

CEP Discussion Paper No 866

April 2008

**Multinational Firms, Monopolistic Competition
and Foreign Investment Uncertainty**

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Abstract

This is a model of multinational firms, which introduces option value of foreign direct investment, into a framework of Dixit-Stiglitz type monopolistic competition. Starting from a pure trading equilibrium and solving for the optimal investment rule gives a scale-up factor which implies existence of a wedge between markup revenues and foreign investment costs. Greater volatility and risk aversion increase this scale-up over foreign investment costs implying a delay in the exercise of FDI option, while growing market size and national income facilitate early exercise. The model is extended to include a Poisson jump process, which has policy implications for FDI reforms and explains 'wait and watch' behaviour of multinational firms better than a pure comparative advantage-trade cost framework does. While investment under uncertainty literature is based on the theory of call options, I solve 'FDI option' as a put option, thereby also enriching the theory of real options.

Keywords: Multinational firm, monopolistic competition, foreign investment uncertainty, FDI option

JEL Classifications: F21, F23

This paper was produced as part of the Centre's Globalisation Programme. The Centre for Economic Performance is financed by the Economic and Social Research Council.

Acknowledgements

I am grateful to Stephen Redding, Tony Venables, Henry Overman and Jean Pierre Zigrand for excellent guidance, feedback and advice.

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Published by
Centre for Economic Performance
London School of Economics and Political Science
Houghton Street
London WC2A 2AE

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ISBN 978-0-85328-262-4

1 Introduction

More than two-thirds of world trade today is determined by activities of multinational enterprises, a phenomenon not well explained by the traditional trade theory. For many years the economics of relationship between trade and investment were studied as complements or substitutes (Mundell, 1957; Lipsey and Weiss, 1981; Blomstrom et al, 1988). More recent studies indicate such a generalization is not possible, and that both trade and investment flows are determined simultaneously by the location decisions of multinational firms (Markusen, 2002).

‘New trade theory’ has developed general equilibrium models for multinational firms in the presence of imperfect competition (Markusen and Venables, 1998; Markusen, 2002; Helpman, 1984). While the vertical multinationals are explained by factor proportion analysis, horizontal multinational activity is explained by the proximity-concentration trade-off (Brainard, 1993; Brainard, 1997). More recently, Export vs FDI cut off has been derived in the presence of firm heterogeneity (Helpman, Melitz and Yeaple, 2004).

Real life behaviour is somewhat more complex than what can be explained by a conventional cost-benefit analysis. While most of the work above is based on a standard Marshallian kind of economic analysis, evidence indicates that firm’s investment decisions, and even more so, foreign investment decisions are undertaken in the face of uncertainty. Studies indicate that if we fail to take this into account, then, even for reasonable parameter values, we may be up to two times off the mark as compared to a routine cost-benefit analysis (Dixit and Pindyck, 1994).

Even when comparative advantage indicates foreign investment should flow in or when economic liberalization removes barriers to such investments, they do not automatically flow in. There is a considerable wait-and-watch, a kind of time lag or inertia followed by herd behaviour when foreign investments actually start flowing in. This implies existence of a value-for-waiting or in other words, some opportunity cost beyond what is accounted for in a pure comparative advantage-trade cost framework.

Almost simultaneously as the new trade theory was taking shape, a strand of literature was growing out of the core finance theory to model investment decisions of individual firms under uncertainty. Starting with the work of McDonald and Siegel (1986) on the value of waiting to invest and Dixit’s (1989) work on firm’s investment decisions under uncertainty, a rich literature has developed on investment under uncertainty a la Dixit and Pindyck

(1994). Baldwin and Krugman (1989) modelled hysteresis in trade under assumptions of large exchange rate shocks. Dixit (1989) applied it to exchange rate pass-through under perfect competition. Rafael and Vettas (2003) modelled Export and FDI for a single seller in the presence of growing demand. Their model showed that FDI is the preferred mode of production for proven demand, while exports is the preferred mode of production for uncertain demand.

I take a step forward from the existing literature and develop a model of many sellers (multinational firms) operating under foreign investment uncertainty within the framework of a Dixit-Stiglitz type monopolistic competition. I use option theory to derive an optimal foreign investment rule and model policy driven FDI liberalization as a mixed Poisson jump-Brownian motion stochastic process. Another useful feature of this model is that while investment under uncertainty literature is based on the theory of call options, I solve ‘FDI option’ as a put option, thereby also enriching the theory of real options.

Section 2 presents the underlying static model. Section 3 introduces foreign investment uncertainty and develops its inter-temporal counterpart. Section 4 elaborates some comparative experiments. Section 5 extends the model by introducing a mixed Poisson jump-Brownian motion stochastic process. Section 6 concludes.

2 The Static Model

There are two countries: i and j . There are two goods: Y , a homogeneous good, and X , a differentiated good with imperfectly substitutable varieties in the Dixit-Stiglitz fashion. Skilled labour (S) is the only factor of production and all costs are expressed in units of this factor.

Good Y (the homogeneous good) is produced by a perfectly competitive industry using a constant returns to scale technology:

$$Y_i = \frac{1}{a_y} S_{iy} \tag{1}$$

where a_y is the unit labour requirement and S_{iy} is the amount of skilled labour used in production of good Y . Its transport is costless and it is treated as numeraire (price normalized to one).

Good X, the differentiated good, is produced by a monopolistically competitive industry. Varieties of this good can be produced by national (indexed by n) or multi-national (indexed by m) firms. Let $N_i^n, N_i^m, N_j^n, N_j^m$ stand for the number of firms of each type operating in equilibrium, headquartered respectively in country i and j.

Sector X has a linear cost function with both fixed and variable costs.³ Total production costs for a national firm in country j are given by:

$$S_{jx}^n = H^n + G^n + cX_{jj}^n + cX_{ji}^n \quad (2)$$

where c are the constant marginal costs, X_{jj}^n and X_{ji}^n are respectively the outputs for domestic and foreign markets. Further, H^n are the headquarter level fixed costs and G^n are the plant level fixed costs, both of which are at least partly irreversible or sunk.

Similarly, total production costs for a multinational firm located in country j are given by:

$$S_{jx}^m + S_{ix}^m = H^m + G^n + cX_{jj}^m + G_i^m + cX_{ji}^m \quad (3)$$

where first part on the right hand side are the costs of operation within the home country and the second part on right hand side are the costs of operating in the foreign country.

Headquarter costs of multi-national operation, H^m , are typically different from headquarter costs of national operation, H^n , because of the need for greater headquarter services and additional costs of creating trans-national networks. Similarly, costs of foreign investment, G_i^m or G_j^m , are different from a similar initiative by domestic firms, G^n , as almost all countries have specific FDI regimes, meaning thereby that routes and mechanisms prescribed for foreign investment are different from those prescribed for domestic firms. Costs of foreign investment are also different between the two countries ($G_i^m \neq G_j^m$) and depend on their local foreign investment environments.

Assuming diversified production,⁴ wage (w) is pinned down by the numeraire sector in both the countries:

$$w = \frac{1}{a_y} \quad (4)$$

³Presence of fixed costs in the cost function implies economies of scale.

⁴I assume labour endowment and value of demand parameters is such that both countries produce both the goods. Symmetric countries always have diversified production.

Total incomes in both the countries depend on their respective factor endowments:

$$M_i = wS_i \quad (5)$$

$$M_j = wS_j \quad (6)$$

On the demand side, there is a representative consumer in each country with a Cobb-Douglas utility function, which for countries i and j are:

$$U_i = X_{ic}^\beta Y_{ic}^{1-\beta} \quad (7)$$

$$U_j = X_{jc}^\beta Y_{jc}^{1-\beta} \quad (8)$$

Here, X_{ic} is the CES aggregate of x-varieties in the familiar Dixit-Stiglitz fashion given by:

$$X_{ic} = [N_i^n (X_{ii}^n)^\alpha + N_j^n (X_{ji}^n)^\alpha + N_i^m (X_{ii}^m)^\alpha + N_j^m (X_{ji}^m)^\alpha]^\frac{1}{\alpha}; \quad (9)$$

such that $\{0 < \alpha < 1\}$.

Define $\epsilon = \frac{1}{1-\alpha}$ as the elasticity of substitution between any two x-varieties.⁵ N , as before, is the number of firms of each type operating in equilibrium. This utility function permits two stage budgeting:

In the first stage budgeting, the consumer allocates his total income to goods X and Y through the following demand functions:

$$Y_{ic} = (1 - \beta)M_i \quad (10)$$

$$X_{ic} = \beta \frac{M_i}{e_i} = \frac{M_{ix}}{e_i} \quad (11)$$

Here, M_{ix} is the amount of country i's national income spent on good X, e_i is the unit expenditure function for X_{ic} (also called the price index) and as already mentioned, good Y is used as numeraire (i.e. its price is equal to one).

In the second stage budgeting, the consumer solves the sub-utility maximization problem for individual varieties. Demand for an individual variety is then a solution to the sub-utility maximization problem given by:

$$x_k = p_k^{-\epsilon} e_i^{\epsilon-1} M_{ix} \quad (12)$$

⁵I assume symmetry within each category of x-varieties in the CES function above.

where subscript k stands for an individual variety and the remaining variables are as defined above.

In large group monopolistic competition each individual firm takes the price index e_i and country income M_i as given. The proportional mark-up of price over variable costs is given by:

$$p = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{c}{a_y} \quad (13)$$

where c is the units of labour used in producing one unit of the good (x-variety) and $1/a_y$ is the wage rate. This pricing equation comes from the first order condition, called the ‘marginal revenue equals marginal cost condition’. The proportional markup of price over marginal costs is constant and independent of market shares. For constant marginal costs and equal wages, this implies each x-variety is produced for the same price $p_x = p$ in equilibrium. However, x-varieties produced by national firms are sold abroad for a higher price $p\tau$, where τ are the iceberg trade costs.

As in the trade cost literature, τ are the inclusive trade costs. They include not only transport costs but all intermediate costs like tariff barriers, non-tariff barriers, border costs, information costs, time costs, currency costs etc. According to the literature, trade costs are fairly large, an average estimate being 170% of the value of the output (Anderson and Wincoop, 2004). Currency costs are only a small proportion of it – about 8-14% out of a total 170%, and this includes both transaction and hedging costs. Trading horizon is typically short-term (few days to few weeks) and currency risks over short periods are easily hedged in financial markets today, say through the spot rate or a short-term forward, which is either costless or has costs which are small. I do not assume large exchange rate shocks as in Baldwin and Krugman (1989), but medium to long term exchange rate risk, which is not easy to hedge against in the forward foreign exchange market, matters for foreign direct investment among other sources of aggregate uncertainty mentioned in section 3 below. This is because the ability to limit risks posed by long term exchange rate shifts is either unavailable or is very expensive (Guay and Kothari, 2003). Further, the foreign exchange futures market is also illiquid beyond the short-term (Layard et al, 2002). As compared to a more straightforward trading decision, FDI typically takes place in the face of foreign investment uncertainty. This will be explicitly modelled later.

Production regime for the X-sector is determined by a set of conditions

called the zero-profit conditions given below:

$$\begin{aligned}
pX_{ii}^n + pX_{ij}^n &\leq wcX_{ii}^n + wcX_{ij}^n + w(H^n + G^n) & (N_i^n) \\
pX_{ii}^m + pX_{ij}^m &\leq wcX_{ii}^m + wcX_{ij}^m + w(H^m + G^m) + wG_j^m & (N_i^m) \\
pX_{jj}^n + pX_{ji}^n &\leq wcX_{jj}^n + wcX_{ji}^n + w(H^n + G^n) & (N_j^n) \\
pX_{jj}^m + pX_{ji}^m &\leq wcX_{jj}^m + wcX_{ji}^m + w(H^m + G^m) + wG_i^m & (N_j^m)
\end{aligned}
\tag{14}$$

These are written as inequalities in the complementary slackness form, meaning thereby that an equation will hold with equality if the output of the corresponding firm is positive, otherwise the output of the corresponding firm is zero. These conditions relate markup revenues to investment costs and number of firms is the endogenous variable. Depending on whether markup revenues cover investment costs or not, firms decide whether to operate as a national firm exporting to the foreign market or undertake a foreign direct investment abroad i.e. become multinational.

Let us assume for a moment that each type of firm is active in equilibrium. Demand functions for varieties produced by each of these firms, as derived from the respective sub-utility maximization problems are given below:

$$\begin{aligned}
X_{ii}^n &= X_{ii}^m = X_{ji}^m = p^{-\epsilon} e_i^{\epsilon-1} M_{ix} = \beta p^{-\epsilon} e_i^{\epsilon-1} M_i = \beta p^{-\epsilon} e_i^{\epsilon-1} w S_i \\
X_{jj}^n &= X_{jj}^m = X_{ij}^m = p^{-\epsilon} e_j^{\epsilon-1} M_{jx} = \beta p^{-\epsilon} e_j^{\epsilon-1} M_j = \beta p^{-\epsilon} e_j^{\epsilon-1} w S_j \\
X_{ji}^n &= p^{-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} M_{ix} = \beta p^{-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} M_i = \beta p^{-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} w S_i \\
X_{ij}^n &= p^{-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} M_{jx} = \beta p^{-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} M_j = \beta p^{-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} w S_j
\end{aligned}
\tag{15}$$

Iceberg trade costs imply, if a quantity X_{ji}^n is shipped by a national (exporting) firm, only $\frac{X_{ji}^n}{\tau}$ arrives in the foreign country and is sold for a price $p\tau$. M_{ix} or M_{jx} is respectively the amount of national income spent on good X, which is further substituted out in terms of the demand parameters, the wage incomes and factor endowments.

It is possible to further simplify the zero-profit conditions above by using the pricing equation and Marshallian demand functions for individual x-varieties. Some algebra (see appendix A) yields a simplified set of conditions, which determine the production regime for the X-sector, and these equations written compactly for country j firms are:

$$\begin{aligned}
\beta p^{1-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \epsilon(H^n + G^n) & (N_j^n) \\
\beta p^{1-\epsilon} e_i^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \epsilon(H^m + G^n) + \epsilon G_i^m & (N_j^m)
\end{aligned}
\tag{16}$$

and similarly, for country i firms are:

$$\begin{aligned} \beta p^{1-\epsilon} e_i^{\epsilon-1} S_i + \beta p^{1-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \epsilon(H^n + G^n) & (N_i^n) \\ \beta p^{1-\epsilon} e_i^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \epsilon(H^m + G^m) + \epsilon G_j^m & (N_i^m) \end{aligned} \quad (17)$$

As before, all equations will not hold with equality at one time. If one of them holds with equality, the corresponding number of firms is positive, otherwise the corresponding number of firms is zero.

3 The Inter-temporal Model

For inter-temporal analysis it is necessary to specify the starting and the end points. Let us start at time $t = 0$ with a national production regime, where only exporting firms are operating in both the countries and there is diversified production. This implies existence of a pure trading equilibrium (no FDI) with both intra-industry and inter-industry trade. Let us assume the representative consumer lives, and that firms potentially operate, forever. National firms in the pure trading equilibrium have an option to undertake foreign direct investment and start multinational production abroad. At some time t^* between $t = 0$ and $t = \infty$ this option could be exercised and the production regime would switch from national to multinational, provided it is optimal to do so. I will explicitly solve for this optimal foreign investment rule.

As compared to a more straight-forward trading (export) decision, FDI is typically undertaken in the face of foreign investment uncertainty. This is an aggregate uncertainty arising from the foreign environment and could be because of statutory FDI policies, corporate governance/tax regimes, medium-to-long-term exchange rate risks, policy shifts like economic liberalization, incentive competition,⁶ industry or economy-wide macro shocks, political instability etc. This uncertainty cannot be easily hedged and exists even when a multinational firm undertakes a foreign direct investment into a seemingly similar economy. It is this uncertainty which is of interest here and is represented by a stochastic shift variable R_i , multiplicative with the costs of

⁶Incentive competition refers to national governments competing with each other to offer investment incentives to multinational firms so as to attract FDI into their respective countries.

foreign investment, say for country i:

$$G_i^m = GR_i \quad (18)$$

Notice the two components of foreign investment – a certain part (G) and an uncertain part (R_i). This being a real model (there is no money here), costs of uncertainty associated with ‘trade-in-invisibles’, which are an integral part of any foreign direct investment, are included in the process R_i . Such ‘trade-in-invisibles’ includes head-quarter services, royalty payments, repatriation of profits and cross-border investment flows, which are subject not only to the regulatory/capital controls, but also to medium to long term exchange rate risks.

In the first instance, I assume this stochastic shift variable follows a geometric Brownian motion, whereby the stochastic process underlying foreign investment uncertainty is given by:⁷

$$dR_i = \mu_i R_i dt + \sigma_i R_i dW_t \quad (19)$$

Here, μ_i is the drift, σ_i is the volatility and dW_t is a Gauss-Wiener process representing Brownian motion and at any instant satisfying $E(dW) = 0$ and $E(dW^2) = dt$.

An uncertainty which is equally faced by both national and multinational firms does not generate an option value between trading and FDI, but foreign investment uncertainty which is faced only by multi-national firms, implies existence of a real option, say for country j’s exporting firms to either undertake a foreign direct investment in country i or to keep exporting as national firms as they were doing at time $t = 0$. I will henceforth call this the ‘FDI option’.

To simplify exposition of this model, I will focus on country i and assume that country j follows a restrictive FDI regime and does not permit any foreign direct investment within its borders. Relaxing this assumption is trivial and the same formulation would apply to the other country.

For an *individual* firm in country j, production decision at any time t^*

⁷This is an important theoretical benchmark. I will later extend the model by introducing Poisson jumps and formulating a mixed Brownian motion-Poisson jump process, which provides a better way of modelling uncertainty related to FDI policy.

between $t = 0$ and $t = \infty$ is given by the present value formulation:

$$\begin{aligned}
& \int_{t^*}^{\infty} e^{-\rho t} (\beta p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j) dt & (20) \\
= & \int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^n + G^n)] dt & \text{(with no FDI)} \\
& \text{OR} \\
& E \int_{t^*}^{\infty} e^{-\rho t} (\beta p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j) dt & \text{(with FDI)} \\
= & E \left[\int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^m + G^m)] dt + \epsilon GR_0 + \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i e^{(\mu_i - \frac{1}{2}\sigma_i^2)t} e^{\sigma W_t}) dt \right]
\end{aligned}$$

where ρ is the (riskless) discount rate, β is the demand parameter, τ are the trade costs, $e_{i \text{ or } j}$ is the aggregate price index, e_{i_0} to e_{i_1} is the expected change in price index when FDI is undertaken in country i, $S_{i \text{ or } j}$ are the factor endowments, ϵ is the elasticity of substitution between any two x-varieties and H^n or m are the headquarter fixed costs. In writing the present value formulation, I use the stochastic differential equation 19 from above. Foreign investment costs are split into two parts - GR_0 , the setup costs which are revealed at time t^* (hence $E[GR_{t^*}] = GR_0$, no discounting needed) and GR_i , the subsequent costs over which expectations are formed and need to be explicitly solved for.

An individual firm takes prices and incomes as given. It is already operating as a national firm in country j (the first equality above), but forms expectations over what would happen if it decided to switch from national to a multinational mode of production (the second equality above). ‘OR’ between the two equalities indicates existence of a ‘real option’ between the two choices.

The firms know their operating characteristics and market structure well. Demand parameters and total factor endowments are given and assumed not to change with time. All firms are identical (i.e. homogeneous) and rational. Thus, when it becomes optimal for one firm in country j to switch from a national to multinational mode of production, it also becomes optimal for other national firms in country j to do so. Under assumption of rational

expectations, this forward looking behaviour implies an individual firm can fully anticipate the change in aggregate price index that would be caused by this switch in production regime from national to multinational, as the producer price of an individual x-variety, p , and the trade costs, τ , remain unchanged. This implies

$$E\{e_{i1}\} = e_{i1} = [N_i^n p^{1-\epsilon} + N_j^m p^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (21)$$

Thus, expectation on left hand side of the second equation is easily taken care of, as rational expectation implies expected present discounted value of mark-up revenues is same as its present discounted value:

$$\begin{aligned} & E \int_{t^*}^{\infty} e^{-\rho t} (\beta p^{1-\epsilon} e_{i1}^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j). dt \\ &= \int_{t^*}^{\infty} e^{-\rho t} (\beta p^{1-\epsilon} e_{i1}^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j). dt \\ &= \frac{\beta}{\rho} p^{1-\epsilon} e_{i1}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j \end{aligned} \quad (22)$$

The expectation on right hand side of the second equation is more tricky as it involves solving the stochastic integral:

$$E \int_{t^*}^{\infty} e^{-\rho t} \epsilon \{GR_i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2)t}\} \{e^{\sigma W_t}\} . dt \quad (23)$$

Using stochastic calculus (for details see Appendix B), this expectation can be simplified because:

$$E\{GR_i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2)t}\} \{e^{\sigma W_t}\} = GR_i \cdot e^{\mu t} \quad (24)$$

The expected present discounted value of fixed costs for a multinational firm potentially operating in country j can therefore be simplified to:

$$E \int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^m + G^n)]. dt + \epsilon GR_0 + E \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2)t} \cdot e^{\sigma W_t}). dt$$

$$= \frac{\epsilon}{\rho}(H^m + G^m) + \epsilon GR_0 + \frac{\epsilon GR_i}{\rho - \mu} \quad (25)$$

Present value of markup revenues and fixed costs for a national firm operating in country j (first part of equation 20 above) can be written as:

$$\int_{t^*}^{\infty} e^{-\rho t} (\beta p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \beta p^{1-\epsilon} e_j^{\epsilon-1} S_j) dt = \frac{\beta}{\rho} p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j \quad (26)$$

and

$$\int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^n + G^m)] dt = \frac{\epsilon}{\rho}(H^n + G^m) \quad (27)$$

respectively.

Having simplified the integrals, we still need to take into account the opportunity cost of real option between exporting and FDI, as indicated by the term ‘OR’ in the decision making problem of an individual firm above. While an operating national firm knows the trade costs it saves fairly well (from its account books), the potential foreign investment costs are at best only an estimate.

Let us call the first state V(ex) and the second state V(fdi). By moving from state V(ex) to V(fdi) the firm not only gains mark-up revenues due to the trade costs saved, but also expects to lose the present value of foreign investment costs that potentially need to be incurred. The exercise of this option can be interpreted as a trade-off between the expected gain and loss in the value of the firm in moving from one state (exporting as a national firm) to the other (undertaking foreign investment as a multinational firm).

To simplify notation, let T be the present value of gain in mark-up revenues due to the saving of trade costs when a country j’s exporting firm undertakes foreign direct investment into country i and let \widehat{R} be the present value of foreign investment costs that will need to be incurred when this happens. Formally:

$$T = \frac{\beta}{\rho} p^{1-\epsilon} (\tau^{1-\epsilon} e_{i_0}^{\epsilon-1} - e_{i_1}^{\epsilon-1}) S_i \quad (28)$$

$$\widehat{R} = \epsilon GR_0 + \frac{\epsilon GR_i}{\rho - \mu} \quad (29)$$

Let $F(\widehat{R}, t; T)$ be the value of this option to switch production regime from exporting to FDI. Payoff from exercising this option at any time t is given by the function:

$$g(\widehat{R}, t; T) = \max[T - \widehat{R}, 0] \quad (30)$$

Taking an analogy from the finance theory, this is like an American Put Option, a class of options that are typically harder to solve and do not have closed form solutions. Such option functions are called free boundary problems and they are essentially variational problems in stochastic mathematics. Fortunately, this real option is not exactly like its financial counterparts. I will use an original idea from Merton (1973), which states that if time to maturity is infinite, the option pricing function becomes time independent and a closed form solution exists. Such options are called ‘perpetual puts’ and its option pricing function is written as $F(\widehat{R}, t = \infty; T)$ or simply, $F(\widehat{R}; T)$.

There are two equivalent ways of solving this problem - either through contingent claim analysis using the arbitrage theory or through stochastic dynamic programming. Because of its expositional neatness I will hereby use the arbitrage theory.

Using the second order Taylor series and Ito’s lemma gives us the following partial differential equation for the option pricing function:

$$(\rho - \delta)RF'(R)dt + \frac{1}{2}\sigma^2R^2F''(R)dt = \rho Rdt \quad (31)$$

where $\mu = \rho - \delta$ (a la Dixit, Pindyck,1994). δ , which is sometimes called the convenience yield, is the difference between the drift term and the riskless rate of return.

To solve for the option value, the partial differential equation is combined with the following boundary conditions:

1. $F(\infty; T) = 0$, a ‘terminal’ condition, which means this option is of no value, if the foreign investment costs tend to infinity.
2. $F(R^*; T) = T - R^*$, a ‘value matching’ condition, which describes the payoffs when it becomes optimal to undertake FDI abroad.
3. $\frac{\partial F(R^*; T)}{\partial R} = -1$, a ‘smooth-pasting’ or ‘high contact’ boundary condition, which implies that slope of payoff and option pricing function match at the exercise boundary and if not, it would not be optimal to exercise. There is thus a continuity or smooth pasting at the optimal exercise boundary.

The general solution to this differential equation is :

$$F(R, \infty; T) = a_1 R + a_2 R^{-\gamma} \quad (32)$$

where

$$\gamma = \frac{\frac{1}{2}\sigma^2 - (\rho - \delta) + \sqrt{\left((\rho - \delta) - \frac{1}{2}\sigma^2\right)^2 + 2\rho\sigma^2}}{\sigma^2} \quad (33)$$

is the positive root of the fundamental quadratic equation:

$$Q = \frac{1}{2}\sigma^2 (\gamma) (\gamma - 1) + (\rho - \delta) (\gamma) - \rho = 0 \quad (34)$$

The first boundary condition implies $a_1 = 0$.

The second or value matching condition implies $a_2 = (T - R^*) R^{*\gamma}$

Now, from the smooth pasting condition, general solution evaluated at the optimal value, R^* is :

$$\frac{\partial F(R^*, \infty; T)}{\partial R} = -\gamma a_2 R^{*\gamma-1} = -1 \quad (35)$$

Substituting for a_2 :

$$-\gamma (T - R^*) R^{*\gamma} R^{*\gamma-1} = -1 \quad (36)$$

This can be simplified to:

$$T = \frac{1 + \gamma}{\gamma} \widehat{R}^* \quad (37)$$

This gives us the ‘optimal foreign investment rule’, which is a deterministic, time-independent solution. As described earlier, T is the present value of gain in mark-up revenues (due to saving of the trade costs) and R^* is the present value of foreign investment costs that would need to be incurred at the optimal exercise boundary for this FDI option.

Intuitively, this rule says, ‘uncertainty combined with irreversibility drives a wedge between the present value of gain in mark-up revenues due to the trade costs saved and a critical value of the foreign investment costs that need

to be incurred'. The size of this wedge is equal to $\frac{1+\gamma}{\gamma}$. The wedge implies 'hysteresis', because by lowering the critical value of foreign investment costs it makes exercise of the FDI option less likely.

If volatility $\sigma \rightarrow 0$ (which implies no uncertainty), the positive root $\gamma \rightarrow \infty$ and the optimal scale-up factor $\frac{1+\gamma}{\gamma} \rightarrow 1$, implying there is no 'hysteresis'. If on the other hand volatility $\sigma \rightarrow \infty$, the positive root $\gamma \rightarrow 0$ and the optimal scale-up factor $\frac{1+\gamma}{\gamma} \rightarrow \infty$, implying the FDI option would not be exercised, no matter how small the foreign investment costs are or how big the gains from cutting trade costs are.

Thus, parameterized in time, the total effect of foreign investment uncertainty is determined through its effect on present discounted values of trade costs saved, the foreign investment costs incurred and through the opportunity cost of real option between exporting and FDI. 'Perpetual Put' makes our life simple, because we can solve for the equilibrium recursively at any point in time and the optimal foreign investment rule remains unchanged (a closed form solution exists).

Equilibrium conditions for country j firms can now be written in the complementary slackness form as follows:

$$\begin{aligned} \frac{\beta}{\rho} p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \frac{\epsilon}{\rho} (H^n + G^n) & (N_j^n) \\ \frac{\beta}{\rho} p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \frac{\epsilon}{\rho} (H^m + G^n) + \left(\frac{1+\gamma}{\gamma}\right) [\epsilon G R_0 + \frac{\epsilon G R_i}{\rho - \mu}] & (N_j^m) \end{aligned} \quad (38)$$

Aggregate price index, which is endogenous, may be further substituted out using the expressions below:

$$\begin{aligned} e_{i_0} &= [N_i^n p^{1-\epsilon} + N_j^n (p\tau)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \\ e_{i_1} &= [N_i^n p^{1-\epsilon} + N_j^m p^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \\ e_j &= [N_i^n (p\tau)^{1-\epsilon} + N_j^n p^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \end{aligned} \quad (39)$$

We can see that while foreign investment uncertainty is driven by an exogenous stochastic shift variable, the real option between exporting and FDI is an indicator function, meaning thereby that optimal foreign investment rule is deterministic, and so are the other endogenous variables, which after substituting out the price index, essentially mean the number of firms of

each type operating in equilibrium. This defines the equilibrium production regime. If there was no uncertainty, there would be no option value, and hence, no scale-up over foreign investment costs, and we would be back to a standard Marshallian kind of revenue-cost analysis.

It is also pertinent to mention that, what is modelled here is only the option decision relating to switching of production regime from trading to FDI, after having started at time $t = 0$ with exporting (national) firms in both the countries. This has been called the ‘FDI option’. It ceases to have an option value after it is exercised. The reverse is usually not a symmetric phenomenon, but rather a pure exit decision for which a separate option problem needs to be formulated. Such exit options have been adequately modelled in the literature (Dixit and Pindyck, 1994).

To summarize, we start at time $t = 0$ with a pure trading equilibrium and diversified production. Let us say that foreign investment environment in country i is not conducive to start with and trading (exporting) firms from country j , which are identical and rational, optimally decide to continue as national firms. These firms wait and watch, thereby retaining an option to undertake FDI abroad, which is valued in terms of its opportunity cost. Say later, because of some FDI reforms, foreign investment environment in country i becomes favourable and at some point in time, in accordance with the foreign investment rule, it becomes optimal for country j ’s exporting firms to exercise this option. Being identical and rational, they all rush to undertake FDI in country i . There will be both partial and general equilibrium effects. While partial equilibrium effects are reflected in the costs and savings for individual firms, general equilibrium effects are reflected in the changes in the aggregate price index and the type of firms operating in equilibrium. The producer price of an individual x -variety remains unchanged, as also the equilibrium wage, which is pinned down by the numeraire sector. The factor market undergoes a simultaneous adjustment as part of the skilled labour in country j freed up by its national firms starting multinational production abroad is used up in headquarter services, while the remaining shifts to the numeraire good sector Y whose production expands. Exactly the opposite happens in country i , where multinational production by country j firms attracts additional skilled labour to the X sector and the numeraire good sector contracts. I maintain the assumption that labour endowment and value of demand parameters is such that production remains diversified.

4 Comparative Experiments

We have seen above that foreign investment uncertainty drives a wedge between the trade costs saved and the foreign investment costs incurred, thereby delaying FDI into country i , beyond what is predicted by riskless cost-benefit analysis. This is 'hysteresis'.

I will now conduct some thought experiments to answer the questions of "when"- that is to analyze the effect of uncertainty on timing of foreign investment given comparative advantage and trade costs; and of "where", that is, in the presence of foreign investment uncertainty, where would it be optimal to undertake FDI amongst alternative locations, given some comparative advantage and trade costs?

Effect of Volatility : Let us say the foreign investment environment in country i is more risky as compared to the foreign investment environment in country j , that is $\sigma_i > \sigma_j$.

Volatility affects the optimal decision rule through scale-up factor

$$\frac{1 + \gamma}{\gamma}. \quad (40)$$

By totally differentiating the fundamental quadratic with respect to volatility parameter σ holding drift constant (a mean preserving spread):

$$\frac{\partial Q}{\partial \gamma} \frac{\partial \gamma}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0 \quad (41)$$

Now, $Q(1) = -\delta$, i.e. fundamental quadratic valued at $\gamma = 1$ is negative. This implies the positive root γ is greater than one.

Further, $Q(0) = -\rho$, i.e. fundamental quadratic valued at $\gamma = 0$ is negative.

This helps us plot the fundamental quadratic, which is itself is a function of γ .

For $\sigma = 0.2, \rho = 0.05$ and $\delta = 0.03$ this plot is as follows:

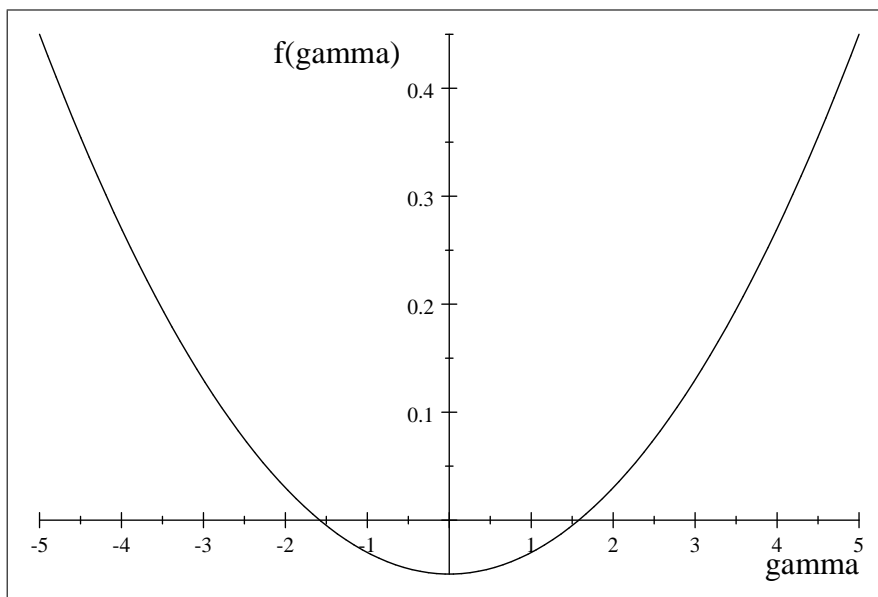


Figure 1: Graph for the fundamental quadratic equation (the stochastic process is geometric Brownian motion with drift).

The partial derivative $\frac{\partial Q}{\partial \sigma} = (\gamma)(\gamma - 1)\sigma$ is positive, given that σ is a positive parameter and γ , the positive root of the fundamental quadratic, is greater than 1. The partial derivative $\frac{\partial Q}{\partial \gamma} > 0$ (is also positive) as the fundamental quadratic is increasing at its positive root (see graph above). Thus, for the total differentiation equality to hold, $\frac{\partial \gamma}{\partial \sigma} < 0$, that is, the partial derivative of gamma with respect to the volatility parameter is negative. Therefore, if σ increases, γ decreases and the optimal scale-up factor $\frac{1+\gamma}{\gamma}$ increases.

In other words, comparing two countries i and j , if $\sigma_i > \sigma_j$, the cost of FDI option is higher in country i and we need either a higher saving of trade costs from country j to i , or in the presence of falling foreign investment costs, firms in country j need to wait longer for foreign investment costs in country i to fall low enough to trigger the FDI option.

We have proved the following:

Proposition 1 : A mean preserving increase (higher volatility) in foreign investment uncertainty drives a greater wedge between trade costs that need to be saved and foreign investment costs that need to be incurred at the optimal trigger point between exporting and FDI.

Effect of drift : The effect of drift is two fold - effect on the expected

present value and effect on the opportunity cost of real option between exporting and FDI.

Again, by total differentiation,

$$\frac{\partial Q}{\partial \gamma} \frac{\partial \gamma}{\partial \delta} + \frac{\partial Q}{\partial \delta} = 0 \quad (42)$$

where $\delta = \rho - \mu$ is the difference between the discount factor (riskless rate) and the drift of the stochastic process as described above.

The partial derivative $\frac{\partial Q}{\partial \delta} = -\gamma$ is negative because γ is the positive root of fundamental quadratic defined above. That partial derivative $\frac{\partial Q}{\partial \gamma}$ is positive has already been proved above. Hence, for the total differentiation equality to hold, the partial derivative $\frac{\partial \gamma}{\partial \delta}$ must be positive.

Thus, a lower drift (lower μ) implies a higher δ , which from the partial derivative above means a higher γ . This in turn implies a lower wedge or a lower scale-up ($\frac{1+\gamma}{\gamma}$) between the trade costs saved and the foreign investment costs incurred. This is the opportunity cost or implicit insurance premium of holding the ‘FDI option’.

Further, a lower μ also implies a lower $\frac{\epsilon GR_i(t_0)}{\rho - \mu}$ and hence lower expected present value of foreign investment costs given that both ρ and μ are positive fractions less than one (they are percentages) and that $\rho > \mu$ given the assumption of a convergent solution.

We have thus established the second proposition:

Proposition 2 : A lower drift of foreign investment uncertainty has two fold effect on the FDI decision of exporting firms. It has a direct effect through a decrease in the expected present value of foreign investment costs incurred and an indirect effect through a decrease in the size of wedge or optimal scale-up between the trade costs saved and the foreign investment costs incurred. Put together, they imply an increase in chances of early exercise of the FDI option.

Effect of Risk aversion: Till now the FDI-option was analyzed assuming firms are risk-neutral, a procedure called risk-neutral valuation. Suppose now that firms (investors and/or managers) are risk averse. While earlier, volatility affected the optimal decision of risk-neutral firms directly, it now has an additional effect through the ‘drift’ term.

The simplest way to allow risk-aversion is to replace riskless discount rate (ρ) with an appropriate risk-adjusted discount rate ($\bar{\rho}$) from the capital asset pricing model (Dixit and Pindyck, 1994). Risk-aversion essentially

means allowing for returns to adjust as σ changes. Each unit increase in σ now requires an increase in risk-adjusted discount rate by a coefficient term containing the correlation coefficient and the market price of risk as given by the equation below (for details see Appendix C):

$$\delta = \bar{\rho} - \mu = \rho + \phi\eta_{fm}\sigma - \mu \quad (43)$$

Here, ϕ is the market price of risk, η_{fm} is the correlation coefficient between the value of FDI option and the whole market portfolio, while μ , ρ and σ are the parameters as defined above. The meaning of the term δ now becomes clearer. It is the opportunity cost of delaying foreign investment and keeping the option to undertake FDI alive.

While volatility of the foreign investment costs mattered even for risk-neutral firms (Proposition 1 above), in the presence of risk-aversion it matters even more. For a ‘put option’ (unlike a call) the correlation coefficient is negative, as the positive deviations of uncertainty decrease (and not increase) the payoffs from exercising this option. Thus, a higher volatility, σ , implies a lower δ , which in accordance with the result above, further increases the wedge or optimal scale-up $\frac{1+\gamma}{\gamma}$ of trade costs over the foreign investment costs. We have thus established the third proposition:

Proposition 3 : If firms are risk-averse, as compared to when they are risk-neutral, they will need either lower foreign investment costs or greater saving of trade costs before they can undertake FDI abroad. In the presence of falling foreign investment costs, this implies a greater wait and watch before the FDI option is exercised.

Country Size and Income:

Since income is endogenous in general equilibrium, the variable of interest here is the total factor endowment S_i . Let us say, skilled factor endowment of country i is growing with time at a defined rate η :

$$S_i(t^*) = S_{io}e^{\eta t} \quad (44)$$

Equilibrium conditions for country j firms, in complementary slackness form, can now be written as follows:

$$\frac{\beta}{\rho - \eta} p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_{io} + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j \leq \frac{\epsilon}{\rho} (H^n + G) \quad (N_j^n)$$

$$\frac{\beta}{\rho - \eta} p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_{i_0} + \frac{\beta_j}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j \leq \frac{\epsilon}{\rho} (H^m + G) + \left(\frac{1+\gamma}{\gamma}\right) \left[\epsilon G R_0 + \frac{\epsilon G R_i}{\rho - \mu}\right] \quad (N_j^m) \quad (45)$$

This raises the strike price, and hence, payoffs from exercising the FDI option:

$$T = \frac{\beta}{\rho - \eta} p^{1-\epsilon} (\tau^{1-\epsilon} e_{i_0}^{\epsilon-1} - e_{i_1}^{\epsilon-1}) S_{i_0} \quad (46)$$

Such a country then naturally becomes a more attractive destination for FDI. We have established the following:

Proposition 4 : Country with a larger market size/greater endowment of skilled factors is more likely to attract foreign direct investment given a level of the trade costs saved and the foreign investment costs incurred. This is because it raises the strike price and hence, payoffs from exercising the FDI option.

Compare this result with the conventional model, where positive effects of market size on foreign direct investment come through the scale economies (Markusen and Venables, 1998). Here, we not only have the usual scale effects due to increasing returns at the firm level, but also an additional effect due to increase in strike price and hence payoffs from exercising the FDI option.

I have focussed on the foreign investment uncertainty here for the simple reason that it is relevant to the facts-in-issue and is closely related to FDI policy in the real world. For simplicity, I have assumed there are no demand or productivity shocks. I will now proceed to extend the model by introducing Poisson jumps in the stochastic process, which offer a better way of modelling uncertainty related to policy and the impact of FDI reforms, as foreign investment liberalisation is popularly known in emerging economies today.

5 Poisson Jump Process

Geometric Brownian motion (with drift) is an important theoretical benchmark, but for more comprehensive analysis I will extend this model by formulating a mixed Brownian motion-Poisson jump process. Foreign investment uncertainty arises from a variety of sources, including but not limited to private sector expectations of public policy, sudden policy shifts like economic liberalisation/FDI reforms, changes in corporate tax/governance

regimes; exchange rate costs related to repatriation of profits, royalty payments, headquarter services or cross-border investment flows; industry or economy wide macro-shocks, political instability etc. Among alternative stochastic processes mixed Brownian motion-Poisson jump process is the closest one can get to such policy related uncertainties.

Let us allow for the possibility of a downward jump ϕ that can suddenly bring down the foreign investment costs in country i. Let λ be the probability that such a downward jump can arrive in any time-period. If it arrives, the foreign investment costs would fall irreversibly to $1 - \phi$ times the original value. The stochastic shift process driving the foreign investment costs will now be represented by a mixed Brownian motion-Poisson jump process given below:

$$dR_i = \mu_i R_i dt + \sigma_i R_i dW_t - R_i dq, \quad (47)$$

where $dq = \phi$ with probability λ

$dq = 0$ with probability $1 - \lambda$ in any time-period dt

and other terms are as described before.

The expected percentage change in foreign investment costs in country i is now given by :

$$E[dGR_i] = (\mu_i - \lambda\phi)GR_i \cdot dt \quad (48)$$

and expected variance of this change is :

$$Var[dGR_i] = \sigma^2(GR_i)^2 dt + \lambda\phi^2(GR_i)^2 dt \quad (49)$$

As before we need to solve for the optimal foreign investment rule for an individual firm in two steps. First, find the expected present value of foreign investment costs and Second, find the scale-up factor that implies implicit insurance premium of holding the FDI option.

First, the expected present value of fixed costs for a firm holding this FDI option is given by (for details see Appendix D):

$$\frac{\epsilon}{\rho}(H^m + G) + \epsilon GR_0 + \frac{\epsilon GR_i(t_0)}{\rho - \mu + \lambda\phi} \quad (50)$$

Intuitively it says, a higher probability of FDI reforms (higher λ) or a higher impact of FDI reforms (higher ϕ implying a greater percentage fall

in foreign investment costs) decrease the present value of foreign investment costs making foreign direct investment more likely.

Secondly, we need to solve for the opportunity cost of holding this FDI option.

The partial differential equation can now be written as:

$$(\rho - \delta)RF'(R)dt + \frac{1}{2}\sigma^2R^2F''(R)dt - \lambda[F(R) - F(R(1 - \phi))]dt = \rho Rdt \quad (51)$$

The boundary conditions remain the same:

$$1. F(\infty; T) = 0 \quad (52)$$

$$2. F(R^*; T) = T - R^* \quad (53)$$

$$3. \frac{\partial F(R^*; T)}{\partial R} = -1 \quad (54)$$

The general solution is again of the form

$$F(R, \infty; T) = a_1R + a_2R^{-\gamma} \quad (55)$$

As before, the first boundary condition implies

$$a_1 = 0 \quad (56)$$

The second or the value matching condition implies

$$a_2 = (T - R^*) R^{*\gamma} \quad (57)$$

γ is now the positive solution (negative solution is ruled out by the boundary conditions) to the following characteristic non-linear equation:

$$\frac{1}{2}\sigma^2(\gamma)(\gamma - 1) + (\rho - \delta)(\gamma) + \lambda(1 - \phi)^\gamma - (\rho + \lambda) = 0 \quad (58)$$

This equation does not have an analytic solution and so it needs to be solved numerically. For $\sigma = 0.2$, $\rho = 0.05$, $\delta = 0.03$, $\lambda = 0.05$ and $\phi = 0.2$, the graph of this equation is drawn in Figure 2 (on page 36 in the appendix). $\gamma = 1.8267$ is its positive solution.

This is the only part of this model that needs to be solved numerically. The remaining derivation required in obtaining the optimal foreign investment rule can be done analytically.

Smooth pasting implies

$$\frac{\partial F(R^*, \infty; T)}{\partial R} = -\gamma a_2 R^{*\ -\gamma-1} = -1 \quad (59)$$

Substituting for a_2 gives:

$$-\gamma (T - R^*) R^{*\gamma} R^{*\ -\gamma-1} = -1 \quad (60)$$

Solving for the optimal scale-up factor again gives:

$$T = \frac{1 + \gamma}{\gamma} \widehat{R}^* \quad (61)$$

As we can see, the optimal foreign investment rule or the scale-up over foreign investment costs remains the same. As before, it is a deterministic, time independent solution. However, γ now has different values obtained as a numerical solution to the characteristic non-linear equation 58 above.

As before, equilibrium conditions for country j's firms can be written in the complementary slackness form as follows:

$$\frac{\beta}{\rho} p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_{j_0}^{\epsilon-1} S_j \leq \frac{\epsilon}{\rho} (H^n + G) \quad (N_j^n)$$

$$\frac{\beta}{\rho} p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_{j_1}^{\epsilon-1} S_j \leq \frac{\epsilon}{\rho} (H^m + G) + \left(\frac{1 + \gamma}{\gamma}\right) \left[\epsilon G R_0 + \frac{\epsilon G R_i(t_0)}{\rho - \mu + \lambda \phi}\right] \quad (N_j^m) \quad (62)$$

I will again perform some comparative experiments. The difference is that now I will solve numerically for γ each time an exogenous parameter changes.

Effect of Volatility: As volatility of foreign investment uncertainty increases, the value of γ decreases. This solution for various values of σ is given in Table 2.1 below:

σ	ρ	δ	λ	ϕ	γ
0.05	0.05	0.03	0.05	0.2	3.3263
0.10	0.05	0.03	0.05	0.2	2.5772
0.15	0.05	0.03	0.05	0.2	2.1146
0.20	0.05	0.03	0.05	0.2	1.8267
0.25	0.05	0.03	0.05	0.2	1.6367
0.30	0.05	0.03	0.05	0.2	1.5046
0.35	0.05	0.03	0.05	0.2	1.4091
0.40	0.05	0.03	0.05	0.2	1.3378

This result is the same as in Proposition 1 above. Higher volatility implies a lower γ and therefore a higher wedge or higher scale-up over foreign investment costs i.e. to undertake FDI the firms need either a higher saving of transport costs or in the presence of falling foreign costs, they wait longer before the FDI option can be exercised.

Effect of Drift: An increase in δ , which implies a lower drift, leads to an increase in the value of γ as shown in Table 2.2 below:

σ	ρ	δ	λ	ϕ	γ
0.20	0.05	0.01	0.05	0.2	1.3432
0.20	0.05	0.02	0.05	0.2	1.5666
0.20	0.05	0.03	0.05	0.2	1.8267
0.20	0.05	0.04	0.05	0.2	2.1223

This decreases the optimal scale-up $\frac{1+\gamma}{\gamma}$ over the foreign investment costs. Besides, it also implies a lower expected present value of foreign investment costs (equation 50 above). Both these effects together imply the result in Proposition 2 above, meaning thereby that falling foreign investment costs increase the chances of an early exercise of the FDI option.

Probability of FDI reforms: A higher probability of FDI reforms, as measured by the factor λ , increases the value of γ as shown in Table 2.3 below:

σ	ρ	δ	λ	ϕ	γ
0.20	0.05	0.03	0.00	0.2	1.5811
0.20	0.05	0.03	0.05	0.2	1.8267
0.20	0.05	0.03	0.10	0.2	2.0887
0.20	0.05	0.03	0.15	0.2	2.3602
0.20	0.05	0.03	0.20	0.2	2.6356

This decreases the optimal scale-up $\frac{1+\gamma}{\gamma}$ over the foreign investment costs. Further, it decreases the expected present value of foreign investment costs (equation 50 above). The combined effect is summarized in Proposition 5 below:

Proposition 5: An increase in the probability of a sudden drop in foreign investment costs decreases the optimal scale-up over foreign investment costs and also decreases the expected present value of foreign investment costs. This dual effect facilitates foreign direct investment by increasing chances of an early exercise of the FDI option.

Impact of FDI reforms: A larger impact of FDI reforms as measured by the percentage parameter ϕ also increases the value of parameter γ as shown in Table 2.4 below:

σ	ρ	δ	λ	ϕ	γ
0.20	0.05	0.03	0.05	0.00	1.5811
0.20	0.05	0.03	0.05	0.10	1.7063
0.20	0.05	0.03	0.05	0.20	1.8267
0.20	0.05	0.03	0.05	0.30	1.9356
0.20	0.05	0.03	0.05	0.40	2.0280
0.20	0.05	0.03	0.05	0.50	2.1018

This decreases the optimal scale-up $\frac{1+\gamma}{\gamma}$ over foreign investment costs. Further, it decreases the expected present value of foreign investment costs (equation 50 above). The combined effect on foreign direct investment is summarized in Proposition 6 below.

Proposition 6: An increase in size of the percentage downward jump in foreign investment costs decreases the optimal scale-up over foreign investment costs and also decreases the expected present value of foreign investment costs. This dual effect facilitates foreign direct investment by increasing chances of an early exercise of the FDI option.

6 Conclusion

Real life behaviour of multinational firms is somewhat more complex than what can be explained by conventional comparative advantage-trade cost analysis. Almost all countries, including the developed ones, have specific FDI regimes which are driven by policy changes. Even when FDI reforms bring down foreign investment costs, multinational firms do not immediately rush in. There is a considerable ‘wait and watch’, a kind of time lag or inertia before investments actually start flowing in. This is more commonly seen in developing economies, where foreign investment uncertainty is expected to be high. This model shows that, by indulging in such cautious behaviour, multinational firms are actually seeking additional compensation for some real costs over and above what can be accounted for in a conventional cost-benefit analysis. Although, for rigorous exposition, this model is solved in a Dixit-Stiglitz framework, the idea behind this model is more general. For example, it could be applied to a sub-national context, where State Governments compete for inward investment within a framework of competitive federalism.

In this paper, I first solve for the optimal foreign investment rule, which is a deterministic, time-independent solution. Foreign investment costs are scaled up by a factor, which depends on the parameters of foreign investment uncertainty. This implies ‘hysteresis’, because in the presence of falling foreign investment costs, multinational firms will wait longer than they would have in the absence of such uncertainty. Similarly, given comparative advantage and trade costs between alternative locations, firms prefer the ones with less uncertain foreign investment environments.

Greater volatility and risk aversion delay the exercise of FDI option, while growing market size (national income) facilitates its early exercise. A particularly interesting aspect of this paper is the mixed Poisson jump-Brownian motion process, which explicitly models policy driven FDI reforms. It shows how a sudden drop in foreign investment costs brought about by a policy shift, as also a greater probability of it, can facilitate early exercise of the FDI option.

To summarize, this paper enriches the existing general equilibrium models of multinational firms by providing a better explanation for their observed behaviour in uncertain foreign environments. It embeds the theory of real options into a framework of Dixit-Stiglitz type monopolistic competition. It explicitly solves for the policy driven FDI liberalization as a mixed Poisson

jump-Brownian motion stochastic process. And further, while the investment under uncertainty literature is based on the theory of call options, I solve the ‘FDI option’ as a put option, thereby also enriching the theory of real options.

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Appendix A : Solving the Static Model

1) Zero profit condition for national firms in country j:

$$pX_{jj}^n + pX_{ji}^n \leq wcX_{jj}^n + wcX_{ji}^n + w(H^n + G) \quad (N_j^n)$$

Bring the quantities produced of each variety to the left hand side:

$$(p - wc)X_{jj}^n + (p - wc)X_{ji}^n \leq w(H^n + G) \quad (N_j^n)$$

Substitute using the mark-up or pricing equations :

$$\frac{p}{\epsilon}X_{jj}^n + \frac{p}{\epsilon}X_{ji}^n \leq w(H^n + G) \quad (N_j^n)$$

Substitute using the marshallian demand functions:

$$\beta p^{1-\epsilon} e_j^{\epsilon-1} w S_j + \beta p^{1-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} w S_i \leq w \epsilon (H^n + G) \quad (N_j^n)$$

Cancelling out wages from both sides gives the required equation :

$$\beta p^{1-\epsilon} e_j^{\epsilon-1} S_j + \beta p^{1-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} S_i \leq \epsilon (H^n + G) \quad (N_j^n)$$

2) Zero profit condition for multinational firms in country j:

$$pX_{jj}^m + pX_{ji}^m \leq wcX_{jj}^m + wcX_{ji}^m + w(H^m + G) + wG_i^m \quad (N_j^m)$$

Bring the quantities produced of each variety to the left hand side:

$$(p - wc)X_{jj}^m + (p - wc)X_{ji}^m \leq w(H^m + G) + wG_i^m \quad (N_j^m)$$

Substitute using the pricing or mark-up equations:

$$\frac{p}{\epsilon}X_{jj}^m + \frac{p}{\epsilon}X_{ji}^m \leq w(H^m + G) + wG_i^m \quad (N_j^m)$$

Substitute using the marshallian demand functions:

$$\beta p^{1-\epsilon} e_j^{\epsilon-1} w S_j + \beta p^{1-\epsilon} e_i^{\epsilon-1} w S_i \leq w \epsilon (H^m + G) + w \epsilon G_i^m \quad (N_j^m)$$

Cancelling out wages from both sides gives us the required equation :

$$\beta p^{1-\epsilon} e_j^{\epsilon-1} S_j + \beta p^{1-\epsilon} e_i^{\epsilon-1} S_i \leq \epsilon (H^m + G) + \epsilon G_i^m \quad (N_j^m)$$

The same can now be repeated for national and multinational firms in country i to get the results stated in equation 17.

Appendix B : Solving the Stochastic Integral

To solve for the expected present value of multinational operation, we need to solve the following stochastic integral:

$$E \int_{t^*}^{\infty} e^{-\rho t} \epsilon \{GR_i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2)t}\} \{e^{\sigma W_t}\} \cdot dt$$

A crucial part of this expression is the expectation of the Wiener process $E_t\{e^{\sigma W_t}\}$,

I undertake a change of variable by defining $Z_t = e^{\sigma W_t}$.

By Ito's lemma

$$dZ_t = \sigma e^{\sigma W_t} dW_t + \frac{1}{2} \sigma^2 e^{\sigma W_t} dt$$

Writing it in the integral form :

$$Z_t = Z_0 + \sigma \int_0^t e^{\sigma W_s} dW_s + \int_0^t \frac{1}{2} \sigma^2 e^{\sigma W_s} ds$$

Taking expectation of both the sides and using the fact that $E[Z_0] = 1$ (because by definition $W_0 = 0$); and that $E[\int_0^t e^{\sigma W_s} dW_s] = 0$ (increments of Wiener process are independent of the observed past), this expectation simplifies to :

$$E[Z_t] = 1 + \int_0^t \frac{1}{2} \sigma^2 E[e^{\sigma W_s}] ds$$

I now define another change of variable $E[Z_t] = x_t$.

The expression is now equivalent to an ordinary differential equation $\frac{dx_t}{x_t} = \frac{1}{2} \sigma^2 x_t ds$ with initial condition $x_0 = 1$.

And its solution is $x_t = E[Z_t] = e^{\frac{1}{2}\sigma^2 t}$.

Substituting the changed variables back into the stochastic integral, the $\frac{1}{2}\sigma^2 t$ terms cancel out and the stochastic expectation simplifies to:

$$E\{GR_i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2)t}\} \{e^{\sigma W_t}\} = GR_i \cdot e^{\mu t}$$

Appendix C : Effect of Risk Aversion

The simplest way to allow for risk-aversion is to replace riskless discount rate (ρ) with an appropriate risk-adjusted discount rate ($\bar{\rho}$) from the capital asset pricing model. If the firms undertaking FDI are risk-averse, then from the CAPM formula, the risk-adjusted rate of return for holding the FDI option would be:

$$\bar{\rho} = \rho + [E(\rho_m) - \rho] \frac{Cov\left(\frac{dF}{F}, \rho_m\right)}{Var(\rho_m)}$$

where ρ_m is the rate of return on the whole market portfolio, ρ is the risk-neutral rate and F is the option value function. The covariance-variance term is the correlation coefficient between the value of option and the whole market portfolio (also called the systematic risk or market beta). I will denote this by η_{fm} . The sign of covariance (correlation coefficient) could be positive or negative.

Multiplying and dividing the second term on right hand side by the volatility parameter σ gives:

$$\bar{\rho} = \rho + \frac{[E(\rho_m) - \rho]}{\sigma} \frac{Cov\left(\frac{dF}{F}, \rho_m\right)}{Var(\rho_m)} \cdot \sigma$$

Since, $\frac{[E(\rho_m) - \rho]}{\sigma}$ is, by definition, the market price of risk (let us call it ϕ), the above equation becomes:

$$\bar{\rho} = \rho + \phi \eta_{fm} \sigma$$

Since, parameter δ has already been defined as the difference between the drift and the discount rate

$$\delta = \bar{\rho} - \mu = \rho + \phi \eta_{fm} \sigma - \mu$$

This gives us equation 43, which forms the basis for Proposition 3 above.

Appendix D : Present Value of Fixed Costs for the Poisson Jump Process

The stochastic shift process with Poisson jumps is given by

$$dR_i = \mu_i R_i dt + \sigma_i R_i dW_t - R_i dq$$

This implies dR_i could take the following different values, which along with their respective probabilities, are :

$$\begin{aligned} dR_i &= \mu_i + \sigma R_i \sqrt{dt} \text{ with probability } \frac{1}{2} (1 - \lambda) dt \\ &\mu_i - \sigma R_i \sqrt{dt} \text{ with probability } \frac{1}{2} (1 - \lambda) dt \\ &\mu_i - \phi R_i \text{ with probability } \lambda dt \end{aligned}$$

The Poisson jump is assumed to be much bigger than a single increment in the Wiener process.

We know from the properties of Brownian motion that $E[dW_t] = 0$ i.e. expected change in Wiener process for any time period dt is zero.

Expected change in foreign fixed costs over any time period dt is therefore given by the equation:

$$E[GR_i] = (\mu_i - \lambda\phi) GR_i \cdot dt$$

The (strong) solution to the stochastic differential equation for mixed Poisson jump-Brownian motion process is:

$$GR_i = GR_{t=0} \int_0^t \exp\left[\left(\mu_i - \lambda\phi - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right] dt$$

To solve for the expected foreign fixed costs, we need to solve the expectation of the stochastic exponential on right hand side:

$$E_t[GR_i] = E_t\left[GR_{t=0} \int_0^t \exp\left\{\left(\mu_i - \lambda\phi - \frac{1}{2}\sigma^2\right) t\right\} \cdot \exp\{\sigma W_t\} \cdot dt\right]$$

which implies

$$E_t [GR_i] = GR_{t=0} \int_0^t \exp\left\{\left(\mu_i - \lambda\phi - \frac{1}{2}\sigma^2\right) t\right\} \cdot E_t[\exp\{\sigma W_t\}] dt$$

Solving the stochastic exponential as in the Appendix B above gives:

$$E_t[\exp\{\sigma W_t\}] = \exp\left(\frac{1}{2}\sigma^2 t\right)$$

We can now get rid of the expectation term on the right hand side as everything else is deterministic. Since

$$E_t [GR_i] = GR_{t=0} \int_0^t \exp\{(\mu_i - \lambda\phi) t\} dt$$

starting at a time t^* between $t = 0$ and $t = \infty$ and potentially operating forever, expected present value of fixed costs, for a country j multinational firm undertaking foreign direct investment in country i , is given by:

$$\begin{aligned} & E_{t^*} \left\{ \int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^m + G)] \cdot dt + \epsilon GR_0 + \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \lambda\phi - \frac{1}{2}\sigma_i^2)t} e^{\sigma W_t}) dt \right\} \\ &= \int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^m + G)] \cdot dt + \epsilon GR_0 + E_{t^*} \left\{ \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \lambda\phi - \frac{1}{2}\sigma_i^2)t} e^{\sigma W_t}) dt \right\} \\ &= \frac{\epsilon}{\rho} (H^m + G) + \epsilon GR_0 + \frac{\epsilon GR_i}{\rho - \mu + \lambda\phi} \end{aligned}$$

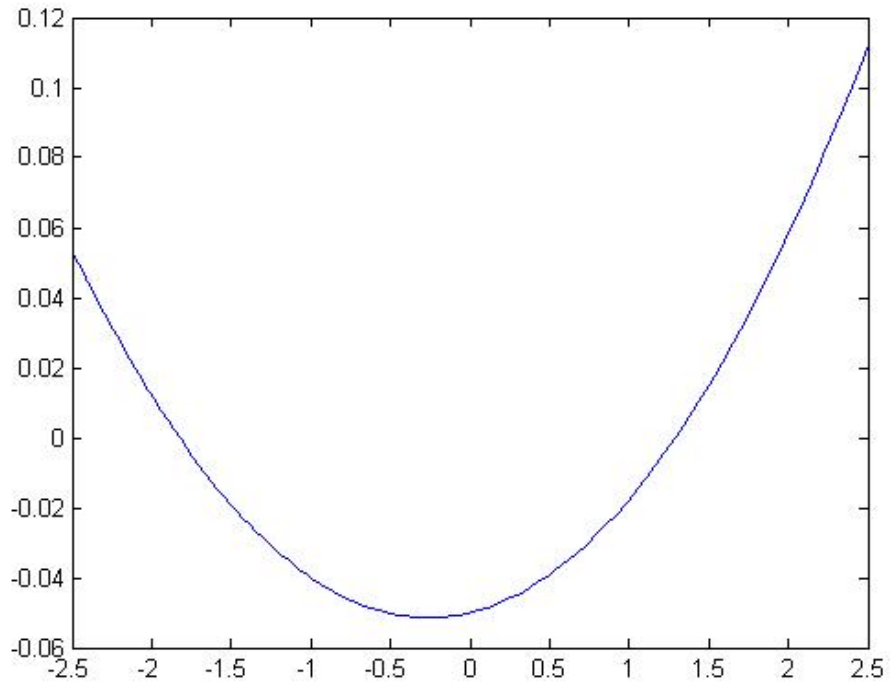


Figure 2: Graph for the characteristic equation of the mixed Poisson jump-Brownian motion stochastic process [parameter γ on the x-axis and function $f(\gamma)$ on the y-axis].

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