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**Information Costs, Networks and  
Intermediation in International Trade**

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## **Abstract**

This paper presents a pairwise matching model with two-sided information asymmetry to analyse the impact of information costs on endogenous network building and matching by information intermediaries. The framework innovates by examining the role of information costs on incentives for trade intermediation, thereby endogenising the pattern of direct and indirect trade. Intermediation is shown to unambiguously raise expected trade volume and social welfare by expanding the set of matching technologies available to traders. Moreover, convexity in network-building costs is necessary for both direct and indirect trade to arise in equilibrium while the pattern of trade is shown to depend on the level of information costs as well as the relative effectiveness of direct and indirect matching technologies with changing information costs. The model sheds light on the relationship between information frictions and aggregate trade volume, which may be non-monotonic as a result of conflicting effects of information costs on the incentives for direct and indirect trade.

Keywords: International Trade, Pairwise Matching, Information Cost, Intermediation, Networks

JEL Classifications: F10, C78, D43, D82, D83, L10

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# 1 Introduction

There is a broad literature addressing the many functions of middlemen. They have been shown to reduce search costs (Rubinstein and Wolinsky, 1987; Yavas, 1992, 1994), to offer expertise in markets with adverse selection (Biglaiser, 1993), to operate as guarantors of quality under producer moral hazard (Biglaiser and Friedman, 1994), as well as to operate as investors in quality-testing technology (Li, 1998). More recently, Shevchenko (2004) endogenises the number of intermediaries who buy and sell goods and examines the optimality of the size and composition of their inventories. Common to all of these works is the exploration of the role of middlemen as buyers and sellers of goods. In contrast, this paper explores the role of intermediaries as brokers of information.

Information is required to identify profitable trading opportunities and locate suitable trading partners, particularly where goods are differentiated and information about product characteristics is important. Information asymmetries, coupled with costs of acquiring information, can hinder the matching of agents with opportunities and prevent prices from allocating scarce resources across countries. Portes and Rey (2005) point to a lack of information about international trading opportunities and the need to tap into ‘deep knowledge’. In such a setting, international trade can be facilitated through intermediaries who invest in information networks or contacts and match agents with suitable opportunities for a fee.

Rauch and Watson (2002) present some summary statistics from a pilot survey of international trade intermediaries based in the US. Despite the small number of observations, their evidence suggests that 50% of trade intermediation in differentiated products does not involve taking title of goods and reselling, as compared to only 1% for homogeneous-goods. Moreover, 36% of the revenue from trade intermediation of differentiated products is reported to come from success fees based on the value of transactions, while the figure for homogeneous-good intermediation is only 1%. This is consistent with the search based or network view of trade, pioneered by Rauch (2001), Rauch and Trindade (2000) and others, that posits that the information requirements for differentiated goods are much greater due to the need to match specific characteristics. The evidence to date supports this, pointing to a more pronounced role for information intermediaries in the trade of differentiated goods.

The facilitation of trade through information networks has only recently begun to be formally developed. Recent literature on networks in international trade (Casella and Rauch, 2002) focuses on gaining insight on how information-sharing networks among internationally dispersed ethnic minorities or business groups can overcome informal trade barriers such as inadequate information about trading opportunities and weak enforcement of international contracts (Anderson and Marcouiller, 2002).

Casella and Rauch (2002) develop a model where output is produced through a joint venture and agents cannot judge the quality of their match abroad. They show that introducing a subset of agents with social ties, who have complete information when it comes to matching with other group members, increases

aggregate trade and income, but hurts the anonymous market. More recently, Rauch and Watson (2002) model the supply of ‘network intermediation’ where agents endogenously choose whether to be producers or intermediaries, depending on their endowment of contacts. The emphasis of the existing literature has largely been the effects of pre-existing social ties or contacts on trade. This paper contributes to the literature by analysing the incentives for contact-building and exploring how trade intermediation can offer a more efficient means of trade matching, without relying on any pre-existing ties between agents.

This paper analyses the role of information costs on the incentives for information intermediaries to emerge as trade facilitators and addresses a broad range of issues in a tractable, unified, theoretical framework. First, the model sheds light on how barriers to information flow can affect trade patterns and the organisation of trade, either directly, or indirectly through an intermediary. Second, it explores the incentives for contact-building and intermediation with varying levels of information costs and for a broad range of parameter values reflecting different network-building technologies. The pairwise matching model developed contributes to the literature by showing how information costs affect the realisation and organisation of trade transactions, for a given set of trade opportunities, in a framework where the pattern of information intermediation is determined endogenously.

The model is particularly applicable to international trade in differentiated goods for which information about product characteristics is important. The model can also be applied more broadly to intermediated markets where contact-building and matching are key. Examples may include headhunters in the job market, real estate agents in the housing or rental market, charterers in the transportation market, matchmakers in the marriage market (in some cultures), among others.

The remainder of this paper is organised as follows. Section 2 introduces the intermediation model. Section 3 extends the network-building cost specification giving rise to a richer set of results. Section 4 concludes.

## 2 The Model

This section introduces a pairwise matching model with a continuum of importers and exporters, and a single trade intermediary. The framework captures the incentives for network-building and intermediation when there are barriers to the flow of information and sheds light on the role information costs play on the organisation of trade.

### 2.1 Model Set-up

Consider a two-sided market where importers and exporters match in pairs to exchange a single unit of output. Let there be a continuum of exporters ( $X$ ), distributed uniformly and with unit density, over the interval  $[0, 1]$  and a continuum of importers ( $M$ ), also distributed uniformly with unit density over

$[0, 1]$ . Suppose that for each trader there is a unique partner on the other side of the market with whom they can trade. Each trade transaction generates a joint surplus  $S > 0$ , but if agents fail to locate their match they receive a payoff of 0. Moreover, assume all market participants are risk-neutral.

The framework best reflects trade in differentiated goods where specific characteristics have to be matched, whether these are feature of the product, timing of delivery etc. In the absence of trade frictions, importers and exporters identify each other costlessly and all trade opportunities are exploited generating a total surplus of  $S$ .

Suppose there is two-sided information asymmetry such that traders on both sides of the market do not know the location of their partner on the other side of the market. Within the set of infinitely many traders, the probability of each exporter (importer) locating her partner by selecting a random trader from the measure of importers (exporters) is zero. Any pair  $j$  of trade partners  $(X_j, M_j)$  is assumed to be able to match through a direct matching technology, however, which achieves successful matching with probability  $q(i)$ , where parameter  $i \in [0, 1]$  reflects the level of information costs or barriers to information flow between the two sides of the market. Let  $q'(i) < 0$ , so a higher prevailing level of information costs implies a lower probability of matching for each pair. Parameter  $i$  may be interpreted as reflecting the state of information and communication technology (ICT). An ICT improvement reflects a decline in  $i$ , which in turn implies a higher probability of a direct match. Further, let  $q(1) = 0$  and  $q(0) = 1$ , so information cost level  $i = 1$  prohibits any matching, while  $i = 0$  corresponds to the full information case where all trade opportunities are exploited.  $q(i)$  is also the expected trade volume and  $q(i)S$  the expected joint surplus from direct trade. The two-sided market is represented in figure (1).

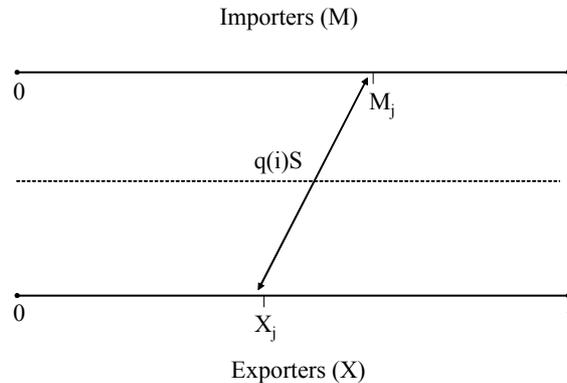


Figure 1: The two-sided market with pairwise trade matches.

Suppose the market has a single intermediary ( $I$ ) with access to a technology

for developing contacts with importers and exporters and finding out their trade characteristics (location, product features etc.). The intermediary incurs a set up cost,  $F$ , for creating a network and a marginal cost of network expansion,  $c(i)$ , where  $c'(i) > 0$  and  $c(0) = 0$ . The cost of making contacts is assumed to increase monotonically with the level of information costs but assumed to be entirely costless when  $i = 0$ .

The intermediary's network is denoted by a measure of importers,  $P_M$ , and a measure of exporters,  $P_X$ , where  $P_M \in [0, 1]$  and  $P_X \in [0, 1]$ , contacted by incurring cost  $c(i) P_M$  and  $c(i) P_X$ , respectively. Let  $C(P_X, P_M)$  denote the intermediary's total investment cost for building a network of contacts of size  $\{P_X, P_M\}$ , where this is linear and described by (1):

$$C(P_X, P_M) = F + c(i)(P_X + P_M) \quad (1)$$

Once network investment costs are sunk, it is assumed costless for the intermediary to match trade pairs from within his network of contacts. The intermediary's marginal cost of trade intermediation is zero. The proportion  $P_X$  also reflects the *ex ante* probability that any particular exporter  $X_j$  is a network member. Similarly,  $P_M$  is the *ex ante* probability that any particular importer  $M_j$  is a network member. Thus,  $P_X P_M$  describes the *ex ante* joint probability that both trade partners in pair  $(X_j, M_j)$  are contacted by the intermediary, for given network size. Once uncertainty regarding the identity of network members is resolved, the intermediary is able to match trade pairs from within his network<sup>1</sup> with probability 1.

The intermediary raises revenue by charging a commission for matching trading partners through his network. Let  $\alpha_I$  denote the share of trade surplus, or commission rate, that the intermediary demands for successful intermediation of trade. The intermediary's power to extract trade surplus through  $\alpha_I$  is constrained by the traders' outside option to trade directly with probability  $q(i)$ . In particular, as direct matching prospects worsen with  $i$ , the highest commission rate consistent with trader participation increases.

### 2.1.1 Timing of the Game

The timing of the game between traders and intermediary  $I$  is as follows:

**Stage 1 - Network investment:** The intermediary invests in a network of size  $\{P_X, P_M\}$  by contacting a proportion of importers and exporters. Network investment costs,  $C(P_X, P_M)$ , are sunk. The intermediary offers contacts a take-it-or-leave-it contract specifying commission rate  $\alpha_I$  for successful matching.

**Stage 2 - Contracting:** Traders in receipt of a contract accept or reject it.

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<sup>1</sup>This assumption can easily be relaxed so that indirect matching takes place with a probability less than or equal to 1 but higher than the probability of a direct match.

**Stage 3 - Indirect trade:** Uncertainty over which trade matches are feasible through the network is resolved. The intermediary matches pairs of traders in his network, provided both parties accepted in stage 2.

**Stage 4 - Direct trade:** Any unmatched traders trade directly with probability  $q(i)$ .

### 2.1.2 Equilibrium Concept

The solution concept used is subgame perfect equilibrium (SPE) and the method employed is backward induction. A strategy for intermediary  $I$  is a set  $\{P_X(i), P_X^*(i), \alpha_I(i)\}$  that describes network size and commission rate, given information costs  $i$ . A strategy for trader  $j$  is described by a rule  $R_a$  for accepting or rejecting a contract in stage 2, if such a contract is received. A set of strategies  $\{P_X^*(i), P_X^*(i), \alpha_I^*(i), R_a^*\}$  can be said to form a subgame perfect equilibrium of the game if under these strategies the expected profit of the intermediary and the expected trade surplus of each trader are maximised, given the strategies of all other players.

## 2.2 Direct and Indirect Trade

The pool of unmatched traders in the final stage of the game includes three groups of traders: (a) those not contacted in stage 1, (b) those contacted but who rejected the contract in stage 2, and (c) those who were contacted and accepted, but could not be matched through the network in stage 3. Unmatched traders can expect to match directly with probability  $q(i)$  in the final stage of the game. Each direct match generates  $S$ , so the *ex ante* expected surplus from the direct trade route is  $q(i)S$ . Let  $\alpha_X$  and  $\alpha_M$  denote the surplus shares of exporters and importers, respectively, where  $\alpha_X + \alpha_M = 1$ . For simplicity, assume both parties have equal bargaining power so gains from any transaction are split evenly<sup>2</sup>, such that  $\alpha_X = \alpha_M = \frac{1}{2}$ . The expected payoffs from direct trade for importers and exporters, denoted by  $E^{DT}(\Pi_M)$  and  $E^{DT}(\Pi_X)$ , respectively, can thus be expressed as:

$$E^{DT}(\Pi_X) = E^{DT}(\Pi_M) = \frac{1}{2}q(i)S \quad (2)$$

Intermediated trade transactions in stage 3 between network members who accept in stage 2 also generate  $S$  per match. Since traders are identical in terms of their future trade prospects, they all either accept or reject the take-it-or-leave-it offer in stage 2. The intermediary maximises stage 1 expected profit subject to participation constraints, thereby ensuring that all traders contacted by the intermediary find it optimal to accept in equilibrium. Let  $\alpha_j$  denote the share of trade surplus captured by  $j$ , given information costs  $i$ , where

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<sup>2</sup>The particular values of  $\alpha_X$  and  $\alpha_M$  have no bearing on the intermediary's investment decision, or choice of commission rate. Symmetry is assumed for simplicity.

$j = \{X, M, I\}$ . As with direct trade, exporters and importers are assumed to split (residual) surplus equally, so  $\alpha_X = \alpha_M \equiv \alpha_T$ . It follows that:

$$2\alpha_T + \alpha_I = 1 \quad (3)$$

Traders' expected payoffs from indirect trade, denoted by  $E^{IT}(\Pi_M)$  and  $E^{IT}(\Pi_X)$ , respectively, can thus be expressed as:

$$E^{IT}(\Pi_X) = E^{IT}(\Pi_M) = \frac{1}{2}(1 - \alpha_I)S \quad (4)$$

The measure of intermediated transactions, for any given network size, will vary in stage 3 depending on the degree of overlap between the two groups of contacts. Hence, the measure of intermediated trade matches, denoted by  $T_I$ , is a random variable. For any network of size  $(P_X, P_M)$ , the largest measure of matches possible through the network is  $\min\{P_X, P_M\}$ , reflecting the maximal measure of overlap between importer and exporter contacts. Similarly, the smallest measure of matches that may arise is  $\max\{P_X + P_M - 1, 0\}$ , where mismatch between the two contact groups is greatest.

For any pair  $(X_j, M_j)$ , the *ex ante* probability of matching through the intermediary is given by the joint probability of both partners being contacted by the intermediary in stage 1,  $P_X P_M$ . The probability of any pair  $j$  matching is integrated over the range of possible pairs to give the expected measure of intermediated matches  $E(T_I) = P_X P_M$ .

In equilibrium, the intermediary builds contacts symmetrically in order to maximise  $E(T_I)$ , for any given network investment. Thus,  $P_X = P_M \equiv P$ . Hence, the expected measure of intermediated matches is  $E(T_I) = P^2$ .

Proposition (1) establishes the optimality of a symmetric network, allowing the subgame perfect equilibrium strategy set to be redefined as  $\{P^*(i), \alpha_I^*(i), R_a^*\}$ .

**Proposition 1** *It is optimal for the trade intermediary to invest symmetrically in network-building on both sides of the market, such that  $P_X = P_M \equiv P$ , where  $P \in [0, 1]$ .*

**Proof.** Consider a network of size  $(P_X, P_M)$  from which a measure of trade matches  $E(T_I) = P_X P_M$  is expected. The intermediary can maximise the return from his network investment by choosing  $P_X$  and  $P_M$  to maximise  $E(T_I)$ , given  $C(P_X, P_M) = F + c(i)(P_X + P_M)$ . The first order conditions of the constrained optimisation yield  $P_X = P_M$  as the trade maximising network configuration. Proposition (1) follows directly. ■

For any exporter (importer) evaluating whether to sign up with the intermediary in stage 2, the probability of her partner also being in the network is  $P$ . Each exporter (or importer) can expect to receive  $E^{IT}(\Pi_X)$  (or  $E^{IT}(\Pi_M)$ ) with probability  $P$  and  $E^{DT}(\Pi_X)$  (or  $E^{DT}(\Pi_M)$ ) with probability  $1 - P$ . Hence, the expected payoff of exporter  $X_j$  (or importer  $M_j$ ), conditional on being contacted by the intermediary in stage 1, is given by:

$$E(\Pi_{X_j} | X_j \in P) = E(\Pi_{M_j} | M_j \in P) = \frac{1}{2}[P(1 - \alpha_I) + (1 - P)q(i)]S \quad (5)$$

Contrasting the expected payoffs described by equations (2) and (5) yields proposition (2).

**Proposition 2** *In equilibrium, the intermediary offers contracts demanding commission rate  $\alpha_I^*(i) = 1 - q(i)$ . All contracts are accepted.*

**Proof.** To ensure trader participation in stage 2, the intermediary must set  $\alpha_I$  sufficiently low so that expected payoff from signing up to the network, described by equation (5), is at least as large as the expected payoff from an exclusively direct trade route, described by (2). The highest commission rate consistent with trader participation is thus:

$$\alpha_I = 1 - q(i) \tag{6}$$

Hence, traders' optimal acceptance rule  $R_a^*$  in stage 2 is 'accept the contract if  $\alpha_I \leq 1 - q(i)$ ; reject otherwise'. Anticipating the traders' incentives in stage 2, the intermediary sets the largest participation-consistent commission rate<sup>3</sup>. Hence, the intermediary  $\alpha_I^*(i) = 1 - q(i)$  in stage 1 and all contracts offered are accepted in stage 2. ■

The intermediary is constrained by traders' outside option to trade directly, which in turn depends on the level of information costs. The worse are the traders' prospects in the market, the higher the commission rate the intermediary can charge and still ensure participation. Even though a larger network improves the chances of an indirect trade match, the option to trade directly remains available, so  $\alpha_I^*(i)$  is independent of  $P$ . Moreover, since all surplus over and above that generated through direct trade is appropriated by the intermediary, all traders are indifferent between trading directly or the possibility of trading through the network.

**Proposition 3** *In equilibrium, importers and exporters are indifferent ex ante between the prospect of direct matching only and having the opportunity to trade both directly and indirectly.*

**Proof.** At the outset of the game, anticipating a network of size  $P$ , any pair  $(X_j, M_j)$  can expect to find themselves in one of four possible positions: (i) with probability  $(1 - P)^2$ , both trade partners are outside the network; (ii) with probability  $P(1 - P)$ ,  $M_j$  is inside the network and  $X_j$  outside; (iii) with probability  $P(1 - P)$ ,  $X_j$  is inside the network and  $M_j$  outside, and (iv) both partners are members of the network, with probability  $P^2$ . The expected payoff for each partner is  $\frac{1}{2}q(i)S$  in (i)-(iii) and  $\frac{1}{2}(1 - \alpha_I)S$  in (iv). Weighing the expected payoffs with their respective probabilities yields the *ex ante* expected payoff to any trader  $j$  at the outset of the game, given  $P$ . This is summarised by:

$$E(\Pi_{X_j} | P) = E(\Pi_{M_j} | P) = \frac{1}{2} [q(i)(1 - P^2) + (1 - \alpha_I)P^2] S \tag{7}$$

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<sup>3</sup>Assume that when indifferent between the two modes of trade, traders sign up with the intermediary. Alternatively, the intermediary could offer an infinitesimally small additional amount,  $\epsilon$ , to ensure traders sign up to the network.

Anticipating that  $\alpha_I^*(i) = 1 - q(i)$ , equation (7) simplifies to give  $E(\Pi_X | P) = E(\Pi_M | P) = \frac{1}{2}q(i)S = E^{DT}(\Pi_X) = E^{DT}(\Pi_M)$ . Hence, traders are indifferent between having the prospect of intermediated trade, or not. ■

### 2.3 Equilibrium Network Size

The intermediary chooses  $P \in [0, 1]$  to maximise expected profits (net of network investment cost),  $E(\Pi_I)$ , subject to  $\alpha_I^*(i) = 1 - q(i)$  and  $R_a^*$ , where expected profit can be expressed by:

$$E(\Pi_I) = [1 - q(i)] SP^2 - 2c(i)P - F \quad (8)$$

The specification does not yield an interior equilibrium for  $P$ , as verified by the non-negative second order condition<sup>4</sup>. Hence, the intermediary chooses to develop contacts with all traders, or none, depending on the level of information costs. When profitable at the margin, the network expands to include all traders, provided the measure of trade matches is sufficiently large to cover set up costs. Otherwise, no contacts are developed at all, and the intermediary is inactive. Which of the two corner equilibria prevails hinges on the relative magnitude of two conflicting effects of  $i$  on expected profits. The greater the prevailing barriers to information flow, the higher are the costs of network development. At the same time, traders' direct matching prospects worsen, thereby allowing a higher commission rate to be charged. The net effect of information costs on  $E(\Pi_I)$  thus depends on the relative impact of  $i$  on  $q(i)$  and  $c(i)$ . This is summarised formally in proposition (4).

**Proposition 4** *Expected profit from intermediation is monotonically increasing with the level of information costs if  $c'(i) < -\frac{PS}{2}q'(i)$ , for  $P > 0$ .*

**Proof.** Partially differentiating (8) with respect to  $i$  yields:

$$\frac{\partial E(\Pi_I)}{\partial i} = -[2c'(i) + PSq'(i)]P \quad (9)$$

It follows directly from (9) that  $E(\Pi_I)$  is monotonically increasing with  $i$ , if:

$$c'(i) < -\frac{PS}{2}q'(i), \text{ for } P > 0 \quad (10)$$

■  
Let  $P^*(i)$  describe the intermediary's optimal network investment strategy for any  $i \in [0, 1]$ . This defines the equilibrium network path  $P^*(i)$  in the subgame perfect equilibrium. The parameter space can be partitioned into two sets; the set of parameters for which condition (10) is satisfied, denoted by (A0), and the set for which it is not, denoted by (B0). For each set there exists a unique equilibrium pattern of intermediation.

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<sup>4</sup>The second order condition is non-negative for all values of information cost  $i$  and network size  $P$ :  $\frac{\partial^2 E(\Pi_I)}{\partial^2 P} = 2S[1 - q(i)] \geq 0$ .

Equilibrium path (A0) arises for parameter values that satisfy condition (10) and thus for which the intermediary's expected profit is increasing in information costs  $i$ . Hence, for sufficiently small network set-up costs,  $F$ , relative to trade surplus,  $S$ , there is a threshold level of information costs,  $\hat{i} \in [0, 1]$ , above which the intermediary finds it profitable to invest in an information network spanning the entire market and below which the intermediary is inactive. Moreover, the higher the trade surplus relative to fixed costs, the lower the threshold above which the intermediary is active.

Equilibrium path (B0) arises where expected profit fails to satisfy condition (10), so expected profit is decreasing with information costs. This describes the case where the negative effect of higher  $i$  on network investment cost outweighs the positive effect on revenue from the ability to set a higher commission rate. Hence, for sufficiently small network set-up costs relative to trade surplus, there is a threshold level of information costs,  $\bar{i} \in [0, 1]$ , below which the intermediary finds it profitable to invest in an information network that covers the entire market. The intermediary's network investment is constrained by market size, yielding a constrained expected profit  $E(\Pi_I)|_{P=1}$ , which is non-monotonic in  $i$ , and which yields a threshold level  $\underline{i} < \bar{i}$ , below which the market size constraint is so restrictive that positive profits cannot be attained. Hence, in equilibrium (B0), a trade network is only viable for values of  $i$  that lie between the two thresholds.

Propositions (5) and (6) characterise the two equilibrium patterns of intermediation.

**Proposition 5** *If expected profits are monotonically increasing in  $i$ , then equilibrium network size,  $P^*$ , expected trade,  $E^*(T)$ , and expected welfare,  $E^*(W)$ , are:*

$$P^* = \begin{cases} 0 & \text{if } i \in [0, \min(\hat{i}, 1)] \\ 1 & \text{if } i \in [\min(\hat{i}, 1), 1] \end{cases}$$

$$E^*(T) = \begin{cases} q(i) & \text{if } i \in [0, \min(\hat{i}, 1)] \\ 1 & \text{if } i \in [\min(\hat{i}, 1), 1] \end{cases}$$

$$E^*(W) = \begin{cases} q(i)S & \text{if } i \in [0, \min(\hat{i}, 1)] \\ S - 2c(i) - F & \text{if } i \in [\min(\hat{i}, 1), 1] \end{cases}$$

where  $\hat{i}$  is the positive threshold level of information costs that solves  $c(i) + \sqrt{c(i)^2 + [1 - q(i)]SF} = [1 - q(i)]S$ , above which  $E(\Pi_I) > 0$ .

**Proof.** Setting expected profit in equation (8) to zero,  $E(\Pi_I) = 0$  simplifies to give the following quadratic expression in  $P$ :

$$[1 - q(i)]SP^2 - 2c(i)P - F = 0 \quad (11)$$

Equation (11) describes the combinations of  $i$  and  $P$  for which  $E(\Pi_I) = 0$ . Hence, equation (11) implicitly defines the iso-profit contour in  $(i, P)$  space,

along which expected profits are zero. Let  $\widehat{P}(i)$  denote the positive, real root of (11), given  $i$  and in terms parameters of the model, where:

$$\widehat{P}(i) = \frac{c(i) + \sqrt{c(i)^2 + [1 - q(i)] SF}}{[1 - q(i)] S} > 0 \quad (12)$$

$\widehat{P}(i)$  gives a measure of the market size that would, given  $i$ , generate exactly enough revenue to cover the network set-up cost and variable costs. It can be interpreted as the minimum network size consistent with  $E(\Pi_I) \geq 0$ , given  $i$ . If  $\widehat{P}(i) \leq 1$ , then the revenue generated from the unit measure of traders is sufficient to cover network costs so the intermediary invests in a trade network spanning the entire market ( $P^* = 1$ ). Conversely, for  $i$  where  $\widehat{P}(i) > 1$ , the measure of traders is not large enough for a viable network, so  $P^* = 0$ .

If condition (10) holds for all values of  $i > 0$ , then there is a unique value of  $i$ ,  $\widehat{i}$ , that solves  $\widehat{P}(i) = 1$  at which  $E(\Pi_I) = 0$ . It follows that  $E(\Pi_I) \geq 0$  for  $i \geq \widehat{i}$  and  $E(\Pi_I) < 0$  for  $i < \widehat{i}$ . Hence,  $P^* = 1$  for values of  $i$  where  $\widehat{P}(i) \leq 1$  and 0 otherwise.

If  $P^* = 0$ , then there is no intermediated trade. Expected trade volume is thus  $q(i)$  direct matches, generating an expected surplus of  $q(i)S$ . If  $P^* = 1$ , then the intermediary can match all pairs. It follows that all trade is intermediated and trade volume is 1. The expected welfare is the surplus generated from trade,  $S$ , less the network costs incurred by the intermediary. Hence,  $E^*(W) = S - 2c(i) - F$  when the intermediary is active. ■

**Proposition 6** *If expected profits are non-monotonic in  $i$ , then equilibrium network size,  $P^*$ , expected trade,  $E^*(T)$ , and expected welfare,  $E^*(W)$ , are:*

$$P^* = \begin{cases} 0 & \text{if } i \in [0, \underline{i}] \\ 1 & \text{if } i \in [\underline{i}, \bar{i}] \\ 0 & \text{if } i \in [\bar{i}, 1] \end{cases}$$

$$E^*(T) = \begin{cases} q(i) & \text{if } i \in [0, \underline{i}] \\ 1 & \text{if } i \in [\underline{i}, \bar{i}] \\ q(i) & \text{if } i \in [\bar{i}, 1] \end{cases}$$

$$E^*(W) = \begin{cases} q(i)S & \text{if } i \in [0, \underline{i}] \\ S - 2c(i) - F & \text{if } i \in [\underline{i}, \bar{i}] \\ q(i)S & \text{if } i \in [\bar{i}, 1] \end{cases}$$

where  $\underline{i}$  and  $\bar{i}$  are positive roots of  $c(i) + \sqrt{c(i)^2 + [1 - q(i)] SF} = [1 - q(i)] S$ , between which  $E(\Pi_I) > 0$ .

**Proof.** If  $c'(i) < -\frac{PS}{2}q'(i)$  for  $i \in [0, \underline{i}]$  and  $c'(i) \geq -\frac{PS}{2}q'(i)$  for  $i \in [\bar{i}, 1]$ , given  $P > 0$ , then expected profit is non-monotonic in  $i$  and there are, in general, two positive, real roots of  $\widehat{P}(i) = 1$ . Let the two roots be defined as  $\underline{i}$  and  $\bar{i}$ , respectively, where  $\bar{i} > \underline{i} > 0$  and  $\bar{i} \in [\underline{i}, 1]$ . It follows that  $E(\Pi_I) \geq 0$  for  $i \in [\underline{i}, \bar{i}]$ , and  $E(\Pi_I) < 0$ , otherwise. Hence,  $P^* = 1$  for  $i \in [\underline{i}, \bar{i}]$ , and 0 otherwise.  $E^*(T)$  and  $E^*(W)$  follow directly. ■

### 2.3.1 Trade and Welfare

Since any unmatched network members in stage 3 continue to have the opportunity to trade directly in stage 4, expected trade can never be lower with an active intermediary in the market than without. This is formalised in proposition (7).

**Proposition 7** *An active intermediary raises expected trade volume unambiguously compared to expected trade when only direct trade is possible.*

**Proof.** Let  $E(T)$  denote expected trade volume. Investment in a network of size  $P$ , where  $P \in [0, 1]$ , generates  $P^2$  expected indirect matches in stage 3. A proportion  $q(i)$  of the remaining  $1 - P^2$  pairs trade directly in stage 4. It follows that:

$$\begin{aligned} E(T) &= q(i) + P^2 [1 - q(i)] \\ &\geq q(i) = E^{DT}(T) \end{aligned} \tag{13}$$

Expected trade volume with an intermediary is thus at least as large as when only direct trade is possible, for any choice of network size  $P$ . Moreover, expected trade is unambiguously higher when the intermediary is active ( $P > 0$ ).

■

The intermediary exploits his monopoly power and sets a commission rate that leaves traders as well off (in expected terms) under the intermediation contract as through direct trade. Hence, the intermediary's expected profit represents a pure welfare gain. The gain arises from the fact that the intermediary expands the set of possible production technologies for matching, while his exclusive appropriation of these welfare gains stems from his market power from being a monopolist provider<sup>5</sup> of the indirect matching technology. Proposition (8) formalises this discussion.

**Proposition 8** *An active intermediary raises expected welfare unambiguously compared to expected welfare when only direct trade is possible.*

**Proof.** Let  $E^{DT}(W)$  denote expected welfare arising from direct trade, without an intermediary. This mirrors expected trade, and is given by:

$$E^{DT}(W) = q(i)S \tag{14}$$

Further, let  $E(W)$  denote expected welfare with a trade network of any size  $P$ , where  $P \in [0, 1]$ . The total surplus generated from direct and indirect trade is  $P^2S$  and  $q(i)(1 - P^2)S$ , respectively. Subtracting the intermediary's network costs gives:

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<sup>5</sup>Chapter 2 analyses the competitive interaction of two intermediaries in the two-sided market. For equilibria where both intermediaries are active, traders with access to intermediation services are, on average, strictly better off than those with access to direct trade only.

$$E(W) = (1 - P^2) q(i)S + P^2 S - 2c(i)P - F \quad (15)$$

Rearranging (15) gives:

$$E(W) = q(i)S + [1 - q(i)] SP^2 - 2c(i)P - F \quad (16)$$

In equilibrium,  $P^* \geq 0$  if  $E(\Pi_I) \geq 0$ , in which case (8) implies that  $[1 - q(i)] S (P^*)^2 \geq 2c(i)P^* - F$ . Moreover, for all values of  $i$  where  $E(\Pi_I) < 0$ ,  $P^* = 0$ . Hence:

$$\begin{aligned} E^*(W) &= q(i)S + [1 - q(i)] S (P^*)^2 - 2c(i)P^* - F \\ &\geq q(i)S = E^{DT}(W) \end{aligned} \quad (17)$$

Equilibrium expected welfare with an intermediary is thus at least as large as expected welfare when only direct trade is possible. Moreover, for levels of information costs where  $P^* > 0$ , expected welfare is unambiguously higher with the intermediary. Proposition (8) follows directly. ■

### 2.3.2 Illustrative Examples

To provide further intuition an illustrative example is provided for each of the two equilibrium patterns of intermediation. To add structure to the discussion, let marginal cost of network expansion  $c(i)$  and direct matching probability  $q(i)$  be specified as  $c(i) = i^\alpha$  and  $q(i) = 1 - i^\delta$ , respectively, where  $\alpha, \delta \geq 1$ . For these specifications, equilibrium pattern (A0) arises where  $\delta \geq \alpha$ , while equilibrium pattern (B0) arises for parameter values where  $\alpha > \delta$ . For sufficiently large trader surplus  $S$  relative to network set-up cost  $F$ , the lower threshold levels of information cost above which the intermediary is active lie within  $i \in [0, 1]$ . Consider the following illustrative examples for each case.

**Equilibrium Intermediation Path (A0)** Figure (2) illustrates the equilibrium path of network size with information costs, for which<sup>6</sup>  $\delta \geq \alpha$  and thus where condition (10) is satisfied. A map of iso-profit contours is depicted where the lowest corresponds to zero profits, and illustrates  $\hat{P}(i)$ , the minimum network size that allows the intermediary to break even. Threshold  $\hat{i}$  corresponds to  $\hat{P}(\hat{i}) = 1$ , below which the intermediary is inactive and above which network size is 1. The higher cost implications of higher prevailing information costs are dominated by the commission effect through condition (10), so the intermediary is active for all  $i \in [\hat{i}, 1]$ .

The corresponding expected trade pattern,  $E(T)$ , is illustrated in figure (3).  $E^{DT}(T)$  depicts the declining expected trade path that would prevail without an intermediary. Despite the relatively low level of information costs that prevail

<sup>6</sup>All figures for equilibrium (A0) are illustrated for  $S = 9$ ,  $q(i) = 1 - i^4$ ,  $c(i) = i^2$  and  $F = 0.001$ .

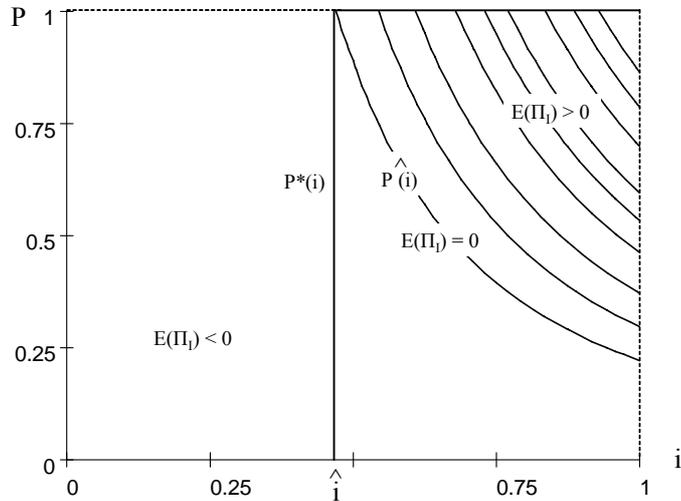


Figure 2: Equilibrium A0: path of network size with information costs.

when the intermediary is inactive, a proportion of trade matches  $1 - q(i)$  is lost due to information frictions. As barriers to information flow worsen, an increasing measure of transactions fail to materialise, enabling the intermediary to become active beyond threshold  $\hat{i}$ . The network enables all trading pairs to match indirectly, raising trade volume to 1, despite the larger frictions that a higher  $i$  implies. Moreover, Figure (4) illustrates the positive welfare effect of the intermediary's investment. Since profits from intermediation are monotonically increasing in  $i$ , the welfare gain from intermediation increases as the barriers to information flow become more severe.

**Equilibrium Intermediation Path (B0)** Consider the iso-profit map<sup>7</sup> in figure (5) that reflects the intermediary's incentives where  $\delta < \alpha$ . For relatively low levels of information costs  $i$ , expected profit is increasing with  $i$ . For this range of information costs the revenue effect of increasing information costs outweighs the cost effect. The trade-off between the two effects worsens with  $i$ , however, for any given network size  $P > 0$ , until the threshold is reached above which the cost effect outweighs the revenue effect. A trade network is thus unviable when information costs are very low ( $i < \underline{i}$ ), or very high ( $i > \bar{i}$ ).

Figures (6) and (7) illustrate the corresponding expected trade volume and welfare effects. The trade network represents a more efficient information technology than direct matching, thereby improving welfare, but over a limited range of  $i$ . The pattern of trade in equilibrium (B0) indicates that even small

<sup>7</sup>All figures for equilibrium (B0) are illustrated for  $S = 1.2$ ,  $q(i) = 1 - i^3$ ,  $c(i) = i^6$  and  $F = 0.005$ .

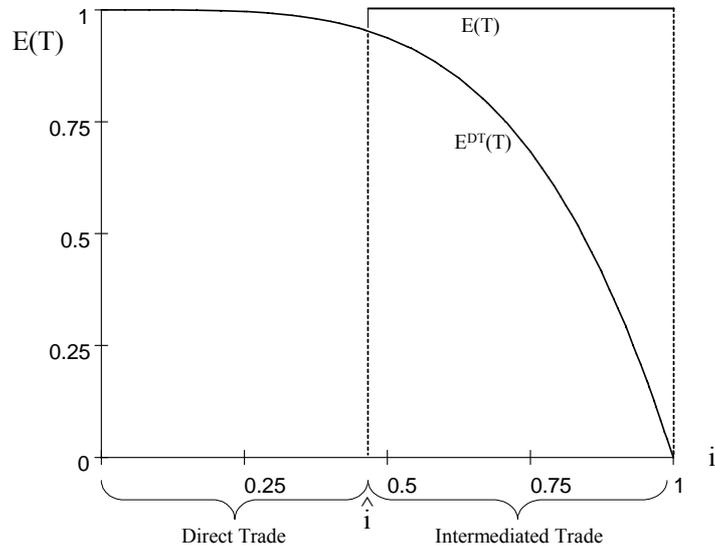


Figure 3: Equilibrium A0: expected trade path.

changes in information costs may have dramatic implications for the organisation of trade between direct and indirect as a result of pivotal thresholds that trigger network investment or, indeed, network collapse.

The model points to the possibility of a complete reorganisation of trade beyond threshold levels of information costs. The dramatic swings between direct trade and intermediated trade result from the linear network cost specification. Since both direct and intermediated trade is observed in practice, it is important to examine the conditions under which an interior equilibrium exists and how it may be affected by information costs. In the next section, network size is introduced as an argument of the intermediary's cost function and the interior equilibrium solved analytically under convexity in network-building costs. Note that core propositions (7) and (8) do not rely on any assumptions on costs and  $q(i)$ , so continue to hold.

### 3 Convex Network-Building Costs

This section allows the intermediary's costs to depend on network size,  $P$ , in addition to information costs  $i$ . In particular, let marginal costs of network

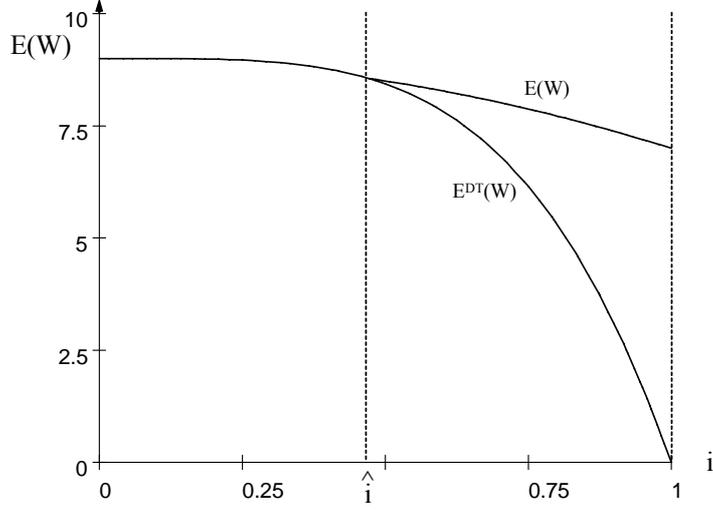


Figure 4: Equilibrium A0: expected welfare path.

expansion be denoted by  $c(i, P)$ , where:

$$\begin{aligned}
 c(0, \cdot) &= 0 ; c(\cdot, 0) = 0 \\
 c_i(i, P) &> 0 ; c_{ii}(i, P) \geq 0 \\
 c_p(i, P) &> 0 ; c_{pp}(i, P) > 0 \\
 c_{ip}(i, P) &= c_{pi}(i, P) > 0
 \end{aligned} \tag{18}$$

As described in (18),  $c(i, P)$  is monotonically increasing in  $i$ , for any given network size  $P$ , and monotonically increasing in  $P$ , for any given level of information costs. Convexity in network size  $P$  (but not  $i$ ) is necessary in order to generate an interior equilibrium. Let  $c(i, P)$  be specified by equation (19), which satisfies the conditions in (18):

$$c(i, P) = \gamma i^\alpha P^\beta, \text{ where } \alpha \geq 1, \beta \geq 2 \text{ and } \gamma > 0 \tag{19}$$

Parameter  $\alpha$  is the elasticity of cost  $c(i, P)$  with respect to information costs  $i$  and  $\beta$  is the elasticity of cost  $c(i, P)$  with respect to network size  $P$ . Coefficient  $\gamma$  is a shift factor, which raises (or lowers) network investment cost for given  $i$  and  $P$ . Total network investment cost  $C(P) = F + 2\gamma i^\alpha P^{\beta+1}$  is thus convex in  $P$ .

Further, let  $q(i)$  be described by:

$$q(i) = 1 - i^\delta, \text{ where } \delta \geq 1 \tag{20}$$

Hence, from proposition (2), the commission rate demanded by the intermediary in equilibrium is  $\alpha_i^*(i) = i^\delta$ . Parameter  $\delta$  thus denotes the elasticity of

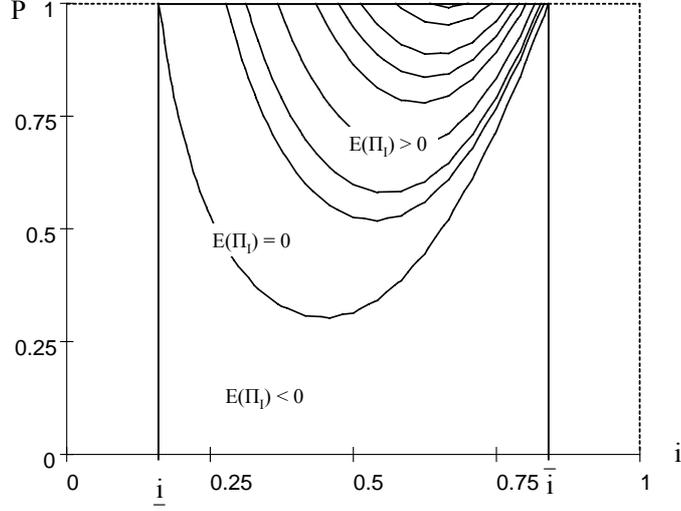


Figure 5: Equilibrium B0: path of network size with information costs.

the equilibrium commission rate with respect to information cost  $i$ .

Substituting (19) and (20) into equation (8) yields the following expression for expected profits

$$\begin{aligned} E(\Pi_I) &= [1 - q(i)] SP^2 - 2Pc(i, P) - F \\ &= Si^\delta P^2 - 2\gamma i^\alpha P^{\beta+1} - F \end{aligned} \quad (21)$$

Maximising (21) with respect to  $P$  yields equilibrium network size in terms of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$  and  $S$ . Analytically, this can be expressed <sup>8</sup> by:

$$\tilde{P} = \left[ \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}} > 0 \quad (22)$$

Equilibrium network size is given by (22), provided  $E(\Pi_I) \geq 0$  and  $\tilde{P} \leq 1$ , which shows that the equilibrium pattern of intermediation and trade depend on the relative values of  $\delta$  and  $\alpha$ . The convexity of network investment costs gives rise to an interior equilibrium, subject to the constraint imposed by the size of the market and provided set-up costs  $F$  are sufficiently low relative to trade surplus  $S$ .

Proposition (9) describes the necessary condition for expected profit in the interior equilibrium to be increasing in  $i$ .

<sup>8</sup>A derivation of (22) is included in Appendix B.

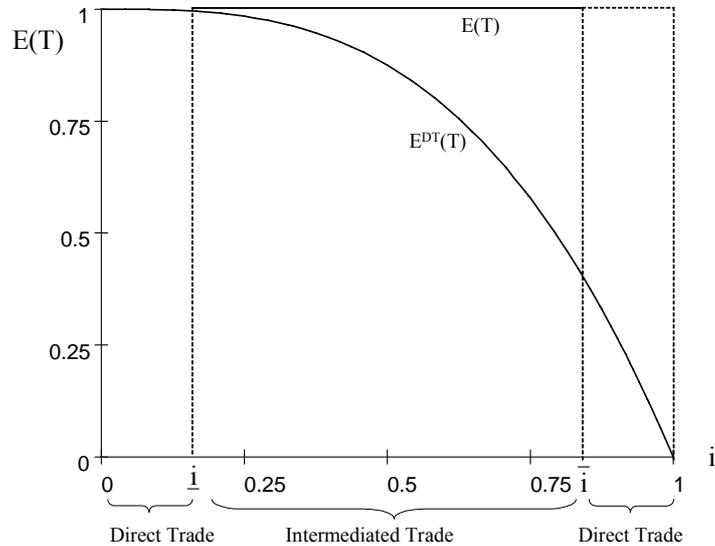


Figure 6: Equilibrium B0: expected trade path.

**Proposition 9** *Unconstrained expected profit is monotonically increasing with the level of information costs if  $(\beta + 1)\delta > 2\alpha$ .*

**Proof.** For proof see Appendix A. ■

Condition  $(\beta + 1)\delta > 2\alpha$  implies that as information costs increase, the direct matching route worsens relatively more than the cost of network provision. This gives rise to higher expected profits for the intermediary by relaxing the constraint on the commission fee the intermediary can demand.

The analysis proceeds by distinguishing between four distinct equilibrium patterns of network investment. The parameter space can be split into four ranges, denoted by (A1)-(D1), each corresponding to a different set of incentives for network investment. These are discussed in turn.

**(A1)**  $\delta > \alpha \geq 1$ : For this parameter range, the elasticity of the intermediary's optimal commission rate with respect to information costs,  $\delta$ , exceeds the elasticity of the intermediary's marginal cost of network expansion with respect to information costs, given by  $\alpha$ . Hence, as information costs worsen, the increase in the commission rate the intermediary can command exceeds the increase in networking cost  $c(i, P)$ , making a network expansion profitable. For this parameter range, optimal network size is increasing with  $i$ .

**(B1)**  $\delta = \alpha \geq 1$ : If the elasticities of the commission rate and  $c(i, P)$  are exactly equal, then the effects of changing information cost  $i$  on the intermediary's cost and expected revenue exactly offset each other. Hence the

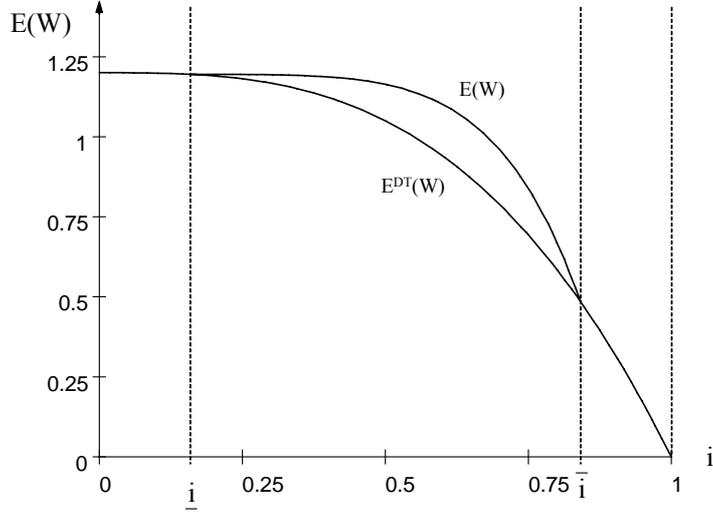


Figure 7: Equilibrium B0: expected welfare path.

intermediary's optimal choice of network size is unchanging with  $i$ . Note, however, that while the intermediary's investment decision is unaffected at the margin, it follows from proposition (9) that unconstrained profits are increasing with  $i$ .

- (C1)  $\frac{2\alpha}{\beta+1} < \delta < \alpha$ : If the elasticity of marginal networking cost  $c(i, P)$  exceeds the elasticity of the commission rate with respect to  $i$ , then it is optimal for the intermediary to contract network size as information costs worsen. Despite the contracting network size, unconstrained expected profits are increasing with  $i$ . Recall that  $\beta$  is the elasticity of  $c(i, P)$  with respect to network size  $P$ . Since  $(\beta + 1)\delta > 2\alpha$  holds, then within this range of parameter values, cost  $c(i, P)$  is sufficiently elastic with respect to network size  $P$ , so as to offset the effects of information cost  $i$  on  $c(i, P)$ , thereby raising equilibrium profit overall.
- (D1)  $\delta \leq \frac{2\alpha}{\beta+1}$ : For this range of elasticities, the commission rate is less responsive to information cost  $i$  than is networking cost  $c(i, P)$  and moreover, the responsiveness of  $c(i, P)$  with respect to  $P$  is not sufficient so as to allow a contraction to offset the negative effect on expected profit. Hence, equilibrium (unconstrained) expected profit is decreasing with  $i$ .

The four equilibrium patterns of intermediation, (A1)-(D1), shed light on how information frictions affect direct and indirect matching technologies. The model thus suggests that we can learn about the relative elasticities of the costs of network provision and the probability of direct matching from an empirical

examination of the impact of changing information costs on intermediation.

The rest of the section formally characterises the interior equilibrium path of network size, expected trade and expected welfare for parameter ranges, (A1)-(D1). Further intuition is provided through the discussion of illustrative examples.

### 3.1 Equilibrium Pattern of Intermediation (A1)

**Proposition 10** *If  $\delta > \alpha \geq 1$ , then the interior equilibrium is characterised by the following:*

- (a) *Network size is increasing in the level of information costs  $i$  and trade surplus  $S$  and decreasing in cost parameters  $\beta$  and  $\gamma$ .*
- (b) *The proportion of indirect trade to total trade is increasing in the level of information costs  $i$ . The relationship between total expected trade and information costs is non-monotonic.*
- (c) *The contribution of intermediation to social welfare is positive and increasing in the level of information costs  $i$ .*

**Proof.** Formally, network size,  $P^*$ , expected trade volume,  $E^*(T)$ , and expected welfare,  $E^*(W)$ , are described by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ \left[\frac{Si^{\delta-\alpha}}{\gamma(\beta+1)}\right]^{\frac{1}{\beta-1}} & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{\hat{i}}, 1\} \\ 1 & \text{if } \min\{\hat{\hat{i}}, 1\} \leq i \leq 1 \end{cases}$$

$$E^*(T) = \begin{cases} 1 - i^\delta & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ 1 - i^\delta + \left[\frac{S}{\gamma(\beta+1)}\right]^{\frac{2}{\beta-1}} i^{\frac{\delta(\beta+1)-2\alpha}{\beta-1}} & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{\hat{i}}, 1\} \\ 1 & \text{if } \min\{\hat{\hat{i}}, 1\} \leq i \leq 1 \end{cases}$$

$$E^*(W) = \begin{cases} (1 - i^\delta)S & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ (1 - i^\delta)S + i^\delta SA^{\frac{2}{\beta-1}} - 2\gamma A^{\frac{\beta+1}{\beta-1}} i^\alpha - F & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{\hat{i}}, 1\} \\ S - 2\gamma i^\alpha - F & \text{if } \min\{\hat{\hat{i}}, 1\} \leq i \leq 1 \end{cases}$$

where  $A = \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)}, \hat{i} = \left[\gamma^{\frac{2}{\beta-1}} \left(\frac{F}{\beta-1}\right) \left(\frac{\beta+1}{S}\right)^{\frac{\beta+1}{\beta-1}}\right]^{\frac{1}{\delta(\beta+1)-2\alpha}} > 0$

and  $\hat{\hat{i}} = \left[\frac{\gamma(\beta+1)}{S}\right]^{\frac{1}{\delta-\alpha}} > 0$

For a full proof of the above see Appendix B.

It follows from the interior equilibrium that:

$$\begin{aligned} \frac{\partial P^*}{\partial i} &= \frac{(\delta - \alpha)S}{\gamma(\beta - 1)(\beta + 1)} \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{2-\beta}{\beta-1}} i^{\frac{\beta+1+(\delta-\alpha)}{\beta-1}} > 0 \text{ when } \delta > \alpha \\ \frac{\partial P^*}{\partial S} &> 0; \quad \frac{\partial P^*}{\partial \gamma} < 0; \quad \frac{\partial P^*}{\partial \beta} < 0 \end{aligned} \quad (23)$$

Moreover,  $E^*(T)$  can be decomposed into direct and indirect equilibrium trade. Let direct<sup>9</sup> and indirect trade in equilibrium be denoted by,  $E_D^*(T)$ , and  $E_I^*(T)$ , respectively, where:

$$E_D^*(T) = (1 - i^\delta) \left[ 1 - \left( \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right)^{\frac{2}{\beta-1}} \right] \quad (24)$$

$$E_I^*(T) = \left( \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right)^{\frac{2}{\beta-1}} \quad (25)$$

Let the equilibrium direct and indirect trade shares be denoted by  $s_D$  and  $s_I$ , respectively, where:

$$s_D \equiv \frac{E_D^*(T)}{E^*(T)} = \frac{q(i) \left[ 1 - (P^*)^2 \right]}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2} \quad (26)$$

$$s_I \equiv \frac{E_I^*(T)}{E^*(T)} = \frac{(P^*)^2}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2} \quad (27)$$

It is straightforward to show that  $\frac{\partial s_D}{\partial i} < 0$  and  $\frac{\partial s_I}{\partial i} > 0$ . Higher information costs correspond to both a larger network size and a lower probability of direct matching. Both effects drive the result that the proportion of indirect trade to total trade is increasing in the level of information costs. Moreover, for  $i \in [\hat{i}, 1]$ , where  $P^* = 1$ , all trade is intermediated, so  $s_D = 0$  and  $s_I = 1$ .

Recall that  $E^{DT}(W)$  is the expected welfare that would prevail if there were no intermediary in the market. It follows from (17) that  $E^*(W) - E^{DT}(W) = E^*(\Pi)$  is a measure of the intermediary's contribution to social welfare. Moreover, since  $\delta > \alpha \geq 1$ , it follows that  $\delta > \frac{2}{(\beta+1)}\alpha$ . Hence, from proposition (9),  $E^*(\Pi_I)$  is increasing in  $i$  in the interior equilibrium, so the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active. ■

### 3.1.1 Illustrative Example

Figures (8) - (10) illustrate equilibrium network size, expected trade and expected welfare, respectively, for  $i \in [0, 1]$ , for parameter values  $\beta = 2$ ,  $\gamma = 1$ ,

<sup>9</sup>  $E_D^*(T)$  is not to be confused with  $E^{DT}(T)$ .  $E_D^*(T)$  represents the equilibrium measure of direct trade matches, as a component of equilibrium total trade  $E^*(T)$ . In contrast,  $E^{DT}(T)$  represents the measure of equilibrium total trade if there were no intermediary in the market.

$\delta = 4, \alpha = 2, F = 0.001$  and  $S = \{2.5, 3, 4\}$ , which satisfy  $\delta > \alpha \geq 1$  and the convexity assumption  $\beta \geq 2$ .

Figure (8) illustrates the positive relationship between optimal network size and prevailing information costs where the elasticity of the intermediary's commission exceeds the elasticity of cost  $c(i, P)$  with respect to  $i$ . The fixed set-up cost  $F$  implies that information costs must be above a threshold level for intermediation to be profitable in the two-sided market. The optimal network path is illustrated for (a)  $S = \gamma(\beta + 1)$ , (b)  $S > \gamma(\beta + 1)$  and (c)  $S < \gamma(\beta + 1)$ , verifying that network size and threshold level  $\hat{i}$  are increasing in  $S$  relative to cost parameters  $\beta$  and  $\gamma$ .

Figure (9) illustrates the effect of intermediation on total expected trade between the two sides of the market. The intermediary's network investment provides access to a more efficient matching technology than direct trade, thereby raising total trade relative to access to direct matching only. The relationship between expected trade volume and information cost  $i$  is non-monotonic due to the conflicting effects of information cost  $i$  on the constituent parts of expected trade. For this range of parameters, the intermediary finds it optimal to increase network size with  $i$ , thereby increasing the expected measure of intermediated trade matches. The impact on direct trade is twofold. First, higher information cost worsens the probability of a direct match, and second, the expansion in network size results in a smaller expected pool of unmatched traders in stage 4. The net effect is ambiguous, giving rise to a non-monotonic relationship between  $i$  and total expected trade  $E(T)$  in equilibrium.

Figure (10) shows that intermediation is welfare improving and that it more so when information cost is higher.

### 3.2 Equilibrium Pattern of Intermediation (B1)

**Proposition 11** *If  $\delta = \alpha \geq 1$  and  $S < \gamma(\beta + 1)$ , then there exists an interior equilibrium characterised by the following:*

- (a) *Network size is independent of the level of information costs  $i$ , increasing in trade surplus  $S$  and decreasing in cost parameters  $\beta$  and  $\gamma$ .*
- (b) *The measure of intermediated transactions is independent of the level of information costs but represents an increasing proportion of total trade, which is unambiguously decreasing in information costs  $i$ .*
- (c) *The contribution of intermediation to social welfare is positive and increasing in the level of information costs  $i$ .*

**Proof.** If  $\delta = \alpha \geq 1$  and  $S \leq \gamma(\beta + 1)$ , then equilibrium network size,  $P^*$ , expected trade volume,  $E^*(T)$ , and expected welfare,  $E^*(W)$ , are described by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\ \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}} & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 \end{cases}$$

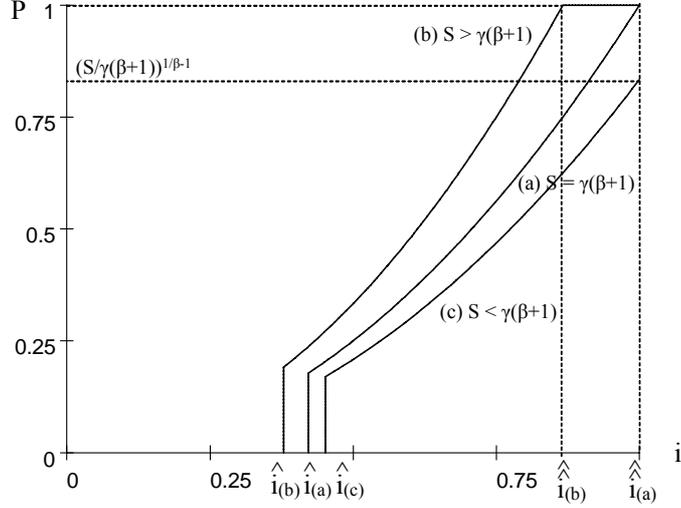


Figure 8: Equilibrium A1: path of network size with information costs.

$$E^*(T) = \begin{cases} 1 - i^\delta & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ 1 - i^\delta \left[1 - B^{\frac{2}{\beta-1}}\right] & \text{if } \min\{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

$$E^*(W) = \begin{cases} (1 - i^\delta) S & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ (1 - i^\delta) S + i^\delta S B^{\frac{2}{\beta-1}} - 2i^\delta \gamma B^{\frac{\beta+1}{\beta-1}} - F & \text{if } \min\{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

where  $B = \frac{S}{\gamma(\beta+1)}$  and  $\hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{\delta(\beta-1)}} > 0$ .

For a full proof of the above see Appendix C.

If  $\delta = \alpha \geq 1$  and  $S > \gamma(\beta + 1)$ , then the unit measure of market size poses a binding constraint. The constrained optimum network size is thus  $P^* = 1$ , provided  $E(\Pi_I) \geq 0$ . The equilibrium is analogous to that of proposition (5), with cost given by  $c(i, 1)$ .

Whether constrained or unconstrained, the equilibrium network size is constant over the range of values of  $i$  where  $E(\Pi_I) \geq 0$ . It follows that the measure of intermediated trade is also constant. Where  $P^* < 1$ , Equations (24) and (25) simplify to give:

$$E_D^*(T) = (1 - i^\delta) \left[ 1 - \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{2}{\beta-1}} \right] \quad (28)$$

$$E_I^*(T) = \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{2}{\beta-1}} \quad (29)$$

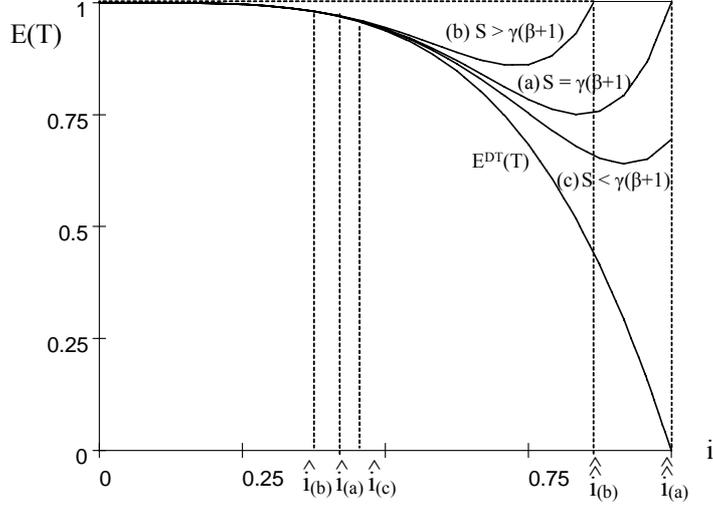


Figure 9: Equilibrium A1: expected trade path.

It follows immediately from (28) and (29) that indirect trade is constant and direct trade decreases with  $i$  as the probability of successful matching declines. Hence,  $\frac{\partial s_D}{\partial i} < 0$  and  $\frac{\partial s_I}{\partial i} > 0$ . At the limit where  $P^* = 1$ , all trade is intermediated, so  $E_D^*(T) = s_D = 0$  and  $E_I^*(T) = s_I = 1$ .

Furthermore, since  $\delta = \alpha \geq 1$ , it follows that  $\delta > \frac{2}{(\beta+1)}\alpha$ . Hence, from proposition (9),  $E^*(\Pi_I)$  is increasing in  $i$  in the interior equilibrium, so the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active.

The constrained profit path, where  $P^* = 1$  is lower than if the intermediary could expand the trade network further, but increasing in  $i$  nonetheless, since  $E(\Pi_I)|_{P=1} = i^\delta (S - 2\gamma) - F$ . ■

### 3.2.1 Illustrative Example

Figures (11) and (12) illustrate<sup>10</sup> the equilibrium network size where  $\delta = \alpha \geq 1$ . Figure (11) shows that optimal network size is unaffected by the level of information cost  $i$ . The intermediary's optimal investment is again increasing in  $S$  relative to cost parameters  $\beta$  and  $\gamma$ . Figure (12) shows that expected trade volume decreases monotonically with  $i$ , but lies above the expected trade path that prevails with access to direct matching only.

<sup>10</sup>Figures (11) and (12) are illustrated for  $\beta = 2$ ,  $\gamma = 1$ ,  $\alpha = \delta = 3$ ,  $F = 0.001$  and  $S = \{2, 3\}$ .

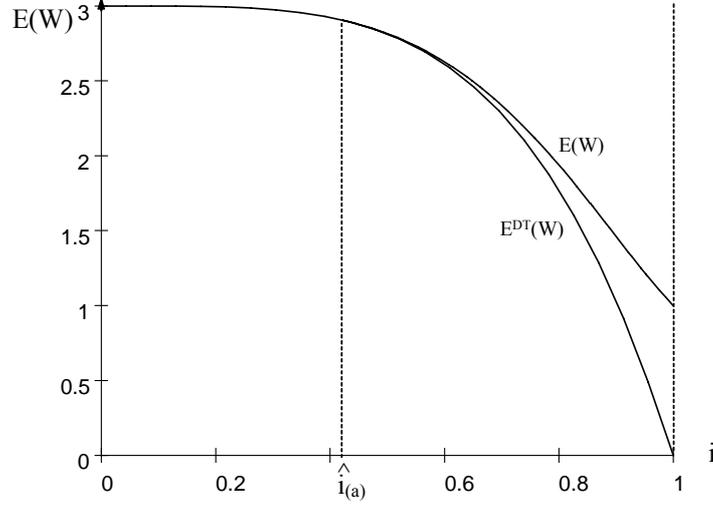


Figure 10: Equilibrium A1: expected welfare path.

### 3.3 Equilibrium Pattern of Intermediation (C1)

**Proposition 12** *If  $\frac{2}{\beta+1}\alpha < \delta < \alpha$ , then the interior equilibrium is characterised by the following:*

- (a) *Network size is decreasing in the level of information costs  $i$  and cost parameters  $\beta$  and  $\gamma$  and increasing in trade surplus  $S$ .*
- (b) *Indirect trade is decreasing and direct trade increasing in information costs  $i$ . Total expected trade is unambiguously decreasing in information costs  $i$ .*
- (c) *The contribution of intermediation to social welfare is positive and increasing in the level of information costs  $i$ .*

**Proof.** If  $\frac{2}{\beta+1}\alpha < \delta < \alpha$ , then equilibrium network size,  $P^*$ , expected trade volume,  $E^*(T)$ , and expected welfare,  $E^*(W)$ , are described by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ 1 & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{i}, 1\} \\ \left[\frac{S}{\gamma(\beta+1)i^{\alpha-\delta}}\right]^{\frac{1}{\beta-1}} & \text{if } \min\{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

$$E^*(T) = \begin{cases} 1 - i^\delta & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ 1 & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{i}, 1\} \\ 1 - i^\delta + \left[\frac{S}{\gamma(\beta+1)}\right]^{\frac{2}{\beta-1}} i^{\frac{\delta(\beta+1)-2\alpha}{\beta-1}} & \text{if } \min\{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

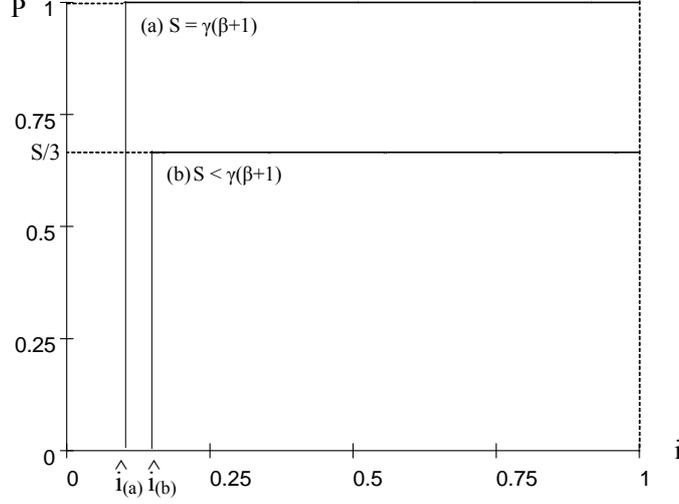


Figure 11: Equilibrium B1: path of network size with information costs.

$$E^*(W) = \begin{cases} (1 - i^\delta) S & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ S - 2\gamma i^\alpha - F & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{i}, 1\} \\ (1 - i^\delta) S + i^\delta S G^{\frac{2}{\beta-1}} - 2\gamma G^{\frac{\beta+1}{\beta-1}} i^\alpha - F & \text{if } \min\{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

where  $G = \frac{S}{\gamma(\beta+1)i^{\alpha-\delta}}$  and  $\hat{i}$  is the<sup>11</sup> smaller positive root of

$$E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0 \text{ and } \hat{i} = \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\alpha-\delta}} > 0.$$

The proof of the above is in Appendix D.

It follows from the interior equilibrium that:

$$\begin{aligned} \frac{\partial P^*}{\partial i} &= -\frac{(\alpha - \delta) S}{\gamma(\beta - 1)(\beta + 1)} \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{2-\beta}{\beta-1}} i^{\frac{\beta+1+(\delta-\alpha)}{\beta-1}} < 0 \text{ when } \alpha > \delta \\ \frac{\partial P^*}{\partial S} &> 0; \quad \frac{\partial P^*}{\partial \gamma} < 0; \quad \frac{\partial P^*}{\partial \beta} < 0 \end{aligned} \quad (30)$$

<sup>11</sup>This is the threshold above which the intermediary can attain a positive profit. It is computed based on the constrained profit equation, where  $P = 1$ . If  $F$  is sufficiently high, however, market size is not a binding constraint in the region where  $E(\Pi_I) = 0$ . so the

threshold which applies is:  $\hat{i}|_{P=\hat{P}} = \left[ \left( \frac{1}{\gamma} \right)^{\frac{2}{\beta-1}} \left( \frac{\beta-1}{F} \right) \left( \frac{S}{\beta+1} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{2\alpha-\delta(\beta+1)}} > 0$ .

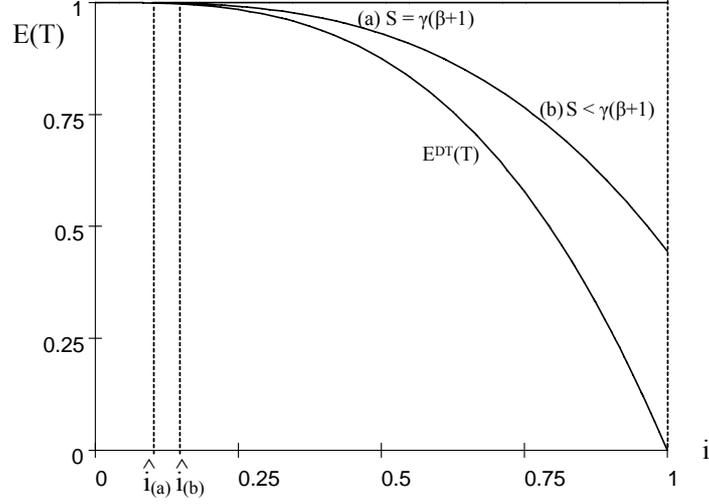


Figure 12: Equilibrium B1: expected trade path.

Hence, optimal network size is decreasing in  $i$  and cost parameters, but increasing in  $S$ . Moreover,  $E_D^*(T)$ , and  $E_I^*(T)$ , are given by:

$$E_D^*(T) = (1 - i^\delta) \left[ 1 - \left( \frac{S}{\gamma(\beta + 1)i^{\alpha - \delta}} \right)^{\frac{2}{\beta - 1}} \right] \quad (31)$$

$$E_I^*(T) = \left( \frac{S}{\gamma(\beta + 1)i^{\alpha - \delta}} \right)^{\frac{2}{\beta - 1}} \quad (32)$$

The decline in network size with information cost  $i$  is mirrored by  $E_I^*(T)$  when  $\alpha > \delta$ . The decline in intermediated matches with  $i$  increases the measure of traders seeking a direct match in stage 4. At the same time, a higher  $i$  implies a lower probability of successful direct matching.

Furthermore, since  $\delta > \frac{2}{(\beta+1)}\alpha$ , it follows from proposition (9) that  $E^*(\Pi_I)$  is increasing in information cost  $i$  in the interior equilibrium. Hence, the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active. ■

### 3.4 Equilibrium Pattern of Intermediation (D1)

**Proposition 13** *If  $\delta \leq \frac{2}{\beta+1}\alpha$ , then the interior equilibrium is characterised by the following:*

- (a) *Network size is decreasing in the level of information costs  $i$  and cost parameters  $\beta$  and  $\gamma$  and increasing in trade surplus  $S$ .*
- (b) *Indirect trade is decreasing and direct trade increasing in information costs*

- i.* Total expected trade is unambiguously decreasing in information costs  $i$ .  
*(c)* The contribution of intermediation to social welfare is positive but decreasing in information costs  $i$ .

**Proof.** If  $\delta \leq \frac{2}{\beta+1}\alpha$ , then equilibrium network size,  $P^*$ , expected trade volume,  $E^*(T)$ , and expected welfare,  $E^*(W)$ , are described by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min \left\{ \widehat{i}_{|P=1}, 1 \right\} \\ 1 & \text{if } \min \left\{ \widehat{i}_{|P=1}, 1 \right\} \leq i \leq \min \left\{ \widehat{i}, 1 \right\} \\ \left[ \frac{S}{\gamma(\beta+1)i^{\alpha-\delta}} \right]^{\frac{1}{\beta-1}} & \text{if } \min \left\{ \widehat{i}, 1 \right\} \leq i \leq \min \left\{ \widehat{i}_{|P=\tilde{P}}, 1 \right\} \\ 0 & \text{if } \min \left\{ \widehat{i}_{|P=\tilde{P}}, 1 \right\} \leq i \leq 1 \end{cases}$$

$$E^*(T) = \begin{cases} 1 - i^\delta & \text{if } 0 \leq i \leq \min \left\{ \widehat{i}_{|P=1}, 1 \right\} \\ 1 & \text{if } \min \left\{ \widehat{i}_{|P=1}, 1 \right\} \leq i \leq \min \left\{ \widehat{i}, 1 \right\} \\ 1 - i^\delta + \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{2}{\beta-1}} i^{\frac{\delta(\beta+1)-2\alpha}{\beta-1}} & \text{if } \min \left\{ \widehat{i}, 1 \right\} \leq i \leq \min \left\{ \widehat{i}_{|P=\tilde{P}}, 1 \right\} \\ 1 - i^\delta & \text{if } \min \left\{ \widehat{i}_{|P=\tilde{P}}, 1 \right\} \leq i \leq 1 \end{cases}$$

$$E^*(W) = \begin{cases} (1 - i^\delta) S & \text{if } 0 \leq i \leq \min \left\{ \widehat{i}_{|P=1}, 1 \right\} \\ S - 2\gamma i^\alpha - F & \text{if } \min \left\{ \widehat{i}_{|P=1}, 1 \right\} \leq i \leq \min \left\{ \widehat{i}, 1 \right\} \\ (1 - i^\delta) S + i^\delta S A^{\frac{2}{\beta-1}} - 2\gamma A^{\frac{\beta+1}{\beta-1}} i^\alpha - F & \text{if } \min \left\{ \widehat{i}, 1 \right\} \leq i \leq \min \left\{ \widehat{i}_{|P=\tilde{P}}, 1 \right\} \\ (1 - i^\delta) S & \text{if } \min \left\{ \widehat{i}_{|P=\tilde{P}}, 1 \right\} \leq i \leq 1 \end{cases}$$

where  $A = \frac{S i^{\delta-\alpha}}{\gamma(\beta+1)}$ ,  $\widehat{i} = \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\alpha-\delta}} > 0$ ,

$\widehat{i}_{|P=1}$  is the smaller positive root of  $E(\Pi_I)_{|P=1} = S i^\delta - 2\gamma i^\alpha - F = 0$

and  $\widehat{i}_{|P=\tilde{P}} = \left[ \left( \frac{1}{\gamma} \right)^{\frac{2}{\beta-1}} \left( \frac{\beta-1}{F} \right) \left( \frac{S}{\beta+1} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{2\alpha-\delta(\beta+1)}} > 0$ .

For a proof of the above see Appendix E.

The trade effects follow from the proof of Proposition (13). Expected profit is unconstrained in the interior equilibrium. Since  $\delta \leq \frac{2}{\beta+1}\alpha$  then it follows from proposition (9) that expected profit and thus the contribution of intermediation to social welfare is decreasing in the level of information costs  $i$ . ■

### 3.4.1 Illustrative Example

Figure (13) illustrates<sup>12</sup> the pattern of network investment where  $\delta \leq \frac{2}{\beta+1}\alpha$ . For this range of elasticities, the commission rate is less responsive to information cost  $i$  than is networking cost  $c(i, P)$ , giving rise to a negative relationship between network size and information costs along the interior path. Moreover, as illustrated in figure (14), unconstrained expected profit, denoted by  $E^U(\Pi_I)$

<sup>12</sup>Illustrated for parameter values  $\alpha = 6$ ,  $\delta = 3$ ,  $\beta = 2$ ,  $\gamma = 1$ ,  $F = 0.1$ , and  $S = 2$ .

rises without limit as  $i \rightarrow 0$ , which implies that in the absence of a binding market size constraint, the intermediary finds it profitable to invest in an ever-increasing network size as information costs tend to zero. Thus below threshold  $\hat{i}$ , equilibrium network size is constrained by the size of the market. For interval  $i \in [0, \hat{i}]$  the intermediary's expected profits follow the constrained path, denoted by  $E^C(\Pi_I)_{|P=1}$  in figure (14). While unconstrained expected profit is increasing, constrained expected profit is declining as information costs tend to zero, rendering the network unviable below some threshold level  $\hat{i}$ .

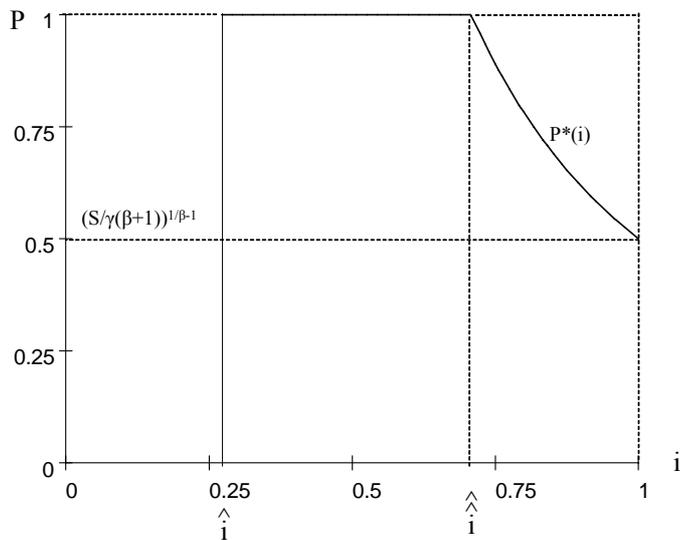


Figure 13: Equilibrium D1: path of network size with information costs.

## 4 Conclusion

This paper presents a pairwise matching model with two-sided information asymmetry between trade partners, where an intermediary has the opportunity to invest in a network of contacts and facilitate trade matching for a success fee. The framework innovates by examining the role of information costs on incentives for trade intermediation, thereby endogenising the pattern of direct and indirect trade.

The framework delivers four key results. First, intermediation unambiguously raises expected trade volume and social welfare by expanding the set of matching technologies available to traders. Second, convexity in network-building costs is necessary for both direct and indirect trade to arise in equilib-

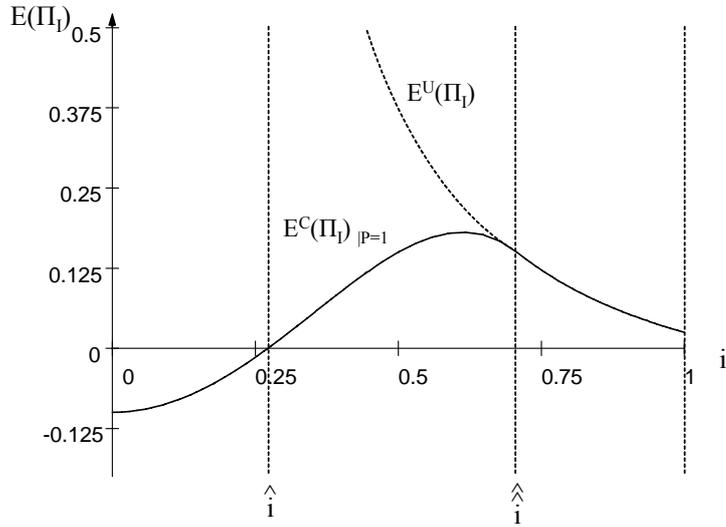


Figure 14: Equilibrium D1: constrained and unconstrained profits.

rium; otherwise, the level of information costs determines whether all trade is routed through the intermediary or takes place directly.

Third, under assumptions of convexity in the intermediary's technology, optimal network size and hence the equilibrium pattern of trade is shown to depend on the level of information costs as well as the relative effectiveness of direct and indirect matching technologies with changing information costs. In particular, if the probability of direct matching is more responsive to changing information costs than is the cost of network expansion, then indirect trade offers a relatively more attractive matching technology than direct trade as information costs rise. Hence, the proportion of indirect trade to total trade is increasing in the level of information frictions. Conversely, if networking costs are more responsive than the probability of a direct match, then the intermediary has an incentive to contract network size with the opposite trade implications. The model thus suggests that we can learn about the relative elasticities of direct and indirect matching technologies from an empirical examination of the impact of changing information costs on intermediation.

Finally, the model sheds light on the relationship between information frictions and aggregate trade volume, which may be non-monotonic as a result of conflicting effects of information costs on the incentives for direct and indirect trade. Higher information costs worsen direct matching prospects but can, at the same time, provide an incentive for network-building and thus indirect trade through a trade network.

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## Appendix A. Proof of Proposition 9.

Differentiating (21) partially with respect to  $i$  yields:

$$\frac{\partial E(\Pi_I)}{\partial i} = -P [2c_i(i, P) + PSq'(i)] \quad (33)$$

It follows directly that expected profits are increasing with  $i$ , if:

$$c_i(i, P) < -\frac{PS}{2}q'(i) \quad (34)$$

Substituting for  $c_i(i, P)$  and  $q'(i)$  simplifies the condition to:

$$P^{\beta+1} < \frac{S\delta}{2\alpha\gamma}i^{\delta-\alpha} \quad (35)$$

Substituting the expression for (interior) equilibrium network size,  $P^* = \left[ \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}}$ , and rearranging, yields the necessary and sufficient condition for unconstrained equilibrium profits,  $E^*(\Pi_I)$ , to be increasing in  $i$ :

$$(\beta+1)\delta > 2\alpha \quad (36)$$

## Appendix B. Proof of Proposition 10.

Maximising (21) with respect to  $P$  yields the first order condition:

$$\frac{\partial E(\Pi_I)}{\partial P} = 2P [Si^\delta - \gamma(\beta+1)P^{\beta-1}i^\alpha] = 0 \quad (37)$$

Solving yields the interior profit-maximising network size,  $\tilde{P}$ , where:

$$\tilde{P} = \left[ \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}} > 0 \quad (38)$$

The second order condition is found to be:

$$\frac{\partial^2 E(\Pi_I)}{\partial P^2} = 2 [Si^\delta - \gamma\beta(\beta+1)P^{\beta-1}i^\alpha] \quad (39)$$

The second order condition is negative provided  $P > \left[ \frac{Si^{\delta-\alpha}}{\gamma\beta(\beta+1)} \right]^{\frac{1}{\beta-1}}$ . Since  $\beta \geq 2$ ,  $\tilde{P} > \left[ \frac{Si^{\delta-\alpha}}{\gamma\beta(\beta+1)} \right]^{\frac{1}{\beta-1}}$  and so corresponds to an interior maximum.

The intermediary sets  $P = \tilde{P}$  provided  $E(\Pi_I) \geq 0$  and  $\tilde{P} \leq 1$ . Let  $\hat{i}$  denote the threshold level of information costs at which  $E(\Pi_I)|_{P=\tilde{P}} = 0$ . Since  $\delta > \alpha \geq 1$ , it follows that  $(\beta+1)\delta > 2\alpha$ , so, from proposition (9),  $E(\Pi_I)$  is increasing in  $i$  in the interior equilibrium. Hence,  $E(\Pi_I) \geq 0$  when  $i \geq \hat{i}$ . Solving  $E(\Pi_I)|_{P=\tilde{P}} = 0$

for  $i$  yields:

$$\hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{\delta(\beta+1)-2\alpha}} \quad (40)$$

Equilibrium network size is thus  $P^* = 0$  for  $i \in [0, \min \{\hat{i}, 1\}]$ .

Furthermore,  $\tilde{P}$  is increasing in  $i$  since  $\delta > \alpha \geq 1$ , but network size is constrained by market size.

Let  $\hat{i}$  denote the threshold level of information costs, at which  $\tilde{P} = 1$ . Solving  $\tilde{P} = 1$  for  $i$  yields:

$$\hat{i} = \left[ \frac{\gamma(\beta+1)}{S} \right]^{\frac{1}{\delta-\alpha}} \quad (41)$$

Hence, equilibrium network size is  $P^* = 1$  for  $i \in [\min \{\hat{i}, 1\}, 1]$ .

For values  $i \in [\min \{\hat{i}, 1\}, \min \{\hat{i}, 1\}]$ , where  $E(\Pi_I) \geq 0$  and  $\tilde{P} \leq 1$ , network size follows the interior path  $P^* = \tilde{P} = \left[ \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}}$ . These results are summarised by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min \{\hat{i}, 1\} \\ \left[ \frac{Si^{\delta-\alpha}}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}} & \text{if } \min \{\hat{i}, 1\} \leq i \leq \min \{\hat{i}, 1\} \\ 1 & \text{if } \min \{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

where  $\hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{\delta(\beta+1)-2\alpha}} > 0$  and  $\hat{i} = \left[ \frac{\gamma(\beta+1)}{S} \right]^{\frac{1}{\delta-\alpha}} > 0$ .

If  $0 \leq i \leq \min \{\hat{i}, 1\}$ , then the intermediary does not invest in a trade network and all trade takes place directly. The expected trade volume is thus  $q(i) = 1 - i^\delta$ . If  $\min \{\hat{i}, 1\} \leq i \leq 1$ , then the intermediary's network spans the entire market so all transactions are intermediated and trade volume is 1. For values of  $i$ ,  $\min \{\hat{i}, 1\} \leq i \leq \min \{\hat{i}, 1\}$ , both direct and indirect trade are observed in equilibrium. Substituting  $P^*$  into equation (13) yields the equilibrium expected (total) trade path over this range of information costs. These results are summarised by:

$$E^*(T) = \begin{cases} 1 - i^\delta & \text{if } 0 \leq i \leq \min \{\hat{i}, 1\} \\ 1 - i^\delta + \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{2}{\beta-1}} i^{\frac{\delta(\beta+1)-2\alpha}{\beta-1}} & \text{if } \min \{\hat{i}, 1\} \leq i \leq \min \{\hat{i}, 1\} \\ 1 & \text{if } \min \{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

where  $\hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{\delta(\beta+1)-2\alpha}} > 0$  and  $\hat{i} = \left[ \frac{\gamma(\beta+1)}{S} \right]^{\frac{1}{\delta-\alpha}} > 0$ .

Finally, the piece-wise function  $E^*(W)$  follows directly from substitution of

$$P^* = 0, \left[ \frac{S i^{\delta-\alpha}}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}} \text{ and } 1, \text{ respectively, into equation (17). This yields:}$$

$$E^*(W) = \begin{cases} (1-i^\delta) S & \text{if } 0 \leq i \leq \min\{\hat{i}, 1\} \\ (1-i^\delta) S + i^\delta S A^{\frac{2}{\beta-1}} - 2\gamma A^{\frac{\beta+1}{\beta-1}} i^\alpha - F & \text{if } \min\{\hat{i}, 1\} \leq i \leq \min\{\hat{i}, 1\} \\ S - 2\gamma i^\alpha - F & \text{if } \min\{\hat{i}, 1\} \leq i \leq 1 \end{cases}$$

where  $\hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{\delta(\beta+1)-2\alpha}} > 0$  and  $\hat{i} = \left[ \frac{\gamma(\beta+1)}{S} \right]^{\frac{1}{\delta-\alpha}} > 0$  and  $A = \frac{S i^{\delta-\alpha}}{\gamma(\beta+1)}$

### Appendix C. Proof of Proposition 11.

The equilibrium path if  $\delta = \alpha \geq 1$  follows directly from equation (22). If  $\delta - \alpha = 0$ , then  $\tilde{P}$  simplifies to:

$$\tilde{P}_{|\delta=\alpha} = \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}} > 0 \quad (42)$$

$\tilde{P}_{|\delta=\alpha}$  is a positive constant, that represents the profit maximising network size. From (42) it follows that equilibrium network size, trade and welfare depend on whether (i)  $S \leq \gamma(\beta+1)$  or (ii)  $S > \gamma(\beta+1)$ :

(i) If  $S \leq \gamma(\beta+1)$ , then  $\tilde{P}_{|\delta=\alpha} \leq 1$ . Let  $\hat{i} > 0$  denote the threshold level of  $i$  at which  $E^*(\Pi_I) = 0$ . Since  $\delta = \alpha$ , it follows from proposition (9) that  $(\beta+1)\delta > 2\alpha$ , so  $E(\Pi_I)$  is increasing in  $i$  in the interior equilibrium. Hence,  $E(\Pi_I) < 0$  when  $i < \hat{i}$ . Solving  $E(\Pi_I)|_{P=\tilde{P}} = 0$  for  $i$  and simplifying yields:

$$\hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{\frac{\beta+1}{\beta-1}} \right]^{\frac{1}{\delta(\beta-1)}} > 0 \quad (43)$$

Equilibrium network size is thus  $P^* = 0$  for  $i \in [0, \min\{\hat{i}, 1\}]$ .

For all values of  $i \geq \hat{i}$ , expected profits are positive, so the intermediary invests in contact-building to  $\tilde{P}_{|\delta=\alpha} < 1$ . Equilibrium network size is thus  $P^* = \tilde{P}_{|\delta=\alpha}$  for  $i \in [\min\{\hat{i}, 1\}, 1]$ . Furthermore, substituting both  $P^* = 0$  and  $\left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\beta-1}}$  into equations (13) and (17), respectively, yields the piece-wise functions  $E^*(T)$  and  $E^*(W)$ .

(ii) If  $S > \gamma(\beta+1)$ , then  $\tilde{P}_{|\delta=\alpha} > 1$ , so the constraint imposed by market size is binding. The constrained optimum is thus  $P^* = 1$ , provided  $E(\Pi_I) \geq 0$ . The equilibrium is analogous to that described in proposition (5).  $P^* = 0$  below a threshold value  $\hat{i}$ , that solves  $E(\Pi_I)|_{P=1} = i^\delta (S - 2\gamma) - F = 0$ , and  $P^* = 1$  otherwise.

## Appendix D. Proof of Proposition 12.

The equilibrium path if  $\frac{2}{\beta+1}\alpha < \delta < \alpha$  follows directly from equation (22). Since  $\delta < \alpha$ , then  $\tilde{P}$  can be rearranged to give:

$$\tilde{P}_{|\delta < \alpha} = \left[ \frac{S}{\gamma(\beta+1)i^{\alpha-\delta}} \right]^{\frac{1}{\beta-1}} > 0 \quad (44)$$

From (44) it follows that  $\frac{\partial \tilde{P}}{\partial i} < 0$ , so the interior equilibrium path of network size is declining with information cost  $i$ . Moreover, the second order condition in equation (39) is negative provided  $P > \left[ \frac{S}{\gamma\beta(\beta+1)i^{\alpha-\delta}} \right]^{\frac{1}{\beta-1}}$ . Since  $\beta \geq 2$  it must be true that  $\tilde{P}_{|\delta < \alpha} > \left[ \frac{S}{\gamma\beta(\beta+1)i^{\alpha-\delta}} \right]^{\frac{1}{\beta-1}}$ . Hence (44) corresponds to an interior maximum.

Let  $\hat{i}$  denote the threshold level of information costs, at which  $\tilde{P}_{|\delta < \alpha} = 1$ . Solving for  $i$  yields:

$$\hat{i} = \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\alpha-\delta}} \quad (45)$$

Let  $\hat{i}$  denote the threshold level of information costs at which  $E(\Pi_I) = 0$ . Since the interior equilibrium path of network size is declining with information cost  $i$ , then for sufficiently low  $F$ , the threshold  $\hat{i}$  corresponds to a range where  $P = 1$ . If so, then  $\hat{i}$  solves  $E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0$  and  $\hat{i} \leq \hat{i}$ , where  $\hat{i}$  is described by equation (45). If (for sufficiently high  $F$ ) threshold  $\hat{i}$  corresponds to a range where  $P = \tilde{P}_{|\delta < \alpha}$ , however, then  $\hat{i}$  solves  $E(\Pi_I)|_{P=\tilde{P}} = 0$ . This yields the threshold level in equation (40) and must exceed  $\hat{i}$ , where  $\hat{i}$  is described by equation (45). If the value of  $\tilde{P}_{|\delta < \alpha}$  at  $E(\Pi_I)|_{P=\tilde{P}} = 0$  exceeds 1, then this indicates that the constrained optimisation applies and the relevant threshold is  $\hat{i}$  solves  $E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0$ .

These results are summarised by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\ 1 & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{i}, 1 \} \\ \left[ \frac{S}{\gamma(\beta+1)i^{\alpha-\delta}} \right]^{\frac{1}{\beta-1}} & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 \end{cases}$$

where  $\hat{i}$  is as above and  $\hat{i} = \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\alpha-\delta}} > 0$ .

The piece-wise functions  $E^*(T)$  and  $E^*(W)$  follow directly from  $P^*$  and equations (13) and (17), respectively.

## Appendix E. Proof of Proposition 13.

If  $\delta \leq \frac{2}{\beta+1}\alpha$ , then from proposition (9) it follows that  $E(\Pi_I)$  is decreasing in information cost  $i$  in the interior equilibrium. Moreover, since  $\delta < \alpha$ , the interior

path is described by  $\tilde{P}_{|\delta < \alpha}$ , where  $\tilde{P}_{|\delta < \alpha}$  is given by equation (44).

The declining profits along the equilibrium path imply that as  $i \rightarrow 0$ ,  $\tilde{P} \rightarrow \infty$ , hence the constraint that  $P = \min \{ \tilde{P}, 1 \}$  is binding. Let  $\hat{i}$  denote the threshold level of information costs, at which  $\tilde{P}_{|\delta < \alpha} = 1$ . This corresponds to the threshold given by equation (45).

Further, let  $\hat{i}$  denote the threshold level of information costs at which  $E(\Pi_I) = 0$ . While unconstrained profit is decreasing with  $i$ , constrained profit  $E(\Pi_I)|_{P=1}$  is increasing for low values of  $i$  (hence, expected profit is non-monotonic with information cost  $i$ . Case D under convex network-building costs is analogous to Equilibrium B described in proposition (6) under the linear cost specification).

Let  $\hat{i}_{|P=1}$  solve  $E(\Pi_I)|_{P=1} = 0$  and  $\hat{i}_{|P=\tilde{P}}$  solve  $E(\Pi_I)|_{P=\tilde{P}} = 0$ . It follows from  $\delta \leq \frac{2}{\beta+1}\alpha$  and the definition of  $\hat{i}$  that  $\hat{i}_{|P=1} < \hat{i} < \hat{i}_{|P=\tilde{P}}$ . Thus,  $E(\Pi_I)$  is non-negative between these thresholds. Hence, the intermediary is inactive for low levels of information cost  $i \leq \hat{i}_{|P=1}$  and also for  $i \geq \hat{i}_{|P=\tilde{P}}$ .

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min \{ \hat{i}_{|P=1}, 1 \} \\ 1 & \text{if } \min \{ \hat{i}_{|P=1}, 1 \} \leq i \leq \min \{ \hat{i}, 1 \} \\ \left[ \frac{S}{\gamma(\beta+1)i^{\alpha-\delta}} \right]^{\frac{1}{\beta-1}} & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{i}_{|P=\tilde{P}}, 1 \} \\ 0 & \text{if } \min \{ \hat{i}_{|P=\tilde{P}}, 1 \} \leq i \leq 1 \end{cases}$$

The piece-wise functions  $E^*(T)$  and  $E^*(W)$  follow directly from  $P^*$  and equations (13) and (17), respectively.

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