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Optimal Unemployment Insurance Over the Business Cycle

Camille Landais, Pascal Michaillat and Emmanuel Saez

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

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Abstract
This paper characterizes optimal unemployment insurance (UI) over the business cycle using a model of equilibrium unemployment in which jobs are rationed in recession. It offers a simple optimal UI formula that can be applied to a broad class of equilibrium unemployment models. In addition to the usual statistics (risk aversion and micro-elasticity of unemployment with respect to UI), a macro-elasticity appears in the formula to capture the macroeconomic impact of UI on unemployment. In a model with job rationing, the formula implies that optimal UI is countercyclical. This result arises because in recession, jobs are lacking irrespective of job search. Therefore (1) a higher aggregate search effort cannot reduce aggregate unemployment much; and (2) individual search effort creates a negative externality by reducing other jobseekers' probability of finding a job as in a rat race. Hence the social benefits of job search are low. In a calibrated model, optimal UI increases significantly in recession. This quantitative result holds whether the government adjusts the level or duration of benefits; whether it balances its budget each period or uses deficit spending.

Keywords: Unemployment insurance, business cycle, job rationing, matching frictions
JEL Classifications: E24; E32; H21; H23

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Camille Landais is a Post-Doctoral Fellow at Stanford Institute for Economic Policy Research (SIEPR). Pascal Michaillat is an Associate of the Centre for Economic Performance and Lecturer in Economics, London School of Economics. Emmanuel Saez is E. Morris Cox Professor of Economics and Director, Center for Equitable Growth, University of California at Berkeley.

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1 Introduction

This paper studies optimal unemployment insurance (UI) when workers cannot insure themselves against unemployment risk, and unemployed workers' job search cannot be monitored. The government chooses unemployment benefits by trading off their insurance value with their cost in terms of additional unemployment caused by reduced job-search efforts. A large literature studies this trade-off [Baily, 1978; Chetty, 2006a; Hopenhayn and Nicolini, 1997; Shavell and Weiss, 1979]. In these models, unemployment depends solely on job-search effort. But the long queues of unemployed workers at factory gates observed during the Great Depression suggest that jobs are lacking in recessions, however intensively unemployed workers search. Hence, existing models seem inadequate to explain recessionary unemployment and analyze UI in recession.

To study optimal UI in recession, this paper uses the equilibrium unemployment model of Michaillat [forthcoming]. This model combines real wage rigidity and a downward-sloping labor demand to capture two critical aspects of recessions. First, unemployment is high and above its socially efficient level in recessions. Second, jobs are rationed in recessions, in the sense that some unemployment would remain even if unemployed workers devoted arbitrarily large efforts to job search. A key property of the model is that, although the labor market always sees vast flows of workers and a great deal of matching activity, recessions are periods of acute job shortage during which job search has little influence on labor market outcomes.

We build on the model of Michaillat [forthcoming] by introducing risk-averse workers who choose their job-search effort when unemployed. Unemployment benefits are financed by a labor tax. Some frictions impede matching on the labor market, hence equilibrium wages are indeterminate and labor market tightness acts as a price equilibrating labor supply and labor demand. Our model is quite general. If we make labor demand perfectly elastic, unemployment depends solely on search effort and we obtain the model of Baily [1978] and Chetty [2006a]. At the polar opposite if we make labor demand perfectly inelastic, unemployment is completely independent of search effort and we obtain a rat-race model.

Our first contribution is to derive an optimal UI formula in a one-period model of equilibrium unemployment. Our formula presents two departures from the classical Baily-Chetty formula. First, while the Baily-Chetty formula expresses the optimal replacement rate as a function of risk aversion and micro-elasticity of unemployment with respect to net reward from work, our formula replaces the micro-elasticity by a macro-elasticity. In an equilibrium unemployment model, only the macro-elasticity is able to capture the budgetary costs incurred by the government when increasing UI. Micro- and macro-elasticity are different. The micro-elasticity is the elasticity of the probability of unemployment for a worker whose individual unemployment benefits change. The macro-elasticity is the elasticity of aggregate unemployment when the generosity of UI changes for all workers. The macro-elasticity accounts for the equilibrium adjustment in labor market tightness that follows a change in UI, whereas the micro-elasticity takes labor market tightness as given. Second, our formula includes an additional term increasing with the wedge between micro- and macro-elasticity. This wedge captures the first-order welfare effects of the adjustment of aggregate employment that arises from the equilibrium adjustment of labor market tightness after a change in UI.[1] Last, our formula is robust to changes in the primitives of the model because it is expressed in terms of *sufficient statistics* [Chetty, 2006a]. It is easily adapted to a broader class of models: models in which wages respond to UI, such as the Pissarides [2000] model with Nash bargaining; or models in which workers can partially insure themselves against unemployment.

Our second contribution is to prove that there exists a positive wedge between micro- and macro-elasticity in our model. When jobs are rationed, searching more to increase one's probability of finding a job mechanically decreases others' probability of finding one of the jobs left, thus reducing the macro-elasticity compared to the micro-elasticity. Indeed since unemployed workers choose their effort taking the per-unit job-finding probability as given, they do not internalize their influence on others' employment probability, thus imposing a negative *rat-race externality*. We also prove that this wedge is countercyclical and the macro-elasticity is procyclical. Intuitively in recession, jobs are lacking irrespective of job search. Efforts of jobseekers have little influence on aggregate unemployment, and the rat-race externality is exacerbated. Thus the macro-elasticity is small and the wedge between micro- and macro-elasticity is large. Last, the positive wedge

[1]In contrast, jobs destroyed through reduced search efforts have no first-order welfare effects as the unemployed set their search efforts to maximize expected utility.

between micro- and macro-elasticity is a testable implication of our model that distinguishes it from standard models of equilibrium unemployment. For instance, the wedge is nil in the Hall [2005] model with rigid wages, and negative in the Pissarides [2000] model with Nash bargaining.

Our third and most important contribution is to prove that the optimal generosity of UI is countercyclical. The first reason is that the macro-elasticity decreases sharply in recession. Hence a more generous UI, while reducing aggregate search effort, has smaller budgetary cost because it only increases unemployment negligibly. The second reason is that the wedge between micro- and macro-elasticity, which measures the welfare cost of the rat-race externality, increases in recession. Accordingly UI, which corrects this externality by discouraging job search, is more desirable. Although we model only technology shocks, we conjecture that other shocks affecting labor demand such as credit shocks or aggregate demand shocks would affect optimal UI in the same way.

Finally, we use numerical methods to assess the robustness of our theoretical results in an infinite-horizon, stochastic model under various arrangements for the administration of UI. We calibrate the model with US data. In the baseline case, in which the government balances its budget each period and unemployment benefits never expire, we find large variations in the optimal replacement rate: from 67% when unemployment is as low as 4% to 85% when unemployment reaches 9%. Next, we allow the government to borrow and save. After an adverse economic shock the optimal replacement rate responds as in the baseline case, although the government provides higher consumption to both employed and unemployed workers. Lastly, we make the UI system more realistic by allowing the government to adjust the duration of unemployment benefits. In a model calibrated to match an optimal duration of 26 weeks when unemployment is at 5.9%, as in the US, the optimal duration of unemployment benefits is strongly countercyclical: it increases from less than 10 weeks to over 100 weeks when unemployment increases from 4% to 8%.

The paper is organized as follows. Section 2 presents a one-period model in which we derive optimal UI formulas in terms of estimable sufficient statistics. Section 3 specializes this model to introduce job rationing, and characterizes optimal UI over the business cycle. Section 4 verifies the robustness of our theoretical results in a calibrated infinite-horizon model. Section 5 discusses empirical evidence. Derivations, proofs, and robustness checks are collected in the Appendix.

2 Optimal Unemployment Insurance Formula

This section derives an optimal UI formula in a generic one-period model of equilibrium unemployment. The formula is expressed in terms of sufficient statistics (curvature of the utility function, micro- and macro-elasticity of unemployment with respect to net reward from work) and does not require more structure on the primitives of the model. We extend the formula if workers can partially insure themselves, and if UI influences wages. This static model transparently captures the economic mechanisms at play; it is embedded in a more realistic dynamic setting in Section 4.

2.1 Labor market

There is a unit mass of workers. Initially, $u \in (0,1)$ workers are unemployed and search for a job with effort e, while $1-u$ workers are employed. Firms post o job openings to recruit unemployed workers. The number of matches h made is given by a constant-returns matching function $h = h(e \cdot u, o)$ of aggregate search effort $e \cdot u$ and vacancies o, differentiable and increasing in both arguments, with the restriction that $h(e \cdot u, o) \leq \min\{u, o\}$. Conditions on the labor market are summarized by labor market tightness $\theta \equiv o/(e \cdot u)$. A jobseeker finds a job with probability $f(\theta) \equiv h(e \cdot u, o)/(e \cdot u) = h(1, \theta)$ per unit of search effort; hence a jobseeker searching with effort e finds a job with probability $e \cdot f(\theta)$. A vacancy is filled with probability $q(\theta) \equiv h(e \cdot u, o)/o = h(1/\theta, 1)$. In a tight market it is easy for jobseekers to find jobs—the per-unit job-finding probability $f(\theta)$ is high—and difficult for firms to hire—the job-filling probability $q(\theta)$ is low.

2.2 Worker

A worker's utility is $v(c) - k(e)$, where $v(c)$ is an increasing and concave function of consumption c and $k(e)$ is an increasing and convex function of effort e. Employed workers earn a wage $w(a)$ that is taxed at rate t to finance unemployment benefits $b \cdot w(a)$. The parameter a proxies for the position in the business cycle, and is fixed throughout Section 2. Workers neither borrow nor save, so consumption is $c^e = w(a) \cdot (1-t)$ when employed and $c^u = b \cdot w(a)$ when unemployed.[2] We

[2] We relax the assumptions that wages do not respond to UI and workers cannot self-insure in Sections 2.7 and 2.8.

denote by $\Delta c = c^e - c^u$ and $\Delta v = v(c^e) - v(c^u)$ the net reward from work in terms of consumption and utility, respectively. Given labor market tightness θ and net reward from work Δv, a jobseeker chooses effort e to maximize expected utility

$$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e).$$

The optimal job-search effort satisfies the following first-order condition:

$$k'(e) = f(\theta) \cdot \Delta v. \tag{1}$$

Equation (1) implicitly defines the optimal effort $e(\theta, \Delta v)$, which increases with θ—as the per-unit job-finding probability $f(\theta)$ increases with θ—and with the net utility gain from working Δv.

For a given labor market tightness θ and average job-search effort e, a fraction $e \cdot f(\theta)$ of the u unemployed workers finds a job during matching. These $u \cdot e \cdot f(\theta)$ new hires add to the $1-u$ workers already employed before matching, to give aggregate employment after matching

$$n^s(e,\theta) = (1-u) + u \cdot e \cdot f(\theta). \tag{2}$$

$n^s(e,\theta)$ increases mechanically with e and θ, so that labor supply $n^s(e(\theta,\Delta v),\theta)$ increases with θ and Δv. θ affects labor supply through the optimal provision of job-search effort $e(\theta,\Delta v)$, and mechanically, through the per-unit job-finding probability $f(\theta)$. $n^s(e(\theta,\Delta v),\theta)$ is a labor supply because it gives the number of employed workers after matching when jobseekers choose search effort optimally for a given labor market tightness θ.

2.3 Labor demand and equilibrium

In a model of equilibrium unemployment, labor market tightness θ equalizes labor demand and labor supply:

$$n^s(e(\theta,\Delta v),\theta) = n^d(\theta; a) \equiv n(\Delta v; a), \tag{3}$$

where Δv is fixed by the UI policy, a is the fixed parameter determining the position in the business cycle, $n^d(\theta;a)$ is a general function that summarizes firms' demand for labor, and $n(\Delta v;a)$ denotes equilibrium employment. We assume that equilibrium labor market tightness $\theta(\Delta v;a)$ is uniquely defined by equation (3). We put more structure on $n^d(\theta;a)$ in Section 3 when we characterize optimal UI over the business cycle using a model with job rationing.

Equation (3) is the key departure from the canonical Baily-Chetty model of optimal UI. The Baily-Chetty framework is a partial-equilibrium model of unemployment in the sense that it fixes labor market tightness θ and per-unit job-finding probability $f(\theta)$. In contrast, our framework is a general-equilibrium model of unemployment in the sense that labor market tightness θ is determined endogenously in equation (3) to equilibrate supply and demand for labor. While the Baily-Chetty framework studies the partial-equilibrium response $\partial n^s[e(\theta,\Delta v),\theta]/\partial\Delta v|_\theta$ of labor supply to a change in unemployment benefits, we focus on the general-equilibrium response of aggregate employment $dn/d\Delta v$ to a change in unemployment benefits.

A cut in benefits increases the utility gain from work by $d\Delta v > 0$, which increases effort by $de = [\partial e(\theta,\Delta v)/\partial\Delta v|_\theta]\cdot d\Delta v > 0$ and labor supply by $dn^s_e = [\partial n^s/\partial e|_\theta]\cdot de > 0$ in partial equilibrium with θ constant. However in general equilibrium, θ adjusts so that (3) continues to hold. The response of aggregate employment takes into account the partial-equilibrium response of labor supply dn^s_e as well as the equilibrium adjustment of labor market tightness $d\theta$, which affects equilibrium employment by $dn^s_\theta = [\partial n^s/\partial\theta|_e]\cdot d\theta$. Our framework nests the Baily-Chetty framework as a special case in which labor demand n^d is perfectly elastic and determines θ independently of UI. But as long as labor demand is not perfectly elastic, the implications of our model differ from those of the Baily-Chetty model because the general-equilibrium response of aggregate employment $dn = dn^s_e + dn^s_\theta$ differs from the partial-equilibrium response dn^s_e of labor supply.

2.4 Government

The government chooses consumption levels c^e and c^u to maximize social welfare

$$n^s(e,\theta)\cdot v(c^u+\Delta c)+[1-n^s(e,\theta)]\cdot v(c^u)-u\cdot k(e) \tag{4}$$

where $e(\theta, \Delta v)$ is given by the worker's optimal choice of effort (1); $\theta(\Delta v; a)$ clears the labor market as imposed by (3); and consumptions c^e, c^u satisfy the government's budget constraint:

$$n \cdot c^e + (1-n) \cdot c^u = n \cdot w. \tag{5}$$

2.5 Micro-elasticity and macro-elasticity

To solve the government's problem, we need to characterize the response of jobseekers (through a change in effort) and of the aggregate labor market (through a change in tightness) to a change in UI. To this end, we define two elasticities.

DEFINITION 1. The *micro-elasticity* of unemployment with respect to net reward from work is

$$\varepsilon^m \equiv \frac{\Delta c}{1-n} \cdot \left.\frac{\partial n^s}{\partial e}\right|_\theta \cdot \left.\frac{\partial e}{\partial \Delta v}\right|_\theta \cdot \frac{d\Delta v}{d\Delta c}. \tag{6}$$

The *macro-elasticity* of unemployment with respect to net reward from work is

$$\varepsilon^M \equiv \frac{\Delta c}{1-n} \cdot \frac{dn}{d\Delta c} = \varepsilon^m + \frac{\Delta c}{1-n} \cdot \left(\left.\frac{\partial n^s}{\partial \theta}\right|_e + \left.\frac{\partial n^s}{\partial e}\right|_\theta \cdot \left.\frac{\partial e}{\partial \theta}\right|_{\Delta v} \right) \cdot \frac{d\theta}{d\Delta v} \cdot \frac{d\Delta v}{d\Delta c}. \tag{7}$$

If labor demand is perfectly elastic, θ is determined by firms independently of UI and $\varepsilon^M = \varepsilon^m$.

Both elasticities are normalized to be positive. The micro-elasticity measures the percentage increase in unemployment $1-n$ when the net reward from work Δc decreases by 1%, ignoring the equilibrium adjustment of θ on n.[3] This elasticity can be estimated by measuring the reduction in the job-finding probability of an individual unemployed worker whose unemployment benefits are increased, keeping the benefits of all other workers constant such that labor market conditions remain unchanged. The macro-elasticity measures the percentage increase in unemployment when the net reward from work decreases by 1%, assuming all variables adjust. This elasticity can be estimated by measuring the increase in aggregate unemployment following a general increase in

[3]Equations (1) and (2) define labor supply $n^s(e(\theta, \Delta v), \Delta v)$ as a function of Δv and θ, so the natural partial-equilibrium elasticity of labor supply is defined relative to Δv. To obtain an elasticity with respect to Δc, we need to include the term $d\Delta v / d\Delta c$ that specifies the increase in Δv in response to a budget-balanced increase in Δc.

unemployment benefits. Section 5 proposes empirical strategies to estimate these elasticities.

Critically, as long as labor demand is not perfectly elastic, these two elasticities differ in a model of equilibrium unemployment. As an illustration, consider a pure rat-race model in which there are u jobseekers, and a fixed number $o < u$ of job openings. For a given job-finding probability f per unit of search effort, the unconditional probability to be employed after the matching process for a worker searching with effort e is $n^s(e,f) = (1-u) + u \cdot e \cdot f$. At the micro level, searching harder increases employment probability so that micro-elasticity $\varepsilon^m > 0$. But firms only need to fill a fixed number of vacant jobs, so that equilibrium employment is fixed, independent of aggregate search effort: $n = 1 - u + o < 1$. Hence macro-elasticity $\varepsilon^M = 0$. The discrepancy between ε^m and ε^M arises because, as a result of the job shortage, per-unit job-finding probability f falls when aggregate search effort e rises to equilibrate labor supply $n^s(e,f)$ with the fixed labor demand $1-u+o$. Indeed in equilibrium, $f = o/(u \cdot e)$.

2.6 Formula

Following optimal income tax theory, the government chooses the net consumption gain from work Δc, which determines $c^u = n \cdot (w - \Delta c)$ and $c^e = c^u + \Delta c$ through the budget constraint.[4] Denoting average marginal utility by $\bar{v}' \equiv n \cdot v'(c^e) + (1-n) \cdot v'(c^u)$, and using the envelope theorem as workers choose effort e optimally, the first-order condition of the government's problem (4) with respect to Δc is[5]

$$n \cdot v'(c^e) + \bar{v}' \cdot \frac{dc^u}{d\Delta c} + \Delta v \cdot \left.\frac{\partial n^s}{\partial \theta}\right|_e \cdot \frac{d\theta}{d\Delta c} = 0. \tag{8}$$

To gain intuition, consider a small increase $d\Delta c > 0$ in the net reward from work—equivalent to a cut in unemployment benefits. The first term in (8) captures the utility gain of the n employed workers, whose consumption $c^e = c^u + \Delta c$ increases by $d\Delta c$: $dS_1 = n \cdot v'(c^e) \cdot d\Delta c$. To satisfy the budget constraint, increasing Δc requires cutting unemployment benefits $c^u = n \cdot (w -$

[4] Optimal income tax theory always expresses optimal tax rates as a function of the elasticity of earnings with respect to one minus the marginal tax rate. The optimal UI problem is isomorphic to an optimal tax problem where (i) the implicit tax rate on work is $t^* = t + b$, the sum of labor tax and benefits rate, and (ii) there are two earning levels, "working" and "not working". Δc is directly related to t^*: $\Delta c = (1-t^*) \cdot w$.

[5] To apply the envelope theorem, we notice that social welfare (4) is $(1-u) \cdot v(c^e) + u \cdot [v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)]$.

Δc), which reduces by dc^u the consumption of all workers, including the employed as $c^e = c^u + \Delta c$. The second term in (8) captures this utility loss: $dS_2 = -\bar{v}' \cdot dc^u$. Since $dc^u = -n \cdot d\Delta c + (w - \Delta c) \cdot dn = -\left\{n - (1-n) \cdot [(w - \Delta c)/\Delta c] \cdot \varepsilon^M\right\} \cdot d\Delta c$, then we can rewrite $dS_2 = -\bar{v}' \cdot \left\{n - (1-n) \cdot [(w - \Delta c)/\Delta c] \cdot \varepsilon^M\right\} \cdot d\Delta c$. The macro-elasticity ε^M appears in this expression of dS_2 to capture the budgetary cost of the increase in equilibrium unemployment caused by higher UI.

In our model, the per-unit job-finding probability $f(\theta)$ depends on labor market tightness θ, which is determined in equilibrium by (3) as the intersection of demand and supply for labor. The increase $d\Delta c > 0$ in net reward from work increases the incentive to search by $d\Delta v > 0$, which shifts labor supply $n^s(e(\theta, \Delta v), \theta)$ outwards. Hence, a small increase $d\Delta c > 0$ leads to a small equilibrium adjustment $d\theta$ of labor market tightness. This change $d\theta$ in turn leads to a small change dn_θ in aggregate employment through two channels: (i) a change $(\partial n^s/\partial e) \cdot (\partial e/\partial \theta) \cdot d\theta$ in employment through a reduction in search effort—this reduction, however, does not have any welfare effect by the envelope theorem as workers choose effort to maximize expected utility; and (ii) a change $(\partial n^s/\partial \theta) \cdot d\theta$ in employment through a change in per-unit job-finding probability $f(\theta)$. Each new job created through (ii) generates a first-order utility gain $\Delta v > 0$ as finding a job discretely increases consumption. The third term in (8) captures the welfare change from this equilibrium adjustment $d\theta$. As indicated by the definition (7) of the macro-elasticity ε^M, the employment change dn_θ can be measured by the wedge between micro-elasticity ε^m and macro-elasticity ε^M. In fact, we can even relate the change $(\partial n^s/\partial \theta) \cdot d\theta$ in employment, which is the only relevant change from a welfare perspective, to the wedge $\varepsilon^m - \varepsilon^M$, as showed in Lemma 1.

LEMMA 1. *The partial derivative of equilibrium labor market tightness satisfies:*

$$\frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c} = -\frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{h} \cdot \left[\varepsilon^m - \varepsilon^M\right],$$

$$\frac{\Delta c}{1-n} \cdot \left.\frac{\partial n^s}{\partial \theta}\right|_e \cdot \frac{d\theta}{d\Delta c} = -\frac{\kappa}{\kappa+1} \cdot \left[\varepsilon^m - \varepsilon^M\right],$$

where $\kappa = e \cdot k''(e)/k'(e)$ *is the elasticity of the marginal disutility of effort* $k'(\cdot)$*,* $1 - \eta = \theta \cdot f'(\theta)/f(\theta)$ *is the elasticity of the per-unit job-finding probability* $f(\cdot)$*, and* $h = u \cdot e \cdot f(\theta)$ *is the number of new hires.*

Using this Lemma, we can rewrite $dS_3 = -\Delta v \cdot [\kappa/(1+\kappa)] \cdot [(1-n)/\Delta c] \cdot [\varepsilon^m - \varepsilon^M] \cdot d\Delta c$. At the optimum the sum of the three effects $dS_1 + dS_2 + dS_3$ is zero, yielding first-order condition (8). We rewrite (8) in terms of elasticities in Proposition 1.

PROPOSITION 1. *The optimal replacement rate $\tau = c^u/c^e$ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \left[n + (1-n) \cdot \frac{v'(c^u)}{v'(c^e)}\right]^{-1} \cdot \left\{\frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{\Delta v}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right]\right\}. \quad (9)$$

If $n \approx 1$, and if the third and higher order terms of $v(\cdot)$ are small, the optimal formula simplifies to

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\varepsilon^M} \cdot (1-\tau) + \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right] \cdot \frac{\kappa}{1+\kappa} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right], \quad (10)$$

where $\rho = -c^e \cdot v''(c^e)/v'(c^e)$ is the coefficient of relative risk aversion.

If labor demand is perfectly elastic, $\varepsilon^m = \varepsilon^M$, the second term in the right-hand side of (9) *and* (10) *vanishes, and the formulas reduce to those in Baily [1978] and Chetty [2006a].*

The proposition provides a formula for the optimal replacement rate $\tau = c^u/c^e$, which measures the generosity of the UI system. Equation (9) provides an exact formula while equation (10) provides a simpler formula using the approximation method of Chetty [2006a]. The approximated formula (10) is expressed in terms of sufficient statistics, which means that the formula is robust to changes in the primitives of the model. Indeed the formula is valid for: any utility over consumption with coefficient of relative risk aversion ρ; any marginal disutility of effort with elasticity κ and associated micro-elasticity ε^m; any labor demand, function only of labor market tightness and an exogenous shock, yielding a macro-elasticity ε^M; and any constant-returns matching function. Since these four statistics are estimable, the formula can be used to assess the current UI system.[6] Admittedly, the statistics are endogenous functions of the replacement rate τ, so we cannot infer directly the optimal replacement rate from current estimates of the statistics. Nevertheless, we can infer that increasing the replacement rate is desirable if the current $\tau/(1-\tau)$ is lower than the

[6] Section 5 discusses how to estimate micro- and macro-elasticity. In the Appendix, we explain how to estimate κ from the micro-elasticity of unemployment with respect to benefits. Many studies estimate the coefficient of relative risk aversion [Chetty, 2004, 2006b].

right-hand side of formula (10) evaluated using current estimates of the four statistics.

The first term in the optimal replacement rate (10) increases with the coefficient of relative risk aversion ρ, which measures the value of insurance. Absent any wedge between micro- and macro-elasticity ($\varepsilon^m = \varepsilon^M$), our formulas reduce to the classical Baily-Chetty formula. For instance, the approximated formula (10) becomes $\tau/(1-\tau) \approx (\rho/\varepsilon^m) \cdot (1-\tau)$. In this formula, the trade-off between need for insurance (captured by the coefficient of relative risk aversion ρ) and need for incentives to search (captured by the micro-elasticity ε^m) appears transparently. In a model of equilibrium unemployment, there is generally a wedge between micro- and macro-elasticity, and our formula presents two departures from the Baily-Chetty formula.

The first term in the right-hand side of formulas (9) and (10) involves the macro-elasticity ε^M and not the micro-elasticity ε^m that has been conventionally used to calibrate optimal benefits [Chetty, 2008; Gruber, 1997]. What matters for the government is the cost of UI in terms of higher aggregate unemployment and hence higher outlays of unemployment benefits. Only the macro-elasticity ε^M is able to capture this cost of moral hazard in general equilibrium. The optimal replacement rate naturally decreases with the elasticity ε^M.

A second term, increasing with the ratio $\varepsilon^m/\varepsilon^M$, also appears in the right-hand side of formulas (9) and (10) when $\varepsilon^m \neq \varepsilon^M$. This term is a correction that accounts for the first-order welfare effects of the adjustment of aggregate employment that arises from the equilibrium adjustment of labor market tightness after a change in UI. Even in the absence of any concern for insurance—for instance, if workers are risk neutral—some unemployment insurance should be provided as long as this correction term is positive.

2.7 Workers are able to partially insure themselves

We now extend our model to include partial self-insurance by workers. Chetty [2006a] shows that the Baily formula carries over to models with savings, borrowing constraints, private insurance, or leisure benefits of unemployment. Similarly, formulas (9) and (10) carry over with minor modifications. Introducing self-insurance through borrowing and saving would require a fully dynamic model. Instead, we consider the simpler case of self-insurance through home production. In addition to unemployment benefits c^u received from the government, unemployed workers who have not been matched to a job consume an amount y of good produced at home at a utility cost $m(y)$, increasing, convex, and normalized so that $m(0) = 0$. We denote by $\hat{c}^u = c^u + y$ the total consumption when unemployed, and by $\hat{\Delta} v = v(c^e) - [v(c^u + y) - m(y)]$ the utility gain from work. Jobseekers choose effort e and home production y to maximize

$$[1 - e \cdot f(\theta)] \cdot [v(c^u + y) - m(y)] + [e \cdot f(\theta)] \cdot v(c^e) - k(e).$$

Home production y is chosen so that $v'(c^u + y) = m'(y)$. It provides additional insurance that is partially crowded out by UI, as y decreases with c^u. The government chooses Δc to maximize

$$n^s(e,\theta) \cdot v(c^u + \Delta c) + [1 - n^s(e,\theta)] \cdot [v(c^u + y) - m(y)] - u \cdot k(e),$$

where both e and y are chosen optimally by individuals, subject to the same constraints as in our original problem. Using the envelope theorem as earlier, we derive an optimal UI formula:

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \left[n + (1-n) \cdot \frac{v'(\hat{c}^u)}{v'(c^e)}\right]^{-1} \cdot \left\{ \frac{n}{\varepsilon^M} \cdot \left[\frac{v'(\hat{c}^u)}{v'(c^e)} - 1\right] + \frac{\hat{\Delta} v}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa + 1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right] \right\}.$$

Hence, formula (9) carries over simply by replacing $v'(c^u)$ by $v'(\hat{c}^u)$, and Δv by $\hat{\Delta} v$.[7] Although the structure of the formula does not change, the benefit from consumption smoothing: $v'(\hat{c}^u)/v'(c^e) - 1$ in the first term of the formula is smaller if individuals can partially self-insure using home production, because $\hat{c}^u \geq c^u$. The welfare effect of the equilibrium adjustment of θ is also smaller because $\max_y [v(c^u + y) - m(y)] \geq v(c^u)$ so $\hat{\Delta} v = v(c^e) - [v(c^u + y) - m(y)] \leq \Delta v = v(c^e) - v(c^u)$. Hence, if workers can partially smooth consumption on their own, the optimal replacement rate $\tau = c^e/c^u$ is lower than in our original model without self-insurance. As already noted by Baily [1978] and Chetty [2006a], a UI program is less desirable in this case. This extended formula can be implemented using estimates of the consumption-smoothing benefit of UI [Gruber, 1997]. Finally,

[7] The Appendix derives an approximated optimal UI formula expressed in terms of sufficient statistics as in (10).

it is conceivable that self-insurance technology is not available in recessions as workers exhaust savings or ability to borrow. This absence would provide an additional rationale for increasing UI in recession, over and above the mechanism described in this paper.[8]

2.8 UI influences wages

We now extend our model to account for a possible response of wages to UI. Formula (9) carries over with minor modifications. We assume that the wage $w(t^*;a)$ is a function of the total implicit tax on work $t^* = t + b$. In that case, a change $d\Delta c$ in the generosity of UI affects the government budget's constraint not only through a change dn in employment, but also through a change dw in wages. Let $\varepsilon^w = ([1-t^*]/w) \cdot (dw/dt^*)$ be minus the elasticity of equilibrium wages with respect to one minus the total implicit tax on work. ε^w is typically positive if wages are bargained.[9] The optimal UI formula (9) becomes

$$\frac{1}{n}\frac{\tau}{1-\tau} = \left[n + (1-n)\frac{v'(c^u)}{v'(c^e)}\right]^{-1} \left\{\frac{n}{\varepsilon^M}\left[\frac{v'(c^u)}{v'(c^e)} - 1\right] + \frac{\Delta v}{v'(c^e)\Delta c}\frac{\kappa}{\kappa+1}\left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right]\right\} + \frac{[n + \tau/(1-\tau)]\,\varepsilon^w}{(1-n)(1-\varepsilon^w)\varepsilon^M}.$$

A new term appears on the right-hand side of the formula because wages respond to UI.[10] This term is positive if $\varepsilon^w > 0$, as higher benefits translate into higher wages and hence a bigger tax base. More importantly, the macro-elasticity ε^M is likely to be much higher than in our basic model because higher benefits now increase wages, depress labor demand, and hence increase unemployment further. Therefore, optimal UI is likely to be lower when wages respond to UI.

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[8] Kroft and Notowidigdo [2011] estimate that the consumption-smoothing benefit of UI is acyclical, suggesting that this channel may not be quantitatively important.

[9] Higher unemployment benefits typically strengthen the outside option of workers and raise wages in bargaining.

[10] This formula also applies to any setting in which the government's budget constraint is $n \cdot c^e + (1-n) \cdot c^u = n \cdot x(t^*;a)$, where $x(t^*;a)$ is taxable output per employed worker, by simply replacing the elasticity ε^w by $\varepsilon^x = ([1-t^*]/x) \cdot (dx/dt^*)$. For instance, it applies if the government taxes wages and some fraction of firms' profits.

3 Optimal Unemployment Insurance over the Business Cycle

This section applies formula (9) to a model capturing two key properties of recessions: (i) unemployment is higher in recessions; and (ii) jobs are rationed in recessions, as some unemployment remains even if unemployed workers search for jobs intensively. In this model of job rationing, we characterize micro- and macro-elasticity to infer that the optimal UI is countercylical.

3.1 The job-rationing model of Michaillat [forthcoming]

The representative firm takes prices as given. It takes labor n as input to produce a consumption good according to the production function $a \cdot g(n) = a \cdot n^{\alpha}$. $\alpha > 0$ measures marginal returns to labor. $a > 0$ is the level of technology, which proxies for the position in the business cycle.

ASSUMPTION 1. The production function has diminishing marginal returns to labor: $\alpha < 1$.

This assumption yields a downward-sloping demand for labor in the price θ-quantity n diagram, which has important macroeconomic implications. This assumption is motivated by the observation that, at business cycle frequency, some production inputs are slow to adjust so that a short-run production function exhibits diminishing marginal returns to labor.

As in Pissarides [2000], it costs $r \cdot a$ to open a vacancy, where $r > 0$ denotes the resources spent on recruiting due to matching frictions. We assume away randomness at the firm level: a worker is hired with certainty by opening $1/q(\theta)$ vacancies and spending $r \cdot a/q(\theta)$. When the labor market is tighter, a firm posts more vacancies to fill a job, and recruiting is more costly.

Wages are set once worker and firm have matched. Since the costs of search are sunk at the time of matching, there are always mutual gains from trade. There is no compelling theory of wage determination in such an environment [Hall, 2005]. Given the indeterminacy of wages, we use a simple wage schedule: $w(t^*; a) = \omega(t^*) \cdot a^{\gamma}$. As in Blanchard and Galí [2010], the parameter γ captures the rigidity of wages over the business cycle. If $\gamma = 0$, wages do not respond to technology and are completely fixed over the cycle. If $\gamma = 1$, wages are proportional to technology and are fully flexible over the cycle. The function $\omega(t^*)$ captures the response of wages to a change in the implicit tax on work $t^* = t + b$.

ASSUMPTION 2. The wage schedule is rigid: $\omega(t^*) = \omega > 0$ and $\gamma < 1$.

We assume that wages are rigid, in the sense that (i) they only partially adjust to a change in technology, and (ii) they do not respond to a change in UI. Rigidity (i) generates unemployment fluctuations over the business cycle [Hall, 2005]. Rigidity (ii) makes labor demand independent of UI and allows us to focus on the classical trade-off between insurance and incentive to search. Both assumptions are empirically grounded. Many ethnographic and empirical studies document wage rigidity over the business cycle [Michaillat, forthcoming]. Empirical studies consistently find that re-employment wages of unemployed workers do not respond to changes in unemployment benefits [for example, Card et al., 2007].

The firm starts with $1 - u$ workers, and decides how many additional workers to hire such that employment n^d maximizes real profit:[11]

$$\pi = a \cdot g(n^d) - w(a) \cdot n^d - \frac{r \cdot a}{q(\theta)} \cdot \left[n^d - (1 - u) \right].$$

The first-order condition (after dividing by a) defines implicitly labor demand $n^d(\theta; a)$:

$$g'(n^d) = \frac{w(a)}{a} + \frac{r}{q(\theta)}. \tag{11}$$

Under Assumption 1, $g'(n)$ decreases in n. Thus labor demand $n^d(\theta; a)$ decreases with labor market tightness θ, since the job-filling probability $q(\theta)$ decreases in θ. Intuitively, when the labor market is slack, it is easy and cheap for firms to recruit, stimulating labor demand. Under Assumption 2, $w(a)/a$ decreases with a, and hence $n^d(\theta; a)$ increases with a. When technology is low, wages are relatively high, depressing labor demand.

The equilibrium in the labor market is depicted in Figure 1 in a price θ-quantity n diagram. This figure plots labor demand curves for high (left panel) and low (right panel) technology; it also plots

[11] We assume that technology a is high enough such that it is optimal for the firm to choose positive hiring: $h = n^d - (1 - u) > 0$. This assumption requires $a > (\omega/\alpha) \cdot (1 - u)^{(1-\alpha)/(1-\gamma)}$.

labor supply for low (dotted line) and high (solid line) incentive to search Δv. Equilibrium employment $n(\Delta v; a)$ is given by the intersection of the downward-sloping labor demand curve $n^d(\theta; a)$ with the upward-sloping labor supply curve $n^s(e(\theta, \Delta v), \theta)$. In this frictional labor market wages are indeterminate so labor market tightness θ acts as a price that equalizes labor supply and labor demand. If labor supply is above labor demand, a reduction in θ: increases labor demand n^d by reducing recruiting costs; reduces labor supply n^s by reducing the per-unit job-finding probability as well as optimal search effort; until labor supply and labor demand are equalized.

Jobs are rationed in recessions in the sense that the labor market does not clear and some unemployment remains even as the search effort of unemployed workers becomes arbitrarily large. The mechanism creating this job shortage is quite simple, and is depicted in Figure 1. After a negative technology shock, the marginal product of labor falls but rigid wages adjust downwards only partially, so that the labor demand shifts inward (from the left to the right panel). If the adverse shock is sufficiently large, the marginal product of the least productive workers falls below the wage. It becomes unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$: labor demand cut the x-axis at $n^R < 1$ on the right panel. Even if workers searched infinitely hard, shifting labor supply outwards and pushing the labor market tightness θ to 0, firms would never hire more than $n^R < 1$ workers: jobs are rationed. This property implies that when the shortage of jobs is acute in recessions, the social returns to search are small because an increase in aggregate search effort leads only to a negligible increase in aggregate employment.

Our model is quite general as it nests as polar opposites: (i) the pure rat-race in which the number of jobs is fixed because labor demand is perfectly inelastic; and (ii) the Baily-Chetty model in which jobs are not rationed because labor demand is perfectly elastic and aggregate employment is solely driven by job-search efforts. To obtain the pure rat-race model, we set the job-filling probability as a constant: $q(\theta) = q$.[12] To obtain the Baily-Chetty model, we set constant marginal returns to labor: $\alpha = 1$. In Figure 1, labor demand $n^d(\theta; a)$ is vertical for the pure rat-race model, and horizontal for the Baily-Chetty model.

[12]With a Cobb-Douglas matching function $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^{\eta} \cdot o^{1-\eta}$, we achieve $q(\theta) = q$ by setting $\eta = 0$.

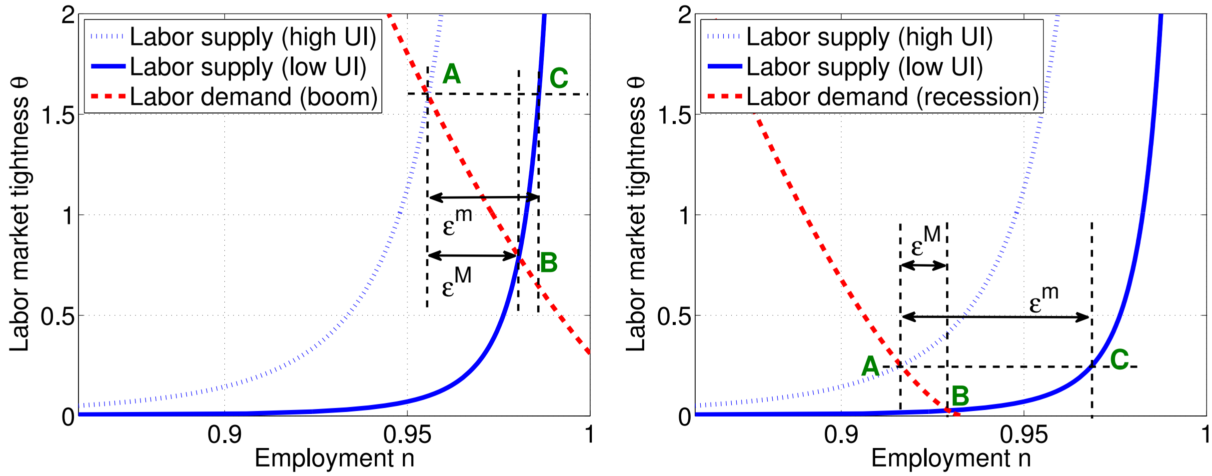

Figure 1: Labor market equilibrium in a price θ–quantity n diagram

3.2 Wedge between micro-elasticity and macro-elasticity

Section 2.5 defined micro- and macro-elasticity ε^m and ε^M. In the standard Baily-Chetty model, $\varepsilon^m = \varepsilon^M$. In contrast, $\varepsilon^m > \varepsilon^M = 0$ in the pure rat-race model with a fixed number of jobs. This section shows that a positive wedge between micro- and macro-elasticity arises in our model with endogenous job rationing.

ASSUMPTION 3. The utility functions are isoelastic: $v(c) = c^{1-\rho}/(1-\rho)$, $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$. The matching function is Cobb-Douglas: $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^{\eta} \cdot o^{1-\eta}$.

$\rho > 0$ is the coefficient of relative risk aversion, $\omega_k > 0$ measures the disutility of searching, $\omega_h > 0$ measures the effectiveness of matching, $1-\eta > 0$ is the elasticity of the per-unit job-finding probability with respect to labor market tightness θ, and as showed in the Appendix, $\kappa > 0$ is the elasticity of effort with respect to net reward from work $\Delta v = v(c^e) - v(c^u)$. Assumption 3 enables us to derive a simple expression for the ratio $\varepsilon^m/\varepsilon^M$, and simplifies the study of formula (9).

PROPOSITION 2. *Under Assumption 3, the ratio $\varepsilon^m/\varepsilon^M$ admits a simple expression*

$$\frac{\varepsilon^m}{\varepsilon^M} = 1 + \chi \cdot q(\theta) \cdot \frac{h}{n} \cdot n^{\alpha-1}$$

where $\chi = \alpha \cdot (1-\alpha) \cdot [(1-\eta)/\eta] \cdot [(1+\kappa)/\kappa] \cdot (1/r)$ *is constant. Under Assumption 1:* $\varepsilon^m/\varepsilon^M > 1$.

This proposition shows that there is a positive wedge between micro- and macro-elasticity when the demand for labor is downward-sloping, as illustrated by Figure 1. To understand where the wedge between these elasticities come from, consider a cut in unemployment benefits $d\Delta c > 0$. This change creates variations in all variables $d\Delta v$, dn, $d\theta$, and de, so that all equilibrium conditions continue to be satisfied. The change in effort can be decomposed as $de = de_{\Delta v} + de_{\theta}$, where $de_{\Delta v} = (\partial e/\partial \Delta v) d\Delta v$ is a partial-equilibrium variation in response to the change in UI, and de_{θ} is a general-equilibrium adjustment following the change $d\theta$ in labor market tightness. Using the labor supply equation (2), we have $dn = dn_e + dn_{\theta}$ where $dn_e = (\partial n^s/\partial e) de_{\Delta v}$ and $dn_{\theta} = [\partial n^s/\partial \theta + (\partial n^s/\partial e)(\partial e/\partial \theta)] d\theta$. Following a cut in benefits an individual jobseeker increases his search effort, increasing his own probability to find a job by $dn_e > 0$. From the jobseeker's perspective, labor market tightness θ remains constant. The interval A–C in Figure 1 represents dn_e. However when the jobseeker finds a job, he reduces the profitability of the marginal jobs left vacant because (1) the productivity of these jobs falls by diminishing returns to labor, but (2) the prevailing wage does not adjust to this drop in marginal productivity . Thus, the firm reduces the number of vacancies posted to fill these less profitable jobs. Labor market tightness falls by $d\theta < 0$, reducing the per-unit job-finding probability $f(\theta)$ of jobseekers who are still unemployed. This is the exact same mechanism as in the pure rat-race model of Section 2.5. $dn_{\theta} < 0$ is the corresponding reduction in employment, represented by interval C–B in Figure 1. As a consequence, the general-equilibrium increase in aggregate employment dn following an increase in aggregate search efforts is smaller than the partial-equilibrium increase dn_e in the individual probability to find a job following an increase in individual search efforts. The interval A–B in Figure 1 represents $dn < dn_e$. The difference between the micro-effect dn_e and the macro-effect dn is $dn_{\theta} < 0$. This difference arises because of job rationing, and is captured by the wedge $\varepsilon^m - \varepsilon^M$ (as formalized by Lemma 1).

Policy implications. Proposition 2 has important implications for the design of UI. It implies that private insurers under-provide UI from a social perspective. Small private insurers would use the Baily-Chetty formula and solely take into account the micro-elasticity of unemployment when they determine the optimal level of insurance for their client. From the perspective of the private insurer's budget, it is optimal to have unemployed workers search hard for jobs to increase their individual probability to find a job. When jobs are rationed this additional search effort reduces the probability of other jobseekers to find a job, but private insurers do not internalize this externality. If the government provides UI instead, it would take into account the macro-elasticity of unemployment and offer a more generous UI.[13]

Testable implication. Proposition 2 shows that there is a positive wedge $\varepsilon^m > \varepsilon^M$ in our model with job rationing. This positive wedge is a testable implication of our model that distinguishes it from standard models of equilibrium unemployment. For instance, the wedge is nil in the Hall [2005] model with rigid wages, and negative in the Pissarides [2000] model with Nash bargaining. Estimating the sign of this wedge empirically would therefore allow us to distinguish between these different models of equilibrium unemployment, which have very different implications for the design of optimal UI. We now briefly discuss the sign of the wedge $(\varepsilon^m/\varepsilon^M) - 1$ in the models of Hall [2005] and Pissarides [2000].

To capture the main features of the model with rigid wages from Hall [2005], we modify the model of Section 3.1 by assuming that the production function is linear: $\alpha = 1$. This model generates large employment fluctuations but does not exhibit job rationing [Michaillat, forthcoming]. In Figure 1, the labor demand $n^d(\theta; a)$ would be horizontal because of constant marginal returns to labor. Hence, points B and C would be superposed: $\varepsilon^m = \varepsilon^M$.

To capture the main features of the canonical model from Pissarides [2000], we modify the model presented in Section 3.1 by assuming that (i) the production function is linear: $\alpha = 1$; and (ii) wages are determined by Nash bargaining and, without loss of generality, workers are risk neutral: $v(c) = c$. The firm's surplus from an established relationship is the hiring cost $r \cdot a/q(\theta)$ since a firm can replace a worker immediately at that cost during the matching period. The worker's surplus from work is $\Delta v = \Delta c = (1 - t^*) \cdot w$. As the bargaining solution divides the surplus of

[13]We are grateful to Guido Lorenzoni for pointing out to us this application of the result of Proposition 2.

the match between worker and firm with the worker keeping a fraction $\beta \in (0,1)$ of the surplus, worker's and firm's surplus are related by

$$(1-t^*)\cdot w = \frac{\beta}{1-\beta}\cdot\frac{r\cdot a}{q(\theta)}. \tag{12}$$

Using the firm's first-order condition (11), we infer that the wage schedule satisfies $w(t^*;a) = \omega(t^*)\cdot a$ with $\omega(t^*) = \beta/[\beta+(1-\beta)\cdot(1-t^*)]$. The equilibrium wage arising from Nash bargaining is fully flexible over the business cycle as it is proportional to technology a. It increases when the implicit tax on work $t^* = t+b$ increases, because a higher t^* implies a better outside option for workers. Increasing $\Delta c = (1-t^*)\cdot w$ by reducing t^* leads workers to search harder but also reduces wages and leads firms to recruit more. In equilibrium, labor market tightness increases. In the diagram of Figure 1, the labor supply shifts outwards and the horizontal labor demand shifts upwards. Hence, the macro-elasticity is higher than the micro-elasticity. Formally, the surplus-sharing condition (12) can be rewritten as $\Delta c = [\beta/(1-\beta)]\cdot(r\cdot a)/q(\theta)$ and therefore the elasticity of θ with respect to Δc is simply $\varepsilon^{\theta}_{\Delta c} = 1/\eta > 0$. From Lemma 1 we infer that the macro-elasticity is larger than the micro-elasticity: $\varepsilon^M > \varepsilon^m$.

3.3 Elasticities and optimal replacement rate over the business cycle

ASSUMPTION 4. Assume that $\rho \geq 1$, $\eta \geq (1+\kappa)/(1+2\cdot\kappa)$, and $\gamma < \bar{\gamma}$ where

$$\frac{1-\bar{\gamma}}{\bar{\gamma}} = (\rho-1)\cdot\frac{\eta}{1-\eta}\cdot\frac{1}{\kappa+1}\cdot\sup_{\Delta v,a}\left\{\frac{a\cdot g'\left[n(\Delta v;a)\right]}{w(a)}-1\right\}. \tag{13}$$

PROPOSITION 3. *Under Assumptions 1, 2, 3, and 4:* $\left.\frac{\partial(\varepsilon^m/\varepsilon^M)}{\partial a}\right|_{\tau} < 0$ *and* $\left.\frac{\partial\varepsilon^M}{\partial a}\right|_{\tau} > 0$.

The proposition shows that the wedge $\varepsilon^m/\varepsilon^M$ between micro- and macro-elasticity is small in good times, but large in recessions when unemployment is high. Furthermore, the macro-elasticity ε^M is high in expansions, but small in recessions. Intuitively, recessions are periods of acute job shortage during which the job-search behavior of unemployed workers has little influence on aggregate unemployment. Hence the macro-elasticity is bound to be small. Furthermore, because of the acute lack of jobs in recessions, searching more to increase one's probability of finding a job mechanically decreases other jobseekers' probability of finding one, as in the pure rat-race model. Hence, the wedge between micro- and macro-elasticity is large.

Assumption 4 gathers a set of technical conditions used to compute the comparative statics with respect to technology a, taking the replacement rate τ as given. These conditions are satisfied by our preferred calibration later presented in Table 1, and are satisfied for a broad range of parameter values. For instance, with log-utility ($\rho = 1$), Assumption 4 boils down to a condition on η. If wages are completely rigid ($\gamma = 0$), it boils down to the conditions on ρ and η. Finally, if technology a is bounded above, there exists a wage rigidity $\bar{\gamma} > 0$ that satisfies equation (13).[14]

Proposition 4 infers the cyclicality of the optimal replacement rate τ using formula (9) and the cyclical properties of elasticities ε^m and ε^M.

PROPOSITION 4. *Assume that formula* (9) *implicitly defines a unique function* $\tau(a)$*, continuous and differentiable. Then under Assumptions 1, 2, 3, and 4,* $d\tau/da < 0$.

This proposition proves that the optimal UI replace rate $\tau = c^u/c^e$ is more generous in recessions than in expansions. The intuition for this result can be seen using approximated formula (10) and the results from Proposition 3. In recessions, ε^M is smaller as job-search has little effect on aggregate unemployment. Hence a more generous UI, while reducing aggregate search effort because of moral hazard, has smaller budgetary cost since it only increases unemployment negligibly (the first term in formula (10) increases). Furthermore, the wedge $\varepsilon^m/\varepsilon^M$ is larger in recession. Since unemployed workers choose their effort taking the per-unit job-finding probability as given, they do not internalize their influence on others' employment probability, thus imposing a negative *rat-race externality*. The wedge between micro- and macro-elasticity measures the welfare cost of the rat-race externality. Accordingly UI, which corrects the rat-race externality by discouraging job search, is more desirable in recession (the second term in formula (10) increases).

[14] $a \cdot g'(n)/w(a) > 1$ is the wedge between the marginal product of labor and the wage (the wedge is > 1 because of the existence of positive recruiting costs $r/q(\theta)$). Since employment $n \in (1-u, 1]$, the marginal product of labor $g'(n)$ is bounded. $a/w = (1/\omega) \cdot a^{1-\gamma}$ is bounded above if technology a is bounded above (which is a natural assumption at business cycle frequency). Thus, the right-hand side of (13) is bounded above if technology is bounded above. In that case there exists a wage rigidity $\bar{\gamma} > 0$ that satisfies (13).

4 Extension to an Infinite-Horizon Model

This section verifies numerically that our central theoretical result (Proposition 4) holds in an infinite-horizon, stochastic extension of the static model of Section 3. In the model calibrated with US data, the increase in the generosity of optimal UI in recession is quantitatively large. This numerical result is robust to various institutional arrangements for the administration of UI that could not be studied in the static model. It holds whether the government adjusts level or duration of benefits; and whether the government balances its budget each period or uses deficit spending.

4.1 The economy

Technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. Together with initial employment n_{-1} in the representative firm, the history of technology realizations $a^t \equiv (a_0, a_1, \ldots, a_t)$ fully describes the state of the economy in period t. The time-t element of the worker's choice, firm's choice, and government policy must be measurable with respect to (a^t, n_{-1}).

The labor market is similar to that in the one-period model. The only difference is that at the end of period $t-1$, a fraction s of the n_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job become unemployed, and start searching for a new job at the beginning of period t. At the beginning of period t, u_t unemployed workers look for a job:

$$u_t = 1 - (1-s) \cdot n_{t-1}.$$

In steady state, inflow to unemployment $s \cdot n$ equals outflow from unemployment $u \cdot e \cdot f(\theta)$, so labor market tightness θ, effort e, and employment n are related through a Beveridge curve

$$n = \frac{e \cdot f(\theta)}{s + (1-s) \cdot e \cdot f(\theta)}. \tag{14}$$

The government chooses $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ subject to the sequence of budget constraints: for all t,

$$n_t \cdot w(a_t) = n_t \cdot c_t^e + (1-n_t) \cdot c_t^u. \tag{15}$$

Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$, the representative worker chooses job-search effort $\{e_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{(1-n_t^s)\cdot v(c_t^u) + n_t^s \cdot v(c_t^e) - \left[1-(1-s)\cdot n_{t-1}^s\right]\cdot k(e_t)\right\}, \tag{16}$$

subject to the law of motion of the probability to be employed in period t,

$$n_t^s = (1-s)\cdot n_{t-1}^s + \left[1-(1-s)\cdot n_{t-1}^s\right]\cdot e_t \cdot f(\theta_t).$$

$\mathbb{E}_0$ denotes the mathematical expectation conditioned on time-0 information, $\delta < 1$ is the discount factor. Let $1+\kappa$ be the elasticity of the disutility from searching $k(\cdot)$. The optimal effort satisfies

$$\left\{\frac{k'(e_t)}{f(\theta_t)} - \delta\cdot(1-s)\cdot\mathbb{E}_t\left[\frac{k'(e_{t+1})}{f(\theta_{t+1})}\right]\right\} + \kappa\cdot\delta\cdot(1-s)\cdot\mathbb{E}_t\left[k(e_{t+1})\right] = v(c_t^e) - v(c_t^u). \tag{17}$$

The representative firm is owned by risk-neutral entrepreneurs. Given labor market tightness and technology $\{\theta_t, a_t\}_{t=0}^{+\infty}$, the firm chooses employment $\left\{n_t^d\right\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{a_t \cdot g(n_t^d) - w(a_t)\cdot n_t^d - \frac{r\cdot a_t}{q(\theta_t)}\cdot\left[n_t^d - (1-s)\cdot n_{t-1}^d\right]\right\}.$$

As in Hall [2005], we require that no worker-firm pair has an unexploited opportunity for mutual improvement. Wages should neither interfere with the formation of an employment match that generates a positive bilateral surplus, nor cause the destruction of such a match.[15] In that case, endogenous layoffs and quits never occur, and $n_t^d - (1-s)\cdot n_{t-1}^d \geq 0$ is the number of hires in period t. The optimal employment satisfies

$$a_t \cdot g'(n_t^d) = w(a_t) + \frac{r\cdot a_t}{q(\theta_t)} - \delta\cdot(1-s)\cdot\mathbb{E}_t\left[\frac{r\cdot a_{t+1}}{q(\theta_{t+1})}\right], \tag{18}$$

which implies that the firm hires labor until marginal revenue from hiring equals marginal cost.

[15] As in Michaillat [forthcoming], we can derive a sufficient condition for the wage process to respect the private efficiency of all worker-firm matches. This condition imposes a lower bound on wage rigidity γ.

Table 1: Parameter values in simulations (weekly frequency)

	Interpretation	Value	Source
s	Separation rate	0.94%	JOLTS, 2000–2010
δ	Discount factor	0.999	Corresponds to 5% annually
ω_h	Efficiency of matching	0.19	JOLTS, 2000–2010
η	Effort-elasticity of matching	0.7	Petrongolo and Pissarides [2001]
γ	Real wage rigidity	0.5	Pissarides [2009], Haefke et al. [2008]
r	Recruiting cost	0.21	Barron et al. [1997], Silva and Toledo [2005]
ω	Steady-state real wage	0.67	Matches unemployment of 5.9%
α	Returns to labor	0.67	Matches labor share of 0.66
ρ	Relative risk aversion	1	Chetty [2004, 2006b]
κ	Elasticity of marginal disutility of effort	2.1	Matches micro-elasticity of 0.9 [Meyer, 1990]
ω_k	Disutility of search effort	0.58	Matches effort of 1 for $t = 7.65\%$, $b = 60\%$

Notes: The calibration of these parameters is detailed in the Appendix.

Wages follow an exogenous stochastic process and cannot equalize labor supply and demand. Hence labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n_t^d\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$:

$$n_t = n_t^d = n_t^s. \tag{19}$$

An *equilibrium with unemployment insurance* is a collection of stochastic processes $\{c_t^e, c_t^u, e_t, n_t, \theta_t\}_{t=0}^{+\infty}$ that satisfy equations (17), (18), (15), (19). The unemployment insurance program is fully contingent on the history of realizations of shocks, and is taken as given by firms and workers. Importantly, we assume that the government can fully commit to the policy plan. The government's problem is to choose a government policy $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (16) over all equilibria with unemployment insurance. An *optimal equilibrium* is an equilibrium that attains the maximum of (16). Finally, we calibrate all parameters of the model at a weekly frequency as shown in Table 1. The calibration strategy is described in the Appendix.[16]

[16] There remains considerable uncertainty about some of the parameters and our model abstracts from a number of relevant issues. Particularly, there is no consensus about the size of the coefficient of relative risk aversion [Chetty, 2004, 2006b]. Thus, this exercise is only illustrative of the magnitudes of the optimal policy.

4.2 Optimal unemployment insurance over the business cycle

This section considers static equilibria where technology $a_t = a$ if fixed (no aggregate shocks) and analyzes how the equilibria vary with technology level a.[17] Environments with lower technology have higher unemployment. Figure 2 displays in six panels, as a function of unemployment: (a) labor market tightness, (b) job-search effort, (c) optimal replacement rate $\tau = c^u/c^e$, (d) optimal consumptions c^e and c^u, (e) optimal labor tax rate $t = 1 - c^e/w$, and (f) optimal benefit rate $b = c^u/w$. Panels (a) is a Beveridge curve, showing that labor market tightness decreases with unemployment. Panel (b) shows that effort decreases with unemployment. Panel (c) displays the critical result of this section: the optimal replacement rate is strongly countercyclical, for it increases from 64% to 86% when unemployment increases from 4% to 11%. The simulation in panel (c) confirms that the theoretical result of Proposition 4 also holds in our calibrated infinite-horizon model. It implies that consumption of unemployed workers increases relative to that of employed workers in recession. Panel (d) goes one step further: it shows that consumption of unemployed workers even increases in absolute terms. Panels (e) and (f) show that both benefit rate and labor tax rate should be countercyclical. In recession, labor tax should increase substantially, not only to finance benefits to a larger number of unemployed workers, but also to finance benefits that are more generous relative to the prevailing wage.

4.3 Formula in sufficient statistics

Figure 2 depicts the optimal replacement rate $\tau(a)$ as a function of the underlying technology level. To obtain such a schedule $\tau(a)$, one needs to specify and calibrate the entire structure of the model. In this section, we present an alternative approach to determining optimal UI, which only requires estimating a few sufficient statistics that summarize the relevant characteristics of the model.

We assume that disutility of effort is isoelastic: $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$. In the infinite-horizon

[17] In a static environment, the labor market is in steady state: the Beveridge curve (14) holds. In search-and-matching models, the comparison of static environments delivers the same qualitative predictions as the study of a stochastic environment [Michaillat, forthcoming; Pissarides, 2009].

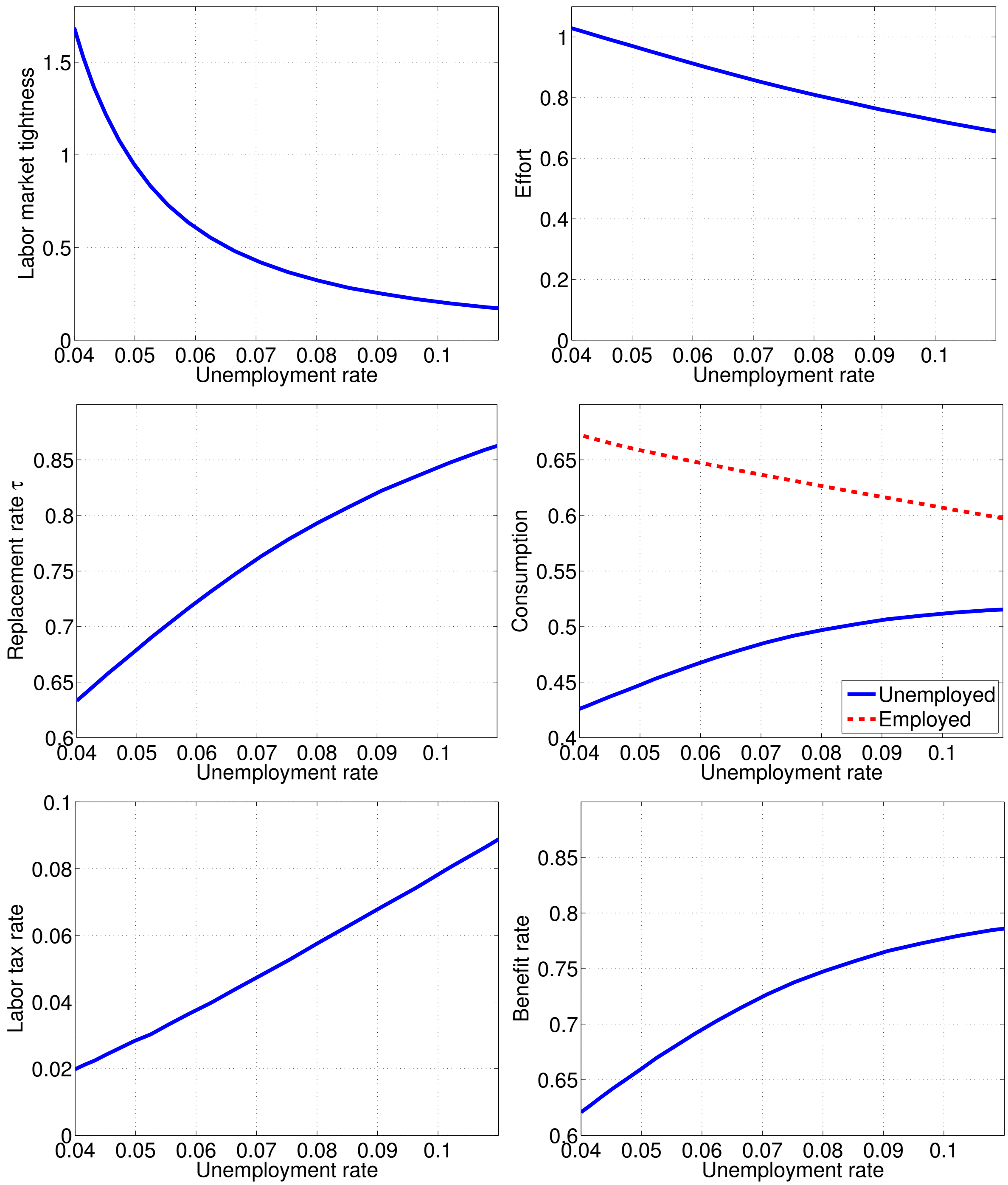

Figure 2: Optimal unemployment insurance over the business cycle

Notes: All computations are based on the infinite-horizon model calibrated in Table 1. Each panel plots a collection of optimal equilibria in static environments characterized by different underlying technology levels: the unemployment rate u spans $[0.04, 0.11]$ for technology $a \in [0.96, 1.04]$. The Appendix characterizes these optimal equilibria, and presents the numerical computations in detail.

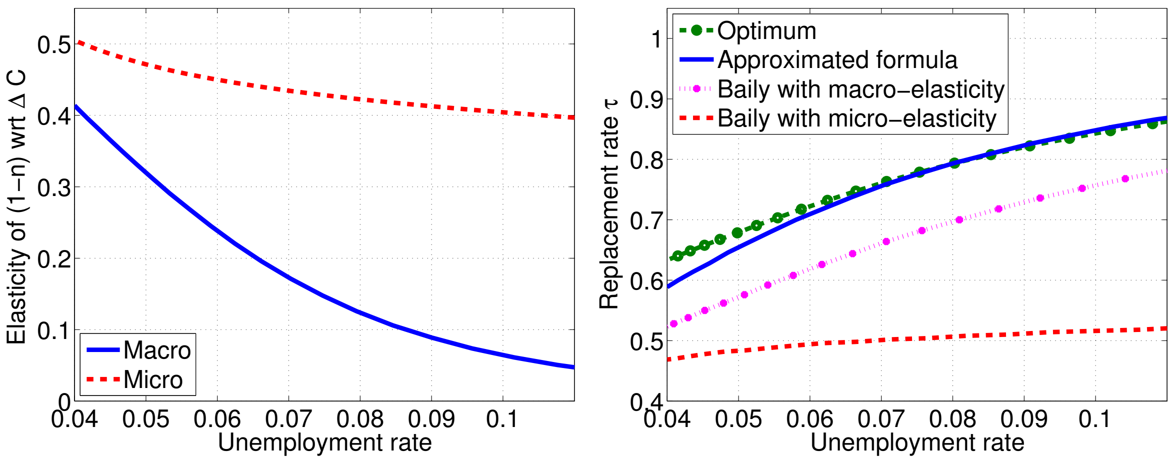

Figure 3: Micro-elasticity, macro-elasticity, and replacement rates

Notes: Both panels are based on the infinite-horizon model calibrated in Table 1. The left panel plots, as a function of unemployment, the elasticities of unemployment $1-n$ with respect to reward from work $\Delta c = c^e - c^u$, obtained for $\tau = 65\%$. Macro-elasticity ε^M (blue, solid line) and micro-elasticity ε^m (red, dashed line) are defined and computed in the Appendix. The right panel plots replacement rates as a function of unemployment. The red, dashed line is the replacement rate obtained with the Baily-Chetty formula using micro-elasticity ε^m: $\tau/(1-\tau) = (\rho/\varepsilon^m) \cdot (1-\tau)$. The magenta, dotted with circles, line is the replacement rate obtained with the Baily-Chetty formula using macro-elasticity ε^M: $\tau/(1-\tau) = (\rho/\varepsilon^M) \cdot (1-\tau)$. The blue, solid line is the replacement rate obtained with formula (20). For comparison, the green, dashed with circles, line is the exact optimal replacement rate plotted in Figure 2. Each point corresponds to a different underlying technology level a: $u \in [0.04, 0.11]$ for $a \in [0.96, 1.04]$.

setting formula (10), obtained in the one-period model of Section 2, becomes:[18]

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\varepsilon^M} \cdot [1-\tau] + \frac{1+\kappa}{\kappa} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right] \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right]. \tag{20}$$

This approximated formula is valid in a static environment if $n \approx 1$, $u \ll \kappa$, $\delta \approx 1$, and the third and higher order terms of $v(\cdot)$ are small. The term $\kappa/(1+\kappa)$ in (10) is replaced by $(1+\kappa)/\kappa$ in (20), capturing an increase in the welfare cost of the rat-race externality in the infinite-horizon model, relative to the one-period model.[19]

The left panel in Figure 3 displays micro-elasticity ε^m and macro-elasticity ε^M of unemployment with respect to net reward from work as a function of the unemployment rate for a constant replacement rate $\tau = 65\%$ (the average replacement rate in the US). The panel confirms that the

[18]The optimal UI formula (10) in the one-period environment is obtained without making any functional-form assumption. The optimal search decision (17) is more complex in the infinite-horizon environment as it involves not only $k'(e)$ as in the static model but also the level $k(e)$. Relating $k(e)$ to $k'(e)$ requires the isoelasticity assumption.

[19]The Appendix details reasons why the rat-race externality has higher welfare costs in the infinite-horizon model.

results from Propositions 2 and 3 extend to this infinite-horizon environment: (1) macro-elasticity is always smaller than micro-elasticity and the wedge between the two elasticities increases in recessions; and (2) the macro-elasticity decreases in recessions. Furthermore, these cyclical fluctuations are quantitatively large: the ratio $\varepsilon^m/\varepsilon^M$ increases from 5/4 when unemployment is 4% to 8 when unemployment is 11%; the macro-elasticity decreases from 0.40 when unemployment is 4% to 0.05 when unemployment is 11%. At the same time, the micro-elasticity remains broadly constant. It stays in the narrow 0.4–0.5 range when unemployment varies between 4% and 11%.

The right panel in Figure 3 displays the replacement rate obtained from three alternative formulas, as a function of unemployment. This panel illustrates the discussion of the optimal UI formula presented in Section 2.6. The green dotted line plots the exact optimal replacement rate of Figure 2. The blue solid curve is the replacement rate obtained with the approximated optimal UI formula (20). Those two curves are almost identical showing that formula (20) delivers an excellent approximation to the exact optimum. Next, the magenta dotted line is the replacement rate obtained from a Baily-Chetty formula, similar to (20) but excluding the term correcting for the rate-race externality. This replacement rate is lower than the full optimum because the correction term is positive as there is a positive wedge between micro- and macro-elasticity. Finally, the red dashed line is the replacement rate obtained from a standard Baily-Chetty formula, similar to (20) but excluding the correction term and replacing macro-elasticity ε^M by micro-elasticity ε^m in the first term. As micro-elasticity is almost acyclical, this replacement rate is almost acyclical as well: it varies within the narrow 48%–52% range. While this replacement rate, used in the public economics literature [for example, Gruber, 1997], is close to the optimum when unemployment is low, it departs significantly from it in recession.[20]

4.4 The government can borrow and save

So far, we constrained the government to balance its budget each period. The government could not use deficit spending to shift resources intertemporally from expansions to recessions and smooth

[20]The micro-elasticity would be slightly more cyclical with higher risk aversion. A higher risk aversion would also increase significantly the optimal replacement rate and would quantitatively reduce the difference in replacement rates between our formula and the standard Baily-Chetty formula.

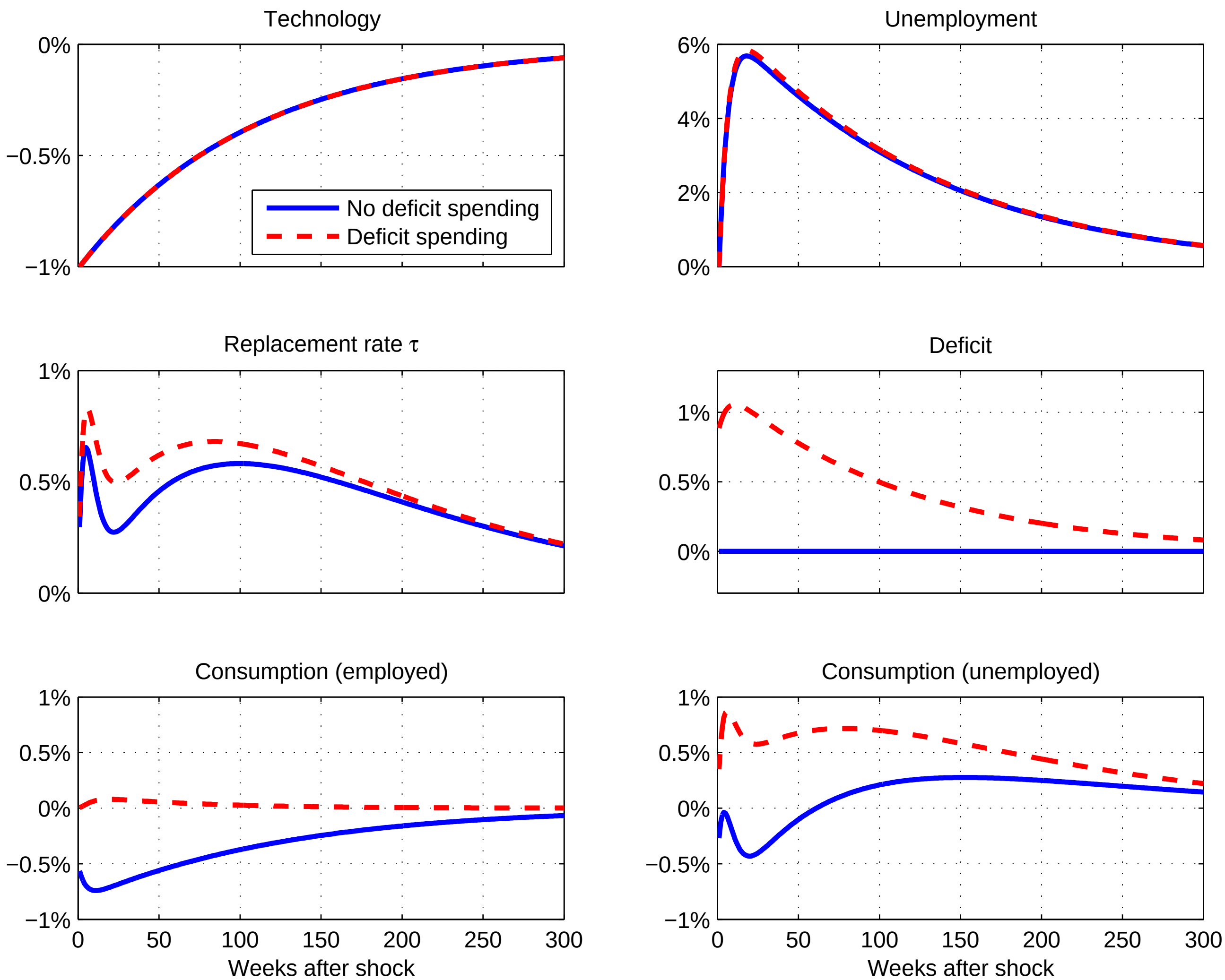

Figure 4: Impulse response of optimal unemployment insurance to a negative technology shock

Notes: This figure displays impulse response functions (IRFs), which represent the percentage-deviation from steady state for each variable. We assume that the log-deviation of technology $\check{a}_t \equiv d\ln(a_t)$ follows an AR(1) process: $\check{a}_{t+1} = \nu \cdot \check{a}_t + z_{t+1}$ where $z_t \sim N(0, \sigma^2)$ is an innovation to technology. As in Michaillat [forthcoming], we estimate this AR(1) process using BLS data for 1964:Q1–2010:Q2 and find $\nu = 0.991$ and $\sigma = 0.0026$ at weekly frequency. IRFs are obtained by imposing an unexpected negative technology shock $z_1 = -0.01$ to the log-linear infinite-horizon model. The time period displayed on the x-axis is 300 weeks. The blue solid IRFs are responses of the optimal equilibrium when the government is constrained by (15) to balance its budget each period. The red dashed IRFs are responses of the optimal equilibrium when the government is subject to a single intertemporal budget constraint (21). Both log-linear systems and the IRFs computations are described in the Appendix.

workers' consumption. This modelling choice allowed us to focus on the trade-off between insurance and incentive to search within each period. However, it is important to understand how our results change when the government is able to borrow and save as is the case in practice.

In this section, we show that our results are robust to assuming that the government has access to a complete market for Arrow-Debreu securities. We assume that the government faces risk-neutral investors with discount factor δ on the security market. An Arrow-Debreu security pays one unit of consumption good after history a^t. The price of this security is $\delta^t \cdot p(a^t)$, where $p(a^t)$ is the probability of history a^t based on time-0 information. The government trades securities at time 0 to finance UI in all histories, and faces a unique intertemporal budget constraint:

$$0 = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot [n_t \cdot w(a_t) - n_t \cdot c_t^e - (1 - n_t) \cdot c_t^u] . \tag{21}$$

We solve the government's problem by log-linearization as described in the Appendix. To confirm the comovements of technology with optimal UI in a stochastic environment, we compute impulse response functions. Figure 4 depicts the response of optimal UI to a negative technology shock in two cases: (1) the blue solid lines are responses in the baseline case in which the government is constrained by (15) to balance his budget each period; and (2) the red dashed lines are responses when the government is subject to a single intertemporal budget constraint (21). The response of the optimal replacement rate to an adverse economic shock is almost identical whether the government uses deficit spending or not. On impact, the replacement rate increases by 0.5%; it then falls slightly, before building again for 100 weeks; at its peak, it increases by about 0.7% in both cases. While the generosity of UI is similar in both cases, consumption of *both* employed and unemployed workers is higher when the government can borrow. In that case, the government smoothes consumption of employed workers almost perfectly. In contrast, the consumption of employed workers falls by about 0.7% on impact when the government must balance its budget each period. Indeed when the government is able to borrow, its budget deficit—defined as benefit outlays minus tax revenue in the period—increases by about 1% on impact, a consequence of the additional consumption smoothing provided to workers in recessions. Finally, unemployment responds similarly in both cases: it builds slowly and peaks after about 20 weeks.

4.5 Unemployment benefits have finite duration

For simplicity, we assumed that unemployment benefits were available to all unemployed workers, independently of the length of their unemployment spell, and that the government adjusted the level of unemployment benefits over the business cycle. In practice, unemployment benefits have finite duration and governments often modulate the generosity of UI over the business cycle by adjusting the duration rather than the level of benefits.[21] While we could not account for the duration of UI in a one-period model, we build on our infinite-horizon model to analyze quantitatively this option.

In this section, we assume that the replacement rate of UI is fixed, that unemployment benefits have finite duration, and that the government can adjust the duration of UI over the business cycle. We confirm that the optimal duration of UI is countercyclical. For tractability, we follow Fredriksson and Holmlund [2001] and assume that workers exhaust their unemployment benefits with probability λ_t at the end of each period t. Eligible unemployed workers receive consumption c_t^u from unemployment benefits, and ineligible unemployed workers receive consumption $c_t^a < c_t^u$ from social assistance until they find a job. At the beginning of period t, there are x_t^u jobseekers exerting job-search effort e_t^u, and x_t^a ineligible jobseekers exerting job-search effort e_t^a. The matching process is the same as in the baseline model of Section 4.1, except that we redefine labor market tightness $\theta_t \equiv o_t / (e_t^a \cdot x_t^a + e_t^u \cdot x_t^u)$. After the matching, z_t^u eligible jobseekers and z_t^a ineligible jobseekers are still unemployed. The stocks of workers are linked by the following relations: $z_t^u = x_t^u \cdot (1 - e_t^u \cdot f(\theta_t))$, $z_t^a = x_t^a \cdot (1 - e_t^a \cdot f(\theta_t))$, $n_t = 1 - (z_t^a + z_t^u)$, $x_t^u = z_{t-1}^u \cdot (1 - \lambda_{t-1}) + s \cdot n_{t-1}$, $x_t^a = z_{t-1}^a + \lambda_{t-1} \cdot z_{t-1}^u$. Worker's and firm's problems are very similar to those in Section 4.1, and are described in the Appendix. We assume that the generosity of unemployment benefits: $\tau^{u,e} = c_t^u / c_t^e$, as well as the generosity of social assistance: $\tau^{a,e} = c_t^a / c_t^e$, are constant over time. The government chooses the rate λ_t at which eligible unemployed workers become ineligible, in

[21] US unemployment benefits have a maximum duration of 26 weeks in normal times. Duration is automatically extended by up to 20 weeks in states where unemployment is above 8%. Duration is often further extended by the government in severe recessions. For example, the federal Emergency Unemployment Compensation program, enacted in 2008, extends durations by an additional 53 weeks when state unemployment is above 8.5%.

order to maximize social welfare

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left[-x_t^u \cdot k(e_t^u) - x_t^a \cdot k(e_t^a) + n_t \cdot v(c_t^e) + z_t^u \cdot v(c_t^u) + z_t^a \cdot v(c_t^a)\right],$$

subject to a budget constraint in each period t:

$$n_t \cdot w(a_t) = n_t \cdot c_t^e + z_t^u \cdot c_t^u + z_t^a \cdot c_t^a.$$

We calibrate the model so that an expected duration of 26 weeks is optimal when the unemployment rate is at its average level of 5.9%. The left panel in Figure 5 shows how unemployment and its composition varies with technology. When technology increases, total unemployment falls, the number of eligible jobseekers falls, but the number of ineligible jobseekers increases because the expected duration of benefits falls drastically. In fact, all unemployed workers should be eligible when unemployment reaches 9%, whereas only 60% of them should be eligible when unemployment falls to 4%. The government chooses the arrival rate λ of ineligibility, and the expected duration of unemployment benefits is $1/\lambda$. The right panel shows that quantitatively, the optimal expected duration of benefits is strongly countercyclical. When unemployment is 4%, the optimal arrival rate of ineligibility is 15% and the optimal expected duration of benefits is 7 weeks. When unemployment reaches 5.9%, the optimal arrival rate falls to 3.9%, and the optimal duration of benefits increases to 26 weeks. Finally, when unemployment reaches 8.0%, the optimal arrival rate drops to 0.5%, and the optimal duration of benefits increases to 200 weeks.[22]

5 Some Empirical Evidence

To assess the current UI system over the business cycle using formula (20), we need estimates of micro- and macro-elasticity at various points of the business cycle. Although a large empirical literature examines the effects of UI on unemployment duration, no study has estimated separately

[22] As the government chooses the instantaneous arrival rate of ineligibility, durations would not last 200 weeks if a recession ends quickly. The key point is that jobseekers hardly ever loose their eligibility to UI during deep recessions.

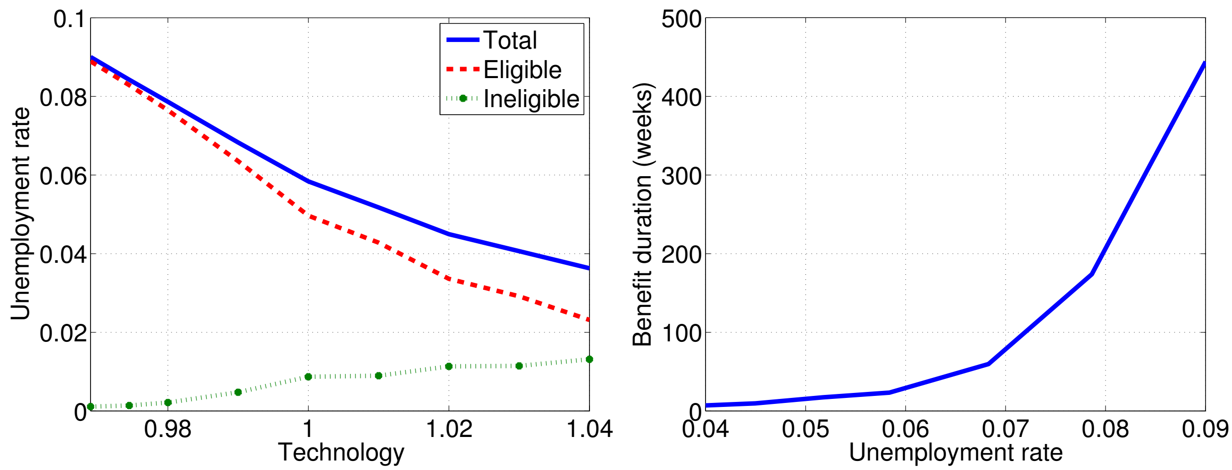

Figure 5: Optimal duration of unemployment benefits

Notes: Both panels are obtained with the infinite-horizon model in which unemployment benefits have finite duration. The model is calibrated according to Table 1 (except that $\omega_k = 0.43$ here). These panels plot optimal equilibria in static environments corresponding to different underlying technology levels. In the right panel, $u \in [0.04, 0.09]$ for $a \in [0.96, 1.04]$. The Appendix characterizes the optimal equilibria, and describes calibration and simulations.

micro- and macro-elasticity, let alone the cyclicality of these two elasticities. This section discusses the ideal experiments to estimate micro- and macro-elasticity, reviews the existing findings of the literature, and reports our own new estimates of micro-elasticity over the US business cycle.

5.1 Estimating the micro-elasticity of unemployment

The ideal experiment to estimate the micro-elasticity is to offer higher UI benefits to a randomly selected *small* subset of individuals within a labor market and compare unemployment durations between these treated individuals and the rest of the unemployed. Studies in the literature comparing individuals with different benefits in the same labor market at a given time, while controlling for individual characteristics, estimate primarily micro-elasticities. To investigate the cyclicality of the micro-elasticity, it is necessary to replicate this estimation across labor markets with different unemployment levels. The closest empirical setting to the ideal experiment is that of Schmieder et al. [2011]. They use sharp variations in the potential duration of unemployment benefits by age in Germany, population-wide administrative data, and a regression discontinuity approach to identify compellingly the micro-elasticity of unemployment duration with respect to the *potential duration* of benefit entitlement. Their elasticity estimates are broadly constant over the German business cycle. The estimates are also small in magnitude relative to estimates of elasticities with respect to benefit *levels* such as Meyer [1990].

To estimate the micro-elasticity of unemployment duration with respect to benefit *levels*, we use administrative data from the Continuous Wage and Benefit History (CWBH) that record employment and unemployment history for all workers in 8 US states from 1976 to 1983. To identify the micro-elasticity, we estimate the effect of benefits using only within state$\times$year variations in individual benefits. We fit a Cox proportional hazard model with state and year fixed effects interacted, and controlling for observable characteristics of the unemployed (age, education, marital status, ethnicity, number of dependents). We also introduce a series of non-parametric controls for previous wage and previous work experience. When adding this rich set of controls, the residual variation in benefits is likely to be exogenous, and comes primarily from non-linearities in the benefit schedule. We estimate this model for low- and high-unemployment regimes.[23] The Appendix provides all the details. Our main finding is that the elasticity of duration with respect to benefits is 0.34 (0.04) for low-unemployment regimes, and 0.32 (0.04) for high-unemployment regimes.[24] These estimates are very close, suggesting that the micro-elasticity is acyclical as in the simulation of our calibrated model presented in Figure 3. These findings imply that the conventional Baily-Chetty formula would recommend a constant replacement rate over the business cycle, in sharp contrast with the optimal UI in our calibrated model, displayed in Figure 2.

5.2 Estimating the macro-elasticity of unemployment

The ideal experiment to estimate the macro-elasticity is to offer higher UI benefits to all individuals in a randomly selected subset of labor markets and compare unemployment durations across treated and control labor markets. Estimating the macro-elasticity is inherently more difficult than

[23] A spell is in a low unemployment regime if the quarterly unemployment rate of the state is below the median unemployment rate of all states in the US at the beginning of the spell.

[24] Those estimates are lower than the classic estimates of Meyer [1990]. As shown in Table A1 in the Appendix, we can replicate almost perfectly the higher magnitude of the estimates of Meyer [1990] when using his exact set of controls (i.e., not including state$\times$year fixed effects nor non-parametric controls in prior wages and experience). This suggests that the discrepancy in magnitude is likely due to omitted variable bias in Meyer [1990] estimates.

estimating the micro-elasticity because it requires finding exogenous variation in benefits across a large set of otherwise comparable labor markets, instead of exogenous variation across individuals within a single labor market. Estimating the cyclicality of the macro-elasticity would require repeating the same experiment for different initial levels of labor market tightness. Although no existing study offers compelling identification of the macro-elasticity, studies comparing individuals with different benefits across labor markets—for example across US states or within state over time—capture mainly macro-elasticities. Moffitt [1985] finds that estimates of the elasticity of unemployment duration with respect to unemployment benefits decline significantly with state unemployment rates. More recently, Valletta and Kuang [2010] find modest effects of unemployment benefit extensions on average unemployment in the US Great Recession. Using survey data, Kroft and Notowidigdo [2011] use variation in average benefits within states over time, controlling for state fixed effects. They provide the most convincing evidence to date that the elasticity of duration with respect to benefits is smaller when state unemployment is higher, suggesting that the macro-elasticity is countercyclical. In contrast to our basic job-rationing theory, their estimates are larger than our micro-elasticity estimates presented above. This could be due to differences in time periods and data, potential endogeneity in the variation of average state benefits over time in Kroft and Notowidigdo [2011], or other factors increasing the macro-elasticity (such as wage bargaining discussed at the end of Section 3.2). Unfortunately the CWBH data used above do not span a long enough time period, and therefore do not include sufficient variation in average benefits within state over time, for us to investigate the cyclicality of the macro-elasticity.[25]

5.3 Alternative: estimating the wedge between micro- and macro-elasticity

As it is difficult to obtain comparable estimates of both the micro- and macro-elasticity, it may be easier to estimate directly the wedge between micro- and macro-elasticity. This could be done by analyzing whether there are search spillovers. The ideal experiment is to offer higher benefits to a large fraction of randomly chosen individuals in a randomly selected subset of labor mar-

[25] The elasticity of duration with respect to average benefits in each state$\times$quarter found fitting our Cox proportional hazard model without state fixed effects is higher for low- than for high-unemployment regimes. The validity of such estimates, however, is questionable because they suffer from a potentially serious omitted variable bias.

kets and compare unemployment durations of *untreated* individuals across treated labor markets and control labor markets. Because a change in benefits for a *large* fraction of workers within a labor market affects aggregate search effort, it affects labor market tightness and ultimately unemployment durations of workers who did not experience a change in benefits. A small body of work finds evidence of such a rat-race externality, although identification is not fully satisfactory. Levine [1993] finds that an increase in benefits for insured unemployed workers results in a reduction of unemployment duration among uninsured workers. Burgess and Profit [2001] also find evidence of such externalities across neighboring areas. Policy variation could be used to credibly test for spillover effects. For example, the Regional Extended Benefit Program (REBP) in Austria dramatically increased the duration of benefits from 30 to 209 weeks for workers aged above 50 in some regions of Austria during 1988–1993. Lalive [2008] shows that this program led to a large decrease in job-search effort for treated workers. Evaluating whether comparable untreated workers in treated regions experience a reduction in unemployment duration could provide a compelling estimate of spillover effects. This project is left for future research.

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Appendix

A Proofs

A.1 Notations

We define the following functions, which we study in the Appendix:

- Labor supply: $n^s(e,\theta)$ is increasing in e and θ, and is defined by

$$n^s(e,\theta) = (1-u) + u \cdot e \cdot f(\theta).$$

- Labor demand: for $\alpha < 1$, $n^d(\theta,a)$ is increasing in a, decreasing in θ, and is defined by

$$n^d(\theta,a) = \left\{ \frac{1}{\alpha} \left(\omega \cdot a^{\gamma-1} + \frac{r}{q(\theta)} \right) \right\}^{1/(\alpha-1)}.$$

- Effort supply: $e^s(\theta,\Delta v)$ is increasing in θ and Δv, and is defined implicitly by

$$k'(e^s) = f(\theta) \cdot \Delta v.$$

- A useful constant: χ is defined by

$$\chi = (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{1-\eta}{\eta} \cdot \frac{1}{r}$$

- Ratio of elasticities $\left(\varepsilon^m/\varepsilon^M - 1\right)$: $Q(n,\theta)$ is defined by

$$Q(n,\theta) = \chi \cdot q(\theta) \cdot \left[\frac{n-(1-u)}{n} \right] \cdot n^{\alpha-1} = \chi \cdot q(\theta) \cdot \left(\frac{h}{n} \right) \cdot n^{\alpha-1} \tag{A1}$$

- Another ratio of elasticities $n \cdot \left(\varepsilon^m/\varepsilon^M - 1\right)$: $T(n,\theta)$ is defined by

$$T(n,\theta) = \chi \cdot q(\theta) \cdot [n-(1-u)] \cdot n^{\alpha-1} = \chi \cdot q(\theta) \cdot h \cdot n^{\alpha-1}$$

- Equilibrium labor market tightness: $\theta(a,\Delta v)$ is defined implicitly by

$$n^d(\theta,a) = n^s\left(e^s(\theta,\Delta v),\theta\right)$$

- Equilibrium effort: $e(a,\Delta v)$ is defined by

$$e(a,\Delta v) = e^s(\theta(a,\Delta v),\Delta v)$$

- Equilibrium employment: $n(a, \Delta v)$ is defined by

$$n(a, \Delta v) = n^s(e(a, \Delta v), \theta(a, \Delta v))$$

- Ratio of elasticities $\left(\varepsilon^m/\varepsilon^M - 1\right)$: $R(a, \Delta v)$ is defined by

$$R(a, \Delta v) = Q(n(a, \Delta v), \theta(a, \Delta v))$$

- Another ratio of elasticities $n \cdot \left(\varepsilon^m/\varepsilon^M - 1\right)$: $X(a, \Delta v)$ is defined by

$$X(a, \Delta v) = T(n(a, \Delta v), \theta(a, \Delta v)) \tag{A2}$$

- Incentive to search: $\Delta v^*(a, \tau)$ is defined implicitly by the system:

$$\Delta v^* = v(c^e) - v(c^u)$$
$$n(a, \Delta v^*) \cdot w(a) = n(a, \Delta v^*) \cdot c^e + [1 - n(a, \Delta v^*)] \cdot c^u$$
$$\tau = c^u/c^e$$

- Equilibrium labor market tightness: $\theta^*(a, \tau)$ is defined by

$$\theta^*(a, \tau) = \theta(a, \Delta v^*(a, \tau))$$

- Equilibrium effort: $e^*(a, \tau)$ is defined by

$$e^*(a, \tau) = e(\theta^*(a, \tau), \Delta v^*(a, \tau))$$

- Equilibrium employment: $n^*(a, \tau)$ is defined by

$$n^*(a, \tau) = n(a, \Delta v^*(a, \tau))$$

- Ratio of elasticities $\left(\varepsilon^m/\varepsilon^M - 1\right)$: $R^*(a, \tau)$ is defined by

$$R^*(a, \tau) = R(a, \Delta v^*(a, \tau))$$

- Another ratio of elasticities $n \cdot \left(\varepsilon^m/\varepsilon^M - 1\right)$: $X^*(a, \tau)$ is defined by

$$X^*(a, \tau) = X(a, \Delta v^*(a, \tau))$$

- Incentive to search: $\Delta v^{\dagger}(a, \Delta c)$ is defined implicitly by the system:

$$\Delta v^{\dagger} = v(c^e) - v(c^u)$$
$$n(a, \Delta v^{\dagger}) \cdot w(a) = n(a, \Delta v^{\dagger}) \cdot c^e + \left[1 - n(a, \Delta v^{\dagger})\right] \cdot c^u$$
$$\Delta c = c^e - c^u$$

- Equilibrium labor market tightness: $\theta^{\dagger}(a, \Delta c)$ is defined by

$$\theta^{\dagger}(a, \Delta c) = \theta(a, \Delta v^{\dagger}(a, \Delta c))$$

- Equilibrium effort: $e^{\dagger}(a, \Delta c)$ is defined by

$$e^{\dagger}(a, \Delta c) = e(\theta^{\dagger}(a, \Delta c), \Delta v^{\dagger}(a, \Delta c))$$

- Equilibrium employment: $n^{\dagger}(a, \Delta c)$ is defined by

$$n^{\dagger}(a, \Delta c) = n(a, \Delta v^{\dagger}(a, \Delta c))$$

- Equilibrium consumption: $c^{u\dagger}(a, \Delta c)$ is defined by

$$c^{u\dagger}(a, \Delta c) = n^{\dagger}(a, \Delta c) \cdot (w(a) - \Delta c)$$

As in the text, we formally define the elasticities of unemployment with respect to UI "in consumption":

$$\varepsilon^M \equiv \frac{\Delta v}{1-n} \cdot \frac{\partial n^{\dagger}}{\partial \Delta c}$$
$$\varepsilon^m \equiv \frac{\Delta v}{1-n} \cdot \left[\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta v}\right] \cdot \frac{\partial \Delta v^{\dagger}}{\partial \Delta c}.$$

We define the elasticities of unemployment with respect to UI "in utility":

$$\varepsilon_v^M \equiv \frac{\Delta v}{1-n} \cdot \frac{\partial n}{\partial \Delta v} \tag{A3}$$
$$\varepsilon_v^m \equiv \frac{\Delta v}{1-n} \cdot \left[\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta v}\right]. \tag{A4}$$

We also define the following elasticities:

$$\varepsilon_a^n \equiv \frac{a}{n} \cdot \frac{\partial n}{\partial a}$$
$$\varepsilon_{\Delta v}^{\theta} \equiv \frac{\Delta v}{\theta} \cdot \frac{\partial \theta}{\partial \Delta v}$$
$$\varepsilon_a^{\Delta v*} \equiv \frac{a}{\Delta v} \cdot \frac{\partial \Delta v^*}{\partial a}$$
$$\varepsilon_a^{n*} \equiv \frac{a}{n} \cdot \frac{\partial n^*}{\partial a}$$
$$\varepsilon_{\Delta c}^{\Delta v \dagger} \equiv \frac{\Delta c}{\Delta v} \cdot \frac{\partial \Delta v^\dagger}{\partial \Delta c}.$$

A.2 Proof of Lemma 1

We prove that the partial derivative of equilibrium labor market tightness $\theta^\dagger(a, \Delta c)$ satisfies:

$$\varepsilon_{\Delta c}^{\theta \dagger} \equiv \frac{\Delta c}{\theta} \cdot \frac{\partial \theta^\dagger}{\partial \Delta c} = \frac{\kappa}{\kappa + 1} \cdot \frac{1}{1 - \eta} \cdot \frac{1-n}{h} \cdot \left(\varepsilon^M - \varepsilon^m\right).$$

By definition:

$$\frac{1-n}{\Delta v}\left(\varepsilon_v^M - \varepsilon_v^m\right) = \frac{\partial n}{\partial \Delta v} - \frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta v}$$
$$\frac{\partial n}{\partial \Delta v} = \frac{\partial n^s}{\partial e} \cdot \left(\frac{\partial e^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta v} + \frac{\partial e^s}{\partial \Delta v}\right) + \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta v}$$
$$\frac{\partial n}{\partial \Delta v} - \frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta v} = \left(\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} + \frac{\partial n^s}{\partial \theta}\right) \cdot \frac{\partial \theta}{\partial \Delta v}$$
$$\frac{1-n}{\Delta v}\left(\varepsilon_v^M - \varepsilon_v^m\right) = \left(\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} + \frac{\partial n^s}{\partial \theta}\right) \cdot \frac{\partial \theta}{\partial \Delta v}.$$

LEMMA A1. *Denote $\kappa = e \cdot k''(e)/k'(e)$ the elasticity of the marginal disutility of effort $k'(e)$, and denote $1 - \eta = \theta \cdot f'(\theta)/f(\theta)$ the elasticity of the per-unit job-finding probability. The partial derivatives of effort supply $e^s(\theta, \Delta v)$ satisfy:*

$$\frac{\Delta v}{e^s} \cdot \frac{\partial e^s}{\partial \Delta v} = \frac{1}{\kappa}$$
$$\frac{\theta}{e^s} \cdot \frac{\partial e^s}{\partial \theta} = \frac{1-\eta}{\kappa}.$$

Proof. The worker's optimal choice of effort (1) gives $k'(e^s) = f(\theta) \cdot \Delta v$. Thus, differentiating with respect to Δv (keeping θ constant):

$$k''(e^s) \cdot \frac{\partial e^s}{\partial \Delta v} = \frac{k'(e^s)}{\Delta v}$$
$$\frac{\Delta v}{e^s} \cdot \frac{\partial e^s}{\partial \Delta v} = \frac{1}{\kappa}$$

And differentiating with respect to θ (keeping Δv constant):

$$k''(e^s) \cdot \frac{\partial e^s}{\partial \theta} = (1-\eta) \cdot \frac{k'(e^s)}{\theta}$$
$$\frac{\theta}{e^s} \cdot \frac{\partial e^s}{\partial \theta} = \frac{1-\eta}{\kappa}.$$

□

LEMMA A2. *Denote $\kappa = e \cdot k''(e)/k'(e)$ the elasticity of the marginal disutility of effort $k'(e)$, and denote $1-\eta = \theta \cdot f'(\theta)/f(\theta)$ the elasticity of the per-unit job-finding probability. Denote $h = u \cdot e \cdot f(\theta) = n^s(e,\theta) - (1-u)$ the number of new hires. The partial derivatives of labor supply $n^s(e,\theta)$ satisfy:*

$$\frac{\partial n^s}{\partial \theta} = (1-\eta) \cdot \frac{h}{\theta}$$
$$\frac{\partial n^s}{\partial e} = \frac{h}{e}$$
$$\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} = \frac{1}{\kappa} \cdot \frac{\partial n^s}{\partial \theta}.$$

Proof. The first two results are obvious using the definition (2) of labor supply: $n^s(e,\theta) = (1-u) + u \cdot e \cdot f(\theta)$. Then using Lemma A1:

$$\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} = \frac{1-\eta}{\kappa} \cdot \frac{h}{\theta} = \frac{1}{\kappa} \cdot \frac{\partial n^s}{\partial \theta}.$$

□

Using Lemma A2, we write:

$$\frac{1-n}{\Delta v}\left(\varepsilon_v^M - \varepsilon_v^m\right) = \frac{1+\kappa}{\kappa} \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta v}$$
$$\Delta v \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta v} = \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot \left(\varepsilon_v^M - \varepsilon_v^m\right)$$
$$\frac{\Delta v}{\theta} \cdot \frac{\partial \theta}{\partial \Delta v} = \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{h} \cdot \left(\varepsilon_v^M - \varepsilon_v^m\right).$$

Multiplying each equation by $\frac{\Delta c}{\Delta v} \cdot \frac{\partial \Delta v^\dagger}{\partial \Delta c}$ yields our result. The second result in the lemma is obtained by combining the first result with the result of Lemma A2. Note that these results can be rewritten in terms of the partial derivative of equilibrium labor market tightness $\theta(a, \Delta v)$:

$$\varepsilon^{\theta}_{\Delta v} \equiv \frac{\Delta v}{\theta} \cdot \frac{\partial \theta}{\partial \Delta v} = \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{h} \cdot \left(\varepsilon^M_v - \varepsilon^m_v\right)$$
$$\frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta v} = -\frac{1-n}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\varepsilon^m_v - \varepsilon^M_v\right).$$

A.3 Proof of Proposition 1

The government chooses Δc to maximize:

$$(1-u) \cdot v(c^e) + u \cdot [v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)] = n^s(e,\theta) \cdot v(c^u + \Delta c) + (1 - n^s(e,\theta)) \cdot v(c^u) - u \cdot k(e)$$

Using the envelope theorem (as workers choose search effort e to maximize $v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$), the first-order condition becomes

$$0 = \left[n \cdot v'(c^e) + (1-n) v'(c^u)\right] \cdot \frac{\partial c^{u\dagger}}{\partial \Delta c} + n \cdot v'(c^e) + \Delta v \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta^\dagger}{\partial \Delta c}$$
$$0 = \overline{v'} \cdot \frac{\partial c^{u\dagger}}{\partial \Delta c} + n \cdot v'(c^e) + \Delta v \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta^\dagger}{\partial \Delta c}$$

where we define $\overline{v'} \equiv [n \cdot v'(c^e) + (1-n) \cdot v'(c^u)]$.

Fist step. Lemma 1 allows us to write:

$$\Delta v \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta^\dagger}{\partial \Delta c} = \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot \left(\varepsilon^M - \varepsilon^m\right) < 0.$$

This negative term is the cost of the job-rationing externality.

LEMMA A3.

$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n$$

Proof. We start from the budget constraint, which defines $c^{u\dagger}(a, \Delta c)$:

$$c^{u\dagger}(a, \Delta c) = n^\dagger(a, \Delta c) \cdot [w(a) - \Delta c]$$
$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{\Delta c} \cdot [w(a) - \Delta c] \cdot \varepsilon^M - n$$
$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{n} \cdot \frac{c^u}{\Delta c} \cdot \varepsilon^M - n$$
$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n.$$

□

Second step. Lemma A3 exploits the budget constraint to yield:

$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n.$$

Third step. We come back to the formula:

$$0 = \overline{v'} \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \varepsilon^M - n \right] + n \cdot v'(c^e) + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot \left(\varepsilon^M - \varepsilon^m\right)$$
$$0 = \overline{v'} \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \varepsilon^M \right] + n \cdot \left[v'(c^e) - \overline{v'}\right] + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot \left(\varepsilon^M - \varepsilon^m\right)$$
$$0 = \overline{v'} \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \varepsilon^M \right] + n(1-n) \left[v'(c^e) - v'(c^u)\right] + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot \left(\varepsilon^M - \varepsilon^m\right)$$

Dividing the equation by $(1-n) \cdot \varepsilon^M \cdot \overline{v'}$ yields:

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{n}{\varepsilon^M} \cdot \frac{1}{\overline{v'}} \cdot \left[v'(c^u) - v'(c^e)\right] + \frac{\Delta v}{\overline{v'}\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right)$$
$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{v'(c^e)}{\overline{v'}} \left[\frac{n}{\varepsilon^M} \cdot \left\{ \frac{v'(c^u)}{v'(c^e)} - 1 \right\} + \frac{\Delta v}{v'(c^e)\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right]$$
$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \left[n + (1-n) \cdot \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \cdot \left[\frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e)\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right]$$

Approximation: Assuming $n \approx 1$ allows us to simplify the optimal formula to

$$\frac{\tau}{1-\tau} = \frac{1}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right)$$

If the third and higher order terms of $v(\cdot)$ are small ($v'''(c) \approx 0$), we can make the following approximations:

$$\frac{\Delta v}{v'(c^e) \cdot \Delta c} \approx 1 - \frac{1}{2} \cdot \frac{v''(c^e)}{v'(c^e)} \cdot \frac{c^e}{c^e} \cdot [c^e - c^u] = 1 + \frac{1}{2} \cdot \rho \cdot (1-\tau)$$
$$\frac{v'(c^u)}{v'(c^e)} \approx \frac{v'(c^e) - v''(c^e) \cdot c^e \cdot \frac{\Delta c}{c^e}}{v'(c^e)} = 1 + \rho(1-\tau).$$

ρ is the coefficient of relative risk aversion of the utility function measured at c^e. The optimal UI formula becomes:

$$\frac{\tau}{1-\tau} = \frac{1}{\varepsilon^M} \cdot \rho \cdot [1-\tau] + \frac{\kappa}{\kappa+1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right] \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right].$$

A.4 Proof of Proposition 2

Step 1: Equivalence with ratio "in utility". Notice that $n^\dagger(a,\Delta c) = n(a,\Delta v^\dagger(a,\Delta c))$. Hence the micro- and macro-elasticity "in consumption" defined in the text can be rewritten as:

$$\varepsilon^M = \frac{\Delta c}{\Delta v} \cdot \left(\frac{\Delta v}{1-n} \cdot \frac{\partial n}{\partial \Delta v}\right) \cdot \frac{\partial \Delta v^\dagger}{\partial \Delta c} \tag{A5}$$

$$\varepsilon^m = \frac{\Delta c}{\Delta v} \cdot \left(\frac{\Delta v}{1-n} \cdot \frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta v}\right) \cdot \frac{\partial \Delta v^\dagger}{\partial \Delta c} \tag{A6}$$

The ratio of micro- and macro-elasticity are the same with both definitions:

$$\frac{\varepsilon^m}{\varepsilon^M} = \frac{\varepsilon^m_v}{\varepsilon^M_v}.$$

In this step, we work with elasticities "in utility" as they are easier to manipulate.

Step 2: Differentiate the firm's first-order condition. The firm's first-order condition, evaluated at the equilibrium, is

$$g'\left(n(a,\Delta v)\right) - w(a)/a = \frac{r}{q\left(\theta(a,\Delta v)\right)}$$

Under Assumption 3, we differentiate the first-order condition with respect to Δv keeping a constant, and we use Lemma 1:

$$\begin{aligned}
(\alpha-1) \cdot \frac{g'(n)}{n} \cdot \frac{\partial n}{\partial \Delta v} &= \eta \cdot \frac{r}{q(\theta)} \cdot \frac{1}{\theta} \cdot \frac{\partial \theta}{\partial \Delta v} \\
(\alpha-1) \cdot g'(n) \cdot \frac{1-n}{n} \cdot \varepsilon^M_v &= \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1-n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(\varepsilon^M_v - \varepsilon^m_v\right) \\
-(1-\alpha) \cdot g'(n) &= \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(1 - \frac{\varepsilon^m_v}{\varepsilon^M_v}\right) \\
\frac{\varepsilon^m_v}{\varepsilon^M_v} &= 1 + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{1}{r} \cdot \frac{1-\eta}{\eta} \cdot q(\theta) \cdot \left(\frac{h}{n}\right) \cdot n^{\alpha-1}.
\end{aligned} \tag{A7}$$

Then since $\theta > 0$, $h > 0$, $\eta \in (0,1)$: $\varepsilon^m/\varepsilon^M > 1$ iff $\alpha \in (0,1)$.

A.5 Some preliminary comparative-static results

LEMMA A4. *Denote $\kappa = e \cdot k''(e)/k'(e)$ the elasticity of the marginal disutility of effort $k'(e)$, and denote $h = u \cdot e \cdot f(\theta) = n^s(e,\theta) - (1-u)$ the number of new hires. The micro-elasticity "in utility" satisfies:*

$$\varepsilon_v^m = \frac{h}{1-n} \cdot \frac{1}{\kappa}.$$

Proof. The definition (A4) of ε_v^m gives

$$\varepsilon_v^m = \left(\frac{\Delta v}{1-n}\right) \cdot \left(\frac{\partial n^s}{\partial e}\right) \cdot \left(\frac{\partial e^s}{\partial \Delta v}\right).$$

Using Lemma A1 and Lemma A2:

$$\frac{\partial e^s}{\partial \Delta v} = \frac{e}{\Delta v} \cdot \frac{1}{\kappa}$$
$$\frac{\partial n^s}{\partial e} = \frac{h}{e}.$$

Therefore,

$$\varepsilon_v^m = \left(\frac{\Delta v}{1-n}\right) \cdot \left(\frac{h}{e}\right) \cdot \left(\frac{e}{\Delta v} \cdot \frac{1}{\kappa}\right)$$
$$\varepsilon_v^m == \frac{h}{1-n} \cdot \frac{1}{\kappa}.$$

□

LEMMA A5. *If $\gamma \in [0,1)$ and $\alpha \in (0,1)$, we have the following comparative statics for equilibrium variables:*

$$\frac{\partial \theta}{\partial a} > 0, \;\; \frac{\partial e}{\partial a} > 0, \;\; \frac{\partial n}{\partial a} > 0.$$

Proof. We have the following comparative statics:

$$\frac{\partial e^s}{\partial \theta} > 0, \;\; \frac{\partial n^s}{\partial \theta} > 0, \;\; \frac{\partial n^s}{\partial e} > 0. \tag{A8}$$

Assume $\gamma \in [0,1)$ and $\alpha \in (0,1)$. We have $\frac{\partial n^d}{\partial \theta} < 0, \frac{\partial n^d}{\partial a} > 0$. Differentiating equilibrium condition (3) with respect to a , keeping Δv constant:

$$\left[\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial \theta}+\frac{\partial n^s}{\partial \theta}\right]\cdot\frac{\partial \theta}{\partial a}=\frac{\partial n^d}{\partial a}+\frac{\partial n^d}{\partial \theta}\cdot\frac{\partial \theta}{\partial a}$$

$$\frac{\partial \theta}{\partial a}=\underbrace{\frac{\partial n^d}{\partial a}}_{+}\cdot\left[\left\{\underbrace{\frac{\partial n^s}{\partial e}}_{+}\cdot\underbrace{\frac{\partial e^s}{\partial \theta}}_{+}+\underbrace{\frac{\partial n^s}{\partial \theta}}_{+}\right\}-\underbrace{\frac{\partial n^d}{\partial \theta}}_{-}\right]^{-1}.$$

Thus we infer that $\frac{\partial \theta}{\partial a}>0$. We conclude by using the comparative statics (A8) and noting that $e(a,\Delta v)=e^s(\theta(a,\Delta v),\Delta v)$ and $n(a,\Delta v)=n^s(e(a,\Delta v),\theta(a,\Delta v))$. □

LEMMA A6. *If $\alpha\in(0,1)$, we have the following comparative statics for equilibrium variables:*

$$\frac{\partial e}{\partial \Delta v}>0,\ \frac{\partial n}{\partial \Delta v}>0,\ \frac{\partial \theta}{\partial \Delta v}<0.$$

Proof. We have the following comparative statics:

$$\frac{\partial e^s}{\partial \theta}>0,\ \frac{\partial e^s}{\partial \Delta v}>0,\ \frac{\partial n^s}{\partial \theta}>0,\ \frac{\partial n^s}{\partial e}>0. \tag{A9}$$

Notice that $\frac{\partial n^d}{\partial \theta}<0$. Differentiating equilibrium condition on the labor market (3) with respect to Δv, keeping a constant, yields:

$$\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial \Delta v}+\left[\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial \theta}+\frac{\partial n^s}{\partial \theta}\right]\cdot\frac{\partial \theta}{\partial \Delta v}=\frac{\partial n^d}{\partial \theta}\cdot\frac{\partial \theta}{\partial \Delta v}$$

$$\frac{\partial \theta}{\partial \Delta v}=-\left[\underbrace{\frac{\partial n^s}{\partial e}}_{+}\cdot\underbrace{\frac{\partial e^s}{\partial \Delta v}}_{+}\right]\cdot\left[\left\{\underbrace{\frac{\partial n^s}{\partial e}}_{+}\cdot\underbrace{\frac{\partial e^s}{\partial \theta}}_{+}+\underbrace{\frac{\partial n^s}{\partial \theta}}_{+}\right\}-\underbrace{\frac{\partial n^d}{\partial \theta}}_{-}\right]^{-1}.$$

We can conclude that $\frac{\partial \theta}{\partial \Delta v}<0$. We conclude by using (A9), noting that $n(a,\Delta v)=n^d(a,\theta(a,\Delta v))$, and

$$e(a,\Delta v)=\frac{n(a,\Delta v)-(1-u)}{u\cdot f(\theta(a,\Delta v))}.$$

□

LEMMA A7. *Under Assumptions 1, 2,and 3, $\frac{\partial X}{\partial \Delta v}>0$. Furthermore, if $\eta\geq\frac{1+\kappa}{1+2\cdot\kappa}$, then $\frac{\partial X}{\partial a}<0$.*

Proof. We make Assumption 3 such that $T(n,\theta)$ and $X(a,\Delta v)=T(n(a,\Delta v),\theta(a,\Delta v))$ be well defined. Recall that

$$T(n,\theta) = \chi \cdot q(\theta) \cdot [n-(1-u)] \cdot n^{\alpha-1}$$
$$\chi = (1-\alpha)\cdot\alpha\cdot\frac{\kappa+1}{\kappa}\cdot\frac{1-\eta}{\eta}\cdot\frac{1}{r}.$$

We make Assumption 1 such that $T > 0$, $X > 0$.

Step 1. From the definition of $T(n,\theta)$ it is clear that

$$\frac{\partial T}{\partial \theta} < 0, \ \ \frac{\partial T}{\partial n} > 0.$$

From Lemma A6, under Assumptions 1,

$$\frac{\partial \theta}{\partial \Delta v} < 0, \ \ \frac{\partial n}{\partial \Delta v} > 0.$$

We can conclude that $\frac{\partial X}{\partial \Delta v} > 0$ because

$$\frac{\partial X}{\partial \Delta v} = \underbrace{\frac{\partial T}{\partial \theta}}_{-}\cdot\underbrace{\frac{\partial \theta}{\partial \Delta v}}_{-} + \underbrace{\frac{\partial T}{\partial n}}_{+}\cdot\underbrace{\frac{\partial n}{\partial \Delta v}}_{+}.$$

Step 2. Using the equilibrium condition $n(a,\Delta v) = n^s(e^s(\theta(a,\Delta v),\Delta v),\theta(a,\Delta v))$ and Lemma A2:

$$\frac{\partial X}{\partial a} = -\eta\cdot\frac{X}{\theta}\cdot\frac{\partial\theta}{\partial a} + \frac{X}{n}\cdot\left\{(\alpha-1)+\frac{n}{h}\right\}\cdot\left[\frac{\partial n^s}{\partial\theta}+\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial\theta}\right]\cdot\frac{\partial\theta}{\partial a}$$
$$\frac{\partial X}{\partial a} = \frac{X}{\theta}\cdot\frac{\partial\theta}{\partial a}\cdot\left[-\eta+\left\{(\alpha-1)+\frac{n}{h}\right\}\cdot\frac{\theta}{n}\cdot\left[\frac{\partial n^s}{\partial\theta}+\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial\theta}\right]\right]$$
$$\frac{\partial X}{\partial a} = \frac{X}{\theta}\cdot\frac{\partial\theta}{\partial a}\cdot\left\{-\eta+\left[1+(\alpha-1)\cdot\frac{h}{n}\right]\cdot(1-\eta)\cdot\frac{\kappa+1}{\kappa}\right\}.$$

From Lemma A5, under Assumption 2, $\frac{\partial\theta}{\partial a} > 0$. Hence $\frac{\partial X}{\partial a} < 0$ iff

$$-\eta+\left[1+(\alpha-1)\cdot\frac{h}{n}\right]\cdot(1-\eta)\cdot\frac{\kappa+1}{\kappa} < 0$$
$$\frac{\eta}{1-\eta} > \left[1+(\alpha-1)\cdot\frac{h}{n}\right]\cdot\frac{\kappa+1}{\kappa}$$
$$\eta > \frac{1}{1+\frac{\kappa}{\kappa+1}\cdot\left[1-(1-\alpha)\cdot\frac{h}{n}\right]^{-1}}$$

Since $h/n > 0$, a sufficient condition for $\frac{\partial X}{\partial a} < 0$ is $\eta \geq \frac{1+\kappa}{1+2\cdot\kappa}$. □

LEMMA A8. *Under Assumptions 1, 2,and 3,* $\frac{\partial R}{\partial \Delta v} > 0$. *Furthermore,* $\frac{\partial R}{\partial a} < 0$ *if* $\eta \geq \frac{1+\kappa}{1+2\cdot\kappa}$.

Proof. We make Assumption 3 such that $R(a, \Delta v)$ be well defined. We make Assumption 1 such that $R > 0$.

Step 1. Using the equilibrium condition $n(a, \Delta v) = n^s(e^s(\theta(a, \Delta v), \Delta v), \theta(a, \Delta v))$, we differentiate $R(a, \Delta v)$ with respect to a:

$$\frac{\partial R}{\partial a} = -\eta \cdot \frac{R}{\theta} \cdot \frac{\partial \theta}{\partial a} + \frac{R}{n} \cdot \left\{(\alpha - 2) + \frac{n}{h}\right\} \cdot \left[\frac{\partial n^s}{\partial \theta} + \frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta}\right] \cdot \frac{\partial \theta}{\partial a}$$

$$\frac{\partial R}{\partial a} \Delta v = \frac{R}{\theta} \cdot \frac{\partial \theta}{\partial a} \cdot \left[-\eta + \left\{(\alpha - 2) + \frac{n}{h}\right\} \cdot \frac{\theta}{n} \cdot \left[\frac{\partial n^s}{\partial \theta} + \frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta}\right]\right].$$

As in the proof of Lemma A7, we conclude that

$$\frac{\partial R}{\partial a} = \frac{R}{\theta} \cdot \frac{\partial \theta}{\partial a} \Delta v \cdot \left\{-\eta + \left[1 + (\alpha - 2) \cdot \frac{h}{n}\right] \cdot (1 - \eta) \cdot \frac{\kappa + 1}{\kappa}\right\}$$

Under Assumption 2: $\frac{\partial \theta}{\partial a} > 0$. Hence $\frac{\partial R}{\partial a} < 0$ iff

$$\eta > \frac{1}{1 + \frac{\kappa}{\kappa+1} \cdot \left[1 - (2 - \alpha) \cdot \frac{h}{n}\right]^{-1}}$$

Since $h/n > 0$, a sufficient condition for $\frac{\partial R}{\partial a} < 0$ is $\eta \geq \frac{1+\kappa}{1+2\cdot\kappa}$.

Step 2. Differentiating $R(a, \Delta v)$ with respect to Δv yields:

$$\frac{\partial R}{\partial \Delta v} = -\eta \cdot \frac{R}{\Delta v} \cdot \varepsilon^{\theta}_{\Delta v} + \frac{R}{n} \cdot \left\{(\alpha - 2) + \frac{n}{h}\right\} \cdot \frac{1 - n}{\Delta v} \cdot \varepsilon^M_v.$$

Using the result from Lemma 1:

$$\frac{\partial R}{\partial \Delta v} = \frac{1 - n}{n} \cdot \frac{R}{\Delta v} \cdot \varepsilon^M_v \cdot \left\{\frac{\eta}{1 - \eta} \cdot \frac{n}{h} \cdot \frac{\kappa}{1 + \kappa} \cdot Q(n, \theta) - (1 - \alpha) + \left(\frac{n}{h} - 1\right)\right\}.$$

Notice that, using the firm's FOC, and under Assumption 1:

$$\frac{\eta}{1-\eta}\cdot\frac{n}{h}\cdot\frac{\kappa}{1+\kappa}\cdot Q(n,\theta) = (1-\alpha)\cdot\alpha\cdot n^{\alpha-1}\cdot\frac{q(\theta)}{r}$$
$$\frac{\eta}{1-\eta}\cdot\frac{n}{h}\cdot\frac{\kappa}{1+\kappa}\cdot Q(n,\theta) = (1-\alpha)\left\{\frac{w(a)}{a}\cdot\frac{q(\theta)}{r}+1\right\}$$
$$\frac{\eta}{1-\eta}\cdot\frac{n}{h}\cdot\frac{\kappa}{1+\kappa}\cdot Q(n,\theta)-(1-\alpha) = (1-\alpha)\cdot\frac{w(a)}{a}\cdot\frac{q(\theta)}{r}>0$$

Since $h/n\leq 1$, we conclude that $\frac{\partial R}{\partial \Delta v}>0$. □

LEMMA A9. *Under Assumptions 3:*

$$\varepsilon_a^{\Delta v*} = (1-\rho)\cdot\frac{\gamma+H(n,\tau)\cdot\varepsilon_a^n}{1-(1-\rho)\cdot H(n,\tau)\cdot\frac{1-n}{n}\cdot\varepsilon_v^M},$$

where we define $H(n,\tau)\equiv\tau/\left[(1-\tau)\cdot n+\tau\right]>0$. Under Assumptions 2, and assuming that $\rho\geq 1$:

$$\frac{\partial \Delta v^*}{\partial a}\leq 0.$$

Proof. Under Assumption 3:

$$\Delta v = c_e^{1-\rho}\cdot\frac{1-\tau^{1-\rho}}{1-\rho}$$

From the government's budget constraint $n\cdot c_e+(1-n)\cdot c_u = n\cdot w(a)$:

$$c_e = \frac{w(a)}{(1-\tau)+\tau/n}$$

Combining both equations:

$$\Delta v = \left[(1-\tau)+\tau/n\right]^{\rho-1}\cdot w(a)^{1-\rho}\cdot j(\tau)$$

where we simplify the expression by defining:

$$j(\tau) = \frac{1-\tau^{1-\rho}}{1-\rho},$$

which satisfies $z'(\tau) = -\tau^{-\rho}<0$. Let us fix τ and consider a marginal change da. We denote

$\check{x} = d\ln(x) = dx/x$. Differentiating the expression above:

$$\check{\Delta v} = (1-\rho)\cdot\gamma\cdot\check{a} - (\rho-1)\cdot\check{n}\cdot\left[\frac{\tau}{(1-\tau)\cdot n+\tau}\right]$$
$$\varepsilon_a^{\Delta v*} = (1-\rho)\cdot\gamma - (\rho-1)\cdot\varepsilon_a^{n*}\cdot H(n,\tau).$$

Moreover, using the definition of $n^*(a,\tau)$:

$$\varepsilon_a^{n*} = \frac{a}{n}\left[\frac{\partial n}{\partial a} + \frac{\partial n}{\partial \Delta v}\cdot\frac{\partial \Delta v^*}{\partial a}\right]$$
$$\varepsilon_a^{n*} = \varepsilon_a^{n} + \frac{a}{n}\cdot\frac{1-n}{\Delta v}\cdot\varepsilon_v^{M}\cdot\frac{\Delta v}{a}\cdot\varepsilon_a^{\Delta v*}$$
$$\varepsilon_a^{n*} = \varepsilon_a^{n} + \frac{1-n}{n}\cdot\varepsilon_v^{M}\cdot\varepsilon_a^{\Delta v*},$$

which allows us to write:

$$\varepsilon_a^{\Delta v*}\cdot\left[1-(1-\rho)\cdot H(n,\tau)\cdot\frac{1-n}{n}\cdot\varepsilon_v^{M}\right] = (1-\rho)\cdot\gamma + (1-\rho)\cdot H(n,\tau)\cdot\varepsilon_a^{n}$$
$$\varepsilon_a^{\Delta v*} = (1-\rho)\cdot\frac{\gamma + H(n,\tau)\cdot\varepsilon_a^{n}}{1-(1-\rho)\cdot H(n,\tau)\cdot\frac{1-n}{n}\cdot\varepsilon_v^{M}}.$$

We know from Lemma A5 that under Assumption 2, $\frac{\partial n}{\partial a} > 0$ and from Lemma A6 that $\frac{\partial n}{\partial \Delta v} > 0$ such that $\varepsilon_v^M > 0$. We infer that if $\rho \geq 1$, $\frac{\partial \Delta v^*}{\partial a} \leq 0$. □

LEMMA A10. *Under Assumptions 1, 2, 3, and imposing* $\rho \geq 1$ *and* $\eta \geq (1+\kappa)/(1+2\kappa)$*:* $\frac{\partial R^*}{\partial a} < 0$.

Proof. Under Assumptions 1, 2, and 3, using Lemmas A8 and A9 when $\rho \geq 1$ and $\eta \geq \frac{1+\kappa}{1+2\cdot\kappa}$, then $\frac{\partial R^*}{\partial a} < 0$ because

$$\frac{\partial R^*}{\partial a} = \underbrace{\frac{\partial R}{\partial a}}_{-} + \underbrace{\frac{\partial R}{\partial \Delta v}}_{+}\cdot\underbrace{\frac{\partial \Delta v^*}{\partial a}}_{-}.$$

□

LEMMA A11. *Under Assumptions 1, 2, 3, and imposing* $\rho \geq 1$ *and* $\eta \geq (1+\kappa)/(1+2\kappa)$*:* $\frac{\partial X^*}{\partial a} < 0$.

Proof. Under Assumptions 1, 2, and 3, using Lemmas A7 and A9 when $\rho \geq 1$ and $\eta \geq \frac{1+\kappa}{1+2\cdot\kappa}$, then $\frac{\partial X^*}{\partial a} < 0$ because

$$\frac{\partial X^*}{\partial a} = \underbrace{\frac{\partial X}{\partial a}}_{-} + \underbrace{\frac{\partial X}{\partial \Delta v}}_{+}\cdot\underbrace{\frac{\partial \Delta v^*}{\partial a}}_{-}.$$

□

LEMMA A12. *We make Assumptions 1 and 3. We impose $\rho \geq 1$ as well as*

$$\frac{1-\gamma}{\gamma} > (\rho-1)\cdot\frac{\eta}{1-\eta}\cdot\frac{1}{\kappa+1}\cdot\Omega,$$

where the constant $\Omega \in (0,+\infty)$ satisfies

$$\Omega < \left[\frac{\alpha}{\omega}\cdot\frac{\overline{a}^{1-\gamma}}{(1-u)^{1-\alpha}}\right]-1,$$

where $\overline{a} = \sup\mathcal{A}$. Then:

$$\frac{\partial n^*}{\partial a} > 0.$$

Proof. By definition and using Lemma A9:

$$\begin{aligned}
\frac{\partial n}{\partial a} &= \frac{n}{a}\cdot\varepsilon_a^n > 0\\
\frac{\partial n}{\partial \Delta v} &= \frac{1-n}{\Delta v}\cdot\varepsilon_v^M > 0\\
\frac{\partial \Delta v^*}{\partial a} &= (1-\rho)\cdot\frac{\Delta v}{a}\cdot\frac{\gamma+H(n,\tau)\cdot\varepsilon_a^n}{1-(1-\rho)\cdot H(n,\tau)\cdot\varepsilon_v^M\cdot\frac{1-n}{n}}
\end{aligned}$$

Recall that $n^*(a,\tau) = n(a,\Delta v^*(a,\tau))$. Given that $\rho \geq 1$, the sign of $\frac{\partial n^*}{\partial a}$ is also the sign of

$$\begin{aligned}
&n\cdot\varepsilon_a^n\cdot\left[1-(1-\rho)\cdot H(n,\tau)\cdot\varepsilon_v^M\cdot\frac{1-n}{n}\right]+\gamma\cdot(1-\rho)\cdot(1-n)\cdot\varepsilon_v^M+(1-\rho)\cdot(1-n)\cdot\varepsilon_v^M\cdot H(n,\tau)\cdot\varepsilon_a^n\\
&= n\cdot\varepsilon_a^n+\gamma\cdot(1-\rho)\cdot(1-n)\cdot\varepsilon_v^M\\
&= \left[\varepsilon_a^n+\gamma\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot\varepsilon_v^M\right]\cdot n
\end{aligned}$$

Under Assumption 3, using Lemma A4 and Proposition 2:

$$\begin{aligned}
\frac{n}{\varepsilon_v^M} &= \kappa\cdot\frac{(1-n)\cdot n}{h}\cdot\left[\frac{\varepsilon_v^m}{\varepsilon_v^M}\right]\\
\frac{1-n}{n}\cdot\varepsilon_v^M &= \frac{h}{n}\cdot\frac{1}{\kappa}\cdot\frac{1}{1+Q(n,\theta)}.
\end{aligned} \tag{A10}$$

Under Assumption 1, we evaluate the firm's first-order condition (11) at the equilibrium employment $n(a,\Delta v) = n^s(e^s(\theta(a,\Delta v),\Delta v),\theta(a,\Delta v))$ and labor market tightness $\theta(a,\Delta v)$. We differentiate the first-order condition with respect to a, holding Δv constant:

$$(\alpha-1)\cdot\frac{\theta}{n}\cdot\left[\frac{\partial n^s}{\partial\theta}+\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial\theta}\right]\cdot\varepsilon_a^\theta = (\gamma-1)\cdot[1-j(\theta,n)]+j(\theta,n)\cdot\eta\cdot\varepsilon_a^\theta$$

where we define to simplify the notations:

$$j(\theta,n) = \frac{r}{\alpha} \cdot \frac{n^{1-\alpha}}{q(\theta)}.$$

Lemma A2 says that:

$$\left[\frac{\partial n^s}{\partial \theta} + \frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \theta}\right] = (1-\eta) \cdot \frac{\kappa+1}{\kappa} \cdot \frac{h}{\theta}$$

Hence,

$$\left[(1-\alpha) \cdot \frac{\theta}{n} \cdot \left[(1-\eta) \cdot \frac{\kappa+1}{\kappa} \cdot \frac{h}{\theta}\right] + j(\theta,n) \cdot \eta\right] \cdot \varepsilon_a^\theta = (1-\gamma) \cdot [1 - j(\theta,n)]$$

$$\varepsilon_a^\theta = (1-\gamma) \cdot [1 - j(\theta,n)] \frac{1}{(1-\alpha) \cdot (1-\eta) \cdot \frac{\kappa+1}{\kappa} \cdot \frac{h}{n} + j(\theta,n) \cdot \eta}$$

Furthermore, using the result from Lemma A2 once more,

$$\varepsilon_a^n = \frac{\theta}{n} \cdot \left[(1-\eta) \cdot \frac{\kappa+1}{\kappa} \cdot \frac{h}{\theta}\right] \cdot \varepsilon_a^\theta$$

$$\varepsilon_a^n = \left[(1-\eta) \cdot \frac{\kappa+1}{\kappa} \cdot \frac{h}{n}\right] \cdot \varepsilon_a^\theta$$

We can infer

$$\varepsilon_a^n = (1-\gamma) \cdot [1 - j(\theta,n)] \cdot \frac{1}{(1-\alpha) + j(\theta,n) \cdot \frac{\eta}{1-\eta} \cdot \frac{\kappa}{1+\kappa} \cdot \frac{n}{h}}$$

$$\varepsilon_a^n = \frac{1-\gamma}{1-\alpha} \cdot [1 - j(\theta,n)] \cdot \frac{Q(n,\theta)}{1+Q(n,\theta)},$$

because we simplified, using the definition (A1) of $Q(n,\theta)$:

$$j(\theta,n) \cdot \frac{\eta}{1-\eta} \cdot \frac{\kappa}{1+\kappa} \cdot \frac{n}{h} = \frac{n^{1-\alpha}}{q(\theta)} \cdot \frac{\eta}{1-\eta} \cdot \frac{\kappa}{1+\kappa} \cdot \frac{r}{\alpha} \cdot \frac{n}{h} = \frac{1-\alpha}{Q(n,\theta)}.$$

Under Assumption 3, $Q(n,\theta) \geq 0$. Signing $\frac{\partial n^*}{\partial a}$ is equivalent to signing:

$$\kappa \cdot \frac{1-\gamma}{1-\alpha} \cdot [1 - j(\theta,n)] \cdot Q(n,\theta) \cdot \frac{n}{h} + \gamma \cdot (1-\rho)$$

Using once again the firm's first-order condition we have

$$[1 - j(\theta,n)] \cdot Q(n,\theta) \cdot \frac{n}{h} = \frac{\chi}{\alpha} \cdot q(\theta) \cdot \left[\alpha \cdot n^{\alpha-1} - \frac{r}{q(\theta)}\right] = \frac{\chi}{\alpha} \cdot \frac{w(a)}{a} \cdot q(\theta).$$

Hence we need to sign

$$
\begin{aligned}
&\kappa \cdot \frac{1-\gamma}{1-\alpha} \cdot \frac{\chi}{\alpha} \cdot \frac{w(a)}{a} \cdot q(\theta) + \gamma \cdot (1-\rho) \\
&= (1-\gamma) \cdot \frac{1-\eta}{\eta} \cdot (\kappa+1) \cdot w(a) \cdot \frac{q(\theta)}{r \cdot a} + \gamma \cdot (1-\rho)
\end{aligned}
$$

We define the wedge between wage and marginal product of labor

$$
W(a,\tau) \equiv \frac{w(a)}{mpl(a,\tau)} \in (0,1).
$$

Hence we need to sign

$$
(1-\gamma) \cdot \frac{1-\eta}{\eta} \cdot (\kappa+1) \cdot \frac{W(a,\tau)}{1-W(a,\tau)} - \gamma \cdot (\rho - 1). \tag{A11}
$$

The expression (A11) is positive for all $\tau \in (0,1)$ if

$$
\begin{aligned}
\frac{1-\gamma}{\gamma} &> \frac{\eta}{1-\eta} \cdot \frac{1}{\kappa+1} \frac{1-W(a,\tau)}{W(a,\tau)} \cdot (\rho - 1) \\
\frac{1-\gamma}{\gamma} &> \frac{\eta}{1-\eta} \cdot \frac{1}{\kappa+1} \Omega \cdot (\rho - 1),
\end{aligned}
$$

where we define $\Omega \in (0, +\infty)$ by

$$
\Omega = \max_{a,\tau} \left\{ \frac{1}{W(a,\tau)} - 1 \right\} = \frac{1}{\min_{a,\tau} W(a,\tau)} - 1.
$$

We can express the constant Ω solely as a function of the parameters of the model (more precisely the boundaries of the admissible values for the parameters). Since $n(a,\tau) \in (1-u, 1]$ and technology is bounded: $a \in \mathcal{A} = (a^*, \overline{a})$. We infer that

$$
\begin{aligned}
W(a,\tau) &\geq \frac{w(\overline{a})}{\overline{a}} \cdot \frac{1}{\alpha} \cdot (1-u)^{1-\alpha} \\
\Omega &\leq \left[\frac{\alpha}{\omega} \cdot \frac{\overline{a}^{1-\gamma}}{(1-u)^{1-\alpha}} \right] - 1.
\end{aligned}
$$

□

A.6 Proof of Proposition 3

LEMMA A13. *Under Assumption 3:*

$$\frac{\Delta v}{\Delta c \cdot v'(c^e)} = \frac{1-\tau^{1-\rho}}{(1-\rho)\cdot(1-\tau)}$$
$$\frac{v'(c^u)}{v'(c^e)} = \tau^{-\rho}$$

Proof. Immediate using the fact that under Assumption 3, $v(c) = c^{1-\rho}/(1-\rho)$, $v'(c) = c^{-\rho}$, and $\tau - c^u/c^e$. □

Step 1: cyclicality of $\varepsilon^m/\varepsilon^M$. From Lemma A10, because under Assumption 3, $\varepsilon^m/\varepsilon^M = 1 + R^*(a,\tau)$.

Step 2: cyclicality of ε^M. We express ε^M as a function of ε^M_v and τ. Using Lemma A3 and Lemma A13 (valid under Assumption 3) we get:

$$\begin{aligned}
\frac{\partial \Delta v^\dagger}{\partial \Delta c} &= v'(c^e) + \Delta v' \cdot \frac{\partial c^{u\dagger}}{\partial \Delta c} \\
\frac{\partial \Delta v^\dagger}{\partial \Delta c} &= \left[(1-n)v'(c^e) + nv'(c^u)\right] + \Delta v' \cdot \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M \qquad \text{(A12)} \\
\varepsilon^{\Delta v\dagger}_{\Delta c} &= \frac{\Delta c \cdot v'(c^e)}{\Delta v} \cdot \left\{ \left[(1-n) + n\frac{v'(c^u)}{v'(c^e)}\right] + \left\{1 - \frac{v'(c^u)}{v'(c^e)}\right\} \cdot \frac{1-n}{n} \cdot \frac{1}{1/\tau - 1} \cdot \varepsilon^M \right\} \\
\varepsilon^{\Delta v\dagger}_{\Delta c} &= \frac{(1-\rho)\cdot(1-\tau)}{1-\tau^{1-\rho}} \cdot \left\{ \left[(1-n) + n\cdot\tau^{-\rho}\right] + \left\{1-\tau^{-\rho}\right\} \cdot \frac{1-n}{n} \cdot \frac{1}{1/\tau - 1} \cdot \varepsilon^M \right\} \\
\varepsilon^{\Delta v\dagger}_{\Delta c} &= j(\tau) \cdot \left\{ \left[(1-n) + n\cdot\tau^{-\rho}\right] - i(\tau) \cdot \frac{1-n}{n} \cdot \varepsilon^M \right\}.
\end{aligned}$$

where for all $\tau \in (0,1)$ and all $\rho > 0$ we define to simplify the notations

$$i(\tau) \equiv \left(\tau^{-\rho} - 1\right) \cdot \frac{1}{1/\tau - 1} > 0$$
$$j(\tau) \equiv \frac{(1-\rho)\cdot(1-\tau)}{1-\tau^{1-\rho}} > 0.$$

Equation (A5) shows that $\varepsilon^M = \varepsilon^M_v \cdot \varepsilon^{\Delta v\dagger}_{\Delta c}$ so that

$$\varepsilon^M \cdot \left[1 + j(\tau)\cdot i(\tau)\cdot \frac{1-n}{n}\cdot \varepsilon^M_v\right] = j(\tau)\cdot\left[(1-n)+n\cdot\tau^{-\rho}\right]\cdot\varepsilon^M_v$$

$$\varepsilon^M = \frac{j(\tau)\cdot\left[(1-n)+n\cdot\tau^{-\rho}\right]\cdot\varepsilon^M_v}{1+j(\tau)\cdot i(\tau)\cdot\frac{1-n}{n}\cdot\varepsilon^M_v}$$

$$\varepsilon^M = j(\tau)\cdot\left[(1-n)+n\cdot\tau^{-\rho}\right]\cdot\frac{1}{(\varepsilon^M_v)^{-1}+j(\tau)\cdot i(\tau)\cdot\frac{1-n}{n}} \tag{A13}$$

According to Lemma A12, under Assumptions 1 and 3, and imposing $\rho \geq 1$ as well as $\frac{1-\gamma}{\gamma} > (\rho-1)\cdot\frac{\eta}{1-\eta}\cdot\frac{1}{\kappa+1}\cdot\Omega$, then $\frac{\partial n^*}{\partial a} > 0$. Using Lemma A4: $\varepsilon^m_v = (1/\kappa)\cdot[(n-(1-u))/n]$. Under Assumption 3, the elasticity κ is a constant and therefore ε^m_v is increasing in n. We infer that

$$\left.\frac{\partial\varepsilon^m_v}{\partial a}\right|_\tau > 0.$$

Under Assumption 3, $\varepsilon^m_v/\varepsilon^M_v = 1 + R^*(a,\tau)$. According to Lemma A10, under Assumptions 1, 2, 3, and imposing $\rho \geq 1$ and $\eta \geq (1+\kappa)/(1+2\kappa)$, $\frac{\partial R^*}{\partial a} < 0$. Hence under these assumptions $\left.\frac{\partial\varepsilon^M_v}{\partial a}\right|_\tau > 0$. Finally, using (A13), we conclude that $\left.\frac{\partial\varepsilon^M}{\partial a}\right|_\tau > 0$.

A.7 Proof of Proposition 4

The proof requires using the elasticities of unemployment "in utility" ($\varepsilon^m_v, \varepsilon^M_v$) instead of the elasticities of unemployment "in consumption" ($\varepsilon^m, \varepsilon^M$) used in the text. Therefore, we re-derive our optimal UI formula (9) in terms of the elasticities $\varepsilon^m_v, \varepsilon^M_v$.

LEMMA A14. *The optimal replacement rate τ satisfies*

$$\frac{1}{n}\cdot\frac{\tau}{1-\tau} = \frac{\Delta v}{v'(c^e)\cdot\Delta c}\cdot\left\{\frac{n}{\varepsilon^M_v}\cdot\left[1-\frac{v'(c^e)}{v'(c^u)}\right]+\left[(1-n)\cdot\frac{v'(c^e)}{v'(c^u)}+n\right]\cdot\frac{\kappa}{\kappa+1}\cdot\left(\frac{\varepsilon^m_v}{\varepsilon^M_v}-1\right)\right\}. \tag{A14}$$

Proof.

First step. The government chooses Δv to maximize:

$$\max (1-u)\cdot v(c^e) + u\cdot v(c^u) + u\cdot e\cdot f(\theta)\Delta v - u\cdot k(e) = v(c^u) + n^s(e,\theta)\cdot\Delta v - u\cdot k(e)$$

Using the envelope theorem, the first-order condition becomes

$$0 = v'(c^u)\cdot\frac{dc^u}{d\Delta v} + n + \Delta v\cdot\frac{\partial n^s}{\partial\theta}\cdot\frac{d\theta}{d\Delta v}.$$

Second step. We start from the budget constraint:

$$c^e + \frac{1-n}{n} \cdot c^u = w$$

$$\Delta v = v(w - \frac{1-n}{n} \cdot c^u) - v(c^u)$$

$$1 = v'(c^e) \cdot \left[-\frac{1-n}{n} \cdot \frac{dc^u}{d\Delta v} + \frac{1}{n^2} \cdot c^u \frac{dn}{d\Delta v} \right] - v'(c^u) \cdot \frac{dc^u}{d\Delta v}$$

$$v'(c^e) \cdot \frac{1}{n^2} \cdot c^u \cdot \frac{dn}{d\Delta v} - 1 = \left[v'(c^e) \cdot \frac{1-n}{n} + v'(c^u) \right] \cdot \frac{dc^u}{d\Delta v}$$

$$v'(c^e) \cdot \frac{1-n}{n} \cdot \frac{c^u}{\Delta v} \cdot \varepsilon^M_v - n = \left[(1-n) \cdot \frac{v'(c^e)}{v'(c^u)} + n \right] v'(c^u) \cdot \frac{dc^u}{d\Delta v}$$

Third step. We come back to the formula, and use Lemma 1:

$$0 = \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] v'(c^u) \frac{dc^u}{d\Delta v} + n \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] + \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] \Delta v \frac{\partial n^s}{\partial \theta} \frac{d\theta}{d\Delta v}$$

$$0 = v'(c^e) \frac{1-n}{n} \frac{c^u}{\Delta v} \varepsilon^M_v - n + n \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] + \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] \frac{\kappa}{\kappa+1} (1-n) \left(\varepsilon^M_v - \varepsilon^m_v \right)$$

$$0 = v'(c^e) \frac{1-n}{n} \frac{c^u}{\Delta v} \varepsilon^M_v + n(1-n) \left[\frac{v'(c^e)}{v'(c^u)} - 1 \right] + \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] \frac{\kappa}{\kappa+1} (1-n) \left(\varepsilon^M_v - \varepsilon^m_v \right)$$

Dividing by $(1-n) \cdot v'(c^e) \cdot \varepsilon^M_v$:

$$\frac{1}{n} \frac{c^u}{\Delta v} = \frac{1}{v'(c^e)} \frac{n}{\varepsilon^M_v} \left[1 - \frac{v'(c^e)}{v'(c^u)} \right] + \frac{1}{v'(c^e)} \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] \frac{\kappa}{\kappa+1} \left(\frac{\varepsilon^m_v}{\varepsilon^M_v} - 1 \right)$$

$$\frac{1}{n} \frac{c^u}{\Delta c} = \frac{\Delta v}{\Delta c} \frac{1}{v'(c^e)} \frac{n}{\varepsilon^M_v} \left[1 - \frac{v'(c^e)}{v'(c^u)} \right] + \frac{\Delta v}{\Delta c} \frac{1}{v'(c^e)} \left[(1-n)\frac{v'(c^e)}{v'(c^u)} + n \right] \frac{\kappa}{\kappa+1} \left(\frac{\varepsilon^m_v}{\varepsilon^M_v} - 1 \right)$$

Note that $\frac{c^u}{\Delta c} = \frac{\tau}{1-\tau}$ since $\tau = c^u / c^e$. The exact optimal UI formula in sufficient statistics becomes

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{\Delta v}{v'(c^e) \cdot \Delta c} \left\{ \frac{n}{\varepsilon^M_v} \left[1 - \frac{v'(c^e)}{v'(c^u)} \right] + \left[(1-n) \cdot \frac{v'(c^e)}{v'(c^u)} + n \right] \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m_v}{\varepsilon^M_v} - 1 \right) \right\}.$$

□

LEMMA A15. *Under Assumption 3, the optimal UI formula (A14) simplifies to*

$$(1-\rho) \cdot \frac{\tau}{1-\tau^{1-\rho}} = \kappa \cdot \frac{(1-n) \cdot n}{h} \cdot (n+X) \cdot [1-\tau^\rho] + \frac{\kappa}{1+\kappa} \cdot [(1-n) \cdot \tau^\rho + n] \cdot X.$$

Proof. Using Lemma A13 (valid under Assumption 3), we can re-express various terms as a function of τ:

$$\left[1-\frac{v'(c^e)}{v'(c^u)}\right]=1-\tau^{\rho}$$

$$\left[(1-n)\frac{v'(c^e)}{v'(c^u)}+n\right]=[(1-n)\tau^{\rho}+n]$$

$$\frac{\Delta v}{v'(c^e)\cdot\Delta c}=\frac{1-\tau^{1-\rho}}{(1-\rho)\cdot(1-\tau)}$$

Furthermore, the definition (A2) of $X(a,\Delta v)$ and the result from Proposition 2 imply that $X = n\cdot(\frac{\varepsilon_v^m}{\varepsilon_v^M}-1)$, which in turn implies that $n\cdot\frac{\varepsilon_v^m}{\varepsilon_v^M}=n+X$. Building on equation (A10) we obtain:

$$\frac{n^2}{\varepsilon_v^M}=\kappa\cdot\frac{(1-n)\cdot n}{h}\left(n+X(a,\Delta v)\right).$$

Bringing all these results together, and evaluating all terms at the equibrium, we obtain:

$$(1-\rho)\cdot\frac{\tau}{1-\tau^{1-\rho}}=\kappa\cdot\frac{(1-n)\cdot n}{h}\cdot(n+X)\cdot[1-\tau^{\rho}]+\frac{\kappa}{1+\kappa}\cdot[(1-n)\cdot\tau^{\rho}+n]\cdot X.$$

□

Let us define

$$F(\tau)\equiv(1-\rho)\cdot\frac{\tau}{1-\tau^{1-\rho}}$$

$$Q(\tau,n,X)\equiv\kappa\cdot\frac{(1-n)\cdot n}{n-(1-u)}\cdot(n+X)\cdot[1-\tau^{\rho}]+\frac{\kappa}{1+\kappa}\cdot[(1-n)\cdot\tau^{\rho}+n]\cdot X. \tag{A15}$$

Recall that $h=n-(1-u)$. We can rewrite the optimal UI formula as

$$F(\tau)=Q(\tau,n,X).$$

LEMMA A16. *Assuming $n\in[1/2,1)$, we have the following comparative statics:*

$$\frac{dF}{d\tau}>0,\ \ \frac{\partial Q}{\partial\tau}<0,\ \ \frac{\partial Q}{\partial n}<0,\ \ \frac{\partial Q}{\partial X}>0.$$

Proof. First notice that for any $\rho\geq 0$:

$$\frac{dF}{d\tau}=\frac{1-\rho}{\left[1-\tau^{1-\rho}\right]^2}\cdot\left[1-\rho\cdot\tau^{1-\rho}\right]>0.$$

Moreover:

$$\frac{\partial Q}{\partial \tau} = -\left[\kappa \cdot \rho \cdot \tau^{\rho-1} \cdot (1-n)\right] \cdot \left[\frac{n+X}{h/n} - \frac{X}{1+\kappa}\right]$$
$$\frac{\partial Q}{\partial X} = \kappa \cdot \frac{(1-n)\cdot n}{h} \cdot [1-\tau^{\rho}] + \frac{\kappa}{1+\kappa} \cdot [(1-n)\cdot \tau^{\rho} + n]$$
$$\frac{\partial Q}{\partial n} = \kappa \cdot (1-\tau^{\rho}) \cdot \left[\frac{1-2n}{h/n} - X \cdot \left(\frac{1}{h/n} - \frac{1}{1+\kappa}\right) - \frac{(1-n)}{h^2} \cdot (n+X) \cdot (1-u)\right]$$

Noting that $h/n \leq 1$, and $n \in (0,1)$ allows us to conclude $\frac{\partial Q}{\partial \tau} < 0$. Since $n \in (0,1)$, $\frac{\partial Q}{\partial X} > 0$. Assuming that $n \in [1/2, 1)$, and noting that $0 \leq h/n \leq 1$, $X \geq 0$, we can conclude $\frac{\partial Q}{\partial n} < 0$. □

The optimality condition can be expressed as

$$F(\tau) = Q\left(\tau, n^*(a,\tau), X^*(a,\tau)\right).$$

Let us define

$$G(\tau, a) \equiv Q\left(\tau, n^*(a,\tau), X^*(a,\tau)\right). \tag{A16}$$

We assume that $F(\tau)$ and $G(\tau,a)$ cross only once for $\tau \in (0,1)$, such that the solution to the government's problem is unique. The function $\tau(a)$, which characterizes the optimal replacement rate, is defined implicitly as the unique intersection of these two curves.

The combination of Lemmas A11 and A16, under the appropriate assumptions, implies

$$\frac{\partial Q}{\partial X} \cdot \frac{\partial X^*}{\partial a} < 0,$$

The combination of Lemmas A12 and A16, under the appropriate assumptions, implies

$$\frac{\partial Q}{\partial n} \cdot \frac{\partial n^*}{\partial a} < 0.$$

We are at technology a and optimal replacement rate $\tau(a)$. We consider a marginal change in technology to $a^* > a$. At $\tau(a)$,

$$F(\tau(a)) = Q(\tau(a), n(a, \tau(a)), X(a, \tau(a))) > Q(\tau(a), n(a^*, \tau(a)), X(a^*, \tau(a))).$$

We assume that $F(\tau)$ and $G(\tau,a)$ cross only once for $\tau \in [0,1]$. Moreover, $\lim_{\tau\to 0} F(\tau) = 0$. At the same time $\lim_{\tau\to 0} G(\tau,a) > 0$. To see this, consider the following two cases:

1. $\lim_{\tau\to 0} n = n_0 \in [1-u, 1)$. Since $X \geq 0$ and $h \leq u$, using the definition (A16) of G and equation (A15), we infer that

$$\lim_{\tau\to 0} G(\tau,a) \geq \kappa \cdot n_0^2 \cdot (1-n_0) > 0.$$

2. $\lim_{\tau \to 0} n = 1$. Then $\lim_{\tau \to 0} h = u$. Using the firm's first-order condition (11), we infer that

$$\lim_{\tau \to 0} q(\theta) = \frac{r}{\alpha - w/a} > 0.$$

This implies, using the definition (A2) of X, that

$$\lim_{\tau \to 0} X = \chi \cdot u \cdot \left[\lim_{\tau \to 0} q(\theta) \right] > 0.$$

Thus, using the definition (A16) of G and equation (A15),

$$\lim_{\tau \to 0} G(\tau, a) \geq \frac{\kappa}{1+\kappa} \cdot \left[\lim_{\tau \to 0} X \right] > 0.$$

Hence it must be that $F(\tau)$ crosses $G(\tau, a)$ from below and that $\tau < \tau(a)$ iff $F(\tau) < G(\tau, a)$. We showed that

$$F(\tau(a)) > G(\tau(a), a^*),$$

therefore

$$\tau(a) > \tau(a^*)$$

Accordingly, $\frac{d\tau}{da} < 0$.

B Extensions in the One-Period Model

B.1 Workers can partially insure themselves

An unemployed worker consumes y in addition to the unemployment benefits c^u received from the government. We denote $\hat{c}^u = c^u + y$ the total consumption when unemployed. Unemployed workers now pick effort e and home production y to maximize

$$[1 - e \cdot f(\theta)] \cdot [v(c^u + y) - m(y)] + [e \cdot f(\theta)] \cdot v(c^e) - k(e)$$

The first-order condition with respect to y yields:

$$m'(y) = v'(c^u + y), \tag{A17}$$

which implicitly defines optimal home production $y(c^u)$. The first-order condition with respect to e yields:

$$k'(e) = f(\theta) \cdot \hat{\Delta} v,$$

where we denote $\hat{\Delta} v = v(c^e) - [v(c^u + y(c^u)) - m(y(c^u))]$ the utility difference between being employed and unemployed. This first-order condition implicitly defines optimal effort $e(\theta, \hat{\Delta} v)$.

The government chooses Δc to maximize:

$$\begin{aligned}&(1-u)\cdot v(c^e)+u\cdot\{[1-e\cdot f(\theta)]\cdot[v(c^u+y)-m(y)]+[e\cdot f(\theta)]\cdot v(c^e)-k(e)\}\\&=n^s(e,\theta)\cdot v(c^u+\Delta c)+[1-n^s(e,\theta)]\cdot[v(c^u+y)-m(y)]-u\cdot k(e)\end{aligned}$$

Using the envelope theorem, as workers choose search effort e and home production y to maximize $[1-e\cdot f(\theta)]\cdot[v(c^u+y)-m(y)]+[e\cdot f(\theta)]\cdot v(c^e)-k(e)$, the first-order condition becomes

$$0=\left[n\cdot v'(c^e)+(1-n)\cdot v'(\hat{c}^u)\right]\cdot\frac{\partial c^{u\dagger}}{\partial\Delta c}+n\cdot v'(c^e)+\hat{\Delta}v\cdot\frac{\partial n^s}{\partial\theta}\cdot\frac{\partial\theta^\dagger}{\partial\Delta c}.$$

As in the case without self-insurance, we derive the optimal UI formula in three steps. The first and second step are identical to those in the case without self-insurance (notice that the proof of Lemma 1 would be modified by taking derivatives with respect to $\hat{\Delta}v$ instead of Δv. However, the results regarding the derivatives with respect to Δc would carry over.).

$$\frac{1}{n}\cdot\frac{\tau}{1-\tau}=\left[n+(1-n)\cdot\frac{v'(\hat{c}^u)}{v'(c^e)}\right]^{-1}\cdot\left[\frac{n}{\varepsilon^M}\cdot\left[\frac{v'(\hat{c}^u)}{v'(c^e)}-1\right]+\frac{\hat{\Delta}v}{v'(c^e)\cdot\Delta c}\cdot\frac{\kappa}{\kappa+1}\cdot\left(\frac{\varepsilon^m}{\varepsilon^M}-1\right)\right]$$

Approximation: We assume that $n\approx 1$. The formula simplifies to

$$\frac{\tau}{1-\tau}=\frac{1}{\varepsilon^M}\cdot\left[\frac{v'(\hat{c}^u)}{v'(c^e)}-1\right]+\frac{\hat{\Delta}v}{v'(c^e)\cdot\Delta c}\cdot\frac{\kappa}{\kappa+1}\cdot\left(\frac{\varepsilon^m}{\varepsilon^M}-1\right)$$

We define

$$\xi=\frac{\hat{c}^u}{c^u}=1+\frac{y}{c^u}\geq 1.$$

If the third and higher order terms of $v(\cdot)$ are small ($v'''(c)\approx 0$), we can make the following approximations:

$$\frac{v'(\hat{c}^u)}{v'(c^e)}\approx\frac{v'(c^e)-v''(c^e)\cdot c^e\cdot\left(\frac{c^e-\xi\cdot c^u}{c^e}\right)}{v'(c^e)}=1+\rho\cdot(1-\xi\cdot\tau).$$

ρ is the coefficient of relative risk aversion of the utility function measured at c^e. Next, we need to approximate

$$\frac{\hat{\Delta}v}{v'(c^e)\cdot\Delta c}=\frac{v(c^e)-v(\hat{c}^u)}{v'(c^e)\cdot\Delta c}+\frac{m(y)}{v'(c^e)\cdot\Delta c}.$$

If the third and higher order terms of $v(\cdot)$ are small ($v'''(c) \approx 0$), we can approximate the first term:

$$\begin{aligned}\frac{v(c^e) - v(\hat{c}^u)}{v'(c^e) \cdot \Delta c} &\approx \frac{c^e - \xi \cdot c^u}{c^e - c^u} - \frac{1}{2} \cdot \frac{v''(c^e)}{v'(c^e)} \cdot \frac{c^e}{c^e} \cdot \frac{[c^e - \xi \cdot c^u]^2}{c^e - c^u} \\ &= \left[\frac{1 - \xi \cdot \tau}{1 - \tau}\right] \cdot \left[1 + \frac{1}{2} \cdot \rho \cdot (1 - \xi \cdot \tau)\right].\end{aligned}$$

To approximate the second term, we need to assume that $m(\cdot)$ is isoelastic: $m(y) = \omega_m \cdot \frac{y^{1+\mu}}{1+\mu}$ with $\mu \in (0, +\infty)$, $\omega_m \in (0, +\infty)$. Then

$$m(y) = y \cdot \frac{m'(y)}{1+\mu}.$$

But the unemployed worker's optimality condition (A17) implies

$$\begin{aligned}m(y) &= y \cdot \frac{v'(\hat{c}^u)}{1+\mu} = c^u \cdot (\xi - 1) \cdot \frac{1}{1+\mu} \cdot v'(\hat{c}^u) \\ \frac{m(y)}{v'(c^e) \cdot \Delta c} &= \frac{\tau}{1-\tau} \cdot \frac{\xi - 1}{1+\mu} \cdot \frac{v'(\hat{c}^u)}{v'(c^e)} \\ \frac{m(y)}{v'(c^e) \cdot \Delta c} &= \frac{\tau}{1-\tau} \cdot \left[\frac{\xi - 1}{1+\mu}\right] \cdot [1 + \rho \cdot (1 - \xi \cdot \tau)].\end{aligned}$$

Accordingly, the optimal UI formula in sufficient statistics becomes:

$$\begin{aligned}\frac{\tau}{1-\tau} = \frac{\rho}{\varepsilon^M} \cdot [1 - \xi \cdot \tau] + \frac{\kappa}{\kappa + 1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right] \cdot \frac{1}{1-\tau} \cdot \\ \left\{\tau \cdot \frac{\xi - 1}{1+\mu} + (1 - \xi \cdot \tau) \cdot \left[1 + \rho\left(\frac{1 - \xi \cdot \tau}{2} + \tau \cdot \frac{\xi - 1}{1+\mu}\right)\right]\right\}.\end{aligned}$$

If unemployed workers fully insure themselves without any insurance from the government: $c^u = 0$ and $\hat{c}^u = y = c^e$. This implies that $\tau = 0$ such that the left-hand side of the formula is nil. It also implies that $\xi \cdot \tau = \hat{c}^u / c^e = 1$ (and $\tau \cdot (\xi - 1) = \tau \cdot \xi - \tau = 1$), such that the right-hand side of the formula is positive. Therefore, even though workers can fully insure themselves without UI, it is optimal for the government to provide some UI because of the cost of home production.

B.2 UI influences wages

If UI influences wages, labor demand becomes a function of UI: $n^d = n^d(a, \theta, \Delta c)$, which reflects the influence of UI on firm's recruiting decision through wages. Labor market tightness $\theta^\dagger(a, \Delta c)$ is now characterized by

$$n^d(\theta, \Delta c) = n^s(e^s(\theta, \Delta v^\dagger(a, \Delta c)), \theta).$$

This generalization does not affect the derivations: the macro-elasticity captures the influence of UI on aggregate employment and labor market tightness through all channels, including possibly wages. If the equilibrium wage responds to Δc: $w = w^\dagger(a,\Delta c)$, we amend the budget constraint and modify the end of the derivation of the optimal UI formula.

Second step. We start from the budget constraint:

$$c^{u\dagger}(a,\Delta c) = n^\dagger(a,\Delta c)\cdot\left[w^\dagger(a,\Delta c)-\Delta c\right]$$
$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{\Delta c}\cdot[w-\Delta c]\cdot\varepsilon^M - n\cdot\left(-\frac{\partial w^\dagger}{\partial \Delta c}+1\right)$$
$$\frac{\partial c^{u\dagger}}{\partial \Delta c} = \frac{1-n}{n}\cdot\frac{\tau}{1-\tau}\cdot\varepsilon^M - n + n\cdot\frac{\partial w^\dagger}{\partial \Delta c}$$

Third step. We come back to the formula:

$$0 = \overline{v'}\cdot\left[\frac{1-n}{n}\frac{\tau}{1-\tau}\varepsilon^M - n + n\cdot\frac{\partial w^\dagger}{\partial \Delta c}\right] + nv'(c^e) + \frac{\Delta v}{\Delta c}\cdot\frac{\kappa}{\kappa+1}\cdot(1-n)\cdot\left(\varepsilon^M-\varepsilon^m\right)$$
$$0 = \overline{v'}\cdot\frac{1-n}{n}\cdot\frac{\tau}{1-\tau}\cdot\varepsilon^M + \overline{v'}\cdot n\cdot\frac{\partial w^\dagger}{\partial \Delta c} + n\cdot(1-n)\left[v'(c^e)-v'(c^u)\right] + \frac{\Delta v}{\Delta c}\cdot\frac{\kappa}{\kappa+1}\cdot(1-n)\cdot\left(\varepsilon^M-\varepsilon^m\right)$$

Dividing the equation by $(1-n)\cdot\varepsilon^M\cdot\overline{v'}$ and rearranging the terms: yields:

$$\frac{1}{n}\cdot\frac{\tau}{1-\tau} + \frac{1}{\varepsilon^M}\cdot\frac{n}{1-n}\cdot\frac{\partial w^\dagger}{\partial \Delta c} = \left[n+(1-n)\frac{v'(c^u)}{v'(c^e)}\right]^{-1}\cdot\left\{\frac{n}{\varepsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right] + \frac{\Delta v}{v'(c^e)\Delta c}\cdot\frac{\kappa}{\kappa+1}\cdot\left(\frac{\varepsilon^m}{\varepsilon^M}-1\right)\right\}$$

It is convenient to express the wage $w = w(a,t^*)$ as a function of the implicit total tax on work $t^* = t+b$. We can also write the net reward from work $\Delta c = \Delta c(a,t^*)$ as a function of t^*. By definition

$$\Delta c = w(a,t^*)\cdot(1-t^*)$$
$$\frac{\partial \Delta c}{\partial t^*} = \frac{\partial w}{\partial t^*}\cdot(1-t^*) - w.$$

As $\Delta c/w = (1-t^*)$, we can rewrite the elasticity $\varepsilon^{w\dagger}$:

$$w(a,t^*) = w^\dagger(a,\Delta c(a,t^*))$$
$$\frac{\partial w}{\partial t^*} = \frac{\partial w^\dagger}{\partial \Delta c}\cdot\frac{\partial \Delta c}{\partial t^*}$$
$$\frac{\partial w^\dagger}{\partial \Delta c} = \frac{\frac{\partial w}{\partial t^*}}{\frac{\partial w}{\partial t^*}\cdot(1-t^*)-w}$$
$$\frac{\partial w^\dagger}{\partial \Delta c} = -\frac{\varepsilon^w}{1-\varepsilon^w}\cdot\frac{1}{1-t^*},$$

where ε^w is the minus the elasticity of wages with respect to one minus the total implicit tax on work:

$$\varepsilon^w = \frac{1-t^*}{w} \cdot \frac{\partial w}{\partial t^*}.$$

Notice using the budget constraint that

$$\frac{1}{1-t^*} = \frac{w}{\Delta c} = 1 + \frac{1}{n} \cdot \frac{c^u}{\Delta c} = 1 + \frac{1}{n} \cdot \frac{\tau}{1-\tau}.$$

Combining these results, we obtain the optimal UI formula in the text.

B.3 Estimation of the elasticity of k' : κ, and the elasticity of m' : μ

Estimation of κ. Consider an unemployed workers receiving benefits c^u, expecting to receive c^e if employed, and facing a labor market tightness θ. We denote by $\lambda = e \cdot f(\theta)$ the hazard rate out of unemployment. Assume that the worker receives an increase $dc^u > 0$ in benefits, and reduces his search effort by $de < 0$, which leads to a reduction $d\lambda = f(\theta) \cdot de < 0$ in the hazard rate. If we can measure consumption level, consumption change, hazard rate, and change in hazard rate, we can estimate the following elasticity:

$$\varepsilon^* = \frac{c^u}{\lambda} \frac{d\lambda}{dc^u}.$$

In turn, this elasticity allows us to estimate the coefficient κ. From Lemma A1, $d\ln(e)/d\ln(\Delta v) = 1/\kappa$. Since we are considering a change in benefits for only one worker, labor market conditions are not affected by the policy experiment, and θ remains constant. Hence $d\ln(\lambda) = d\ln(e)$. Furthermore, $d\Delta v = -v'(c^u)dc^u$ such that, if the second and higher order terms of $v(\cdot)$ are small,

$$d\ln(\Delta v) = \frac{d\Delta v}{\Delta v} \approx -\frac{v'(c^u)}{v'(c^u) \cdot \Delta c} \cdot dc^u = \frac{dc^u}{\Delta c}.$$

Therefore,

$$\frac{1}{\kappa} \approx \frac{d\lambda}{\lambda} \cdot \frac{c^u}{dc^u} \cdot \frac{\Delta c}{c^u}$$
$$\frac{1}{\kappa} \approx \varepsilon^* \cdot \frac{1-\tau}{\tau}$$
$$\kappa \approx \frac{1}{\varepsilon^*} \cdot \frac{\tau}{1-\tau}.$$

Estimation of μ. Consider an unemployed workers receiving benefits c^u and consuming a total amount $\hat{c}^u = c^u + y$. Assume that the worker receives an increase $dc^u > 0$ in benefits, and increases his total consumption by $d\hat{c}^u > 0$. If we can measure all the consumptions and consumption changes, we can estimate the following elasticity:

$$\hat{\varepsilon} = \frac{c^u}{\hat{c}^u}\frac{d\hat{c}^u}{dc^u} = \frac{1}{\xi}\cdot\frac{d\hat{c}^u}{dc^u}.$$

In turn, this elasticity allows us to estimate the coefficient μ.

The optimal choice of home production given by (A17), the assumption that $m(\cdot)$ is isoelastic, and the identities $\hat{c}^u = y + c^u = \xi \cdot c^u$ yield:

$$\begin{aligned}
v'(\hat{c}^u) &= m'(y)\\
v''(\hat{c}^u)\cdot\frac{d\hat{c}^u}{dc^u} &= m''(y)\cdot\left(\frac{d\hat{c}^u}{dc^u}-1\right)\\
\frac{d\hat{c}^u}{dc^u} &= \frac{1}{1-\frac{v''(\hat{c}^u)}{m''(y)}}\\
m''(y) &= \mu\frac{m'(y)}{y} = \frac{\mu}{\xi-1}\cdot\frac{v'(\hat{c}^u)}{c^u} = \frac{\mu\cdot\xi}{\xi-1}\cdot\frac{v'(\hat{c}^u)}{\hat{c}^u}\\
\frac{d\hat{c}^u}{dc^u} &= \frac{1}{1-\hat{c}^u\cdot\frac{v''(\hat{c}^u)}{v'(\hat{c}^u)}\cdot\frac{\xi-1}{\mu\cdot\xi}}\\
\hat{\varepsilon} &= \frac{1}{\xi+\rho\cdot\frac{\xi-1}{\mu}}\\
\mu &= \rho\cdot\left[\frac{\xi-1}{1/\hat{\varepsilon}-\xi}\right].
\end{aligned}$$

So by measuring the ratio of consumptions $\xi = \hat{c}^u/c^u$, estimating the elasticity $\hat{\varepsilon}$ (as in Gruber [1997]), and estimating the coefficient of relative risk aversion ρ (as in Chetty [2006b]), we can estimate the coefficient μ.

B.4 Micro-elasticity and macro-elasticity with Nash bargaining

Let L denote the value to a worker of being employed after the matching process. Let U denote the value to a worker of remaining unemployed after the matching process.

$$\begin{aligned}
L &= (1-t)\cdot w\\
U &= b\cdot w.
\end{aligned}$$

Let $t^* = t + b$. The worker's surplus from an established relationship with a firm is therefore:

$$L - U = \{1-(t+b)\}\cdot w = (1-t^*)\cdot w.$$

In our model, the firm's surplus from an established relationship is simply given by the hiring cost $r\cdot a/q(\theta)$ because a firm can immediately replace a worker at that cost during the matching period.

Since the bargaining solution divides the surplus of the match between worker and firm with the worker keeping a fraction $\beta \in (0,1)$ of the surplus, the worker's surplus is related to the firm's surplus:

$$(1-t^*)\cdot w = \frac{\beta}{1-\beta}\cdot\frac{r\cdot a}{q(\theta)}. \tag{A18}$$

The firm's first-order condition is:

$$a = w + \frac{r\cdot a}{q(\theta)},$$

which, combined with (A18), gives both equilibrium labor market tightness $\theta(a,t^*)$ and equilibrium wage $w(a,t^*)$ as a function of technology a and the tax rate t^*:

$$w(a,t^*) = a\cdot\frac{\beta}{\beta+(1-\beta)\cdot(1-t^*)}$$
$$q(\theta(a,t^*)) = r\cdot\left[1+\frac{\beta}{(1-\beta)\cdot(1-t^*)}\right].$$

Notice that the wage obtained from Nash bargaining is proportional to technology a, and that labor market tightness does not depend on a: in the model with Nash bargaining, wages are completely flexible and there are no labor market fluctuations.

Equation (A18) can be rewritten to define implicitly equilibrium labor market tightness $\theta^\dagger(a,\Delta c)$:

$$\Delta c = \frac{\beta}{1-\beta}\cdot\frac{r\cdot a}{q(\theta)}.$$

Thus, the elasticity of equilibrium labor market tightness with respect to Δc is:

$$\varepsilon_{\Delta c}^{\theta\dagger} = \frac{d\ln\theta}{d\ln(\Delta c)} = \frac{1}{\eta} > 0.$$

This is a critical result. Since $\varepsilon_{\Delta c}^{\theta\dagger} > 0$, the macro-elasticity is greater than the micro-elasticity: $\varepsilon^M > \varepsilon^m$. This can be seen using the result from Lemma 1, which is also valid in this model with Nash bargaining, and which implies:

$$\varepsilon_{\Delta c}^{\theta\dagger} > 0 \Leftrightarrow \varepsilon^M > \varepsilon^m.$$

C Derivations in the Infinite-Horizon Model

C.1 Firm's and worker's problem

Representative firm: Endogenous layoffs never occur in equilibrium so the Lagrangian of the firm's problem is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot \left[n_t^d - (1-s) \cdot n_{t-1}^d \right] \right\}.$$

I assume that the firm maximization problem is concave and admits an interior solution (which will always be the case in equilibrium). Immediately, we can show that employment n_t^d is determined by first-order condition (18).

Representative worker: The Lagrangian of the worker's problem is

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ -\left[1-(1-s)n_{t-1}^s\right] \cdot k(e_t) + (1-n_t^s) \cdot v(c_t^u) + n_t^s \cdot v(c_t^e) \right.$$
$$\left. + A_t \left\{ \left[1-(1-s) \cdot n_{t-1}^s\right] \cdot e_t f(\theta_t) + (1-s) \cdot n_{t-1}^s - n_t^s \right\} \right\},$$

where n_t^s is the probability to be employed in period t after period a^t and $\{A_t(a^t)\}$ is a collection of Lagrange multipliers. The first-order condition with respect to effort in the current period e_t gives:

$$k'(e_t) = f(\theta_t) \cdot A_t.$$

The first-order condition with respect to expected employment status n_t^s yields

$$A_t = [v(c_t^e) - v(c_t^u)] + \delta(1-s)\mathbb{E}_t\left[k(e_{t+1})\right] + \delta \cdot (1-s) \cdot \mathbb{E}_t\left[A_{t+1}\left(1 - E_{t+1} f(\theta_{t+1})\right)\right]$$
$$\frac{k'(e_t)}{f(\theta_t)} = [v(c_t^e) - v(c_t^u)] + \delta \cdot (1-s) \cdot \mathbb{E}_t\left[\frac{k'(e_{t+1})}{f(\theta_{t+1})}\right] - \delta \cdot (1-s)(\kappa^* + 1) \cdot \mathbb{E}_t\left[k(e_{t+1})\right] + \delta(1-s)\mathbb{E}_t\left[k(e_{t+1})\right]$$

Thus, the optimal effort function therefore satisfies the Euler equation (17), where we define $(1 + \kappa^*) \equiv d\ln(k(e))/d\ln(e)$, the elasticity of $k(\cdot)$ with respect to e.

C.2 Optimal UI formula in the dynamic model

Assumptions. We consider a static equilibrium of the infinite-horizon model: all variables are constant, technology a is constant, and the net reward from work Δc is constant. We assume that there is no time discounting: $\delta = 1$. In that case, the government chooses Δc to maximize the per-period social welfare.

Notations. We define the following functions, which we study in the Appendix:

- Labor supply: $n^s : \mathbb{R}^+ \times \mathbb{R}^+ \to [0,1]$ is defined by the Beveridge curve:

$$n^s(e,\theta) = \frac{e \cdot f(\theta)}{s + (1-s) \cdot e \cdot f(\theta)}.$$

- effort supply: $e^s : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ defined implicitly by the worker's optimality condition at the limit where $\delta \to 1$:

$$s \cdot \frac{k'(e^s)}{f(\theta)} + \kappa^* \cdot (1-s) \cdot k(e^s) = \Delta v, \tag{A19}$$

  where $(1+\kappa^*)$ is the elasticity of $k(\cdot)$ with respect to e.

- Incentive to search: $\Delta v : \mathbb{R}^{++} \times \mathbb{R}^{++} \to \mathbb{R}^{++}$ is defined implicitly by the system:

$$\Delta v = v(c^e) - v(c^u)$$
$$n(a,\Delta c) \cdot w(a) = n(a,\Delta c) \cdot c^e + [1 - n(a,\Delta c)] \cdot c^u$$
$$\Delta c = c^e - c^u).$$

- Equilibrium labor market tightness: $\theta : \mathbb{R}^{++} \times \mathbb{R}^{++} \to \mathbb{R}^+$ is defined implicitly by

$$n^d(\theta, a) = n^s\left(e^s(\theta, \Delta v(a,\Delta c)), \theta\right)$$

- Equilibrium effort: $e : \mathbb{R}^{++} \times \mathbb{R}^{++} \to \mathbb{R}^+$ defined by

$$e(a,\Delta c) = e^s(\theta(a,\Delta c), \Delta v(a,\Delta c))$$

- Equilibrium employment: $n : \mathbb{R}^{++} \times \mathbb{R}^{++} \to [0,1]$ defined by

$$n(a,\Delta c) = n^s(e(a,\Delta c), \theta(a,\Delta c))$$

- Equilibrium consumption: $c^u : \mathbb{R}^{++} \times \mathbb{R}^{++} \to \mathbb{R}^{++}$ is defined by

$$c^u(a,\Delta c) = n(a,\Delta c) \cdot (w(a) - \Delta c).$$

We also define the elasticities:

$$\varepsilon^M \equiv \frac{\Delta c}{1-n} \cdot \frac{\partial n}{\partial \Delta c} \tag{A20}$$

$$\varepsilon^m \equiv \frac{\Delta c}{1-n} \cdot \left[\frac{\partial n^s}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta v}\right] \cdot \frac{\partial \Delta v}{\partial \Delta c}. \tag{A21}$$

Preliminary results.

ASSUMPTION A1. The disutility of effort is isoelastic: $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$.

LEMMA A17. *Under Assumption A1, the partial derivatives of effort supply $e^s(\theta, \Delta v)$ satisfy:*

$$\frac{\theta}{e^s} \cdot \frac{\partial e^s}{\partial \theta} = (1-\eta) \cdot \frac{u}{\kappa}$$
$$\frac{\Delta v}{e^s} \cdot \frac{\partial e^s}{\partial \Delta v} = \frac{1}{\kappa} \cdot \frac{u+\kappa}{1+\kappa}.$$

Proof. First, we differentiate with respect to θ the optimality condition (A19) of the jobseeker's problem under Assumption A1, keeping Δv fixed:

$$\begin{aligned}
f(\theta) \cdot \Delta v &= s \cdot k'(e) + (1-s) \cdot f(\theta) \cdot \kappa \cdot k(e) \\
(1-\eta) \cdot \frac{f(\theta)}{\theta} \cdot \Delta v &= \left(s \cdot k''(e) + (1-s) \cdot f(\theta) \cdot \kappa \cdot k'(e)\right) \frac{\partial e^s}{\partial \theta} + (1-s) \cdot (1-\eta) \cdot \frac{f(\theta)}{\theta} \cdot \kappa \cdot k(e) \\
(1-\eta) \cdot s \cdot \frac{k'(e)}{\theta} &= \left(s \cdot k''(e) + (1-s) \cdot f(\theta) \cdot \kappa \cdot k'(e)\right) \frac{\partial e^s}{\partial \theta} \\
(1-\eta) \cdot \frac{s}{\theta} &= \left(s \frac{k''(e)}{k'(e)} + (1-s) \cdot f(\theta) \cdot \kappa\right) \frac{\partial e^s}{\partial \theta} \\
(1-\eta) \cdot \frac{s}{\theta} &= \frac{\kappa}{e} \cdot [s + (1-s) \cdot f(\theta) \cdot e] \frac{\partial e^s}{\partial \theta} \\
\frac{\theta}{e} \cdot \frac{\partial e^s}{\partial \theta} &= (1-\eta) \cdot \frac{u}{\kappa}.
\end{aligned}$$

We repeat the exercise by differentiating the optimality condition (A19) with respect to Δv, keeping θ fixed:

$$\begin{aligned}
f(\theta) &= \frac{\kappa}{e} \cdot \frac{s}{u} \cdot \frac{\partial e^s}{\partial \Delta v} \\
\frac{\Delta v}{e} \cdot \frac{\partial e^s}{\partial \Delta v} &= \frac{f(\theta)}{s \cdot k'(e)} \cdot \frac{u}{\kappa} \Delta v \\
\frac{\Delta v}{e} \cdot \frac{\partial e^s}{\partial \Delta v} &= \frac{u+\kappa}{\kappa \cdot (1+\kappa)},
\end{aligned}$$

where the last line derives from Lemma A20. □

LEMMA A18. *The partial derivatives of labor supply $n^s(e, \theta)$ satisfy:*

$$\frac{\partial n^s}{\partial \theta} = (1-\eta) \cdot \frac{u \cdot n}{\theta}$$
$$\frac{\partial n^s}{\partial e} = \frac{u \cdot n}{e},$$

where $1-\eta$ is the elasticity of $f(\cdot)$ with respect to θ. Furthermore under Assumption A1,

$$\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial \theta}=\frac{u}{\kappa}\cdot\frac{\partial n^s}{\partial \theta}.$$

Proof. The Beveridge curve (14), implies that

$$u=1-(1-s)\cdot n=\frac{s}{s+(1-s)\cdot e\cdot f(\theta)},$$

and hence

$$\begin{aligned}\frac{\partial n^s}{\partial e}&=\frac{s\cdot f(\theta)}{[s+(1-s)\cdot e\cdot f(\theta)]^2}=\frac{u\cdot n}{e}\\ \frac{\partial n^s}{\partial \theta}&=\frac{s\cdot e\cdot f'(\theta)}{[s+(1-s)\cdot e\cdot f(\theta)]^2}=(1-\eta)\cdot\frac{u\cdot n}{\theta}\end{aligned}$$

where $1-\eta=\theta\cdot f'(\theta)/f(\theta)$ is the elasticity of $f(\theta)$ with respect to θ. Combining this result with those from Lemma A17, we infer:

$$\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial \theta}=\frac{u}{\kappa}\cdot\frac{\partial n^s}{\partial \theta}.$$

□

LEMMA A19. *Under Assumption A1, the partial derivative of equilibrium labor market tightness $\theta(a,\Delta c)$ satisfies:*

$$\begin{aligned}\varepsilon^{\theta}_{\Delta c}\equiv\frac{\Delta c}{\theta}\cdot\frac{\partial \theta}{\partial \Delta c}&=-\frac{\kappa}{\kappa+u}\cdot\frac{1}{1-\eta}\cdot\frac{1-n}{u\cdot n}\cdot\left(\varepsilon^m-\varepsilon^M\right)\\ \frac{\Delta c}{1-n}\cdot\frac{\partial n^s}{\partial \theta}\cdot\frac{\partial \theta}{\partial \Delta c}&=-\frac{\kappa}{\kappa+u}\cdot\left(\varepsilon^m-\varepsilon^M\right).\end{aligned}$$

Proof. As in the proof of Lemma 1, by definition:

$$\frac{1-n}{\Delta c}\cdot\left(\varepsilon^M-\varepsilon^m\right)=\left[\frac{\partial n^s}{\partial e}\frac{\partial e^s}{\partial \theta}+\frac{\partial n^s}{\partial \theta}\right]\frac{\partial \theta}{\partial \Delta c}.$$

We conclude using Lemma A18. □

LEMMA A20. *The optimal job-search effort, determined by (A19), satisfies two equivalent conditions:*

$$\begin{aligned}s\cdot\frac{k'(e)}{f(\theta)}&=\Delta v\cdot u\cdot\frac{(1+\kappa^*)}{u+\kappa^*}\\ k(e)&=\Delta v\cdot\frac{n}{u+\kappa^*}.\end{aligned}$$

Proof. From the worker's optimality condition (A19):

$$\Delta v = s \cdot \frac{k'(e)}{f(\theta)} + (1-s)\cdot \kappa^* \cdot k(e)$$
$$\Delta v = s \cdot \frac{k'(e)}{f(\theta)} + (1-s)\cdot \kappa^* \cdot \frac{e}{1+\kappa^*} k'(e)$$
$$\Delta v = k'(e) \cdot \left[\frac{s}{f(\theta)} + (1-s)\cdot \frac{\kappa^*}{1+\kappa^*} e\right]$$
$$\Delta v = e \cdot k'(e) \cdot \left[\frac{u}{n} + (1-s)\cdot \frac{\kappa^*}{1+\kappa^*}\right]$$
$$\Delta v = e \cdot k'(e) \cdot \left[\frac{u\cdot(1+\kappa^*) + (1-s)n\cdot\kappa^*}{n(1+\kappa^*)}\right]$$
$$\Delta v = e \cdot k'(e) \cdot \left[\frac{u\cdot(1+\kappa^*) + (1-u)\cdot\kappa^*}{n(1+\kappa^*)}\right]$$
$$\Delta v = \frac{e}{n}\cdot k'(e) \cdot \frac{u+\kappa^*}{(1+\kappa^*)}$$
$$\Delta v = \frac{1}{u}\cdot \left[s \cdot \frac{k'(e)}{f(\theta)}\right] \frac{u+\kappa^*}{(1+\kappa^*)},$$

using $s/f(\theta) = u \cdot e/n$. This allows us to conclude. □

Derivation. The government chooses Δc to maximize

$$n^s(e,\theta)\cdot v(c^u+\Delta c) + (1-n^s(e,\theta))\cdot v(c^u) - [1-(1-s)\cdot n^s(e,\theta)]\cdot k(e).$$

The first-order condition of the government's problem with respect to e is:

$$\frac{\partial n^s}{\partial e}\cdot[\Delta v + (1-s)\cdot k(e)] - u \cdot k'(e) = 0.$$

Using Lemma A18, we can rewrite the condition as:

$$\Delta v = \frac{e}{n}\cdot k'(e) - (1-s)\cdot k(e).$$

Notice that from the Beveridge curve:

$$\frac{n}{e} = \frac{f(\theta)}{s+(1-s)\cdot e \cdot f(\theta)} = f(\theta)\cdot \frac{u}{s}$$

Hence the first-order condition becomes (using $s \cdot n = e \cdot u \cdot f(\theta)$):

$$\Delta v = s \cdot \frac{k'(e)}{f(\theta)} + \frac{s(1-u)}{uf(\theta)e} \cdot \left[e \cdot k'(e)\right] - (1-s) \cdot k(e)$$
$$\Delta v = s \cdot \frac{k'(e)}{f(\theta)} + (1-s) \cdot [(1+\kappa^*) \cdot k(e)] - (1-s) \cdot k(e)$$
$$\Delta v = s \cdot \frac{k'(e)}{f(\theta)} + (1-s) \cdot \kappa^* \cdot k(e),$$

which corresponds to the worker's optimality condition for $\delta \to 1$ given by (A19). The first-order condition is:

$$0 = \overline{v'} \cdot \frac{\partial c^u}{\partial \Delta c} + n \cdot v'(c^e) + [\Delta v + (1-s) \cdot k(e)] \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta c}.$$

Fist step. From Lemma A20:

$$(1-s) \cdot (1+\kappa^*) \cdot k(e) = \Delta v \cdot (1-u) \cdot \frac{(1+\kappa^*)}{u+\kappa^*}$$
$$\Delta v + (1-s) \cdot k(e) = s \cdot \frac{k'(e)}{f(\theta)} + (1-s) \cdot (1+\kappa^*) \cdot k(e)$$
$$\Delta v + (1-s) \cdot k(e) = \Delta v \cdot \frac{(1+\kappa^*)}{u+\kappa^*} \tag{A22}$$

Under Assumption A1, and using the results from Lemma A19:

$$[\Delta v + (1-s) \cdot k(e)] \cdot \frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Delta c} = \frac{\Delta v}{\Delta c} \cdot \frac{\kappa \cdot (1+\kappa)}{(\kappa+u)^2} \cdot (1-n) \cdot \left(\varepsilon^M - \varepsilon^m\right).$$

This negative term is the cost of the job-rationing externality.

Second step. It is identical to the static case:

$$\frac{\partial c^u}{\partial \Delta c} = \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n.$$

Third step. We come back to the formula:

$$0 = \overline{v'} \cdot \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n\right] + nv'(c^e) + \frac{\Delta v}{\Delta c} \cdot \frac{(1+\kappa)\kappa}{(\kappa+u)^2} \cdot (1-n) \cdot \left(\varepsilon^M - \varepsilon^m\right).$$

Dividing the equation by $(1-n)\cdot\varepsilon^M\cdot\overline{v'}$ yields:

$$\frac{1}{n}\frac{\tau}{1-\tau}=\left[n+(1-n)\frac{v'(c^u)}{v'(c^e)}\right]^{-1}\cdot\left\{\frac{n}{\varepsilon^M}\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{\Delta v}{v'(c^e)\Delta c}\frac{(1+\kappa)\kappa}{(\kappa+u)^2}\left(\frac{\varepsilon^m}{\varepsilon^M}-1\right)\right\}. \quad \text{(A23)}$$

Comparison with formula (9) in the one-period model. The sole difference between the optimal UI formula (A23) in the infinite-horizon model and formula (9) in the one-period model is that the term $(1+\kappa)\cdot\kappa/(\kappa+u)^2$ replaces the term $\kappa/(1+\kappa)$. This modification captures differences between the two models in the welfare effects of a change in UI. Recall that job rationing causes a downward adjustment of labor market tightness following a cut in unemployment benefits. The welfare cost of this adjustment differs on two points between infinite-horizon and one-period model.

First, the welfare cost of a job loss caused by the equilibrium adjustment of labor market tightness is $\Delta v+(1-s)k(e)=(1+\kappa)/(u+\kappa)\cdot\Delta v$ in the infinite-horizon model instead of Δv in the one-period model. Therefore the correction term in the optimal UI formula is multiplied by $(1+\kappa)/(u+\kappa)$ in the infinite-horizon model.

The second effect is more subtle. By comparing Lemma A17 to Lemma A1, notice that the elasticity of the optimal effort with respect to θ in the one-period and infinite-horizon model is, respectively,

$$\frac{\theta}{e^s}\cdot\frac{\partial e^s}{\partial\theta}=(1-\eta)\cdot\frac{1}{\kappa}$$
$$\frac{\theta}{e^s}\cdot\frac{\partial e^s}{\partial\theta}=(1-\eta)\cdot\frac{u}{\kappa}.$$

Since $u<<1$, the elasticity of optimal effort $e^s(\theta,\Delta v)$ with respect to θ is lower in the infinite-horizon model. This is because the worker's optimal choice of effort, described by equation (1) in the one-period model and equation (A19) in the infinite-horizon model, involves a mix of $k(\cdot)$ and $k'(\cdot)$ in the infinite-horizon model instead of only $k'(\cdot)$ in the one-period model. Since the optimal effort is less elastic, it falls less for a given reduction in labor market tighness. On the other hand, the changes in employment following an adjustment in effort and an adjustment in labor market tightness are relatively similar in both models, as showed by Lemma A2 and Lemma A18:

$$\frac{\partial n^s/\partial e}{\partial n^s/\partial\theta}=\frac{1}{1-\eta}\cdot\frac{\theta}{e}.$$

By definition, the wedge $\left[\varepsilon^m-\varepsilon^M\right]$ is tied to the change in labor supply n^s following a change in tightness θ:

$$\frac{1-n}{\Delta c}\cdot\left(\varepsilon^M-\varepsilon^m\right)=\left[\frac{\partial n^s}{\partial e}\frac{\partial e^s}{\partial\theta}+\frac{\partial n^s}{\partial\theta}\right]\frac{\partial\theta}{\partial\Delta c}$$

$$\frac{1-n}{\Delta c}\cdot\left(\varepsilon^M-\varepsilon^m\right)=\left[\frac{\partial n^s/\partial e}{\partial n^s/\partial\theta}\cdot\frac{\partial e^s}{\partial\theta}+1\right]\frac{\partial n^s}{\partial\theta}\cdot\frac{\partial\theta}{\partial\Delta c}$$

$$\frac{1-n}{\Delta c}\cdot\left(\varepsilon^M-\varepsilon^m\right)=\left[\frac{1}{1-\eta}\cdot\left(\frac{\theta}{e}\cdot\frac{\partial e^s}{\partial\theta}\right)+1\right]\frac{\partial n^s}{\partial\theta}\cdot\frac{\partial\theta}{\partial\Delta c}.$$

The wedge $\left[\varepsilon^m-\varepsilon^M\right]$ describes the adjustment in employment following a change in Δc. Since the supply of search effort is more inelastic, the elasticity $(\theta/e)\cdot(\partial e^s/\partial\theta)$ is much smaller. To obtain the same adjustment in employment (as a combination of a change in effort $(\partial e^s/\partial\theta)d\theta$ and a change in tightness $d\theta$), it is therefore necessary to have a larger adjustment in labor market tightness $d\theta$, which has a larger welfare cost. In other words, $\partial\theta/\partial\Delta c$ must be larger. In our model, recall that changes in effort have no welfare effect by the envelope theorem, whereas changes in labor market tightness, which affect the per-unit job-finding probability, do have welfare effects.

Comparing the results from Lemma 1 to those of Lemma A19 shows that, for a given wedge $\left[\varepsilon^m-\varepsilon^M\right]$, the adjustment in tightness θ in response to a change in net reward from work Δc is larger in the infinite-horizon model. For a given wedge $\left[\varepsilon^m-\varepsilon^M\right]$ between micro- and macro-elasticity, the amount of job destroyed by the equilibrium adjustment of labor market tightness θ is more important in the infinite-horizon model. The amount of jobs destroyed in the infinite-horizon and one-period model are, respectively:

$$\frac{\Delta c}{1-n}\cdot\frac{\partial n^s}{\partial\theta}\cdot\frac{\partial\theta}{\partial\Delta c}=-\frac{\kappa}{\kappa+1}\cdot\left(\varepsilon^m-\varepsilon^M\right)$$

$$\frac{\Delta c}{1-n}\cdot\frac{\partial n^s}{\partial\theta}\cdot\frac{\partial\theta}{\partial\Delta c}=-\frac{\kappa}{\kappa+u}\cdot\left(\varepsilon^m-\varepsilon^M\right).$$

Hence there are $(\kappa+1)/(\kappa+u)$ times more jobs destroyed in the infinite-horizon model for a given wedge $\left[\varepsilon^m-\varepsilon^M\right]$, which implies that the correction term in the optimal UI formula is once more multiplied by $(1+\kappa)/(u+\kappa)$ in the infinite-horizon model.

To conclude, the $\kappa/(1+\kappa)$ term from the one-period model becomes

$$\left[\frac{\kappa}{1+\kappa}\right]\times\left[\frac{1+\kappa}{u+\kappa}\right]\times\left[\frac{1+\kappa}{u+\kappa}\right]=\frac{(1+\kappa)\cdot\kappa}{(\kappa+u)^2}.$$

Approximation. Assuming $n\approx 1$ and $u<<\kappa$ allows us to simplify the optimal formula to

$$\frac{\tau}{1-\tau}=\frac{1}{\varepsilon^M}\cdot\left[\frac{v'(c^u)}{v'(c^e)}-1\right]+\frac{\Delta v}{v'(c^e)\cdot\Delta c}\cdot\frac{1+\kappa}{\kappa}\cdot\left(\frac{\varepsilon^m}{\varepsilon^M}-1\right)$$

Once we linearize the utility function, the optimal UI formula becomes:

$$\frac{\tau}{1-\tau} = \frac{1}{\varepsilon^M} \cdot \rho \cdot [1-\tau] + \frac{1+\kappa}{\kappa} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1\right] \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau)\right].$$

C.3 Elasticities

In the same way as we derive equation (A7) from the firm's optimality condition, and using Lemma A19 under Assumption A1, we obtain:

$$\frac{\varepsilon^m}{\varepsilon^M} = 1 + (1-\alpha) \cdot \alpha \cdot \frac{1-\eta}{\eta} \cdot \frac{\kappa+u}{\kappa} \cdot u \cdot \frac{q(\theta)}{[1-\delta \cdot (1-s)] \cdot r} \cdot n^{\alpha-1} \equiv Q(u,n,\theta).$$

where $\eta = -\theta \cdot q'(\theta)/q(\theta)$ is minus the elasticity of $q(\cdot)$ and $1-\alpha = -n \cdot g''(n)/g'(n)$ is minus the elasticity of $g'(\cdot)$.

Combining the definition (A21) of the micro-elasticity, and the expressions of various partial derivatives given by (A12), Lemma A17, and Lemma A18, we infer the micro-elasticity:

$$\varepsilon^m = \frac{\Delta c}{\Delta v} \cdot \frac{u \cdot (u+\kappa)}{\kappa \cdot (1+\kappa)} \cdot \left\{ \frac{n}{1-n} \cdot \left[(1-n) \cdot v'(c^e) + n \cdot v'(c^u)\right] + \Delta v' \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M \right\}$$
$$\varepsilon^m = \frac{1}{\mu} \cdot \frac{u \cdot (u+\kappa)}{\kappa \cdot (1+\kappa)} \cdot \left\{ \frac{n}{1-n} \cdot [(1-n) + n \cdot \zeta] + (1-\zeta) \cdot \frac{\tau}{1-\tau} \cdot \frac{\varepsilon^m}{Q(u,n,\theta)} \right\}$$
$$\varepsilon^m = \frac{n}{1-n} \cdot \frac{[1 + n \cdot (\zeta - 1)]}{\mu \cdot \frac{\kappa \cdot (1+\kappa)}{u \cdot (u+\kappa)} + (\zeta-1) \cdot \frac{\tau}{1-\tau} \cdot Q(u,n,\theta)^{-1}},$$

where we define the functions of the consumption levels:

$$\mu \equiv \frac{\Delta v}{\Delta c \cdot v'(c^e)}$$
$$\zeta \equiv \frac{v'(c^u)}{v'(c^e)}.$$

Accordingly, the macro-elasticity is given by:

$$\varepsilon^M = \frac{n}{1-n} \cdot \frac{[1 + n \cdot (\zeta - 1)]}{\mu \cdot \frac{\kappa \cdot (1+\kappa)}{u \cdot (u+\kappa)} \cdot Q(u,n,\theta) + (\zeta-1) \cdot \frac{\tau}{1-\tau}}.$$

Finally, in the calibration of the infinite-horizon model, we use the micro-elasticity of unemployment with respect to benefits (instead of net reward from work) given by

$$\varepsilon^c \equiv -\varepsilon^m \cdot \left[\frac{\Delta c}{c^u} \cdot \frac{\partial c^u}{\partial \Delta c}\right]^{-1}.$$

Given that we know ε^m, we need to determine the elasticity of c^u with respect to Δc to compute this elasticity ε^c. But Lemma A3, which is also valid in the infinite-horizon model, implies that

$$\frac{\Delta c}{c^u} \cdot \frac{\partial c^u}{\partial \Delta c} = \frac{1-n}{n} \cdot \varepsilon^M - \frac{1-\tau}{\tau} \cdot n.$$

Hence this micro-elasticity can be expressed as a function of the micro- and macro-elasticity:

$$\varepsilon^c = \frac{\varepsilon^m}{\frac{1-\tau}{\tau} \cdot n - \frac{1-n}{n} \cdot \varepsilon^M}.$$

C.4 Optimal unemployment insurance: government's problem

The maximization of the government is over a collection of sequences $\{n_t(a^t), e_t(a^t), \theta_t(a^t), c_t^e(a^t), c_t^u(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. We can form a Lagrangian:

$$\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{ (1-n_t) \cdot v(c_t^u) + n_t \cdot v(c_t^e) - [1-(1-s)n_{t-1}] \cdot k(e_t) \\
&+ A_t \left[n_t \cdot w(a_t) - n_t c_t^e - (1-n_t) c_t^u\right] \\
&+ B_t \left[[v(c_t^e) - v(c_t^u)] - \frac{k'(e_t)}{f(\theta_t)} + \delta(1-s)\mathbb{E}_t\left[\frac{k'(e_{t+1})}{f(\theta_{t+1})}\right] - \kappa\delta(1-s)\mathbb{E}_t\left[k(e_{t+1})\right]\right] \\
&+ C_t \left[a_t \cdot g'(n_t) - w_t - \frac{r \cdot a_t}{q(\theta_t)} + \delta(1-s)\mathbb{E}_t\left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right]\right] \\
&+ D_t \left[(1-(1-s) \cdot n_{t-1}) \cdot e_t f(\theta_t) + (1-s) \cdot n_{t-1} - n_t\right] \Bigg\}
\end{aligned}$$

where $\{A_t(a^t), B_t(a^t), C_t(a^t), D_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$ are sequences of Lagrange multipliers. Let $B_{-1} \equiv 0$ and $C_{-1} \equiv 0$. We rewrite the Lagrangian as:

$$
\begin{aligned}
&\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{ (1-n_t)\cdot v(c_t^u) + n_t \cdot v(c_t^e) - [1-(1-s)n_{t-1}]\cdot k(e_t) \\
&+ A_t \left[n_t \cdot w(a_t) - n_t c_t^e - (1-n_t)c_t^u\right] \\
&+ B_t \left[[v(c_t^e) - v(c_t^u)] - \frac{k'(e_t)}{f(\theta_t)} \right] + B_{t-1}\cdot(1-s)\left[\frac{k'(e_t)}{f(\theta_t)} - \kappa \cdot k(e_t)\right] \\
&+ C_t \left[a_t \cdot g'(n_t) - w_t - \frac{r\cdot a_t}{q(\theta_t)} \right] + C_{t-1}\cdot(1-s)\left[\frac{r\cdot a_t}{q(\theta_t)}\right] \\
&+ D_t \left[(1-(1-s)\cdot n_{t-1})\cdot e_t \cdot f(\theta_t) + (1-s)\cdot n_{t-1} - n_t \right] \Bigg\}
\end{aligned}
$$

The first-order conditions of the government's problem with respect to $n_t(a^t)$ for $t \geq 0$ are

$$
\begin{aligned}
0 &= v(c_t^e) - v(c_t^u) + \delta(1-s)\mathbb{E}_t\left[k(e_{t+1})\right] \\
&\quad - D_t + (1-s)\mathbb{E}_t\left[D_{t+1}\cdot(1-E_{t+1}f(\theta_{t+1})\right] \\
&\quad + C_t \cdot a_t \cdot g''(n_t) \\
&\quad + A_t\left\{w(a_t) - (c_t^e - c_t^u)\right\} \\
D_t &= v(c_t^e) - v(c_t^u) + \delta(1-s)\mathbb{E}_t\left[k(e_{t+1})\right] + (1-s)\mathbb{E}_t\left[D_{t+1}\cdot(1-E_{t+1}f(\theta_{t+1})\right] \\
&\quad + C_t \cdot a_t \cdot g''(n_t) + A_t\left\{w(a_t) - (c_t^e - c_t^u)\right\}.
\end{aligned}
$$

The first-order conditions of the government's problem with respect to $e_t(a^t)$ for $t \geq 0$ are

$$
\begin{aligned}
0 &= -u_t \cdot k'(e_t) - B_t \frac{k''(e_t)}{f(\theta_t)} + (1-s)B_{t-1}\frac{k''(e_t)}{f(\theta_t)} - \kappa(1-s)B_{t-1}k'(e_t) + D_t \cdot u_t \cdot f(\theta_t) \\
0 &= -u_t \cdot k'(e_t) + \frac{k''(e_t)}{f(\theta_t)}\left((1-s)B_{t-1} - B_t\right) - \kappa(1-s)B_{t-1}k'(e_t) + D_t \cdot u_t \cdot f(\theta_t) \\
0 &= -(\kappa+1)u_t \cdot k(e_t) + \kappa\frac{k'(e_t)}{f(\theta_t)}\left((1-s)B_{t-1} - B_t\right) - \kappa(\kappa+1)(1-s)B_{t-1}k(e_t) + D_t \cdot \cdot e_t \cdot u_t \cdot f(\theta_t) \\
0 &= -\frac{D_t \cdot h_t}{(\kappa+1)k(e_t)} + u_t + \kappa\frac{1}{e_t f(\theta_t)}\left[B_t - (1-s)B_{t-1}\right] + \kappa(1-s)B_{t-1},
\end{aligned}
$$

where $B_{-1} = 0$. The first-order conditions of the government's problem with respect to $c_t^e(a^t)$ for $t \geq 0$ are

$$
A_t = v'(c_t^e)\cdot\left(1 + \frac{B_t}{n_t}\right).
$$

The first-order conditions of the government's problem with respect to $c_t^u(a^t)$ for $t \geq 0$ are

$$A_t = v'(c_t^u) \cdot \left(1 - \frac{B_t}{1-n_t}\right).$$

The first-order conditions of the government's problem with respect to $\theta_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned}
0 &= (1-\eta)B_t \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} - (1-\eta)(1-s) \cdot B_{t-1} \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} \\
&\quad - C_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + C_{t-1} \cdot (1-s) \cdot \eta \frac{r \cdot a_t}{f(\theta_t)} + D_t \cdot u_t \cdot (1-\eta) \cdot e_t q(\theta_t) \\
0 &= \frac{1-\eta}{\eta} \frac{k'(e_t)}{f(\theta_t)} \left[B_t - (1-s) \cdot B_{t-1}\right] - \frac{r \cdot a_t}{q(\theta_t)} \left[C_t - (1-s) \cdot C_{t-1}\right] + D_t u_t \cdot \frac{1-\eta}{\eta} \cdot e_t f(\theta_t) \\
0 &= h_t \cdot D_t \frac{1-\eta}{\eta} + \frac{1-\eta}{\eta} \frac{k'(e_t)}{f(\theta_t)} \left[B_t - (1-s) \cdot B_{t-1}\right] - \frac{r \cdot a_t}{q(\theta_t)} \left[C_t - (1-s) \cdot C_{t-1}\right] \\
0 &= h_t \cdot D_t q(\theta_t) \frac{1-\eta}{\eta} + \frac{1-\eta}{\eta} \frac{k'(e_t)}{\theta_t} \left[B_t - (1-s) \cdot B_{t-1}\right] - r \cdot a_t \left[C_t - (1-s) \cdot C_{t-1}\right],
\end{aligned}$$

where $B_{-1} = 0$ and $C_{-1} = 0$. To summarize, the optimal equilibrium $\{c_t^e, c_t^u, \theta_t, n_t, E_t\}_{t=0}^{+\infty}$ and the sequences of Lagrange multipliers from the government's problem $\{A_t, B_t, C_t, D_t\}_{t=0}^{+\infty}$ are characterized by the constraints, $\forall t \geq 0$:

$$0 = \left[v(c_t^e) - v(c_t^u)\right] - \frac{k'(e_t)}{f(\theta_t)} + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{k'(e_{t+1})}{f(\theta_{t+1})}\right] - \kappa \cdot \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1})\right] \tag{A24}$$

$$0 = a_t \cdot g'(n_t) - w(a_t) - \frac{r \cdot a_t}{q(\theta_t)} + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right] \tag{A25}$$

$$0 = \left[1 - (1-s) \cdot n_{t-1}\right] \cdot e_t \cdot f(\theta_t) - \left[n_t - (1-s) \cdot n_{t-1}\right] \tag{A26}$$

$$0 = n_t \cdot w(a_t) - n_t \cdot c_t^e - (1-n_t) \cdot c_t^u, \tag{A27}$$

and the first-order conditions from the government's problem, $\forall t \geq 0$:

$$\begin{aligned}
D_t &= v(c_t^e) - v(c_t^u) + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1})\right] + (1-s) \cdot \mathbb{E}_t \left[D_{t+1} \cdot (1 - e_{t+1} f(\theta_{t+1}))\right] \\
&\quad + C_t \cdot a_t \cdot g''(n_t) + A_t \left[w(a_t) - (c_t^e - c_t^u)\right]
\end{aligned} \tag{A28}$$

$$A_t = \left[\frac{n_t}{v'(c_t^e)} + \frac{1-n_t}{v'(c_t^u)}\right]^{-1} \tag{A29}$$

$$B_t = n_t \cdot (1-n_t) \cdot \left[\frac{1}{v'(c_t^e)} - \frac{1}{v'(c_t^u)}\right] \cdot A_t \tag{A30}$$

$$0 = -\frac{D_t \cdot h_t}{(\kappa+1) k(e_t)} + u_t + \kappa \frac{1}{e_t f(\theta_t)} \left[B_t - (1-s) B_{t-1}\right] + \kappa (1-s) B_{t-1} \tag{A31}$$

$$0 = h_t \cdot D_t \cdot q(\theta_t) \cdot \frac{1-\eta}{\eta} + \frac{1-\eta}{\eta} \frac{k'(e_t)}{\theta_t} \left[B_t - (1-s) B_{t-1}\right] - r a_t \left[C_t - (1-s) C_{t-1}\right] \tag{A32}$$

where $B_{-1} = 0$ and $C_{-1} = 0$, $h_t = n_t - (1-s) \cdot n_{t-1}$, $u_t = 1 - (1-s) \cdot n_{t-1}$.

C.5 Optimal equilibrium in a static environment

In a static environment, there are no aggregate shocks ($a_t = a$ for all t), and the labor market in steady state (equation (14) holds). The solution to the government's problem in a static environment is constant: the collection of 9 variables $\{c^e, c^u, n, \theta, e, A, B, C, D\}$ is characterized by the following system of 9 equations:

$$[v(c^e) - v(c^u)] = [1 - \delta \cdot (1-s)] \frac{k'(e)}{f(\theta)} + \delta \cdot (1-s) \cdot \kappa \cdot k(e) \tag{A33}$$

$$0 = g'(n) - \frac{w(a)}{a} - [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)} \tag{A34}$$

$$n = \frac{e \cdot f(\theta)}{s + (1-s) \cdot e \cdot f(\theta)} \tag{A35}$$

$$n \cdot w(a) = n \cdot c^e + (1-n) \cdot c^u \tag{A36}$$

$$\begin{aligned} D\left[1 - (1-s)(1 - e \cdot f(\theta))\right] = {} & [v(c^e) - v(c^u)] + \delta(1-s)k(e) \\ & + C \cdot a \cdot g''(n) + A\left[w(a) - (c^e - c^u)\right] \end{aligned} \tag{A37}$$

$$A = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^u)}\right]^{-1} \tag{A38}$$

$$B = n \cdot (1-n) \left[\frac{1}{v'(c^e)} - \frac{1}{v'(c^u)}\right] \cdot A \tag{A39}$$

$$C/n = \frac{1-\eta}{\eta} \cdot \frac{k'(e)}{r \cdot a \cdot \theta} \left[1 + \frac{B}{n} \cdot \left(\frac{\kappa}{u} + 1\right)\right] \tag{A40}$$

$$D = \frac{k'(e)}{f(\theta)} \cdot \left[1 + \frac{B}{n} \cdot \frac{\kappa}{u}\right]. \tag{A41}$$

This system of equations (A33)–(A40) is obtained directly from the system of 9 equations (A24)–(A32), except that we rewrite the first-order conditions with respect to e and θ (when the labor market is in steady state, $ef(\theta)u = h$):

$$\frac{D \cdot u \cdot f(\theta)}{k'(e)} = u + \kappa(1-s)B + \kappa \frac{s}{ef(\theta)} B$$

$$D = \frac{k'(e)}{f(\theta)} \left\{1 + \kappa(1-s)\frac{B}{u} + \kappa \cdot \frac{B}{n}\right\}$$

$$D = \frac{k'(e)}{f(\theta)} \left\{1 + \frac{B}{n} \cdot \frac{\kappa}{u}\right\},$$

and

$$\begin{aligned}
0 &= s \cdot n \cdot D \frac{1-\eta}{\eta} + \frac{1-\eta}{\eta} \frac{k'(e)}{f(\theta)} s \cdot B - \frac{r \cdot a}{q(\theta)} \cdot s \cdot C \\
\frac{r \cdot a}{q(\theta)} \cdot C &= \frac{1-\eta}{\eta} \left[n \cdot D + \frac{k'(e)}{f(\theta)} \cdot B \right] \\
\frac{r \cdot a}{q(\theta)} \cdot C &= \frac{1-\eta}{\eta} \frac{k'(e)}{f(\theta)} \left[n + B \cdot \left(\frac{\kappa}{u} + 1 \right) \right] \\
C &= \frac{1-\eta}{\eta} \frac{k'(e)}{r \cdot a \cdot \theta} \left[n + B \cdot \left(\frac{\kappa}{u} + 1 \right) \right].
\end{aligned}$$

To solve this system for a given a_j, we perform a grid search over $\Delta v = [v(c^e) - v(c^u)]$. For a sequence $\{\Delta v_i\}_i$, we solve the system of equations (A33)–(A35) to find a collection of sequences $\{n_i, e_i, \theta_i\}_i$. Using (A36) and the definition $\Delta v_i = v(c^e_i) - v(c^u_i)$, we compute a collection of sequences $\{c^e_i, c^u_i\}_i$. From these sequences, we compute a collection of sequences $\{A_i, B_i, C_i, D_i\}_i$ by solving in turn (A38) (to get $\{A_i\}_i$), (A39) (to get $\{B_i\}_i$), (A40) (to get $\{C_i\}_i$),and (A41) (to get $\{D_i\}_i$). Lastly, we pick the index i^* such that equation (A37) be satisfied. The optimal equilibrium in a static environment with technology a_j is $\{c^e_{i*}, c^u_{i*}, n_{i*}, \theta_{i*}, e_{i*}\}$, the optimal replacement rate is $\tau = c^u_{i*}/c^e_{i*}$, the optimal labor tax rate is $1 - c^e_{i*}/w(a)$, and the optimal benefit rate is $c^u_{i*}/w(a)$. We repeat this computation for a sequence of technology $\{a_j\}_j$ to plot the graphs in Figure 2.

C.6 Log-linearization

$\bar{x}$ denotes the steady-state value of variable x_t. $\check{x}_t \equiv d\log(x_t)$ denotes the logarithmic deviation of variable x_t. In steady state, the optimal equilibrium $\{\bar{c}^e, \bar{c}^u, \bar{n}, \bar{\theta}, \bar{e}\}$ and the associated Lagrange multipliers $\{\bar{A}, \bar{B}, \bar{C}, \bar{D}\}$ are characterized by the system of equations (A33)–(A41) in Section C.5 when technology $a = \bar{a} = 1$. Moreover $\bar{h} = s \cdot \bar{n}$ and $\bar{u} = 1 - (1-s) \cdot \bar{n}$. Using the calibration in Table 1, we find that in steady state, when technology $\bar{a} = 1$, the optimal replacement rate $\bar{\tau} = 73\%$, the optimal tax rate $\bar{t} = 4.0\%$, the optimal benefit rate $\bar{b} = 70\%$, and unemployment $\bar{u} = 6.2\%$.

The log-linearized optimal equilibrium $\{\check{c}^e, \check{c}^u, \check{n}, \check{\theta}, \check{e}\}$ and the associated Lagrange multipliers $\{\check{A}, \check{B}, \check{C}, \check{D}\}$ are characterized by the following system of log-linear equations:

- Definition of unemployment $u_t = 1 - (1-s) \cdot n_{t-1}$:

$$\check{u}_t + o_1 \cdot \check{n}_{t-1} = 0$$

  where $o_1 = \frac{1-\bar{u}}{\bar{u}}$.

- Definition of number of hires $h_t = n_t - (1-s) \cdot n_{t-1}$:

$$(1-s) \cdot \check{n}_{t-1} + s \cdot \check{h}_t - \check{n}_t = 0.$$

- Law of motion of employment (A26):

$$\check{u}_t + \check{e}_t + (1-\eta)\cdot\check{\theta}_t - \check{h}_t = 0$$

- Budget constraint (A27):

$$\gamma\cdot\check{a}_t + \check{n}_t - \{p_1\cdot(\check{n}_t + \check{c}_t^e) + p_2\cdot(-q_1\cdot\check{n}_t + \check{c}_t^u)\} = 0,$$

with $q_1 = \overline{n}/(1-\overline{n})$, $p_1 = \frac{\overline{c}^e}{\omega}$, and $p_2 = 1 - p_1$.

- Firm's optimal hiring decision (A25):

$$-\check{a}_t + (1-\alpha)\cdot\check{n}_t + r_1\cdot\gamma\cdot\check{a}_t + r_2\cdot\left(\eta\cdot\check{\theta}_t + \check{a}_t\right) + r_3\mathbb{E}_t\left[\eta\cdot\check{\theta}_{t+1} + \check{a}_{t+1}\right] = 0$$

with $r_1 = \omega\cdot\frac{1}{\alpha\cdot\overline{a}}\cdot\overline{n}^{1-\alpha}$, $r_2 = \frac{c}{q(\overline{\theta})}\cdot\frac{1}{\alpha}\cdot\overline{n}^{1-\alpha}$, and $r_3 = 1 - r_1 - r_2$.

- Worker's optimal search decision (A24):

$$-t_2\left[\frac{1}{1-\delta(1-s)}\left[\kappa\cdot\check{e}_t - (1-\eta)\cdot\check{\theta}_t\right] - \frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\mathbb{E}\left[\kappa\cdot\check{e}_{t+1} - (1-\eta)\cdot\check{\theta}_{t+1}\right]\right] - t_1(1+\kappa)\cdot\mathbb{E}\left[\check{e}_{t+1}\right]$$
$$+\varepsilon_e\cdot s_1\cdot\check{c}_t^e + \varepsilon_u\cdot s_2\cdot\check{c}_t^u = 0$$

where we define the elasticity of $v(\cdot)$ around steady-state $\varepsilon_i = \frac{d\ln(v(x))}{d\ln(x)}\big|_{x=\overline{c_i}}$ and $s_1 = v(\overline{c}^e)/\overline{\Delta v}$, $s_2 = 1 - s_1$, $t_2 = 1 - t_1$, and $t_1 = \frac{\kappa\cdot\delta(1-s)\cdot k(\overline{e})}{\overline{\Delta v}}$.

- Lagrange multiplier A_t defined by equation (A29):

$$\check{A}_t + u_1\cdot\left(\check{n}_t - \varepsilon'_e\cdot\check{c}_t^e\right) + u_2\cdot\left(-q_1\cdot\check{n}_t - \varepsilon'_u\cdot\check{c}_t^u\right) = 0$$

where we define the elasticity of $v(\cdot)$ around steady-state $\varepsilon'_i = \frac{d\ln(v'(x))}{d\ln(x)}\big|_{x=\overline{c_i}}$ and where $u_1 = \frac{\overline{n}/v'(\overline{c}^e)}{\overline{n}/v'(\overline{c}^e)+(1-\overline{n})/v'(\overline{c}^u)}$, and $u_2 = 1 - u_1$.

- Lagrange multiplier B_t defined by equation (A30):

$$\check{B}_t - \left[(1-q_1)\cdot\check{n}_t + \check{A}_t - \left(\varepsilon'_e\cdot\check{c}_{et}\right) - \left(\varepsilon'_u\cdot\check{c}_t^u\right) + \left\{\varepsilon'_e\cdot v_1\cdot\check{c}_t^e + \varepsilon'_u\cdot v_2\cdot\check{c}_t^u\right\}\right] = 0$$

where $v_1 = \frac{v'(\overline{c}^e)}{v'(\overline{c}^e)-v'(\overline{c}^u)}$, and $v_2 = 1 - v_1$.

- Lagrange multiplier D_t defined by equation (A31):

$$\check{D}_t + \check{u}_t + (1-\eta)\check{\theta}_t - \kappa\cdot\check{e}_t - \left[w_2\cdot\check{u}_t + w_3\cdot\check{B}_{t-1} - w_4\left[(1-\eta)\cdot\check{\theta}_t + \check{e}_t - \left\{\frac{1}{s}\cdot\check{B}_t - \frac{1-s}{s}\cdot\check{B}_{t-1}\right\}\right]\right] = 0$$

where $w_1 = \frac{\overline{u}\cdot\overline{D}\cdot f(\overline{\theta})}{k'(\overline{e})}$, and $w_2 = \overline{u}/w_1$, $w_3 = \kappa\cdot(1-s)\cdot\overline{B}/w_1$, $w_4 = 1 - w_2 - w_3$.

- Lagrange multiplier C_t defined by equation (A32):

$$\check{h}_t - \eta \cdot \check{\theta}_t + \check{D}_t - x_6 \left[-\check{\theta}_t + \kappa \cdot \check{e}_t + \frac{1}{s}\check{B}_t - \frac{1-s}{s}\check{B}_{t-1} \right] - x_7 \left[\check{a}_t + \frac{1}{s}\check{C}_t - \frac{1-s}{s}\check{C}_{t-1} \right] = 0$$

where $x_1 = -\overline{h} \cdot q(\overline{\theta}) \cdot \overline{D} \cdot \frac{1-\eta}{\eta}$, $x_2 = \frac{1-\eta}{\eta} \cdot s \cdot \overline{B} \cdot \frac{k'(\overline{e})}{\overline{\theta}}$, and $x_6 = x_2/x_1$, $x_7 = 1 - x_6$.

- First-order condition (A28) with respect to n_t:

$$\check{D}_t - \Big\{ y_1 \left(\varepsilon_e \cdot z_1 \cdot \check{c}^e_t + \varepsilon_u \cdot z_2 \cdot \check{c}^u_t \right) + y_2 (1+\kappa) \mathbb{E}\left[\check{e}_{t+1}\right] + y_3 \cdot \mathbb{E}\left[\check{D}_{t+1} - z_6 \left(\check{e}_{t+1} + (1-\eta)\check{\theta}_{t+1} \right) \right]$$
$$+ y_4 \cdot \left(\check{C}_t + \check{a}_t + (\alpha - 2) \cdot \check{n}_t \right) + y_5 \left(\check{A}_t + \{ z_3 \cdot \gamma \cdot \check{a}_t + z_4 \cdot \check{c}^e_t + z_5 \cdot \check{c}^u_t \} \right) \Big\} = 0$$

where ε_i is defined as above and $z_1 = \frac{v(\overline{c}^e)}{v(\overline{c}^e) - v(\overline{c}^u)}$, $y_1 = \frac{v(\overline{c}^e) - v(\overline{c}^u)}{\overline{D}}$, $y_2 = \delta \cdot (1-s) \cdot \frac{k(\overline{e})}{\overline{D}}$, $y_3 = (1-s) \cdot \left(1 - \overline{e} \cdot f(\overline{\theta})\right)$, $z_3 = \frac{\omega}{\omega - (\overline{c}^e - \overline{c}^u)}$, $z_4 = -\frac{\overline{c}^e}{\omega - (\overline{c}^e - \overline{c}^u)}$, $y_4 = -\alpha \cdot (1-\alpha) \cdot \frac{\overline{C} \cdot \overline{n}^{\alpha - 2}}{\overline{D}}$, $z_6 = \frac{\overline{e} \cdot f(\overline{\theta})}{1 - \overline{e} \cdot f(\overline{\theta})}$, and $z_2 = 1 - z_1$, $z_5 = 1 - z_3 - z_4$, $y_5 = 1 - y_1 - y_2 - y_3 - y_4$.

In addition we assume that the log-deviation of technology $\check{a}_t$ follows an AR(1) process:

$$\check{a}_t = \nu \cdot \check{a}_{t-1} + z_t,$$

where $z_t \sim N(0, \sigma^2)$ is the innovation to technology driving fluctuations in the log-linear model.

We compute the unique stationary rational expectations solution to the log-linear system using the standard Anderson and Moore [1985] method.This solution allows us to compute the IRFs of variables to unexpected technology shocks.

C.7 Calibration

We calibrate all parameters at a weekly frequency as shown in Table 1. The calibration strategy follows closely that in Michaillat [forthcoming], so this section only highlights differences and novelties. We normalize average search effort $\hat{e} = 1$, and average technology $\hat{a} = 1$.

We use a Cobb-Douglas matching function $h(u,o) = \omega_h \cdot u^{\eta} \cdot o^{1-\eta}$ and set $\eta = 0.7$, in line with empirical evidence [Petrongolo and Pissarides, 2001]. To estimate the matching efficiency ω_h, we use the Beveridge curve (14) to find

$$\omega_h = s/(1-s) \cdot (1-\hat{u})/\hat{u} \cdot \hat{\theta}^{\eta - 1}. \tag{A42}$$

We use the seasonally-adjusted, monthly series for the number of vacancies collected by the Bureau of Labor Statistics (BLS) in the Job Openings and Labor Turnover Survey (JOLTS), 2000–2010, and the seasonally-adjusted, monthly unemployment level computed by the BLS from the Current

Population Survey (CPS) over the same period, to compute labor market tightness and unemployment. We find $\hat{\theta} = 0.47$ and $\hat{u} = 5.9\%$. The resulting estimate of matching efficiency is $\omega_h = 0.19$.

We calibrate the wage flexibility γ based on estimates obtained in micro-data. As discussed in Michaillat [forthcoming], the flexibility of wages in newly created jobs, and not that in existing jobs, mostly drives job creation. Furthermore, the estimate closest to this flexibility in US data is provided by Haefke et al. [2008]. They estimate an elasticity of total earnings of job movers with respect to productivity of 0.7, using panel data following a sample of production and supervisory workers over the 1984–2006 period. If there is "cyclical upgrading", a improvement in the composition of jobs accepted by workers in expansions, 0.7 is in fact an upper bound on the elasticity of wages in newly created jobs. A lower bound for the elasticity of wages in newly created jobs is the elasticity of wages in existing jobs, estimated in the 0.1–0.45 range with US data [Pissarides, 2009]. Thus we set $\gamma = 0.5$, in the middle of the range of plausible values.

We choose risk aversion $\rho = 1$, which is on the low side of the most compelling estimates [Chetty, 2004, 2006b]. We choose $\kappa = 2.1$ such that the micro-elasticity of unemployment $1-n$ with respect to benefits c^u, defined by

$$\frac{c^u}{1-n} \cdot \left[\frac{\partial n^s(\theta, e)}{\partial e} \cdot \frac{\partial e(\theta, \Delta v)}{\partial \Delta v} \right] \cdot \frac{d\Delta v}{dc^u} = \varepsilon^m \cdot \left[\frac{\Delta c}{c^u} \cdot \frac{dc^u}{d\Delta c} \right]^{-1}$$

be in line with the elasticity of 0.9 estimated by Meyer [1990].[26]

As summarized by Pavoni and Violante [2007], the state-determined weekly benefits generally replace between 50% and 70% of the individual's last weekly pre-tax earnings. Employee's earnings are subject to a 7.65% payroll taxes: 6.2% is taxed to finance social security and 1.45% is taxed to finance Medicare.[27] Hence, we set the replacement rate to $\hat{\tau} = 0.6/(1-0.0765) = 65\%$. With $\kappa = 2.1$, $\rho = 1$, and $\hat{\tau} = 65\%$, we obtain $\omega_k = 0.58$ to match $\hat{e} = 1$.

C.8 The government can borrow and save

In this section, we characterize the optimal equilibrium when the government has access to a complete market for Arrow-Debreu securities. The worker's and firm's problem are unchanged, and are characterized by (A24) and (A25). The law of motion of unemployment is unchanged, and is given by (A26). However, we remove the period-by-period budget constraint (A27). There is now one less equation in the system characterizing the optimal equilibrium, but there is also one less variable: the Lagrange multiplier A_t.

[26] The elasticity estimated by Meyer [1990] is conceptually close to a micro-elasticity because it either controls for state unemployment rates or uses state fixed effects. Meyer [1990] estimates the elasticity of the hazard rate out of unemployment with respect to benefits, which equals the elasticity of unemployment duration with respect to benefits. In our model, the hazard rate is $e \cdot f(\theta)$, so unemployment duration is $1/(e \cdot f(\theta)) = u/(s \cdot n) \approx u/s \approx (1-n)/s$. Hence the elasticity of unemployment $1-n$ is similar to elasticity of duration with respect to benefits. In the appendix, we express the micro-elasticity of unemployment with respect to benefits as a function of equilibrium variables n, u, θ.

[27] Prior to 1987, benefit income was exempt from income tax. Since 1987 benefits have been fully taxable, so we abstract from the income tax.

The Lagrangian of the government's problem is the same as in Section C.4, except that Lagrange multiplier on the period-by-period budget constraint, A_t, is constant over time and across histories: or all t, a^t, $A_t(a^t) = A$. This is because the government faces the unique intertemporal budget constraint (21), which is weighted by one unique Lagrange multiplier. The first-order conditions of the government's problem simplify accordingly.

We obtain the log-linear system describing the optimal equilibrium by modifying the log-linear system of Section C.6 accordingly. To be able to simulate the log-linear model, however, we need to determine the Lagrange multiplier A on the intertemporal budget constraint. A is determined such that the government's intertemporal budget constraint (21), which replaces the sequence of period-by-period budget constraints, be binding. We define the deficit in period t by

$$\Lambda(S_t) = n_t \cdot c_t^e + (1 - n_t) \cdot c_t^u - n_t \cdot w(a_t).$$

where we define the vector

$$S_t = [a_t, n_t, c_t^e, c_t^u].$$

The intertemporal budget constraint (21) can rewritten as

$$\sum_{t=0}^{+\infty} \delta^t \cdot \mathbb{E}_0 [\Lambda(S_t)] = 0. \tag{A43}$$

We can linearize the deficit around its steady-state value $\Lambda\left(\overline{S}\right)$:

$$\begin{aligned}
\Lambda(S_t) &\approx \Lambda\left(\overline{S}\right) + \overline{a} \cdot \frac{\partial \Lambda}{\partial a}(\overline{S}) \cdot \frac{da_t}{\overline{a}} + \overline{n} \cdot \frac{\partial \Lambda}{\partial n}(\overline{S}) \cdot \frac{dn_t}{\overline{n}} + \overline{c}^e \cdot \frac{\partial \Lambda}{\partial c^e}(\overline{S}) \cdot \frac{dc_t^e}{\overline{c}^e} + \overline{c}^u \cdot \frac{\partial \Lambda}{\partial c^u}(\overline{S}) \cdot \frac{dc_t^u}{\overline{c}^u} \\
\Lambda(S_t) &\approx \Lambda\left(\overline{S}\right) + \Lambda_1 \cdot \check{a}_t + \Lambda_2 \cdot \check{n}_t + \Lambda_3 \cdot \check{c}_t^e + \Lambda_4 \cdot \check{c}_t^u \\
\mathbb{E}_0 [\Lambda(S_t)] &\approx \Lambda\left(\overline{S}\right) + \Lambda_1 \cdot \mathbb{E}_0 [\check{a}_t] + \Lambda_2 \cdot \mathbb{E}_0 [\check{n}_t] + \Lambda_3 \cdot \mathbb{E}_0 [\check{c}_t^e] + \Lambda_4 \cdot \mathbb{E}_0 [\check{c}_t^u],
\end{aligned}$$

where $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4$ are constant. Using (A43), we infer that the intertemporal budget constraint (21) is a linear combination of the expected value of the log-deviations $\{\mathbb{E}_0 \{\check{n}_t\}, \mathbb{E}_0 \{\check{a}_t\}, \mathbb{E}_0 \{\check{c}_t^e\}, \mathbb{E}_0 \{\check{c}_t^u\}\}_{t=0}^{+\infty}$ and of the steady-state deficit $\Lambda\left(\overline{S}\right)$.

We compute the unique stationary rational expectations solution to the log-linear system using the standard Anderson and Moore [1985] method. Let $X_t \in \mathbb{R}^k$ be the vector of log-deviations: $X_t = [\check{a}_t, \check{n}_t, \check{c}_t^e, \check{c}_t^u, \dots]'$. Let $Z_{t+1} \in \mathbb{R}^k$ be a vector of innovations at time $t+1$. In our system there is only one exogenous shock, so there is only one non-zero entry in the vector Z_{t+1}: $Z_{t+1} = [0, 0, \dots, z_{t+1}]'$ where $z_{t+1} \sim N(0, \sigma^2)$. The solution to the log-linear system satisfies

$$X_{t+1} = M_1 X_t + M_2 Z_{t+1},$$

where $M_1 \in \mathbb{R}^{k \times k}, M_2 \in \mathbb{R}^{k \times k}$ are matrices that are constant over time. Taking expectations, and using the fact that X_t is stationary: for all $t \geq 0$,

$$\mathbb{E}_0 [X_t] = \mathbb{E}_0 [X_{t+1}] = M_1 \mathbb{E}_0 [X_t] + M_2 \mathbb{E}_0 [Z_{t+1}] = M_1 \mathbb{E}_0 [X_t].$$

Since all the eigenvalues from M_1 have an absolute value strictly less than one, we infer that for all $t \geq 0$, $\mathbb{E}_0[X_t] = 0$. Hence the log-linear system is such that

$$\mathbb{E}_0[\check{n}_t] = \mathbb{E}_0[\check{a}_t] = \mathbb{E}_0[\check{c}^e_t] = \mathbb{E}_0[\check{c}^u_t] = 0.$$

We conclude that the intertemporal budget constraint is satisfied by the solution to the log-linear system in a stochastic environment as long as it holds in steady-state and $\Lambda\left(\overline{S}\right) = 0$.

Hence to determine A, we need to solve for the steady state of this model, in which the government faces the unique budget constraint (21). This steady state is the same as that of the baseline infinite-horizon model of Section 4.1, in which the government faces a sequence of budget constraints (15). So A can be determined by solving the system of equations (A33)–(A40). Obviously, A is the same as in the steady state of the baseline infinite-horizon model of Section 4.1

C.9 Unemployment benefits of finite duration

Timing.

- beginning of period t, matching process: unemployed workers search for a job with effort e_t
- beginning of period t, end of matching process: jobseekers find a job with probability $e_t \cdot f(\theta_t)$
- middle of period t: production; workers consume transfer c_t from the government
- end of period t, separations: a fraction of employed workers lose their jobs; a fraction λ_t of eligible unemployed workers become ineligible

Notations. We introduce three superscripts: e for Employed; u for unemployed worker eligible to receive Unemployment insurance; a for unemployed worker whose UI expired, and who only receive social Assistance. We now define:

- c^e_t: consumption of an employed worker
- c^u_t: consumption of an unemployed worker who is eligible to receive UI (limited duration)
- c^a_t: consumption of a worker who receives social assistance (unlimited duration)
- x^u_t and x^a_t: probability to be unemployed and receive UI or social assistance at the beginning of period t
- z^u_t and z^a_t: probability to be unemployed and receive UI or social assistance in period t after the matching process, and before the production/consumption process.
- e^u_t and e^a_t: job-search effort of an unemployed worker who receives UI or social assistance in period t

To simplify notation, we define:

$$\Delta v_t^{u,e} \equiv [v(c_t^e) - v(c_t^u)]$$
$$\Delta v_t^{a,u} \equiv [v(c_t^u) - v(c_t^a)]$$
$$\Delta v_t^{a,e} \equiv [v(c_t^e) - v(c_t^a)].$$

Flows of workers. Given the timing of the model and our notations, the various stocks of employed and unemployed workers are related by:

$$z_t^u = x_t^u \cdot (1 - e_t^u \cdot f(\theta_t)) \tag{A44}$$
$$z_t^a = x_t^a \cdot (1 - e_t^a \cdot f(\theta_t)) \tag{A45}$$
$$x_t^u = z_{t-1}^u \cdot (1 - \lambda_{t-1}) + s \cdot n_{t-1} \tag{A46}$$
$$x_t^a = z_{t-1}^a + \lambda_{t-1} \cdot z_{t-1}^u \tag{A47}$$
$$n_t = 1 - (z_t^a + z_t^u). \tag{A48}$$

Worker's problem. The Lagrangian of the worker's problem is

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Bigg\{ -x_t^u \cdot k(e_t^u) - x_t^a \cdot k(e_t^a) + v(c_t^e) - z_t^u \cdot \Delta v_t^{u,e} - z_t^a \cdot \Delta v_t^{a,e}$$
$$+ A_t \{z_t^u - x_t^u \cdot (1 - e_t^u \cdot f(\theta_t))\}$$
$$+ B_t \{z_t^a - x_t^a \cdot (1 - e_t^a \cdot f(\theta_t))\}$$
$$+ C_t \left\{x_t^u - z_{t-1}^u \cdot (1 - \lambda_{t-1}) - s \cdot \left(1 - z_{t-1}^u - z_{t-1}^a\right)\right\}$$
$$+ D_t \left\{x_t^a - \lambda_{t-1} \cdot z_{t-1}^u - z_{t-1}^a\right\} \Bigg\}.$$

The first-order condition with respect to efforts e_t^u and e_t^a in the current period gives:

$$k'(e_t^u) = f(\theta_t) \cdot A_t$$
$$k'(e_t^a) = f(\theta_t) \cdot B_t.$$

The first-order condition with respect to beginning-of-period unemployment probability x_t^u and x_t^a yield:

$$C_t = k(e_t^u) + A_t \cdot (1 - e_t^u f(\theta_t))$$
$$D_t = k(e_t^a) + B_t \cdot (1 - e_t^a f(\theta_t)).$$

The first-order condition with respect to post-matching unemployment probability z_t^u and z_t^a yield:

$$A_t = \Delta v_t^{u,e} + (1 - s) \cdot \delta \cdot \mathbb{E}_t [C_{t+1}] + \lambda_t \cdot \delta \cdot \mathbb{E}_t [D_{t+1} - C_{t+1}]$$
$$B_t = \Delta v_t^{a,e} + (1 - s) \cdot \delta \cdot \mathbb{E}_t [D_{t+1}] + s \cdot \delta \cdot \mathbb{E}_t [D_{t+1} - C_{t+1}].$$

We define $(1+\kappa)$ as the elasticity of the cost function $k(\cdot)$. Combining these equations we have:

$$\frac{\Delta k_t'}{f(\theta_t)} = \Delta v_t^{a,u} + (1-\lambda_t)\cdot\delta\cdot\mathbb{E}_t\left[D_{t+1}-C_{t+1}\right]$$
$$\mathbb{E}_t\left[D_{t+1}-C_{t+1}\right] = \mathbb{E}_t\left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} - \kappa\cdot\Delta k_{t+1}\right]$$

where $\Delta k_t = k(e_t^a) - k(e_t^u)$ and $\Delta k_t' = k'(e_t^a) - k'(e_t^u)$. Combining these equations once more yields:

$$\frac{k'(e_t^u)}{f(\theta_t)} + (1-s)\cdot\delta\cdot\mathbb{E}_t\left[\kappa\cdot k(e_{t+1}^u) - \frac{k'(e_{t+1}^u)}{f(\theta_{t+1})}\right] = \Delta v_t^{u,e} + \lambda_t\cdot\delta\cdot\mathbb{E}_t\left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} - \kappa\cdot\Delta k_{t+1}\right] \quad \text{(A49)}$$
$$\frac{k'(e_t^a)}{f(\theta_t)} + (1-s)\cdot\delta\cdot\mathbb{E}_t\left[\kappa\cdot k(e_{t+1}^a) - \frac{k'(e_{t+1}^a)}{f(\theta_{t+1})}\right] = \Delta v_t^{a,e} + s\cdot\delta\cdot\mathbb{E}_t\left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} - \kappa\cdot\Delta k_{t+1}\right] \quad \text{(A50)}$$

Also, notice that job-search efforts e_t^u and e_t^a are related by:

$$\frac{\Delta k_t'}{f(\theta_t)} + (1-\lambda_t)\cdot\delta\cdot\mathbb{E}_t\left[\kappa\cdot\Delta k_{t+1} - \frac{\Delta k_{t+1}'}{f(\theta_{t+1})}\right] = \Delta v_t^{a,u}.$$

Firm's problem. Even if benefits have finite duration, the firm's problem is similar to that described in Section C.1 in the baseline infinite-horizon model. Hence the optimal hiring behavior of the firm satisfies (18).

Government's problem. The generosity of unemployment insurance and social assistance are parameterized by $\tau_t^{u,e} \equiv c_t^u/c_t^e$, $\tau_t^{a,e} \equiv c_t^a/c_t^e$, $\tau_t^{a,u} \equiv c_t^a/c_t^u = \tau_t^{a,e}/\tau_t^{u,e}$. We assume that the government keep the generosity of the system of transfers constant: for all t, $\tau_t^{u,e} = \tau^{u,e}, \tau_t^{a,e} = \tau^{a,e}, \tau_t^{a,u} = \tau^{a,u}$. Furthermore, we assume that $v(\cdot) = \ln(\cdot)$, consistently with our preferred calibration. This choice allows us to write $\Delta v_t^{u,e} = -\ln(\tau^{u,e})$, $\Delta v_t^{a,e} = -\ln(\tau^{a,e})$, $\Delta v_t^{a,u} = -\ln(\tau^{a,u})$. Under this assumption, the incentives to search provided by government transfers remain constant over the business cycle. The government chooses the arrival rate λ_t of ineligibility to unemployment insurance to maximize social welfare:

$$\mathbb{E}_0\sum_{t=0}^{+\infty}\delta^t\cdot\left\{-x_t^u\cdot k(e_t^u) - x_t^a\cdot k(e_t^a) + \ln(c_t^e) + z_t^u\cdot\ln(\tau^{u,e}) + z_t^a\cdot\ln(\tau^{a,e})\right\},$$

subject to a budget constraint for all t:

$$n_t\cdot w(a_t) = n_t\cdot c_t^e + z_t^u\cdot c_t^u + z_t^a\cdot c_t^a = c_t^e\cdot\left[n_t + z_t^u\cdot\tau^{u,e} + z_t^a\cdot\tau^{a,e}\right];$$

subject to the laws of motion (A44)–(A48) for the stocks $\{x_t^u, x_t^a, z_t^u, z_t^a, n_t\}_{t=0}^{+\infty}$ of employed and unemployed workers; subject to the optimality condition (A49) for the job search $\{e_t^u\}_{t=0}^{+\infty}$ of unemployed workers receiving unemployment benefits, the optimality condition (A50) for the job search $\{e_t^a\}_{t=0}^{+\infty}$ of unemployed workers receiving social assistance, and the optimality condition (18) for firm's employment $\{n_t^d\}_{t=0}^{+\infty}$; and subject to the equilibrium condition on the labor market that labor supply equals labor demand for all t: $n_t = n_t^d$, which determines labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$.

Government's problem in a static environment. We focus on a static environment, , in which there are no aggregate shocks ($a_t = a$ for all t), and the labor market in steady state: $x_t^a = x^a$, $x_t^u = x^u$, $z_t^a = z^a$, $z_t^u = z^u$, $n_t = n$, . In that case, we can simplify the first-order conditions of workers' and firm's problems (3 equations), the laws of motion of the stocks of workers (5 equations), and the budget constraint of the goverment's problem (1 equation). These 9 constraints describe a collection of 9 variables $\{x_u, x_a, z_u, z_a, n, e^u, e^a, \theta, c^e\}$, which constitute an *equilibrium with unemployment insurance*. All other variables of interest can be constructed from these 9 variables (for instance, h, u, c^u, c^a).

Let us construct this system of 9 equations. We first express $\{z_u, x_u, z_a, x_a, n\}$ as a function of $\{\lambda, \theta, e^a, e^u\}$. Outflows from social assistance equal inflows into social assistance:

$$\begin{aligned} x_a e^a f(\theta) &= \lambda x_u (1 - e^u f(\theta)) \\ x_a &= x_u \cdot \lambda \cdot \frac{1 - e^u f(\theta)}{e^a f(\theta)} \end{aligned}$$

Outflows from employment equal inflows into employment:

$$\begin{aligned} s \cdot n &= x_a e^a f(\theta) + x_u e^u f(\theta) \\ n &= \frac{1}{s} \cdot x_u \cdot \left[e^u f(\theta)(1-\lambda) + \lambda \right]. \end{aligned}$$

Writing the stock of unemployment at the beginning of the period in two different ways:

$$\begin{aligned} 1 - (1-s) \cdot n &= x_a + x_u \\ 1 - \frac{1-s}{s} \cdot x_u \cdot [e^u f(\theta)(1-\lambda) + \lambda] &= x_u \left[1 + \lambda \frac{1 - e^u f(\theta)}{e^a f(\theta)} \right] \\ x_u &= \frac{1}{1 + \lambda \cdot \frac{1 - e^u f(\theta)}{e^a f(\theta)} + \frac{1-s}{s} \cdot [e^u f(\theta)(1-\lambda) + \lambda]} \end{aligned}$$

Combining our previous results, we get the first 5 equations:

$$x_u = \left\{1+\lambda\cdot[1-e^u\cdot f(\theta)]\left[\frac{1}{e^a f(\theta)}+\frac{1-s}{s}\right]+\frac{1-s}{s}\cdot e^u\cdot f(\theta)\right\}^{-1} \tag{A51}$$

$$x_a = \left\{1+\frac{1-s}{s}\cdot e^a\cdot f(\theta)\cdot\left\{1+\frac{1}{\lambda}\cdot\left[\frac{1}{e^u\cdot f(\theta)}-1\right]^{-1}\right\}\right\}^{-1} \tag{A52}$$

$$z_u = \left\{1+\lambda\cdot\left[\frac{1}{e^a\cdot f(\theta)}+\frac{1-s}{s}\right]+\frac{1}{s}\cdot\left[\frac{1}{e^u\cdot f(\theta)}-1\right]^{-1}\right\}^{-1} \tag{A53}$$

$$z_a = \left\{1+\left[\frac{1}{e^a\cdot f(\theta)}-1\right]^{-1}\cdot\frac{1}{s}\cdot\left\{1+\frac{1}{\lambda}\cdot\left[\frac{1}{e^u\cdot f(\theta)}-1\right]^{-1}\right\}\right\}^{-1} \tag{A54}$$

$$n = \left\{1+s\cdot\left[\frac{1}{e^a f(\theta)}-1\right]+\frac{s}{(1-\lambda)\cdot e^u\cdot f(\theta)+\lambda}\cdot\left[1-\frac{e^u}{e^a}\right]\right\}^{-1}. \tag{A55}$$

We can also derive c^e from the resource constraint:

$$c^e = [w(a)\cdot n]\cdot[n+z_u\cdot\tau^{u,e}+z_a\cdot\tau^{a,e}]^{-1}. \tag{A56}$$

From the worker's problem, we write $\{e^u, e^a\}$ as functions of $\{\theta, \lambda\}$:

$$[1-(1-s)\cdot\delta]\cdot\frac{k'(e^a)}{f(\theta)}+(1-s)\cdot\delta\cdot\kappa\cdot k(e^a) = -\ln(\tau^{a,e})+s\cdot\delta\cdot\left[\frac{\Delta k'}{f(\theta)}-\kappa\cdot\Delta k\right] \tag{A57}$$

$$[1-(1-\lambda)\cdot\delta]\cdot\frac{\Delta k'}{f(\theta)}+(1-\lambda)\cdot\delta\cdot\kappa\cdot\Delta k = -\ln(\tau^{a,u}). \tag{A58}$$

θ is then determined by the firm's optimality condition:

$$g'(n) = w(a)/a+[1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)}. \tag{A59}$$

In this static environment in which the replacement rates $\tau^{u,e}$ and $\tau^{a,e}$ are fixed, given technology a, the government's problem is to pick the arrival rate λ (or equivalently the expected unemployment benefit duration $1/\lambda$) to maximize per-period social welfare:

$$-x^u\cdot k(e^u)-x^a\cdot k(e^a)+\ln(c^e)+z^u\cdot\ln(\tau^{u,e})+z^a\cdot\ln(\tau^{a,e}), \tag{A60}$$

where $\{z^u, z^a, x^u, x^a, e^u, e^a, c^e\}$, together with $\{n, \theta\}$, solve the system of equations (A51)–(A59).

To find the optimal equilibrium in a static environment for a given a_j, we compute a sequence of equilibria with unemployment insurance for a sequence of arrival rates $\{\lambda_i\}_i$. To solve for an equilibrium with unemployment insurance under technology a_j and arrival rate λ_i, we perform a grid search over θ. For a sequence $\{\theta_k\}_k$, we solve the system of equations (A57)–(A58) to find a collection of sequences $\left\{e_k^u, e_k^a\right\}_k$. Using equations eq:sys1–(A55), we then compute a collection of sequences $\left\{z_k^u, z_k^a, x_k^u, x_k^a, n_k\right\}_k$. We also use equation (A56) to compute the sequence $\left\{c_k^e\right\}_k$. Next, we pick the indice k^* such that equation (A59) be satisfied. An equilibrium with unemployment insurance under technology a_j and arrival rate λ_i in a static environment is $\left\{z_{k*}^u, z_{k*}^a, x_{k*}^u, x_{k*}^a, e_{k*}^u, e_{k*}^a, c_{k*}^e, n_{k*}, \theta_{k*}\right\}$. We repeat this computation for the sequence of arrival rate $\{\lambda_i\}_i$, and we pick the equilibrium with the highest per-period welfare (A60). This gives us the optimal equilibrium and optimal arrival rate under technology a_j. We repeat this computation for a sequence of technology $\left\{a_j\right\}_j$ to plot the graphs in Figure 5.

Calibration. We calibrate this model similarly as the baseline model calibrated in Section C.7. We only need to adjust the calibration of the matching efficiency ω_h and the disutility of effort ω_k. To do so, we set unemployment benefits at 78% of the pre-tax wage, social assistance at $1/2 \cdot 78\% = 39\%$ of the pre-tax wage, such that an expected duration of 26 weeks be optimal when the unemployment rate is at its average level of 5.9%.[28] We normalize $\hat{e}^u \cdot \hat{x}^u + \hat{e}^a \cdot \hat{x}^a = \hat{u}$ to determine $\omega_h = 0.19$ using (A42). Keeping $\kappa = 2.1$, we solve a system of three unknowns: $\hat{e}^a, \hat{e}^u, \omega_k$, and three equations: (A57), (A58), and $\hat{u} = \hat{e}^u \cdot \hat{x}^u + \hat{e}^a \cdot \hat{x}^a$, to find $\omega_k = 0.43$. In this system of three equations, we substituted $\hat{x}^a, \hat{x}^u$ by the functions of $\hat{e}^a, \hat{e}^u$ given by (A51) and (A52). As a byproduct, we find $\hat{e}^a = 1.40$ and $\hat{e}^u = 0.94$.

In steady state, when technology $\overline{a} = 1$, we find that: the optimal arrival rate $\overline{\lambda} = 3.9\%$, corresponding to an expected duration of 26 weeks for unemployment benefits; unemployment $\overline{u} = 5.9\%$; 15% of unemployed workers are ineligible and 85% are eligible.

D Empirical Evidence from the CWBH

The data we use is from the Continuous Wage and Benefit History (CWBH). The dataset records all employment and unemployment history for workers in 8 States from 1976 to 1983.[29] The advantage of CWBH data is that it is administrative data with accurate information on weeks of UI receipt, pre-unemployment earnings, the level of UI benefits, and the potential duration of benefits over time. Since we do not observe individuals after their benefits lapse, we censor their unemployment spells at the time they exhaust their benefits. Our duration outcome of interest is the total number of weeks for which UI was claimed. Since a lot of claims exhibit interruptions, we restrict our sample to individuals for which there is no more than 2 weeks of interruption between

[28] These rates are much higher than what is observed in the US. We saw in Section C.7 that benefits replace 60% of pre-tax earnings, well below 78%. Furthermore, Pavoni and Violante [2007] compute that in 1996, the median monthly allotment of food stamps for a family of four was \$397 per month. Using CPS data, they find that the median monthly post-tax wage for a worker with at most a high-school diploma, eligible to be on welfare rolls, is \$1,540. Hence, if food stamps are the only social assistance available when unemployment benefits are exhausted, the rate of social assistance is roughly $397/1,540 = 26\%$, well below 39%.

[29] We use the exhaustive CWBH files and therefore our data is different from the limited sample used in Moffitt [1985] or Meyer [1990] which contains only 3,365 observations. Our estimation sample contains 39,852 unemployment spells. We thank Patricia Anderson and Bruce Meyer for giving us access to the CWBH data.

each week of benefits.[30] We also eliminate observations with recalls and partial UI claims (*i.e.* people getting UI while still partially at work).

D.1 Graphical Evidence

In order to identify the micro-elasticity, we investigate the effect of UI benefits using only within state×period variations in individual benefits. We begin with some graphical evidence. To make sure that individual variation in Weekly Benefits Amounts (WBA) is not correlated with other characteristics such as previous wage or tenure that might affect unemployment duration, we regress benefits on a series of non parametric controls for previous wage, number of quarters worked in the year prior to unemployment, education, gender, and quarter and state fixed effects interacted. The residual variation in benefit is likely to be exogenous, and comes primarily from non-linearities in the WBA schedule as well as from special state rules regarding total benefit amounts in a given benefit year. We then classify unemployment spells in high and low WBA regimes using the residuals from the previous regression. A spell is in a low WBA regime if the residual WBA is below the 25th percentile of the distribution of residuals in the state for the quarter during which the spell started. A spell is in a high WBA regime if the residual WBA is over the 75th percentile of the distribution of residuals in the state for the quarter during which the spell started.

As in Kroft and Notowidigdo [2011], Figure A1 shows the survival estimates for the duration of unemployment spells in the CWBH dataset for spells broken down by low versus high unemployment regimes as well as by low versus high WBA regimes.[31] We retrieve the non-parametric baseline hazard from a Cox proportional hazard model with State and year fixed effects interacted, also controlling for observable characteristics of the unemployed (previous wage level, age, education, marital status, ethnicity, number of dependents) and stratified in low (dark lines) vs. high (gray lines) individual WBA regimes. Most importantly, we estimate this model separately for low vs high unemployment regimes. To break down spells by low vs high unemployment regimes we use variation in unemployment rate across states as proxies for business cycle conditions. A spell is in a low unemployment regime if at the beginning of the spell, the quarterly unemployment rate of the state is below the median unemployment rate of all the states in the US.[32]

The figure confirms that the baseline survival rate is higher when individual benefits are higher. The effect seems to be very similar in high and low unemployment regimes when we control for local labor market tightness. To investigate the cyclicality of the micro-elasticity, we now turn to semi-parametric estimation methods.

[30] We also looked at the total number of weeks for which UI was paid, as well as the duration of initial spells (total number of weeks claimed without interruption after the initial claim was filed) and found similar results.

[31] We report for each unemployment regime the baseline survival function estimated at the mean of the covariates.

[32] We use variations in unemployment rate across states as proxies for business cycle conditions following Kroft and Notowidigdo [2011]. We find similar results using other measures of labor market conditions. In particular, we find similar results using unemployment rate variations within state over time and defining high unemployment spells as spells that started when the state unemployment rate was over its 1976-2010 median.

D.2 Semi-Parametric Estimation

To identify the micro-elasticity and its cyclical behavior, we estimate the effect of benefits once again using only within state×year variations in individual WBA. We fit a Cox proportional hazard model with State and year fixed effects interacted, and controlling for observable characteristics of the unemployed (age, education, marital status, ethnicity, number of dependents). We control for time to benefit exhaustion by adding a 6-pieces exhaustion spline as in Meyer [1990]. Most importantly, we also introduce a series of non parametric controls for previous wage and previous work experience. In particular, we add 10 dummies for previous wage level, and dummies for the number of quarters worked in the year prior to unemployment. With this rich set of controls, the residual variation in UI benefits comes primarily from non-linearities in the benefits schedule and is more likely to be exogenous. Results are displayed in table A1. Using the approximation that $log(D) = log(1/h)$, where D stands for duration and h is the hazard rate, the duration elasticity and other marginal effects of interest are given by the negative of the coefficient in the estimated hazard model.

Column (1) begins by replicating the specification of Meyer [1990], Table VI, column (7). This specification controls for previous wages using the log of earnings in the base period, and also controls for state fixed effects. Not surprisingly, and even if we use a different (much larger) sample than the one used in Meyer [1990], the results are almost exactly identical, with a duration elasticity of 0.587 versus 0.599 in Meyer [1990]. In order to control for the fact that the benefit level depends on previous earnings and experience, column (2) introduces much richer controls with 10 dummies for previous earnings level and a set of dummy variables for the number of quarters worked in the year preceding unemployment. Interestingly, the duration elasticity is almost divided by two by the introduction of these controls. This suggests that the magnitude of the estimates of Meyer [1990] is actually driven for a large part by the correlation between earnings and UI benefits. When controlling more flexibly for this correlation, the impact of UI benefits on duration becomes significantly smaller. To come closer to the estimation of the micro-elasticity, column (3) uses only within state×year variations in individual benefits by adding state and time fixed effects interacted. This specification has also the advantage of addressing the potential issue of the endogeneity of UI benefit variations over time, if the schedule of state UI benefits is endogenously modified when labor market conditions change over time. Results show that the duration elasticity is actually very similar in magnitude to that in column (2). We now investigate the cyclical behavior of these estimates. To do so, we begin in column (4) by interacting log benefits with a dummy for being in a high unemployment regime, defined, as above, as beginning an unemployment spell in a state whose unemployment rate is above the median unemployment rate in the US. The coefficient on the interaction term is the incremental change in the duration elasticity for spells in high unemployment regimes compared to low unemployment regimes. Our results show that the duration elasticity is very similar in low unemployment regimes: .34 (.038), and high unemployment regimes: .32 (.037). In column (5) we look at an alternate specification where we define state labor market conditions in absolute terms instead of relative terms. We interact log benefits with a dummy for spells beginning in states with unemployment rate superior to 8% (8.1% being the median unemployment rate for all state×quarter cells in our sample). The interaction term in this specification is small and not significantly different from 0. In column (6), we allow for a more flexible interaction between labor market conditions and log benefits. We create four dummies for spells beginning in states with: (1) unemployment rate below the 25th percentile of unemployment rates (for all state*quarter cells in our sample), (2) between the 25th percentile and the median, (3) above the median and below the 75th percentile, and (4) above the 75th percentile. We then interact log benefits with these four dummies. Results show that the duration elasticity is slightly decreasing with higher unemployment regimes, but the duration elasticities are not significantly different from one another.

Overall, this evidence is suggestive that the micro-elasticity of unemployment duration with respect to benefit level is not significantly different in high and low unemployment regimes, and confirms the results obtained by Schmieder et al. [2011] for the micro-elasticity with respect to potential duration.

Unfortunately the CWBH does not span a long time period and therefore does not exhibit enough variations in *average* UI benefits within state over time to investigate the cyclical behavior of the macro-elasticity. The elasticity of unemployment duration with respect to the average benefit level in each state×quarter that we find when fitting the Cox proportional hazard model described above, excluding state fixed effects, is higher for low unemployment regimes than for high unemployment regimes. But the robustness of such estimates is questionable. They suffer from a potentially serious omitted variable bias because benefits are higher in states with unobserved time-invariant characteristics which are correlated with high expected unemployment durations.

Table A1: Semi-Parametric Estimates of Hazard Rates

	(1) Meyer [1990]	(2)	(3)	(4)	(5)	(6)
log(UI)	-0.587***	-0.274***	-0.320***	-0.341***	-0.323***	
	(0.0394)	(0.0365)	(0.0368)	(0.0374)	(0.0370)	
State unemployment rate	-0.0550***	-0.0552***	-0.0207	-0.0226	-0.0251	-0.105***
	(0.00518)	(0.00519)	(0.0142)	(0.0143)	(0.0153)	(0.0209)
log(UI)× (u>median)				0.0248**		
				(0.00812)		
log(UI)×(u> .08)					0.00527	
					(0.00685)	
log(UI)×(u<p25)						-0.363***
						(0.0376)
log(UI)×(p25<u<median)						-0.353***
						(0.0371)
log(UI)×(median<u<p75)						-0.292***
						(0.0371)
log(UI)×(u>p75)						-0.274***
						(0.0378)
Non-param controls for previous wage & experience	NO	YES	YES	YES	YES	YES
Year×state F-E	NO	NO	YES	YES	YES	YES
# Spells	39852	39852	39852	39852	39852	39852
Log-likelihood	-136305.0	-136364.8	-135976.0	-135971.4	-135975.7	-135946.2

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table estimates the effect of UI weekly benefits levels on the hazard rate of leaving UI using the CWBH complete data for 8 US states from the late 1970s to early 1980s. We fit Cox proportional hazard models. All specifications include controls for gender, ethnicity, marital status, year of schooling, a 6-pieces exhaustion spline and state fixed effects. u denotes the state unemployment rate. log(UI) denotes the log-weekly UI benefit amount. p25 and p75 denote the 25th and 75th percentile of unemployment rates (among all state×quarter in our data). Column (1) replicates the specification of Meyer [1990], Table VI, column (7) (Meyer [1990] was using a much smaller dataset). Column (2) further adds non-parametric controls for previous earnings and experience. column (3) further adds year×state fixed effects. Columns (4) and (5) add the interaction of log(UI) and high unemployment dummies (unemployment rate above the median across all US states in the same quarter in column (4) and unemployment rate above 8% in column (5)). Column (6) adds the interaction of log(UI) with quartiles for the level of unemployment (quartiles defined across all state×quarter cells in our sample).

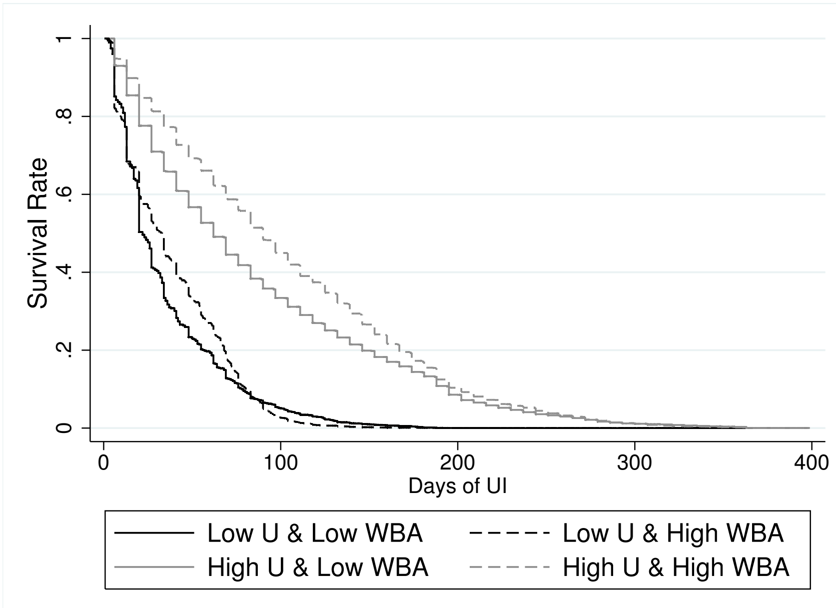

Figure A1: Survival Estimates With State×Year F.-E.

Source: CWBH

Notes: The Figure displays the baseline survival function estimates for the duration of unemployment spells broken down by low (plain lines) vs high (dash lines) unemployment regimes. A spell is in a low unemployment regime if at the beginning of the spell, the monthly unemployment rate of the State is below the median unemployment rate of all the States in the US. We also break down spells in high and low individual WBA regimes. A spell is in a high individual WBA regime if the residual WBA in a regression of WBA on a series of non parametric controls for wage, education, gender plus year and state fixed effects interacted, is below the 25th percentile of the distribution of residuals in the state for the quarter during which the spell started. A spell is in a high UI benefit regime if the residual WBA is over the 75th percentile of the distribution of residuals in the state for the quarter during which the spell started. The baseline survival function estimate is obtained from a Cox proportional hazard model including state and year fixed effects interacted and controlling for observable characteristics of the unemployed (previous wage level, age, education, marital status, ethnicity, number of dependents). The model is stratified in low (dark lines) vs high (gray lines) individual WBA regimes and estimated in high and low unemployment regimes. The model exploits only within State×year variation in benefits and therefore identifies the micro elasticity. The figure shows that higher individual benefits increases unemployment duration but that this effect is almost similar in high and low unemployment regimes.

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