lowers the rents benefit for future governments due to a political compromise, and this effect is stronger when competition is weak (when future rents matter more). It follows that, when electoral and other legislative rules are such that few parties participate in the political process, tight constraints on the public debt undermine the feasibility of a political compromise. If instead the laws are such that numerous political parties actively compete, strong limits on debt issuing tend to foster a compromise. The upshot is that the desirability of tight fiscal rules is inversely related to the stringency of the rules allowing political participation. A caveat is that, when political economy frictions are sufficiently severe, the efficient policy is unfeasible with tight constraints on the debt even under extreme levels of political competition; still, it may be sustainable under limited political competition and with unrestricted government access to the debt.
1.1 Related literature

The impact of political institutions on economic performance has been the focus of a large literature.\(^3\) Yet to date the relationship between the intensity of political competition, debt constraints and economic outcomes has received relatively little attention. Our analysis nevertheless relates to several lines of research.

The reason why a political compromise is useful in our model is closely related to the rationale developed by Alesina (1988) in an early analysis of how cooperation between (two) political parties that are unable to commit to policies can improve economic outcomes.\(^4\) As Alesina elegantly demonstrates, while a party that follows his individually optimal policies when in power obtains a short-run gain, if both parties behave in that way economic performance suffers; with cooperation across the political spectrum, a better outcome for both parties may be achievable.\(^5\) Alesina’s environment and focus is however quite different from ours. For example, in his setting political parties have different preferences but their payoffs do not depend on whether they hold office or not. Alesina (1988) does not study situations where the incumbent’s policy affects the feasible actions of its successors either. Most importantly, in his model the intensity of political competition is fixed, and therefore it says nothing about the effects of different levels of political competition on the feasibility of political compromise, which is our main focus.\(^6\)

Our analysis in the environment without public debt is closest to the recent study of Acemoglu, Golosov and Tsyvinski (2011a). In their setting, political groups alternate in office according to an exogenous probabilistic process. When in office, each group has an incentive to increase its own welfare at the expense of others not in power. In their setup, the incumbent allocates consumption across groups, and would like to increase its own consumption level. In our setup, the incumbent defines the level of public expenditures, and would like to increase it beyond what is efficient from society’s point of view. The essence of the problem is nevertheless the same: the incumbent has the power to make decisions that would help itself at the expense of the rest of the society. Acemoglu et al. (2011a) then study how the degree of power persistence affects the possibility

\(^3\)See for example the comprehensive reviews of Drazen (2000) and Persson and Tabellini (2000).

\(^4\)Acemoglu, Golosov and Tsyvinski (2011b) study instead an infinitely repeated game between a self-interested politician who holds power and consumers. They show that society may be able to discipline the politician and induce him to implement the optimal taxation policy in the long run despite his self-interest. This is possible, however, only if the politician discounts the future as consumers do. Barro (1973) provides early insights on the limits and the workings of voters’ control of politicians.

\(^5\)Dixit, Grossman and Gul (2000) extend Alesina’s (1988) logic to a situation where the political environment evolves stochastically. As a result, the nature of the political compromise between the two parties changes overtime, depending on the electoral strength of the party in office.

\(^6\)A different type of political compromise may arise when power is split. We abstract from such issues here, but see Persson, Roland and Tabellini (1997) for a seminal analysis of the quality of policymaking when the executive and legislative branches share power.
of cooperation among the political groups. Their main finding is that greater turnover helps to reduce political economy distortions and to sustain efficient outcomes. A similar result arises here when public debt is ruled out. Acemoglu et al. do not study, however, situations where current policy affects the set of actions of future governments, which as discussed above effectively reverses our initial results.

The idea that incumbents manipulate the public debt to influence the policies of their successors has of course been studied extensively since the seminal contributions of Persson and Svensson (1989) and Alesina and Tabellini (1990). Our goal is, however, not to explain the dynamics of the public debt. Instead, we build on the insight from that literature to study how the availability of debt shapes the impact of political competition on the feasibility of a welfare-improving political compromise. The key insight we provide is that the public debt can discipline both current and future governments provided that there is some, but not too much, political competition.

Caballero and Yared (2010) study how political economy frictions affect the level of public debt, in an environment with both political turnover and economic volatility. As in this paper, their goal is to study whether rent-seeking politicians spend too much or too little relative to a benevolent social planner. They find that rent-seeking motivations lead to excessive spending when there is high political uncertainty relative to economic uncertainty. If the probability of keeping power is very low, it is optimal to enjoy high rents today and leave the bill for the next government. Yet a rent-seeking incumbent will tend to underspend relative to the social planner during a boom when economic uncertainty is high relative to political uncertainty. The intuition is that an incumbent who has a high probability of keeping power will save during a boom to allow himself higher rents in the future, when the economy is likely to weaken. This result relates to our finding in the debt economy that weak political competition allows for the sustainability of good economic policies because political parties want to preserve their future rents if they return to power. However, the mechanisms here and in Caballero and Yared

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7 Battaglini (2013) departs from those canonical models by extending the analysis to a two-party infinite horizon problem and by explicitly modeling elections. Thus manipulation of public debt by one party affects not only the policy space available to future governments but also electoral probabilities. From a different but insightful angle, Callander and Hummel (2013) emphasize that state variables are not necessary to create intertemporal policy linkages. Linkages can arise also if information about the actual outcomes of a policy is incomplete. Once the party in power decides the initial level of the policy variable, society learns the mapping between policy and outcome at that initial level. Because there is a correlation between policies and outcomes at different levels, the incumbent sometimes engages in preemptive policy experimentation, i.e. uses policy today to affect the policy decision of its successor by manipulating the availability of public information in the policy-outcome space. Bierbrauer and Boyer (2013) also study the relationship between political competition and welfare, but focus on the mode of political competition. Fiva and Natvik (2013) find that strategic manipulation of state variables due to political turnover is not exclusive to public debt, extending also to investment in physical capital.

8 This effect also resembles a force stressed by Azzimonti (2011) when studying how polarization and
(2010) are rather different. They focus on the transitional period toward the steady state of an economy. Instead, our focus is on the feasibility of a political compromise among politicians, a possibility that Caballero and Yared do not address.

Empirically, defining political competition and the quality of policies is a nontrivial task, although one can look at the effect of political turnover on government expenditures and public debt for guidance about the relationship between political competition and economic outcomes. The evidence is nevertheless rather ambiguous. The study of Besley, Persson and Sturm (2010) is more directly related to our question. Using data for US states since the nineteenth century, they find that lack of political competition is strongly associated with “bad,” anti-growth policies. But in their American environment more political competition means simply the difference between elections contested by two parties and elections won by a clearly dominant party. The analogy in our setting is with moving from a single-party (“dictatorship”) to a bipartisan society, in which case more political competition would indeed tend to enhance the efficiency of policies. The empirical investigation that is perhaps closest in spirit to our analysis is Acemoglu, Reed and Robinson’s (2013). They explore the effects of the number of potential local political rulers (“chiefs”) in Sierra Leone, whose number and identity were arguably exogenously determined by the British colonial authorities in late nineteenth century. Acemoglu et al. find that the degree of political competition in a locality, as measured by its number of chiefs, is positively correlated with several measures of economic development. That finding resembles closely our result in the no-debt economy, which is probably a good approximation for those regions, where rulers lack the ability to borrow extensively. A key message from our analysis is, however, that the relationship between political competition and economic outcomes hinges on the ability of governments to manipulate the public debt. More generally, this implies that empirical analyses need to account for the interaction effect between measures of political competition and fiscal constraints, or else risk being misspecified. We return to this point in section 4.

The paper is organized as follows. We study the relationship between party competition, political compromise and economic policy first in a model without public debt (section 2), and then allowing for unrestricted public debt (section 3). In section 4 we discuss more generally how legislative constraints on government borrowing affect the desirability of political compromises. We conclude in section 5.

political instability affects government expenditures, investment and long-run growth. She finds that a greater probability of returning to power puts a break on the inefficiencies due to political uncertainty. For example, in a panel of 19 OECD countries over the 1970-95 period, Perotti and Kontopoulos (2002) find that larger coalition sizes in power (a proxy for the instability of the government) are associated with more public expenditures, but Ricciuti (2004) finds no evidence that faster turnaround in office leads to more government consumption and higher public debt. Similarly, Pettersson-Lidbom (2001) finds that, for Swedish municipalities during 1974-94, a higher probability of political turnover induces right-wing incumbents to accumulate debt, but leads left-wing ones to reduce the debt.
2 A society without public debt

2.1 The economic and political environment

We consider a model that blends economic and political elements. There is a continuum of identical households with Lebesgue measure one, a government, and a single competitive firm producing an homogenous good under constant returns to scale. Production requires only labor. Households are infinitely-lived and enjoy utility from their consumption and leisure, as well as from services generated by the government expenditures \( g \geq 0 \). Given market wages, tax rates on labor income and \( g \), they choose how much to consume and work. As this is a very standard setup, we leave its details to Appendix 1.

What matters for our purposes is how much households enjoy \( g \) relative to its provision cost. Let \( U \) denote the utility that the typical household achieves in a competitive equilibrium. We show in Appendix 1 that \( U \) is a function of \( g \), in a relationship similar to an indirect utility function. The economics underlying its properties is simple: households enjoy an increase in \( g \), but this comes at the cost of higher taxes. That is, \( U \) captures the trade-off between the provision of \( g \) and its funding, concisely describing households’ preferences over consumption, leisure and the public good. Having such a convenient reduced-form representation considerably simplifies the exposition, since it allows us to focus on a single variable (\( g \)) instead of studying the simultaneously determination of all variables of the whole economic model.

Government expenditures are bounded from above by the economy’s maximum feasible output. Without loss of generality, we set this upper bound to one. We show in Appendix 1 that \( U \) is a map from \([0,1]\) into \( \mathbb{R} \). We also show that \( U(1) = -\infty \) only if \( g = 0 \) or \( g = 1 \). Such an unboundedness of \( U \) would lead to a severe but uninteresting problem of equilibrium multiplicity. To prevent that, we assume that \( g \) is bounded from below by a small positive number \( \gamma \) and from above by a number \( \Gamma \) smaller than one.\(^\text{10}\) These bounds can be easily rationalized. Since the economy’s maximum output is one, to achieve \( g = 1 \) the government would need to tax all income while households choose to devote all their available time to work despite the 100% tax. An upper bound on \( g \) below one is therefore a natural consequence of the limits on the government’s ability to raise taxes. The lower bound \( \gamma \) can be understood as the value that the public expenditures would take

\(^{10}\)Although this will become clearer after we describe the game played by the political parties, it is not difficult to see why such a restriction rules out a large family of uninteresting equilibria. Consider the upper bound \( \Gamma \). Since \( U(1) = -\infty \), if \( g = 1 \) at some date, then the household lifetime utility will be equal to \(-\infty\). Hence, as long as political parties care to any extent about household welfare, trigger strategies that specify reversion to the policy \( g = 1 \) could support any policy as an equilibrium outcome. Similar reasoning justifies the introduction of the lower bound \( \gamma \). An alternative to the introduction of the bounds \( \gamma \) and \( \Gamma \) consists in assuming that both \( U(0) \) and \( U(1) \) are larger than \(-\infty\).
if the state were downsized to the minimum dimension allowed by law, since even such a minimalist entity would entail some expenditures.

We assume that \( U \) is strictly concave, twice differentiable, and attains a maximum at a point \( g^* \in (\gamma, \Gamma) \).\(^{11}\) We call \( g^* \) the *efficient policy*.\(^{12}\) A household’s lifetime utility is given by \( \sum_{t=0}^{\infty} \beta^t U(g_t) \), where \( \beta \in (0, 1) \) is the intertemporal discount factor.

A political party is a coalition of agents (“politicians”) who want to achieve power to enjoy some extra utility/rents while in office. There is an exogenous natural number \( n \geq 2 \) of competing and identical political parties. The set of all political parties has measure zero. We denote the set \( \{1, 2, ..., n\} \) of political parties by \( \mathcal{I} \) and use the letter \( i \) to denote a generic party in \( \mathcal{I} \). We refer to the party that holds power in a period by \( p \). We denote by \( \mathcal{O} \) the set of opposition parties, i.e. the difference \( \mathcal{I} - \{p\} \).

The period preferences of party \( i \) are described by

\[
V_i(g) = U(g) + \mathbf{1}_i \lambda g,
\]

where \( \lambda \geq 0 \) and \( \mathbf{1}_i \) denotes an indicator function that is one when party \( i \) is in office and zero otherwise. Since \( \mathbf{1}_i = 0 \) for all \( i \in \mathcal{O} \), the payoff of an opposition party is proportional to \( U \). By contrast, the incumbent party cares about both government expenditures and the welfare of its members/supporters as households. Parameter \( \lambda \) describes the weight that the incumbent places on rents relative to household welfare.

There are at least two possible ways of interpreting the term \( \lambda g \). The first is to understand it as ego rents that increase as the government consumption grows. The second is to interpret it as extra income (e.g., through corruption) that a politician can obtain from public spending. The opportunities to enjoy those additional earnings increase with the level of public expenditures.\(^{13}\)

The period payoff of an opposition party, in turn, is aligned with that of a typical household, \( U(g) \). We adopt this assumption only for simplicity. The feature of representation (1) that really matters is that political parties perceive a higher relative benefit from public expenditures when in power than when out of power.

Although our goal is to study the effect of different degrees of political competition on economic policy, it is useful to define a benchmark where political competition is absent, which is equivalent to having \( n = 1 \). In this case, the function

\[
V(g) = U(g) + \lambda g
\]

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\(^{11}\)As an example, in Appendix 1 we show that \( U \) satisfies all these properties when the household has Cobb-Douglas preferences over consumption, leisure and \( g \).

\(^{12}\)If lump-sum taxes are available in the underlying economy, then \( g^* \) is Pareto efficient. If only distorting taxes are available, then \( g^* \) is efficient in a second-best sense; that is, in the terminology of the optimal fiscal and monetary policy literature, \( g^* \) is a Ramsey policy.

\(^{13}\)Alternatively, one can interpret \( \lambda > 0 \) as a bias to spend public revenue inefficiently, on the “wrong” areas (e.g. on sports stadiums, rather than on basic education).
corresponds to the period payoff of the everlasting ruling party. We define the maximizer \( g^D \) of \( V(g) \) as the *dictatorial policy*. Since \( g \) must lie in the set \([\gamma, \Gamma]\), \( g^D \leq \Gamma \) and \( U'(g^D) \geq -\lambda \); this last condition holds with equality whenever \( g^D < \Gamma \).

It should be clear that \( g^D > g^* \). Hence, a dictator overspends relative to what a benevolent social planner would do. Moreover, \( g^D \) is strictly increasing in \( \lambda \) whenever \( g^D < \Gamma \). Thus, \( \lambda \) reflects the political parties’ degree of profligacy, in the sense that an incumbent who does not strategically interact with other political parties sets \( g = g^D \) and the difference \( g^D - g^* \) is increasing in \( \lambda \).

The interval of time between elections is constant and so is an administration term. We define units so that each period of time corresponds to a term. Political parties cannot commit to specific policies. Furthermore, they share the same preferences before knowing which of them will hold office. Hence, economic policy plays no role in voting decisions and elections are fought over other, non-economic, issues. As our focus is on the intertemporal coordination of policies between current and future governments, we assume that an election is simply a randomizing device that, at the beginning of each period, selects party \( i \) to govern during that period with probability \( \pi_i \geq 0 \), with \( \sum_{i \in I} \pi_i = 1 \). For analytical convenience, with little additional loss of generality we assume further that \( \pi_i = 1/n \) for all \( i \in I \), so that all parties are equally popular. This assumption also helps in directly associating parameter \( n \) with the intensity of political competition, or the frequency of political turnover. It is nevertheless straightforward to generalize the analysis for heterogeneous \( \pi_i \). More generally, what is critical is that neither political parties nor individuals can perfectly predict the outcome of future elections, and that more political competition makes winning elections more difficult.

Our key assumptions are very similar, for example, to those of Aguiar and Amador (2011) in their analysis of investment and growth patterns when governments can expropriate foreign capital. Like here, their political friction stems from incumbents enjoying a higher payoff from government consumption than non-incumbents, the government does not have access to a commitment technology, and political turnover is exogenous (although they allow for—exogenous—incumbency advantage).\(^{14}\)

Our model is fully characterized by the array \((\beta, U, \gamma, \Gamma, \lambda, n)\). Its first four components are economic factors, while the last two are political ones. Hence, we say that \((\beta, U, \gamma, \Gamma)\) is an *economy* and \((\lambda, n)\) is a *polity*. We use the term *society* to denote a combination of an economy and a polity—that is, the entire array \((\beta, U, \gamma, \Gamma, \lambda, n)\).

\(^{14}\)A similar observation applies to Azzimonti (2011), whose setup also features government and society having different objectives, with the former being unable to commit to policies.
2.2 The policy game

To study how political competition impacts policy-making, we consider a game in which the players are the political parties. The incumbent party selects current policies. Future policies are chosen by future governments.

Let \( h^t = (g_0, g_1, \ldots, g_t) \) be a history of policies. At each date \( s \), the incumbent party \( p \) selects a date-\( s \) policy \( g_s \) as a function of the history \( h^{s-1} \). We denote that choice by \( \sigma_{p, s}(h^{s-1}) \). The incumbent also chooses plans \( \{\sigma_{p, t}\}_{t=s+1}^{\infty} \) for future policies in case it later returns to office. An opposition party \( o \) selects only plans \( \{\sigma_{o, t}\}_{t=s+1}^{\infty} \) for future policies. Given an array \( \{\sigma_{i, t}\}_{t=0}^{\infty} \) of policy plans and a history \( h^{t-1} \), the date-\( t \) policy will follow the rule:

\[
g_t = \sum_{i \in \mathcal{I}} 1_i \sigma_i(t^{t-1}).
\]

In words, the actual policy \( g_t \) is the choice of \( g \) for period \( t \) of the incumbent party in period \( t \).

At each date \( s \), the lifetime payoff \( V_{i, s} \) of party \( i \) is given by

\[
V_{i, s} = \sum_{t=s}^{\infty} \beta^{t-s} V_i(g_t).
\]

The incumbent party problem is the following. Given \( h^{s-1} \) and the other parties’ plans \( \{\sigma_{o, t}\}_{t=s+1}^{\infty} \), it chooses a policy plan \( \{\sigma_{p, t}\}_{t=s}^{\infty} \) to maximize the expected value of \( V_{p, s} \). Opposition parties solve a similar problem.

Given the ex-ante symmetry of political parties, it is natural to concentrate on symmetric outcomes. A symmetric political equilibrium is a policy plan \( \{\sigma_t\}_{t=0}^{\infty} \) with the property that, if \( \{\sigma_{o, t}\}_{t=s+1}^{\infty} \), so that all opposition parties follow the proposed policy, then the solution of the incumbent’s problem at every period \( s \) for all histories \( h^{s-1} \) is \( \{\sigma_{p, t}\}_{t=0}^{\infty} = \{\sigma_t\}_{t=s}^{\infty} \). A sequence \( \{g_t\}_{t=0}^{\infty} \) is a symmetric political outcome if there exists a symmetric political equilibrium \( \{\sigma_t\}_{t=0}^{\infty} \) such that \( \sigma_t(g_0, \ldots, g_{t-1}) = g_t \) for all \( t \).

It is easy to see that \( g^D \) is a stationary symmetric political outcome. Define the dictatorial plan \( \{\sigma^D_t\}_{t=0}^{\infty} \) so that, after any history \( h^{t-1} \), every political party sets \( g_t = g^D \) if it holds power. Suppose that, at some date \( t \), party \( P \) believes that all parties in \( O \) will follow the plan \( \{\sigma^D_t\}_{t=0}^{\infty} \). Clearly, the best course of action for party \( P \) is to implement the plan \( \{\sigma^D_t\}_{t=0}^{\infty} \) as well. Therefore, \( \{\sigma^D_t\}_{t=0}^{\infty} \) is a symmetric political equilibrium and the corresponding outcome is \( g_t = g^D \) for every \( t \).

Having identified an equilibrium for the policy game, we can use trigger strategies to characterize other symmetric political outcomes. To do so, we use a revert-to-dictatorship policy plan. It specifies that if all previous governments implemented a certain policy
\( \{g_t\}_{t=0}^{\infty}, \) then the current incumbent does the same; otherwise, the incumbent implements the dictatorial policy \( g^D \) today and whenever it returns to office.

Denote by \( \Omega_s(\{g_t\}_{t=s}^{\infty}) \) the expected value of \( V_{p,s} \) when all parties follow the policy \( \{g_t\}_{t=0}^{\infty}. \) Thus,

\[
\Omega_s(\{g_t\}_{t=s}^{\infty}) = U(g_s) + \lambda g_s + \sum_{t=s+1}^{\infty} \beta^{t-s} \left[ U(g_t) + \frac{\lambda}{n} g_t \right].
\]  

(3)

With some abuse of notation, let \( \Omega(g) \) represent the payoff of party \( i \) when \( g_t = g \) for all \( t. \) It follows that

\[
\Omega(g) = \frac{1}{1 - \beta} \left[ U(g) + \left( 1 - \beta + \frac{\beta}{n} \right) \lambda g \right].
\]  

(4)

If a policy \( \{g_t\}_{t=0}^{\infty} \) satisfies

\[
\Omega_s(\{g_t\}_{t=s}^{\infty}) \geq \Omega(g^D)
\]  

(5)

for every date \( s, \) then \( \{g_t\}_{t=0}^{\infty} \) is a symmetric political outcome. The left-hand side of (5) is the payoff of the date-\( s \) incumbent if \( \{g_t\}_{t=0}^{\infty} \) is implemented from date \( s \) onward, while the right-hand side corresponds to the payoff of that player if the dictatorial policy is implemented from date \( s \) onward.

To see that (5) is a sufficient condition for \( \{g_t\}_{t=0}^{\infty} \) to constitute a symmetric political outcome, suppose that all parties in \( O \) follow the revert-to-dictatorship plan associated with \( \{g_t\}_{t=s}^{\infty}. \) Consider the decision of party \( p \) at some date \( s. \) If the prevailing history is \( \{g_t\}_{t=0}^{s-1}, \) then condition (5) ensures that implementing \( g_t \) is optimal for party \( p. \) If the prevailing history differs from \( \{g_t\}_{t=0}^{s-1}, \) then all parties in \( O \) implement the dictatorial policy \( g^D \) whenever they come to office. As a consequence, the best action for party \( p \) is to implement \( g^D \) as well. Hence, the revert-to-dictatorship plan is a best-response strategy for party \( p. \)

### 2.3 The political feasibility of the efficient policy

Politicians can do better than just following the dictatorial policy if they coordinate policies, i.e. if they forge a political compromise. We study first the conditions under which a political compromise can sustain the efficient policy.

If \( g_t = g^* \) for every \( t, \) then (5) can be written as \( \Omega(g^*) \geq \Omega(g^D). \) This inequality is equivalent to

\[
\frac{\beta}{1 - \beta} \left[ U(g^*) - U(g^D) + \frac{\lambda}{n} (g^* - g^D) \right] \geq V(g^D) - V(g^*).
\]  

(6)
Therefore, the efficient policy is a symmetric political outcome if (6) holds. Its left-hand side represents the present value of the future gains from cooperation for the incumbent, whereas the right-hand side denotes its short-run gain from implementing the dictatorial policy instead of the efficient one.

From the definitions of \( g^* \) and \( g^D \), we have that \( V(g^D) - V(g^*) > 0, U(g^*) - U(g^D) > 0, \) and \( (\lambda/n)(g^* - g^D) < 0 \). Therefore, the right-hand side of (6) is strictly positive but its left-hand side, which is strictly increasing in \( n \), may be negative for small values of \( n \). Intuitively, the gains from cooperation for the incumbent come from preventing excessive public spending when it is not enjoying rents from those expenditures. If the incumbent expects to return often to office, the circumstances under which it would benefit from cooperation become relatively rare and its gain from cooperation may turn negative. This makes clear that the degree of political competition plays a crucial role when it comes to the sustainability of the efficient policy.

If the expression in the left-hand side of (6) were strictly positive, there are two possibilities. Let us study each possibility in turn. Suppose first that

\[
\beta \frac{1}{1 - \beta} \left[ U(g^*) - U(g^D) \right] \leq V(g^D) - V(g^*) \tag{7}
\]

Since \( (\lambda/n)(g^* - g^D) < 0 \), inequality (6) would not hold regardless of the value of \( n \). This happens when a high \( \lambda \) makes the short-run gain from implementing \( g^D \) too large relative to the future gains under coordination. In this case, the efficient policy is unachievable through the revert-to-dictatorship strategy.

**Proposition 1.** For every economy \((\beta, U, \gamma, \Gamma)\), there exists a number \( \lambda_0 \) such that, if a polity \((\lambda, n)\) satisfies \( \lambda \geq \lambda_0 \), then inequality (7) holds. As a result, the efficient policy cannot be implemented by the revert-to-dictatorship strategy for any level of \( n \).

**Proof.** See Online Appendix.\(^{16}\)

Consider now the case in which (7) does not hold:

\[
\beta \frac{1}{1 - \beta} \left[ U(g^*) - U(g^D) \right] > V(g^D) - V(g^*) \tag{8}
\]

It is then possible to place conditions on \( n \) that ensure that (6) holds and, as a consequence, the efficient policy constitutes a symmetric political outcome. Define

\[
N^0(\beta, \lambda) \equiv \frac{\lambda(g^* - g^D)}{1 - \beta [V(g^D) - V(g^*)] - [U(g^*) - U(g^D)]} \tag{9}
\]

It corresponds to the value of \( n \) that makes (6) hold with equality. Observe that (8) implies that \( N^0(\beta, \lambda) > 0 \).

---

\(^{16}\)The Online Appendix is available at http://www.alexbcunha.com/research/papers/paper15oa.pdf.
Proposition 2 If a society \((\beta, U, \gamma, \Gamma, \lambda, n)\) satisfies (8) and \(n \geq N^0(\beta, \lambda)\), then the efficient policy \(g^*\) constitutes a symmetric political outcome.

Proof. The left-hand side of (6) is strictly increasing in \(n\), while its right-hand side does not depend on \(n\). Furthermore, (6) holds with equality for \(n = N^0(\beta, \lambda)\). Thus, if \(n \geq N^0(\beta, \lambda)\), (6) is satisfied. As a consequence, \(g^*\) is a symmetric political outcome.

According to Proposition 2, \(N^0(\beta, \lambda)\) defines the minimum number of parties that can sustain \(g^*\) as an equilibrium with the revert-to-dictatorship plan. Thus, if the efficient policy is sustainable in a polity \((\lambda, n)\), it is also sustainable in a polity \((\lambda, n')\), where \(n' > n\). In that sense, political competition fosters good economic policy.

It is important to stress that Proposition 2 does not depend on the assumption that the period payoff of an opposition party is equal to the utility \(U\) of a representative household. Instead, it relies on the much weaker assumption that politicians have an extra motivation to increase government expenditures when in power.

Our society can suffer from outcomes that differ from the efficient policy \(g^*\) because \(\lambda > 0\) distorts the objectives of politicians away from those of society at large. Proposition 2 shows that a high \(n\) can offset the adverse effects of a positive \(\lambda\). However, as Proposition 1 makes clear, such a conclusion holds only if politicians are not too profligate (i.e., \(\lambda\) is not too large). This result becomes particularly relevant when one observes that the differences \(g^D - g^*\) and \(U(g^*) - U(g^D)\) are weakly increasing functions of \(\lambda\). Hence, exactly when the political distortions can be more severe, competition among the political agents fails to discipline them.

2.4 The politically optimal policy

Our policy game has many equilibrium outcomes. Up to now we have focused on the efficient outcome because it maximizes household welfare. However, even if that outcome were sustainable, the political parties may coordinate on an alternative policy. We present now an equilibrium selection criterion and characterize the resulting equilibrium.

If the players of our game are able to coordinate on outcomes distinct from the dictatorial policy, they will plausibly choose to coordinate on the best policy from their own perspective. Since the incumbent is the only player to implement an action at each date \(t\), it is sensible to consider a stationary outcome where the incumbent proposes the (time-invariant) policy over which the parties coordinate. Due to the symmetry of the political parties, this proposed policy is independent of who holds office. Hence, in this equilibrium all political parties agree on implementing a stationary policy \(g^C\) that maximizes \(\Omega\) in the universe of time-invariant policies—i.e., \(g^C\) maximizes (4). As it depends on \(\beta, \lambda\) and \(n\), we denote it by \(g^C(\beta, \lambda, n)\). We call this most cooperative policy the politically optimal policy. Note that, among stationary policies, \(g^C\) is optimal from the perspective of the incumbent, whereas \(g^*\) is optimal from society’s viewpoint.
Recall that \( g^D \) maximizes \( V(g) = U(g) + \lambda g \). Since \( 0 < 1 - \beta + \beta/n < 1 \), we can substitute \((1 - \beta + \beta/n)\lambda\) for \( \lambda \) in the definition of \( V \) to conclude that it is possible to apply our characterization of \( g^D \) to identify some of the properties of \( g^C \). This reasoning establishes that \( g^* < g^C(\beta, \lambda, n) \leq g^D \). Moreover, \( g^C \) is weakly increasing in \( \lambda \) and weakly decreasing in \( n \). Furthermore, if inequality
\[
g^C(\beta, \lambda, n) < \Gamma
\] holds, \( g^C(\beta, \lambda, n) \) is an interior optimum satisfying
\[
U'(g^C(\beta, \lambda, n)) = -(1 - \beta + \beta/n) \lambda.
\] In that case, \( g^C(\beta, \lambda, n) < g^D \), \( g^C(\beta, \lambda, n) \) is strictly increasing in \( \lambda \), and
\[
\frac{\partial g^C}{\partial n} = \frac{\beta \lambda}{n^2 U''(g^C(\beta, \lambda, n))} < 0.
\]
The intuition for the last result is simple. As \( n \) increases, the incumbent’s future expected rents per period, \((\lambda/n)g\), decrease. Hence, its payoff is maximized at a lower level of \( g \), which is closer to \( g^* \).

Define \( \Delta_0^C(n) \) according to
\[
\Delta_0^C(n) \equiv \Omega(g^C(\beta, \lambda, n), n) - \Omega(g^D, n),
\]
where we emphasize that \( \Omega \) depends on \( n \) both directly and indirectly, through \( g^C \). Expression \( \Delta_0^C(n) \) corresponds to the gain for the incumbent when the parties pursue this cooperative policy, relative to its payoff without cooperation.

Suppose that the political parties establish a political compromise to implement \( g^C \). We show that if political competition increases, then so do household welfare and the politicians’ gain from cooperation.

**Proposition 3** If (10) holds, then both household welfare and politicians’ payoff under the politically optimal policy, respectively \( U(g^C(\beta, \lambda, n)) \) and \( \Delta_0^C(n) \), are strictly increasing in the degree of political competition, \( n \).

**Proof.** Since \( \frac{\partial U(g^C)}{\partial n} = U'(g^C) \frac{\partial g^C}{\partial n} \), combining (11) with (12) implies that \( \frac{\partial U(g^C)}{\partial n} > 0 \). Furthermore,
\[
\frac{d\Delta_0^C}{dn} = \left[ \frac{\partial \Omega(g^C, n)}{\partial g} \frac{\partial g^C}{\partial n} + \frac{\partial \Omega(g^C, n)}{\partial n} \right] - \left[ \frac{\partial \Omega(g^D, n)}{\partial g} \frac{\partial g^D}{\partial n} + \frac{\partial \Omega(g^D, n)}{\partial n} \right].
\]
However, $\frac{\partial \Omega(g^C, n)}{\partial g} = 0$ by the envelope theorem, $\frac{\partial g^D}{\partial n} = 0$ because the degree of political competition does not affect $g^D$, and $\frac{\partial \Omega(g, n)}{\partial n} = -\frac{\beta \lambda}{(1 - \beta)n} g$. Therefore,

$$
\frac{d \Delta^C}{dn} = -\frac{\beta \lambda}{(1 - \beta)n^2} g^C + \frac{\beta \lambda}{(1 - \beta)n^2} g^D = \frac{\beta \lambda}{(1 - \beta)n^2} (g^D - g^C) > 0,
$$

concluding the proof. ■

One can easily grasp the intuition underlying Proposition 3. If $g^C$ is an interior optimum, then the difference $[g^C(\beta, \lambda, n) - g^*]$ decreases as political competition intensifies. The same conclusion applies to $|U(g^C(\beta, \lambda, n)) - U(g^*)|$. Furthermore, by not implementing the dictatorial policy, the incumbent has an expected rent loss per period equal to $(\lambda/n) (g^D - g^C)$. A higher $n$ decreases that loss.

We assume that coordination among the political parties is costless (and there are no collective action-like problems), but this may be unrealistic. For a moment, suppose instead that the parties would need to incur in a fixed organizational cost $F > 0$ if they wanted to implement $g^C$. The feasibility of $g^C$ in such a case would hinge on whether $\Delta^C_0(n) \geq F$. The flavor of our findings would nevertheless remain unchanged: because $d\Delta^C_0/dn > 0$ (Proposition 3), more intense political competition would make that condition more likely to hold (in the sense of increasing the range of parameters where it holds).\textsuperscript{17}

As pointed out before, $g^C(\beta, \lambda, n) > g^*$. However, $g^C$ is weakly decreasing in $n$. Moreover, Proposition 3 implies that if (10) holds, then $U(g^C(\beta, \lambda, n))$ increases with $n$. Therefore, one could be tempted to believe that $g^C(\beta, \lambda, n)$ would converge to $g^*$ as $n$ goes to $\infty$. This would be incorrect, however, as $g^C(\beta, \lambda, n)$ is bounded away from $g^*$.

**Proposition 4** Let $(\beta, U, \gamma, \Gamma)$ be any economy. For every $\lambda > 0$, there exists a number $g^C(\beta, \lambda)$ with the property that $g^* < g^C(\beta, \lambda) \leq g^C(\beta, \lambda, n)$ for every $n$.

**Proof.** See Online Appendix.

We finish this section with a brief summary of our main findings, which are represented in Figure 1 for $\lambda < \lambda_0$. We consider a repeated political game in which the players are competing political parties and the incumbent party has an inherent penchant for allocating more resources to public expenditures than is socially optimal. If politicians are very profligate ($\lambda > \lambda_0$), the efficient outcome is politically infeasible, regardless of the degree of political competition. In such a case, the welfare level $U(g^*)$ illustrated in Figure 1 is unachievable. Otherwise, sufficiently strong competition among political parties can support a political compromise that yields the efficient outcome.\textsuperscript{17}

\textsuperscript{17}The exception is when the organization cost $F$ increases with the number of political parties. In that case, one would need to compare the rate of increase of $F(n)$ and of $\Delta^C_0(n)$.
The political parties may choose instead to coordinate on a policy that maximizes the incumbent’s present value payoff at each date. By construction, such a policy is an equilibrium outcome. Hence, a welfare level of $U(g^C(n = 2))$ is always politically feasible. Furthermore, this policy has the property that an increase in the degree of political competition leads to higher aggregate welfare (converging to $U(g^c)$ as $n \to \infty$), thus making politicians interests increasingly aligned with society’s interests.

Therefore, in the context studied in this section, where the actions of the political party in office have no bearing on the options available to future governments, there is a clear sense in which more political competition fosters the implementation of better policies and improves economic performance. As we will see, this is no longer true when current policies can affect the set of actions available to future governments.

3 A society with unrestricted public debt

We now show how the public debt impacts the strategic interaction between politicians. We will see that it is no longer true that intense political competition fosters the implementation of better economic policies.
### 3.1 The economic and political environment

We extend the model of the previous section by allowing an incumbent to issue public debt. Denote its beginning-of-period $t$ value by $b_t$. This variable is measured in the same units as $g_t$ and its initial value $b_0$ is exogenous and equal to zero.\(^\text{18}\)

In section 2, the period payoff function of a typical household is similar to an indirect utility function that captures the structure of the underlying economy. That function is shaped by the trade-off between the provision of the public good and its funding. The introduction of public debt affects that trade-off. In particular, the vector $(b_t, g_t, b_{t+1})$ defines the level of distortionary taxes required to balance the government period budget constraint.\(^\text{19}\) Accordingly, in this section we describe the payoff of a typical household by a function $U(b_t, g_t, b_{t+1})$. As in the previous section, that function is a convenient representation of a typical household’s utility level in a competitive equilibrium. It is shaped by the trade-off between providing the public good, raising distortionary tax revenues, and managing the public debt. Of course, that trade-off depends on the interest rate, which is a built-in component of $U$ and other elements of our model.

In Appendix 2 we provide an example of a simple dynamic general equilibrium model for which the payoff representation we postulate here either (i) exactly describes or (ii) provides a steady-state approximation of the typical household’s utility. In the latter case, the approximation perfectly matches the household’s lifetime utility for every equilibrium we study.

For notational convenience, we will often denote $b_t$ and $b_{t+1}$ by, respectively, $b$ and $b'$. We denote the partial derivatives of $U$ by $U_b, U_g$ and $U_{g'}$. Similar notation is used for the second-order derivatives. We assume that $U$ is strictly concave in $g$: $U_{gg}(b, g, b') < 0$. We also assume that

$$U_{b'}(b, g, b') \geq 0, \quad U_{bg}(b, g, b') < 0, \quad \text{and} \quad U_{g'}(b, g, b') > 0. \quad (13)$$

If $b$ and $g$ are held constant, an increase in $b'$ reduces the amount of distortionary taxes required to balance the government period budget constraint. This justifies the first inequality. Analogously, if $g$ and $b'$ are held constant, an increase in $b$ leads to an increase in the tax burden, lowering the marginal utility of $g$. Similarly, $U_g$ should be a strictly increasing function of $b'$. Furthermore, if the public debt is held constant over time at a level $b$, then an increase in that level requires, for a fixed $g$, an increase in the

\(^\text{18}\)We show in Appendix 2 that, if $b_0$ were not equal to zero, the efficient policy would change from date zero to date one and be constant thereafter. Such a transition would make the notation heavier and the analysis slightly more complicated, without adding any meaningful insight.

\(^\text{19}\)The results of section 2 do not depend on whether the government has access to lump-sum taxes or not. However, at least since Barro (1974) it is well known that, if lump-sum taxes are available, then the government can relax any constraint imposed by the public debt by simply raising tax revenues that exactly match the value of its outstanding bonds. Hence, our present goal requires us to assume that lump-sum taxes are not available in the underlying economy.
tax burden to fund the debt service; hence, the marginal utility of $g$ should fall. Thus, we require $U_{bg}(b,g,b) + U_{gb'}(b,g,b) < 0$.

The government’s ability to raise tax revenue places bounds on its consumption and interest expenditures. Let $r$ denote the steady-state interest rate. As usual, $r$ and $\beta$ are linked through $\beta = (1 + r)^{-1}$. Let $\bar{B} > 0$ denote the maximum value the public debt can reach at any given date. It has the property that the sum $\gamma + r\bar{B}$ is equal to the maximum amount of tax revenue the government can raise in a single period. Observe that, if $b_{t+1} = \bar{B}$ for some date $t$, then $(g_s, b_{s+1})$ will be equal to $(\gamma, \bar{B})$ for every $s \geq t+1$. That is, if the debt ever reaches its maximum attainable value, the economy becomes locked in the state $(\gamma, \bar{B})$ permanently.

The debt may also take on negative values. In that case, the typical household becomes a debtor. The household’s ability to repay its debt is bounded by the lifetime income that it could obtain by working all available time at every date $t$. Thus, there must be a real number $\bar{b} \geq 0$ such that $b_t \geq -\bar{b}$ for all $t$.

To formalize the mechanism through which the incumbent influences the set of actions available to future governments, let $f^b(b)$ be a strictly increasing and continuously differentiable function. The date-$t$ government choice of $b_{t+1}$ must satisfy

$$b_{t+1} \in [f^b(b_t), \bar{B}].$$

Since $f^b$ is strictly increasing, a rise in $b_t$ shrinks the set $[f^b(b_t), \bar{B}]$. Thus, by increasing the debt it leaves to its successor, the incumbent at date $t - 1$ restricts the choice of $b_{t+1}$ of the next administration. Furthermore, $f^b(\bar{B}) = \bar{B}$, because the economy is locked in the state $(\gamma, \bar{B})$ if the public debt ever reaches the value $\bar{B}$.

We model the constraints that $b_t$ and $b_{t+1}$ place on $g_t$ with the continuously differentiable function $f^g(b, b')$. This function is strictly decreasing in $b$ and strictly increasing in $b'$. The choice of $g_t$ must satisfy

$$g_t \in [\gamma, f^g(b_t, b_{t+1})].$$

The role of the upper bound $\Gamma$ in the previous section is now played by $f^g(b_t, b_{t+1})$. The set $[\gamma, f^g(b_t, b_{t+1})]$ shrinks when $b_t$ increases. The equality $f^g(\bar{B}, \bar{B}) = \gamma$ must hold, because $\gamma$ is the only admissible value for $g_t$ whenever $b_t = \bar{B}$. Suppose now that $b_t$ is equal to some generic value $b$ for every $t$. The higher the value of $b$, the higher the interest to be paid by the government and, as a consequence, the tighter its budget constraint. Hence, the partial derivatives of $f^g$ must satisfy $f^g_b(b, b) + f^g_{b'}(b, b) < 0$.

Whoever is in office at date $t$ can increase $b_{t+1}$ to enlarge the set from which $g_t$ is selected. Such an increase in $b_{t+1}$ also restricts the choices of the next administration by tightening constraints (14) and (15). Hence, the date-$t$ incumbent can increase the end-of-period debt $b_{t+1}$ to achieve simultaneously two goals: first, relax the constraints it faces when selecting $g_t$; second, tighten the constraints the next period government
will face when selecting \((g_{t+1}, b_{t+2})\). In the limiting case in which \(b_{t+1} = \bar{B}\), the date-\(t\) incumbent permanently locks the society in the state \((\gamma, \bar{B})\). These properties of the public debt play a central role in the process of constructing an equilibrium for our game.

Let \(g^*(b, b')\) denote the value of \(g\) that maximizes \(U(b, g, b')\) under the constraint \(\gamma \leq g \leq f^\beta(b, b')\). We assume that, if \(b < \bar{B}\), then \(g^*(b, b') < f^\beta(b, b')\). Furthermore, if the government holds its debt constant at some generic level \(b\), the amount of distortionary revenue needed to balance the government budget will be a strictly increasing function of \(b\):

\[
b < \hat{b} \Rightarrow U(b, g^*(b, b'), b) > U(\hat{b}, g^*(\hat{b}, b), \hat{b}).
\]  

(16)

The efficient policy in this context is the attainable sequence \(\{g^*_t, b^*_t\}_{t=0}^{\infty}\) that maximizes \(\sum_{t=0}^{\infty} \beta^t U(b_t, g_t, b_{t+1})\). For the underlying economy that we consider in Appendix 2, \(b^*_{t+1} = 0\) for every \(t\). Therefore, \(g^*_t = g^*(0, 0)\) for every \(t\). As in Section 2, \(V(b, g, b') = U(b, g, b') + \lambda g\) describes the period preferences of a dictator who stays in power forever. The dictatorial policy \(\{g^D_t, b^D_{t+1}\}_{t=0}^{\infty}\) maximizes \(\sum_{t=0}^{\infty} \beta^t V(b_t, g_t, b_{t+1})\). We show in the Appendix 2 that the dictatorial policy is static, with \(b^D_{t+1} = 0\) for every \(t\). Hence, the efficient and dictatorial debt levels are identical. However, as in the economy without debt, government expenditures are inefficiently high under a dictatorship.

In the present context, an economy is an array \((\beta, U, \gamma, f^\beta, f^\lambda, \bar{B})\). The political structure is exactly as in section 2. Thus, a polity is a vector \((\lambda, n)\) and a society is the combination of an economy and a polity.

### 3.2 The policy game

We modify the game of the previous section as little as possible. The players are the same. A history of policies is now an array \(h^t = ((g_0, b_1), (g_1, b_2), \ldots, (g_t, b_{t+1}))\). After observing \(h^{t-1}\), the date-\(t\) incumbent selects a policy \((g_t, b_{t+1})\). The period payoff of a generic party \(i\) is \(V_i(b, g, b') = U(b, g, b') + 1, \lambda g\), while the probability that any given party will be elected is equal to \(1/n\).

The definition of a symmetric political equilibrium is similar to the one adopted in the previous section. If \(\{g_t, b_{t+1}\}_{t=0}^{\infty}\) is a symmetric political outcome, then the payoff of the date-\(s\) incumbent is

\[
\Omega_s(\{g_t, b_{t+1}\}_{t=s}^{\infty}) = U(b_s, g_s, b_{s+1}) + \lambda g_s + \sum_{t=s+1}^{\infty} \beta^{t-s} \left[ U(b_t, g_t, b_{t+1}) + \frac{\lambda g_t}{n} \right].
\]  

(17)

---

\(^{20}\) This assumption implies that, given \(b\) and \(b'\), the optimal \(b\) is smaller than its attainable upper bound. Hence, a profligate government has room to overspend without increasing the public debt unless \(B = \bar{B}\).

\(^{21}\) Naturally, since our environment is stationary, there is no role for the tax smoothing property of the public debt (as originally indicated by Barro 1979).
3.3 The spendthrift equilibrium

We now turn to the characterization of an equilibrium outcome that we will use to support other equilibria by means of trigger strategies. However, the task here is not as simple as in the previous section because, even if the date-$t$ incumbent believes that all other parties will implement the dictatorial policy regardless of the history $h^{t-1}$, it may find optimal to issue debt to fund a level of $g_t$ above the dictatorial level.

To characterize the equilibria set of our political game, it is convenient to define a function $G(b, b', \lambda)$ that solves

$$
\max_g [U(b, g, b') + \lambda g]
$$

subject to

$$
g \leq f^g(b, b').
$$

Function $G(.)$ defines the level of $g$ that maximizes the incumbent’s period payoff, given $(b, b')$. The first-order condition associated with this problem is

$$
U_g(b, G(b, b', \lambda), b') \geq -\lambda.
$$

This condition holds with equality whenever (19) does not bind.

We show in Lemma 2 of the Online Appendix that $G(.)$ is strictly decreasing in $b$, strictly increasing in $b'$, and weakly increasing in $\lambda$. The intuition behind those properties is simple. If $g$ and $b'$ are held constant, an increase in $b$ requires the government to increase its distortionary revenues. Since the definition of $G$ entails finding an optimal balance between government consumption and distortionary taxation, $G$ decreases as $b$ rises. Similar reasoning implies that $G$ increases in $b'$. Furthermore, a simple inspection of (18) suggests that $G$ should be increasing in $\lambda$.

Suppose that the date-$t$ incumbent believes that all other parties will leave a debt $\bar{B}$ regardless of the debt they inherited. If under this assumption the best strategy for the date-$t$ incumbent is to set $b_{t+1} = \bar{B}$, then we have an equilibrium in which the first incumbent enjoys a relatively high payoff and future governments have no option but to set $g_t = \gamma$ and $b_{t+1} = \bar{B}$. In particular, for the policy plan

$$
\tilde{\sigma}_t(h^{t-1}) = (G(b_t, \bar{B}, \lambda), \bar{B})
$$

to be a symmetric political equilibrium for every $h^{t-1}$, $\lambda$ must be sufficiently large. In this equilibrium, the corresponding outcome is $\{\tilde{g}_t, \tilde{b}_{t+1}\}_{t=0}^\infty$, where $\tilde{g}_{t+1} = \gamma$ and $\tilde{b}_{t+1} = \bar{B}$ for every $t$, while $\tilde{g}_0 = G(0, \bar{B}, \lambda)$. That is, the date-0 incumbent sets a value for $g_0$ high

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22 Another constraint is $g \geq \gamma$, but it never binds.

23 The only hurdle in the process of formalizing that reasoning is that constraint (19) binds at some $(b, b', \lambda)$. As a result, the partial derivatives $G_b$, $G_{b'}$, and $G_{\lambda}$ may be undefined at those points.
enough to drive the economy to a steady state characterized by $\gamma$ and $\bar{B}$. We refer to this equilibrium and its outcome as the spendthrift equilibrium.\(^{24}\)

Since $\lambda$ measures politicians’ degree of profligacy, at first glance it may seem obvious that the spendthrift policy would be an equilibrium outcome if $\lambda$ were large enough. However, this need not be true. For instance, recall that $b_{t+1} = 0$ for every $t$. Hence, regardless of $\lambda$, a dictator would not expand the public debt. The reason is that a high $\lambda$ represents a penchant for rents today but also in the future, and setting $b_1 = \bar{B}$ would dry future rents up to its minimum level.

Hence, for the spendthrift policy to be an equilibrium outcome, two conditions need to be met:

(C1) politicians are sufficiently profligate;

(C2) the rate at which an incumbent can substitute $g_t$ for $g_{t+1}$ is not too small.

To understand these conditions, let $q_t$ denote the price, in units of $g_t$, of $b_{t+1}$. By issuing one unit of $b_{t+1}$ the government can increase $g_t$ by $q_t$ units. To balance its date $t+1$ budget, the government can reduce $g_{t+1}$ by exactly one unit. Hence, an incumbent can use the public debt to substitute $g_t$ for $g_{t+1}$ at a rate equal to $q_t$.

The partial derivatives of the date-$t$ incumbent’s payoff with respect to $g_t$ and $g_{t+1}$ are equal to, respectively, $U_g(b_t, g_t, b_{t+1}) + \lambda$ and $\beta[U_g(b_{t+1}, g_{t+1}, b_{t+2}) + \lambda/n]$. Therefore,

$$-rac{dg_t}{dg_{t+1}} = \beta \frac{U_g(b_{t+1}, g_{t+1}, b_{t+2}) + \lambda/n}{U_g(b_t, g_t, b_{t+1}) + \lambda},$$

where $-dg_t/dg_{t+1}$ is a standard intertemporal marginal rate of substitution. As a consequence, the date-$t$ incumbent has an incentive to increase $g_t$ and to reduce $g_{t+1}$ by issuing debt whenever the following inequality holds:

$$q_t > \beta \frac{U_g(b_{t+1}, g_{t+1}, b_{t+2}) + \lambda/n}{U_g(b_t, g_t, b_{t+1}) + \lambda}. \quad (22)$$

Inequality (22) unwraps how the combination of political competition with conditions (C1) and (C2) brings forth the spendthrift equilibrium. Make $\lambda \to \infty$. Since $U_g$ is bounded, the right-hand side of (22) converges to $\beta/n$. Hence, for $\lambda$ sufficiently large, (22) holds whenever

$$q_t > \frac{\beta}{n}. \quad (23)$$

It is well known from basic Macroeconomics that, if an economy is in a deterministic steady-state, $q_t = \beta$. Thus, if $\lambda$ is large and $q_t$ is not considerably smaller than its

\(^{24}\)The spendthrift equilibrium shares some characteristics with the financial autarky equilibrium of Aguiar and Amador (2011), where a deviation by the government from its promised payments locks the country forever in financial autarky; as a result, the deviating government chooses to set the tax rate at its maximum possible value.
steady-state value, the date-t incumbent will have an incentive to issue debt and increase \( g_t \).

We formally establish in the Online Appendix that if conditions \((C1)\) and \((C2)\) are satisfied, then the spendthrift policy is an equilibrium outcome. For condition \((C1)\), we require that

\[
\lambda > \tilde{\lambda},
\]

where \( \tilde{\lambda} \) is a real number whose existence is established in that Appendix.\(^{25}\)

In turn, condition \((C2)\) entails placing a lower bound on \( q_t \). Since that variable is not an explicit component of our political game, we must disentangle that variable from the whole structure of the game.

To do so, take a policy \( \{g_t, b_{t+1}\}_{t=s}^\infty \) with the property that \( g_t = G(b_t, b_{t+1}, \lambda) \). For simplicity, assume that the partial derivatives \( G_b \) and \( G_{b'} \) are defined at every point \((b, b', \lambda)\). Let \( t \) be any date and \( \delta \) a small positive number. If \( b_{t+1} \) increases by \( \delta \), \( g_t \) will grow by approximately \( \delta G_{b'}(b_t, b_{t+1}, \lambda) \), while \( g_{t+1} \) will fall by approximately \( -\delta G_b(b_{t+1}, b_{t+2}, \lambda) \). Hence, a policymaker can substitute \( g_t \) for \( g_{t+1} \) at the rate

\[
\frac{\delta G_{b'}(b_t, b_{t+1}, \lambda)}{\delta G_b(b_{t+1}, b_{t+2}, \lambda)} = \frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)}.
\]

However, the rate at which a policymaker can substitute \( g_t \) for \( g_{t+1} \) is also equal to \( q_t \). Therefore, we can express \( q_t \) as

\[
q_t = -\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)}.
\]

Substituting back into \((23)\), that condition becomes

\[
-\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)} > \frac{\beta}{n}.
\]

For this condition to hold for all \( n \), we need that

\[
-\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)} > \frac{\beta}{2}.
\]

In the Online Appendix we provide a more general version of \((25)\) that takes into consideration, among other technical issues, that \( G_b \) and \( G_{b'} \) may be undefined at some points \((b, b', \lambda)\).

All that being said, henceforth we assume that conditions \((C1)\) and \((C2)\) are satisfied, so that the spendthrift policy is an equilibrium outcome.

\(^{25}\)When \( \lambda \leq \tilde{\lambda} \) one can show that, if \( V \) satisfies some regularity conditions, then there exists an equilibrium outcome in which the public debt converges, at an increasing rate, to \( \bar{B} \). Given that outcome, it is possible to characterize a subset of all equilibrium outcomes. By contrast, under \((24)\), one can characterize the entire set, in addition to allowing for a faster transition to the state \((\gamma, \bar{B})\).
3.3.1 Example

We provide below examples of functions $U$ and $f^g$, with reasonable and easily understandable features, that lead to a $G$ satisfying (25). Let

$$U(b, g, b') = -\frac{1}{2} \left(g - a_1 e^{-a_2 b}\right)^2 + a_3 (b' - b) g + W(b, b')$$

and

$$f^g(b, b') = \gamma + (2a_4 - a_5) \bar{B} - 2a_4 b + a_5 b',$$

where $a_1, a_2, a_3, a_4,$ and $a_5$ are positive constants and $W$ is any differentiable function increasing in $b'$.

Consider first the function $U$. Observe that its partial derivatives satisfy (13). To grasp the meaning of parameters $a_1, a_2,$ and $a_3$, take debt values $b$ and $b'$ smaller than $\bar{B}$. Then, $g^*(b, b') = a_3 (b' - b) + a_1 e^{-a_2 b}$. Since $g^*(0, 0) = a_1$, we can interpret $a_1$ as a parameter that defines the efficient level of $g$. Parameter $a_2$ defines the impact of equal variations in $b'$ and $b$ over $g^*$, while $a_3$ measures the impact of the public debt growth $(b' - b)$ on $g^*$.

The function $f^g$ is a plain affine relation. In line with our description of the economic features of the model, $f^g_b < 0$, $f^g_{b'} > 0$, and $f^g(\bar{B}, \bar{B}) = \gamma$. Its parameters $a_4$ and $a_5$ define the respective impacts of changes in $b$ and $b'$ on the upper bound of $g$.

The function $G$ induced by (26) and (27) satisfies inequality (25) for several vectors $(a_1, a_2, a_3, a_4, a_5)$. Take a point $(b, b', \lambda)$ such that $G(b, b', \lambda) = f^g(b, b')$. It follows that (25) holds whenever $a_5 > \beta a_4$. Take now a point $(b, b', \lambda)$ such that $G(b, b', \lambda) < f^g(b, b')$. Then (19) does not bind and $G(b, b', \lambda) = a_3 (b' - b) + a_1 e^{-a_2 b} + \lambda$. For simplicity, let the lower bound on $b$ satisfy $\bar{b} = 0$; therefore, (25) holds for every value of $b'$ and $b$ if $(2 - \beta)a_3 > \beta a_1 a_2$.

3.4 The political feasibility of the efficient policy

We use trigger strategies that specify reversion to the spendthrift policy plan $\{\bar{\sigma}_t\}_{t=0}^\infty$ to characterize a set of equilibrium outcomes. Define the revert-to-spendthrift plan associated with a policy $\{g_t, b_{t+1}\}_{t=0}^\infty$ as a plan such that, if the prevailing history is exactly $\{g_t, b_{t+1}\}_{t=s}^{s-1}$, a player sticks to the policy $\{g_t, b_{t+1}\}_{t=0}^\infty$; otherwise, the player implements the policy specified in (21). If the policy $\{g_t, b_{t+1}\}_{t=0}^\infty$ satisfies

$$\Omega_s(\{g_t, b_{t+1}\}_{t=s}^\infty) \geq U(b_s, G(b_s, \bar{B}, \lambda), \bar{B}) + \lambda G(b_s, \bar{B}, \lambda) + \sum_{t=s+1}^\infty \beta^{t-s} \left[U(\bar{B}, \gamma, \bar{B}) + \frac{\lambda}{n} \gamma\right]$$

(28)

for every $s$, then \{g_t, b_{t+1}\}_{t=0}^\infty is a symmetric policy outcome. This is so because inequality (28) ensures that the corresponding revert-to-spendthrift plan is an equilibrium outcome.
strategy. Observe that condition (28) is not only sufficient, but also necessary for a policy \( \{g_t, b_{t+1}\}_{t=0}^{\infty} \) to be an equilibrium outcome. Indeed, if (28) were not satisfied at some date \( s \), the incumbent could implement \((G(b_s, \bar{B}, \lambda), \bar{B})\) and achieve the payoff specified in the right-hand side.

Since (28) is a necessary and sufficient condition that any symmetric policy outcome must satisfy, it provides a complete characterization of the set of all symmetric political outcomes. This allows us to obtain results stronger than those of section 2.

With some abuse of notation, let \( \Omega(g, b) \) denote the payoff of the incumbent party if all parties implement the static policy \((g, b)\). Hence,

\[
\Omega(g, b) = \frac{1}{1-\beta} \left[ U(b, g, b) + \left(1 - \beta + \frac{\beta}{n}\right) \lambda g \right].
\]  

(29)

It follows from (28) that the efficient policy \((g^*(0, 0), 0)\) is a symmetric political outcome if and only if \( \Omega(g^*(0, 0), 0) \geq \Omega_{0}\{(\tilde{g}_t, \tilde{b}_{t+1})\}_{t=0}^{\infty} \). Let \( \Delta U \equiv U(0, g^*(0, 0), 0) - U(\bar{B}, \gamma, \bar{B}) \) and \( \Delta V \equiv V(0, G(0, \bar{B}, \lambda), \bar{B}) - V(0, g^*(0, 0), 0) \). The last inequality is equivalent to

\[
\frac{\beta}{1-\beta} \left[ \Delta U + \frac{\lambda}{n}(g^*(0, 0) - \gamma) \right] \geq \Delta V.
\]  

(30)

The right-hand side is the short-run gain for an incumbent from selecting the spendthrift policy instead of the efficient one. The left-hand side corresponds to its future payoff gain from the implementation of the efficient instead of the spendthrift policy. Since \( g^*(\bar{B}, \bar{B}) = \gamma \), an appeal to (16) establishes that \( U(0, g^*(0, 0), 0) > U(\bar{B}, \gamma, \bar{B}) \). Moreover, \( g^*(0, 0) > \gamma \). Therefore, the left-hand side is strictly positive and strictly decreasing in \( n \). The right-hand side is also positive, since

\[
V(0, G(0, \bar{B}, \lambda), \bar{B}) > V(0, g^*(0, 0), \bar{B}) \geq V(0, g^*(0, 0), 0).
\]

Consider the inequalities

\[
\frac{\beta}{1-\beta} \Delta U \geq \Delta V
\]  

(31)

and

\[
\frac{\beta}{1-\beta} \Delta U < \Delta V.
\]  

(32)

Similarly to section 2, the analysis depends on which of the two inequalities holds. However, the comparison between \( g^* \) and the level of \( g \) achieved in the absence of coordination—\( \gamma \) in the current setting and \( g^D \) in the previous one, where \( \gamma < g^*(0, 0) < g^D \)—has critical implications for the consequences of political competition.

**Proposition 5** If a society \((\beta, U, \gamma, f^g, f^b, \bar{B}, \lambda, n)\) satisfies (31), the efficient policy \((g^*(0, 0), 0)\) constitutes a symmetric political outcome.
Proof. Combine inequalities (31) and $(\lambda/n)(g^*(0,0) - \gamma) > 0$ to conclude that (30) holds. As a consequence, $(g^*(0,0),0)$ is an equilibrium outcome. ■

If politicians are sufficiently profligate and the payoffs satisfy (31), the efficient policy is an equilibrium outcome regardless of the degree of political competition. There was no such a result in section 2. It arises here because the efficient policy yields a level of rents greater than its level under the spendthrift equilibrium after date zero.

The analysis is richer when inequality (32) holds. Define $N^b(\beta, \lambda)$ as

$$N^b(\beta, \lambda) \equiv \frac{\lambda (g^*(0,0) - \gamma)}{1-\beta \Delta V - \Delta U}. \quad (33)$$

Observe that, under (32), $N^b(\beta, \lambda) > 0$.

**Proposition 6** If a society $(\beta,U,\gamma,f^g,f^b,\bar{B},\lambda,n)$ satisfies (32), the efficient policy $(g^*(0,0),0)$ constitutes a symmetric political outcome if and only if $n \leq N^b(\beta, \lambda)$.

**Proof.** We start with the “if” part. Condition (30) holds with equality when $n = N^b(\beta, \lambda)$. Since the left-hand side of (30) is strictly decreasing in $n$, it holds whenever $n \leq N^b(\beta, \lambda)$. Hence, the efficient policy is a symmetric political outcome. For the “only if” part, assume that $(g^*(0,0),0)$ is an equilibrium outcome. Therefore, (30) must hold. As a consequence, $n \leq N^b(\beta, \lambda)$. ■

Some implications of Proposition 6 are quite different from those of its counterpart under no public debt, Proposition 2. First, Proposition 2 lays down a sufficient condition, while Proposition 6 establishes one that is necessary and sufficient. Second, and more importantly, the function $N^b(\beta, \lambda)$ establishes the maximum number of parties that allows for sustaining the efficient policy through trigger strategies. Thus, when the government is free to issue debt, the implementation of the efficient policy requires an upper bound—instead of a lower bound—on the number of competing parties.

The combination of Propositions 5 and 6 implies that, provided that politicians are sufficiently profligate, the efficient policy can be an equilibrium outcome regardless of whether (31) or (32) holds. If the former prevails, the efficient policy is an equilibrium outcome for every value of $n$. But if the latter holds, then political competition cannot be too intense. Therefore, it is important to understand the conditions that determine which of those inequalities will prevail.

**Lemma 1** For every economy $(\beta,U,\gamma,f^g,f^b,\bar{B})$, there exists a number $\lambda^b_0$ such that, if a polity $(\lambda,n)$ satisfies $\lambda > \lambda^b_0$, then inequality (32) holds.

**Proof.** See Online Appendix.
If \( N^b(\beta, \lambda) < 2 \) and \( \lambda > \lambda^b_0 \), Proposition 6 and Lemma 1 would imply that the efficient policy could not be an equilibrium outcome. However, note that \( N^b(\beta, \lambda) > 2 \) if \( \beta \) is sufficiently close to one. Henceforth we concentrate on this more interesting case. It follows that, if politicians are very profligate but consumers are sufficiently patient, then some, but not too much political competition is necessary and sufficient to ensure that the efficient policy is a symmetric political outcome.

**Corollary 1** For every economy \((\beta, U, \gamma, f^g, f^b, \bar{B})\), if a polity \((\lambda, n)\) satisfies \( \lambda > \lambda^b_0 \), the efficient policy \((g^*(0, 0), 0)\) constitutes a symmetric political outcome if and only if \( n \leq N^b(\beta, \lambda) \).

### 3.5 The feasibility of the politically optimal policy

Again, if the parties are able to forge a compromise in which they coordinate policies intertemporally, they will presumably choose such policies optimally. We focus on the static policy that maximizes the payoff of the date-0 incumbent. By construction, it also maximizes (among static policies) the payoff of the date-\( t \) incumbent for each date \( t \).

To characterize this cooperative equilibrium for a given \( b \), we redefine \( g^C \) as the value of \( g \) that maximizes \( \Omega(g, b) \) subject to \( g \leq f^g(b, b) \). If that constraint does not bind, the necessary and sufficient first-order condition is \( \Omega_g(g^C, b) = 0 \), which is equivalent to

\[
U_g(b, g^C(b, \beta, \lambda, n), b) = -(1 - \beta + \beta/n) \lambda. \tag{34}
\]

Observe the similarity between this condition and the first-order condition for \( g^C \) in the model without debt. Furthermore, if the politicians coordinate on a static policy, the cooperative level of the public debt \( b^C \) must be equal to \( b_0 \). Since \( b_0 = 0 \), we have that \( b^C = 0 \). Therefore, the same properties of \( g^C \) discussed in the previous section also apply to \( g^C(b) \equiv g^C(b, \beta, \lambda, n) \).

There are, however, important differences in the payoff implications of \( g^C(b) \). Define the incumbent gain from implementing the cooperative instead of the spendthrift policy:

\[
\Delta^C_0(n) = \Omega(g^C(0, 0), 0) - \Omega_0(\{\tilde{g}_t, \tilde{b}_{t+1}\}_t=0) = \Omega(g^C(0, 0), 0) - \Omega(\gamma, \bar{B}) + V(\bar{B}, \gamma, \bar{B}) - V(0, G(0, \bar{B}, \lambda), \bar{B}).
\]

We have the following.

**Proposition 7** If \( g^C(0, \beta, \lambda, n) < f^g(0, 0) \), then \( U(0, g^C(0, \beta, \lambda, n), 0) \) is strictly increasing in \( n \) and \( \Delta^C_0(n) \) is strictly decreasing in \( n \).

**Proof.** It suffices to apply the reasoning adopted in Proposition 3; a complete proof is available in the Online Appendix.
Intuitively, as $n$ increases, $g^C(0, \beta, \lambda, n)$ moves toward $g^*(0, 0)$. As a consequence, the payoff of the representative household under the cooperative policy approaches its efficient level—although, in the spirit of Proposition 4, one can show that $g^C(0, \beta, \lambda, n)$ is bounded away from $g^*(0, 0)$. On the other hand, by implementing the politically optimal policy the incumbent earns in each future period an expected rent gain of $(\lambda/n)[g^C(0, \beta, \lambda, n) - \gamma]$. This rent clearly decreases with $n$.

By construction $\Omega(g^C(0), 0) \geq \Omega(g, 0)$ for all $g$, so $(g^C(0), 0)$ is the static policy that allows for cooperation under the broadest set of parameters. Thus, if a static policy $(g, 0)$ is a symmetric political outcome, so is $(g^C(0), 0)$. Since the efficient policy is static, if it is an equilibrium outcome, then so is the politically optimal policy. We can then apply Proposition 6 to conclude that if a society $(\beta, U, \gamma, f^g, f^b, B, \lambda, n)$ satisfies (32) and $n \leq N^b(\beta, \lambda)$, then the politically optimal policy is a symmetric political outcome.

Conversely, if $(g^C(0), 0)$ is not an equilibrium policy, then no static policy will be. And in contrast to the $b \equiv 0$ case, $(g^C(0), 0)$ need not be an equilibrium outcome. If we evaluate the left-hand side of (28) at $(g^C(0), 0)$, that inequality becomes

$$\Omega(g^C(0), 0) \geq \Omega_0(\{\tilde{g}_t, \tilde{b}_{t+1}\}_{t=0}^\infty).$$

(35)

We cannot be sure that this inequality holds, since the policy in the right-hand side is not static. In fact, if $\lambda$ is large, then $n$ has to be sufficiently small for $(g^C(0), 0)$ to be an symmetric equilibrium outcome.

**Proposition 8** Let $(\beta, U, \gamma, f^g, f^b, B)$ be any economy. There exist numbers $\lambda^b_1$ and $N^C(\beta, \lambda)$ with the property that, for every polity $(\lambda, n)$ satisfying $\lambda > \lambda^b_1$, the politically optimal policy $(g^C(0, \beta, \lambda, n), 0)$ is a symmetric political outcome if and only if $n \leq N^C(\beta, \lambda)$. Furthermore, $N^C(\beta, \lambda) > N^b(\beta, \lambda)$.

**Proof.** See Online Appendix.

The intuition for this result is simple. The larger is $\lambda$, the higher is the incumbent’s incentive to implement the spendthrift policy. On the other hand, from Proposition 7 we know that the politicians’ gain from implementing the politically optimal policy decreases with $n$. Therefore, the combination of large values for both $\lambda$ and $n$ suffices to rule $(g^C(0, \beta, \lambda, n), 0)$ out as an equilibrium outcome. Hence, for sufficiently profligate parties, intense political competition unequivocally hinders the implementation of the politically optimal policy.

Observe, however, that the payoff of the representative household in the politically optimal equilibrium is strictly increasing in $n$. Thus, when $\lambda > \lambda^b_1$ and politicians seek to implement that policy, household welfare would be maximized when $n = N^C(\beta, \lambda)$. Still, if there were a fixed cost $F > 0$ from coordinating on a policy, the maximum $n$ under which $g^C$ would be an equilibrium would be lower than $N^C(\beta, \lambda)$. And if such a
cost increases with the number of parties—as would be plausible from a collective action perspective—then the possibility of sustaining \( g^C \) would be further reduced, as the gains would decrease while the costs increase when political competition intensifies.

We conclude this section with a synthesis of its results, which are summarized in Figure 2 for \( \lambda > \max\{\bar{\lambda}, \lambda_0^b, \lambda_1^b\} \). We study the strategic interactions of competing political parties in a dynamic political game where the public debt links the incumbent’s actions to the action space of future governments. If politicians are sufficiently profligate (\( \lambda > \bar{\lambda} \)), there is an equilibrium in which the date-0 incumbent sets current public expenditures very high, pushing the public debt up to the point of immiserizing the economy forever, in the sense of leaving welfare stuck at \( U(\bar{B}, \gamma, \bar{B}) \) for \( t \geq 1 \). Adopting that equilibrium as a benchmark, we use trigger strategies to characterize the feasibility of other, welfare-superior outcomes. When political economy motives really matter (\( \lambda > \max\{\lambda_0^b, \lambda_1^b\} \)), the efficient policy can be implemented if there is some \( (n \geq 2) \), but not too much \( (n \leq N^b(\beta, \lambda)) \) political competition. A similar result applies for the feasibility of the politically optimal policy, although it is feasible over a larger range of \( n \) than the efficient policy. By contrast, if political competition is very intense, neither the efficient nor the politically optimal policies are sustainable. As a result, the economy becomes trapped in a bad equilibrium where welfare is \( U(\bar{B}, \gamma, \bar{B}) \) for \( t \geq 1 \). Hence, intense political competition can hurt aggregate welfare significantly when governments have easy access to public debt.
4 Political competition and debt limits

We have found that intense political competition (a high \( n \)) encourages a political compromise when debt is forbidden but discourages it when debt is unrestricted. We now show that there is a more general relationship between political competition, constraints on debt issuing and economic efficiency. We do so by generalizing the model of section 3, so that those of sections 2 and 3 become special cases. Specifically, we let the public debt be constrained by a legal ceiling \( B_L \). If \( B_L = 0 \), we obtain the model without debt of section 2; if \( B_L = \bar{B} \), we have the model of section 3.\(^{26}\)

We assume that the conditions that ensure that the spendthrift policy is an equilibrium outcome are satisfied. One can then extend the reasoning used in section 3 to establish that, for any \( B_L \in (0, \bar{B}) \), the constrained spendthrift policy (i.e., the spendthrift policy with \( B_L \) replacing \( \bar{B} \)) also is an equilibrium outcome. Defining \( \Delta U_L \equiv U(0, g^*(0,0), 0) - U(B_L, G(B_L, B_L, \lambda), B_L) \) and \( \Delta V_L \equiv V(0, G(0, B_L, \lambda), B_L) - V(0, g^*(0,0), 0) \), it follows that the efficient policy \( (g^*(0,0), 0) \) is an equilibrium outcome whenever the net gain from coordination (NGC) is positive:\(^{27}\)

\[
\text{NGC} \equiv \frac{\beta}{1 - \beta} \left[ \Delta U_L + \frac{\lambda}{\alpha}(g^*(0,0) - G(B_L, B_L, \lambda)) \right] - \Delta V_L \geq 0. \quad (36)
\]

To study the conditions under which inequality (36) holds, define \( N^L(\beta, \lambda, B_L) \) analogously to \( N^0(\beta, \lambda) \) and \( N^b(\beta, \lambda) \):

\[
N^L(\beta, \lambda, B_L) \equiv \frac{\lambda[g^*(0,0) - G(B_L, B_L, \lambda)]}{\frac{1 - \beta}{\beta} \Delta V_L - \Delta U_L}. \quad (37)
\]

Now recall that \( G(\bar{B}, \bar{B}, \lambda) = \gamma < g^*(0,0) < g^D = G(0,0, \lambda) \). Since \( G(b, b, \lambda) \) is continuous in \( b \), an appeal to the intermediate value theorem establishes that there exists a debt level \( \hat{B} \in (0, \bar{B}) \) with the property that \( G(\hat{B}, \hat{B}, \lambda) = g^*(0,0) \). The fact that \( G(b, b, \lambda) \) is strictly decreasing in \( b \) ensures that such a \( \hat{B} \) is unique. We then have

\[
\begin{cases}
B_L > \hat{B} \iff g^*(0,0) - G(B_L, B_L, \lambda) > 0 \\
B_L < \hat{B} \iff g^*(0,0) - G(B_L, B_L, \lambda) < 0 \\
B_L = \hat{B} \iff g^*(0,0) - G(B_L, B_L, \lambda) = 0.
\end{cases}
\]

Suppose first that \( B_L > \hat{B} \). It follows from the approach of section 3 that, if

\[
\frac{\beta}{1 - \beta} \Delta U_L \geq \Delta V_L, \quad (38)
\]

\(^{26}\)Observe that any \( B_L > \hat{B} \) is immaterial, as the consequences of any legally defined \( B_L \) are limited by the economic bound on the debt, \( \bar{B} \).

\(^{27}\)For simplicity, we restrict the discussion here to the sustainability of the efficient policy. Naturally, the analysis could also be extended to the politically optimal policy.
then \((g^*(0,0),0)\) is a symmetric political outcome for any \(n\). If instead
\[
\frac{\beta}{1-\beta} \Delta U_L < \Delta V_L,
\]then \((g^*(0,0),0)\) is a symmetric political outcome for any \(n\). If instead
\[
\beta_1 - \beta \Delta U_L < \Delta V_L,
\]then \((g^*(0,0),0)\) is a symmetric political outcome if \(n \leq N^k(\beta,\lambda,B_L)\).

Consider now that \(B_L < \hat{B}\). We use the approach of section 2 to conclude that, if \([\beta/(1 - \beta)] \Delta U_L \leq \Delta V_L\), then \((g^*(0,0),0)\) cannot be a symmetric political outcome. Similarly, if \([\beta/(1 - \beta)] \Delta U_L > \Delta V_L\), then \((g^*(0,0),0)\) is a symmetric political outcome provided that \(n \geq N^k(\beta,\lambda,B_L)\).

Finally, if \(B_L = \hat{B}\), the NGC does not depend on \(n\) and the efficient policy is an equilibrium outcome if (38) holds.

We can now study how a society can use \(B_L\) to improve economic policy. A simple example helps to fix ideas. Consider a society in which \(N^b(\beta,\lambda) = 4\) and \(N^0(\beta,\lambda) = 8\). Suppose that \(n = 3\). Clearly, the efficient policy is sustainable if the government can issue debt without any legal constraint, but is not if debt issuing is forbidden. Now consider that \(n = 10\). The efficient policy becomes feasible if \(B_L = 0\), but may not be if debt is unrestricted. The example illustrates a more general principle: a debt ceiling is deleterious whenever \(n \leq \min\{N^0(\beta,\lambda),N^b(\beta,\lambda)\}\). On the other hand, a society can achieve better economic outcomes by placing a legal constraint on debt issuing if \(n \geq \max\{N^0(\beta,\lambda),N^b(\beta,\lambda)\}\). In sum, the desirability of a legal debt limit hinges on the existing level of political competition. More generally, we have the following result.

**Proposition 9** *The net gain from coordination (NGC) is submodular in the degree of political competition (n) and the ceiling on the public debt (B_L).*

To see this result, it suffices to note that
\[
\frac{\partial^2 \text{NGC}}{\partial n \partial B_L} = \frac{\beta \lambda}{1-\beta} \frac{1}{n^2} [G_b(B_L,B_L,\lambda) + G_b(B_L,B_L,\lambda)] < 0.
\]

It follows from the properties of submodular functions that the value of \(B_L\) that maximizes NGC is decreasing in \(n\). The intuition is as follows. A tighter \(B_L\) lowers the short-run gain from no-cooperation, independently of the degree of political competition. A stricter \(B_L\) also influences the long-run gain from cooperation, affecting the payoffs of households and lowering the parties’ expected future rent gain. The former is independent of \(n\), but the lower future rent gain is more important, the less intense is political competition. Thus, a tighter \(B_L\) is more likely to undermine an otherwise feasible political compromise when competition is limited. Conversely, a stricter \(B_L\) is
more likely to support an otherwise unviable compromise, the more intense political competition is.

At a more fundamental level, observe that a political compromise can both improve economic outcomes and preserve office rents. The latter effect can be critical to make the compromise sustainable, and is present when constraints on the public debt are lax. As preserving rents is more important when there is less political competition, constraints on political competition can improve economic outcomes when the public debt is relatively unconstrained. Conversely, a debt limit is advisable when political competition is intense. Put simply, the feasibility of a political compromise tends to require either a limit on political competition or a cap on the public debt.

Figure 3 illustrates this result. It displays three curves: \( \Delta V_L \) and \( GC_L \equiv \frac{\beta}{1-\beta} [\Delta U_L + \frac{1}{\beta}(g^*(0,0) - G(B_L, B_L, \lambda))] \) for \( n = n_l \) and \( n = n_h, n_l < n_h \). Observe that \( \Delta V_L \) is independent of \( n \). The key to the figure is that, following Proposition 9, \( GC_L \) is steeper in \( B_L \), the lower \( n \) is. In the example of Figure 3, this implies that, under intense political competition \((n = n_h)\), the efficient policy is sustainable if the debt is sufficiently restricted \((B_L \leq B_1)\). In turn, under weak political competition \((n = n_l)\), the efficient policy is sustainable if the debt is left relatively unrestricted \((B_L \geq B_2)\). For moderate levels of the debt ceiling \((B_1 < B_L < B_2)\), the policy \( g^* \) is feasible for neither \( n_l \) nor \( n_h \).

Now, recall from Proposition 1 that, if \( \lambda \) is sufficiently large \((\lambda > \lambda_0)\), \( g^* \) is infeasible with \( B_L = 0 \) regardless of \( n \) (in Figure 3, \( \lambda > \lambda_0 \) would imply \( GC_L(n) < \Delta V_L \) for any \( n \) when \( B_L = 0 \)). In such a case, \( g^* \) may still be sustainable; it would be most likely to be sustainable (in the sense of maximizing \( NGC \)) when \( B_L = \bar{B} \) and \( n = 2 \). That is, when political frictions are severe, efficient policies have the greatest chance of being implemented in a bipartisan polity where the government has unrestricted access to the public debt.

These findings help us to understand the relationship between political competition, debt limits and economic outcomes from a theoretical perspective, but they also have important implications for empirical research. Usually, measures of constraints on debt issuing are not considered in analyses of the effects of political competition, as in most studies discussed in the Introduction. Similarly, measures of political competition do not regularly enter analyses of the effects of fiscal constraints (see Besley 2007). Our model suggests that such omissions tend to bias the relationships in critical ways. In fact, simply controlling for the omitted factor is not enough. Rather, our analysis stresses the interaction between the two measures.

\(^{29}\)It is important to stress that, although so far we have taken the level of political competition as given, in reality it is of course endogenous to the country’s legislative and electoral rules. Morelli (2004), for example, shows how different electoral systems lead to different equilibrium numbers of political parties. Relatedly, Callander (2005) shows how splitting the vote in heterogeneous districts affects the equilibrium level of political competition and the resulting policies.
Specifically, consider a panel regression of the form

$$y_{it} = \alpha_0 + \alpha_n n_{it} + \alpha X_{it} + \epsilon_{it},$$

where $y_{it}$ is an economic outcome (such as GDP) or a proxy for the quality of economic policy, $n_{it}$ measures political competition and $X_{it}$ is a vector of controls. Our analysis indicates that the estimated $\alpha_n$ will be positive when the panel contains mostly cases where the limit on debt issuing ($B_{L_{it}}$) is tight, but negative when the panel contains mostly cases where $B_{L_{it}}$ is lax. If the panel is relatively balanced between the two cases, the estimated $\alpha_n$ will tend to be statistically indistinguishable from zero. Observe that including $B_{L_{it}}$ in $X_{it}$ would not fix the problem. Instead, according to our analysis a properly specified regression to study the effects of $n_{it}$ on $y_{it}$ would be

$$y_{it} = \alpha_0 + \alpha_n n_{it} + \alpha_b B_{L_{it}} + \alpha_{nb} n_{it} B_{L_{it}} + \alpha X_{it} + \epsilon.$$  

Our model establishes that $\alpha_{nb} < 0$ and indicates that the absence of the interaction term can help to explain the conflicting empirical results obtained in the literature. A similar point applies to empirical analyses of debt constraints on economic outcomes.
5 Concluding remarks

We study how the degree of political competition affects the feasibility of efficient policies in a majoritarian political system. We move from the usual emphasis in the literature, on how competition allows voters to discipline politicians, to an environment where political parties may discipline each other. We find that, when the government faces tight limits in its ability to finance its expenditures through debt, strong political competition can facilitate equilibria where socially efficient policies are implemented. The reason is that intense political competition reduces the probability that each individual party will hold power in the future. This lowers the value of future political rents, facilitating a compromise that curbs discretionary spending. Yet the reverse is true when the government is relatively free to finance its expenditures with debt. In that case, efficient policies raise future rents, by preventing equilibria where future governments are constrained to set public expenditures at the minimal level, consistent with serving the maximum feasible level of debt. As a result, intense political competition makes efficient policies harder to sustain. In particular, when politicians are very profligate, efficient policies can be made feasible only under very limited political competition, and only without constraints on the public debt. Overall, our analysis implies that legislation on the ability of the government to issue debt needs to be established in conjunction with legislation that curbs/promotes political competition.

It is worth emphasizing that the forces driving the desirability of a debt limit in our analysis are not the ones usually emphasized in the literature. In general, a debt ceiling can help by restricting how much rent incumbents can extract from future taxpayers. But it is not necessarily helpful. First, a ceiling can prevent the debt from fulfilling its tax smoothing purpose (Barro 1979); second, it may disturb the political equilibrium and aggravate adverse selection problems (as in Besley and Smart 2007). Here neither of these potential costs of debt limits is present, since the model is deterministic and all political parties are assumed to be identical. Instead, society’s dilemma when considering a debt ceiling is between avoiding losses due to incumbents’ short-run “irresponsibility” and reducing the parties’ gain from (and therefore the likelihood of) a political compromise.

A limitation of our analysis is that we treat electoral probabilities as fixed and identical across parties. The latter is unessential. For example, allowing for an exogenous incumbency advantage, as e.g. in Aguiar and Amador (2011), would add another dimension but would not alter the essence of our analysis. Allowing policies to affect reelection prospects, on the other hand, would be significantly more involving. In such a case, we conjecture that the thrust of our results would remain unchanged provided that some electoral uncertainty remains. A full analysis of such a case is, however, beyond the scope of this paper.

Exploring the possibility of an intertemporal compromise among political parties, our goal here is to highlight an important but overlooked mechanism that reveals how
political competition and debt limits interact to shape economic policies. We uncover a novel tradeoff between political competition and debt limits that can guide empirical analyses and may also inform policy recommendations. To our knowledge, this is the first normative result linking restraints on political participation and limits on the public debt. Hopefully, future research will shed light on the robustness of this tradeoff in more general settings.

**Appendix 1: the economy without debt**

**Basic structure and competitive equilibrium**

Consider a society populated by a continuum of infinitely lived households with Lebesgue measure one and a government. Each household is endowed with one unit of time.

A single competitive firm produces a single consumption good. Technology is described by $0 \leq c + g \leq l$, where $l$ is the amount of time allocated to production, $c$ corresponds to household consumption, and $g$ denotes government consumption. Feasibility requires

$$c_t + g_t = l_t, \quad (40)$$

where $t$ denotes time. Since $l_t \leq 1$, we conclude that $g_t \leq 1$.

At each date $t$ a spot market for goods and labor services operates. The government finances its expenditures by taxing labor income at a proportional tax $\tau_t$. Its budget constraint is

$$g_t = \tau_t l_t. \quad (41)$$

The twice differentiable function $u = u(c, l, g)$ describes the typical household period utility function. It is strictly increasing in $c$ and $g$ and strictly decreasing in $l$. For a fixed $g$, $u$ satisfies standard monotonicity, quasi-concavity, and Inada conditions.

Intertemporal preferences are described by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t). \quad (42)$$

A household’s date-$t$ budget constraint is

$$c_t \leq (1 - \tau_t)l_t. \quad (43)$$

Given $\{g_t, \tau_t\}_{t=0}^{\infty}$, at date $t = 0$ a household chooses a sequence $\{c_t, l_t\}_{t=0}^{\infty}$ to maximize (42) subject to (43) and $l_t \leq 1$. 

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A competitive equilibrium for a fiscal policy \( \{g_t, \tau_t\}_{t=0}^\infty \) is a sequence \( \{c_t, l_t\}_{t=0}^\infty \) that satisfies (40) and solves the typical household’s problem. A sequence \( \{g_t\}_{t=0}^\infty \) is attainable if there exist sequences \( \{\tau_t\}_{t=0}^\infty \) and \( \{c_t, l_t\}_{t=0}^\infty \) such that \( \{c_t, l_t\}_{t=0}^\infty \) is a competitive equilibrium for \( \{g_t, \tau_t\}_{t=0}^\infty \).

We now characterize the set of attainable allocations and policies. The household’s first-order necessary and sufficient conditions are (43) taken as equality and

\[
- \frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)} = 1 - \tau_t, \tag{44}
\]

which is equivalent to

\[
\tau_t = 1 + \frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)}. \tag{45}
\]

Combine this expression with (41) to conclude that any attainable outcome \( \{c_t, l_t, g_t\}_{t=0}^\infty \) must satisfy

\[
g_t = \left[ 1 + \frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)} \right] l_t. \tag{46}
\]

We can then use techniques similar to those in Chari and Kehoe (1999) to show that a sequence \( \{c_t, l_t, g_t\}_{t=0}^\infty \) satisfies (40) and (46) if and only if it is attainable.

**The function \( U(g) \)**

At each date \( t \), there are two fiscal variables \( (g_t, \tau_t) \) that the government can select. If the Laffer curve of this artificial economy is monotone, then the government can actually select only one variable. If there are multiple tax rates that fund the same level of government expenditures, for each attainable value of \( g \) we define \( U(g) \) according to

\[
U(g) = \max_{(c,l)} u(c, l, g) \tag{47}
\]

subject to (40) and (46). Hence, whenever we say that a sequence \( \{g_t\}_{t=0}^\infty \) is a policy, we are assuming that \( \tau_t \) is given by the solution of (47) for the corresponding \( g_t \).

It should be clear that \( U \) resembles an indirect utility function. Built in that function is a trade-off between increasing the provision of \( g \) and reducing the tax burden. Although we assume that the government only has distortionary revenues, we could have assumed that lump-sum taxes were available without affecting the results.

The constraints \( l \leq 1 \) and (40) imply that if \( g = 1 \), then \( c = 0 \) and \( l = 1 \). Thus, the Inada conditions on \( u \) imply that \( U(1) \leq U(g) \) for all \( g \leq 1 \). Moreover, it may be the case that \( U(1) = -\infty \). Furthermore, if \( u(c, l, 0) = -\infty \), then \( U(0) = -\infty \). As

\[\footnote{The players of the games considered in the paper are required to select attainable policies.}\]
indicated in section 2, if \( U \) is unbounded from below, then any policy \( \{g_t\}_{t=0}^{\infty} \) can be an equilibrium of our political game. To avoid such an indeterminacy, we assume that there is a lower bound \( \gamma > 0 \) and an upper bound \( \Gamma < 1 \) for \( g \) so that \( g_t \in [\gamma, \Gamma] \).

An inspection of problem (47) shows that the second derivative of \( U \) depends on the third derivatives of \( u \). Thus, unless extra assumptions are placed on \( u \), one cannot ensure that \( U \) is strictly concave, as we assume in section 2. But it is easy to provide conventional examples in which \( U \) is indeed strictly concave. For one, let

\[
U(c, l, g) = \alpha_1 \ln c + \alpha_2 \ln(1 - l) + \alpha_3 \ln g,
\]

where \( \alpha_1, \alpha_2, \) and \( \alpha_3 \) are positive numbers. Then

\[
U(g) = \alpha_1 \ln[\alpha_1 - (\alpha_1 + \alpha_2)g] + \alpha_3 \ln g + \alpha_2 \ln \left[ \frac{\alpha_2}{(\alpha_1 + \alpha_2)^{\alpha_1}} \right] \tag{48}
\]

and

\[
U''(g) = -\left\{ \frac{\alpha_1(\alpha_1 + \alpha_2)}{[\alpha_1 - (\alpha_1 + \alpha_2)g]^2} + \frac{\alpha_3}{g} \right\} < 0. \tag{49}
\]

Appendix 2: the economy with unrestricted debt

Basic structure and competitive equilibrium

Take as starting point the economy described above. We introduce debt on it by assuming that the government issues claims to one unit of the consumption good. These claims are traded at a price \( q_t \). The government period budget constraint is

\[
g_t + b_t = \tau_t l_t + q_t b_{t+1}, \tag{50}
\]

where \( b_t \) is the amount of claims to be redeemed at beginning of date \( t \). Households’ equivalent constraint is

\[
c_t + q_t b_{t+1} \leq (1 - \tau_t) l_t + b_t. \tag{51}
\]

The initial value of the public debt is exogenous and satisfies \( b_0 = 0 \). To avoid Ponzi schemes, the public debt must satisfy the constraints \( |b_{t+1}| \leq M < \infty \), where \( M < \infty \) is large enough so that these constraints never bind. The necessary and sufficient conditions of the household problem are (51) taken as equality plus (44),

\[
\beta u_c(c_{t+1}, l_{t+1}, g_{t+1}) = q_t, \tag{52}
\]

and

\[
\lim_{t \to \infty} \beta^t u_c(c_{t+1}, l_{t+1}, g_{t+1}) b_{t+1} = 0. \tag{53}
\]

A competitive equilibrium for a fiscal policy \( \{g_t, \tau_t\}_{t=0}^{\infty} \) is composed by sequences \( \{c_t, l_t\}_{t=0}^{\infty}, \{b_{t+1}\}_{t=0}^{\infty} \) and \( \{q_{t+1}\}_{t=0}^{\infty} \) that satisfy (40) and the optimal behavior by the
households. A sequence \(\{g_t, b_{t+1}\}_{t=0}^\infty\) is attainable if there exist sequences \(\{\tau_t\}_{t=0}^\infty\), \(\{c_t, l_t\}_{t=0}^\infty\) and \(\{g_{t+1}\}_{t=0}^\infty\) such that \(\{c_t, l_t\}_{t=0}^\infty\), \(\{b_{t+1}\}_{t=0}^\infty\) and \(\{g_{t+1}\}_{t=0}^\infty\) constitute a competitive equilibrium for \(\{g_t, \tau_t\}_{t=0}^\infty\).

Let \(H(c, l, g) \equiv u(c, l, g)c + u_l(c, l, g)l\). Using the reasoning of Chari and Kehoe (1999), we have that the set of attainable sequences is fully characterized by (40) and

\[
\sum_{t=0}^{\infty} \beta^t H(c_t, l_t, g_t) = 0.
\] (54)

Additionally, the public debt sequence must satisfy

\[
\sum_{t=s}^{\infty} \beta^{t-s} H(c_t, l_t, g_t) = u_c(c_s, l_s, g_s)b_s.
\] (55)

**The efficient and the dictatorial policies**

The efficient allocation \(\{c^*_t, b^*_t, g^*_t\}_{t=0}^\infty\) solves the problem of maximizing households’ lifetime utility (42) subject to (40) and (54). The solution is characterized by those constraints plus the first-order conditions

\[
\left\{
\begin{array}{l}
  u_c(c_t, l_t, g_t) - \theta_t + \Theta H_c(c_t, l_t, g_t) = 0 \\
  u_l(c_t, l_t, g_t) + \theta_t + \Theta H_l(c_t, l_t, g_t) = 0 \\
  u_g(c_t, l_t, g_t) - \theta_t + \Theta H_g(c_t, l_t, g_t) = 0,
\end{array}
\right.
\] (56)

where \(\theta_t\) and \(\Theta\) are, respectively, Lagrange multipliers for (40) and (54), while \(H_c, H_l\) and \(H_g\) are partial derivatives.

Equations (56), together with (40), establish that \(\{c^*_t, l^*_t, g^*_t\}_{t=0}^\infty\) is a static sequence. Thus, \(\sum_{t=s}^{\infty} \beta^{t-s} H(c^*_t, l^*_t, g^*_t) = \sum_{t=0}^{\infty} \beta^t H(c^*_t, l^*_t, g^*_t) = 0\) for all \(s\). Hence, (55) implies that \(b^*_s = 0\) for every \(s\).

Finally, observe that if \(b_0 \neq 0\), it would be necessary to add \(u_c(c_0, l_0, g_0)b_0\) to the right-hand side of (54). As a consequence, the date-0 first-order conditions would be slightly different and the efficient allocation would then be static only for \(t \geq 1\). The public debt would be constant for \(t \geq 1\). However, it would not be equal to the initial exogenous value \(b_0\). In synthesis, if \(b_0 \neq 0\), the efficient allocations and the debt levels would change from \(t = 0\) to \(t = 1\) and then reach a steady state.

The characterization of the dictatorial policy \(\{g^D_t, b^D_{t+1}\}_{t=0}^\infty\) requires finding a sequence \(\{c^D_t, l^D_t, g^D_t\}_{t=0}^\infty\) that maximizes \(\sum_{t=0}^{\infty} \beta^t [u(c_t, l_t, g_t) + \lambda g_t]\) subject to (40) and (54). The solution is given by the same equations as the efficient policy except that

\[
u_g(c_t, l_t, g_t) + \lambda - \theta_t + \Theta H_g(c_t, l_t, g_t) = 0
\]

replaces the third equation in (56). Thus, \(b^D_{t+1} = 0\) for all \(t\) and the sequence \(\{g^D_t\}_{t=0}^\infty\) is static.
The function $U(b, g, b')$

As pointed out in Section 3, the function $U(b, g, b')$ is either a perfect representation or a steady-state approximation of the typical household period payoff. In the former case, the approximation is accurate to the point of perfectly matching the household utility in each of the equilibria studied in the paper.

Take an array $[b_0, \{g_t, b_{t+1}\}_{t=0}^\infty]$. Define $\mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^\infty)$ according to

$$\mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^\infty) \equiv \max_{\{c_t, l_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t u(c_t, l_t, g_t)$$

subject to (40), (53), and

$$c_t + \beta \frac{u(c_{t+1}, l_{t+1}; g_{t+1})}{u(c_t, l_t, g_t)} b_{t+1} = -\frac{u_t(c_t, l_t, g_t)}{u(c_t, l_t, g_t)} l_t + b_t.$$ 

This last expression was obtained by combining the equality version of (51) with (44) and (52). By construction, $\mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^\infty)$ is the highest lifetime utility that the household can attain if the government implements the policy $\{g_t, b_{t+1}\}_{t=0}^\infty$.

If there is a function $\mathcal{W}(b, g, b')$ satisfying $\sum_{t=0}^\infty \beta^t \mathcal{W}(b_t, g_t, b_{t+1}) = \mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^\infty)$, then we simply set $U = \mathcal{W}$. Clearly, $U$ will be a perfect measurement of the household period payoff. If $\mathcal{U}$ cannot be decomposed in that way, then we proceed as follows.

Let $\{g, b'\}$ denote a policy $\{g_t, b_{t+1}\}_{t=0}^\infty$ in which $(g_t, b_{t+1}) = (g, b')$ for every $t$. Now take a generic vector $(b, g, b')$. If each of the arrays $[b, \{g, b'\}]$ and $[b', \{g, b'\}]$ is attainable, then define $U(b, g, b')$ so that

$$U(b, g, b') \equiv \mathcal{U}(b, \{g, b'\}) - \beta \mathcal{U}(b', \{g, b'\}).$$

Observe that $\mathcal{U}(b, \{g, b'\})$ is the household lifetime payoff when the economy starts with debt $b$ and reaches the steady-state $(g, b')$ after a single period, while $\mathcal{U}(b', \{g, b'\})$ is the lifetime payoff in such a steady state. Hence, we define $U(b, g, b')$ in such a way that it captures the household utility gain (or loss) associated with that one-period transition.

If $[b, \{g, b'\}]$ or $[b', \{g, b'\}]$ is not attainable, then we need to modify our definition. In that case, we set $U$ according to

$$U(b, g, b') = \mathcal{U}(b, [(\tilde{g}(b, b'), b'), \{\tilde{g}(b', b')\}]) - \beta \mathcal{U}(b', \{\tilde{g}(b', b')\}),$$

where $\tilde{g}(b')$ is the maximum attainable value for $g$ in a steady state with debt $b'$, $\tilde{g}(b, b')$ is the maximum attainable value for $g_0$ when the initial debt is $b$ and the economy will be in state $(\tilde{g}(b', b'), b')$ for every $t \geq 1$, and $[(\tilde{g}(b, b'), b'), \{\tilde{g}(b', b')\}]$ denotes the policy $\{g_t, b_{t+1}\}_{t=0}^\infty$ in which $(g_0, b_1) = (\tilde{g}(b, b'), b')$ and $(g_t, b_{t+1}) = (\tilde{g}(b', b'), b')$ for every $t \geq 1$. Observe that we replaced the values of $g$ specified in the arrays $[b, \{g, b'\}]$ and $[b', \{g, b'\}]$ by the highest
attainable values for that variable. Implicit in our definition is the assumption that $b$ and $b'$ are attainable values for the public debt.

It remains to show that the proposed $U$ will lead to a perfect measurement of the household’s lifetime utility under the the efficient, the politically optimal, and the spendthrift policies (the three equilibrium outcomes discussed in the paper). Let $(g, b)$ be any attainable steady state. We use (58) to conclude that

$$U(b, g, b) = U(b, \{g, b\}) - \beta U(b, \{g, b\}) \Rightarrow$$

$$U(b, g, b) = (1 - \beta) U(b, \{g, b\}) \Rightarrow$$

$$\sum_{t=0}^{\infty} \beta^t U(b, g, b) = U(b, \{g, b\}).$$

Therefore, we have a perfect measurement of the household’s lifetime utility in any steady state. Since both the efficient and politically optimal policies are static, $U$ properly measures their corresponding payoffs.

Finally, denote the household payoff under the spendthrift policy by $X$. Thus,

$$X = U(b_0, G(b_0, \bar{B}, \lambda), \bar{B}) + \frac{\beta}{1 - \beta} U(\bar{B}, \gamma, \bar{B}).$$

The value of $U(\bar{B}, \gamma, \bar{B})$ is given by (58), while $U(b_0, G(b_0, \bar{B}, \lambda), \bar{B})$ follows (59). Hence,

$$X = U(b_0, [(\bar{g}(b_0, \bar{B}), \bar{B}), \{\gamma, \bar{B}\}]) - \beta U(\bar{B}, \{\gamma, \bar{B}\}) + \frac{\beta}{1 - \beta} (1 - \beta) U(\bar{B}, \{\gamma, \bar{B}\}) \Rightarrow$$

$$X = U(b_0, [(\bar{g}(b_0, \bar{B}), \bar{B}), \{\gamma, \bar{B}\}]).$$

From Lemma 3 in the Online Appendix, $G(b_0, \bar{B}, \lambda)$ is equal to the maximum attainable value for $g_0$ when $\lambda$ is large. Hence, $G(b_0, \bar{B}, \lambda) = \hat{g}(b_0, \bar{B})$. Therefore,

$$X = U(b_0, [(G(b_0, \bar{B}, \lambda), \bar{B}), \{\gamma, \bar{B}\}]),$$

as we wanted to show.

References


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