CENTRE *for* ECONOMIC PERFORMANCE

CEP Discussion Paper No 822

August 2007

Search Frictions, Real Rigidities and Inflation Dynamics

Carlos Thomas

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

Abstract
The standard New Keynesian model suffers from the so-called .macro-micro pricing conflict: in order to match the dynamics of inflation implied by macroeconomic data, the model needs to assume an average duration of price contracts which is much longer than what is observed in micro data. Here I show how departing from the standard model's assumption of a perfectly competitive labor market can help resolve the pricing conflict. I do so by assuming search frictions in the labor market. In this framework, labor becomes firm-specific and marginal cost curves become upward-sloping. This mechanism reduces the slope of the New Keynesian Phillips curve given a frequency of price adjustment. Conversely, given an estimate of this slope, my model implies shorter price durations than the standard model. For a plausible calibration and for different slope values, my model consistently delivers price durations that are roughly half of those implied by the standard model.

Keywords: New Keynesian, macroeconomics, micro data, inflation, search and matching
JEL Classifications: E52, E32, J40

This paper was produced as part of the Centre's Macro Programme. The Centre for Economic Performance is financed by the Economic and Social Research Council.

Acknowledgements
I am very grateful to Kosuke Aoki, Nobu Kiyotaki, Francesco Zanetti and Alwyn Young for their comments and suggestions. All remaining errors are mine.

Carlos Thomas is an Associate of the Macro Programme, Centre for Economic Performance, London School of Economics. Email: cthomas1@lse.ac.uk

Published by
Centre for Economic Performance
London School of Economics and Political Science
Houghton Street
London WC2A 2AE

ISBN 978 0 85328 082 8

1 Introduction

The standard New Keynesian model of the business cycle has recently been subject to the following criticism. The key equation of this model is the so-called New Keynesian Phillips curve, which describes the dynamics of inflation. The slope of this curve, which measures the elasticity of inflation with respect to real marginal costs, is a function of the average duration of price contracts in the model. Given an estimate of the slope parameter, one can then infer the average price duration implied by the model. Most estimates of the slope parameter imply average price durations in the range of 5 to 9 quarters.[1] However, recent studies based on micro data show that, at least in the US, firms change prices as often as 1.5 quarters.[2] Altig et al. (2004) refer to this divergence in estimated average frequencies of price adjustment as the 'micro-macro pricing conflict'.

Some authors, including Woodford (2005) and Altig et al. (2004), have pointed out that the origin of this conflict may be found in the absence of *real rigidities* in the standard New Keynesian model. The concept of real rigidities, as defined by Ball and Romer (1990), refers to those real factors that increase the slope of the firm's real marginal cost curve. Real rigidities have the effect of reducing the size of individual price changes in response to aggregate fluctuations. A firm considering for instance a price reduction anticipates that the price cut will increase demand for its product for the duration of the price contract. If marginal cost curves are increasing in output, then the projected rise in marginal costs leads the firm to choose a smaller price cut than the one initially considered. Once pricing decisions are aggregated, real rigidities have the effect of reducing the slope of the New Keynesian Phillips curve, given an average duration of price contracts. The flip side of the coin is that, given an estimate of the slope parameter, a model with real rigidities leads one to infer a shorter average price duration. Therefore, real rigidities become key to resolving the micro-macro pricing conflict.

In the standard New Keynesian model, labor can be costlessly and instantaneously relocated among firms in a perfectly competitive market. This implies that the marginal cost of labor is given simply by the market hourly wage, which is independent of the firm's output. Therefore, the assumption of a perfectly competitive labor market is partly responsible for the lack of real rigidities in the New Keynesian model.

This paper shows how departing from the assumption of a perfectly competitive labor market can introduce real rigidities in the New Keynesian model and thus help resolve the micro-macro pricing conflict. I do so by assuming that the labor market is subject to search

[1] See e.g. Gali and Gertler (1999), Altig et al. (2004) and Eichenbaum and Fisher (2004).

[2] See Bils and Klenow (2004) and Klenow and Kryvtsov (2004).

and matching frictions, as in the framework popularized by Pissarides (2000, ch. 1). Search frictions imply that it is costly and time-consuming for firms to find suitable workers. In this context, employment relationships have a long-term nature, i.e. labor becomes firm-specific. Because it takes time to hire new workers, the only way a firm can expand production in the short-run is by increasing the number of hours worked by its current employees. However, Nash wage bargaining between firms and workers implies that the latter must be compensated for the labor disutility suffered in each period. Under the realistic assumption that labor disutility is convex in hours worked, short-run marginal costs become increasing in hours per employee and therefore in output.[3] That is, search frictions give rise to real rigidities.

I then quantify the extent to which this mechanism contributes to reconciling the model average frequency of price adjustment with the frequency observed in micro data. I show that, for different estimates of the slope parameter that have been provided by the empirical literature, the model with search frictions consistently delivers average price durations that are roughly half of those implied by the standard model. Therefore, search frictions and the resulting firm-specificity of labor prove helpful in bringing the model closer to the data in terms of the average frequency of price changes.

This paper contributes to the existing literature in two ways. On the one hand, Woodford (2005), Altig et al. (2004) and Eichenbaum and Fisher (2005) have shown how departing from the assumption of a perfectly competitive rental market for homogenous capital can create real rigidities in the New Keynesian model. In particular, these authors assume that capital is firm-specific, implying that a firm's capital stock is predetermined and can only by changed by varying the rate of investment. However, Altig et al. (2004) acknowledge that assuming that the firm's entire stock of capital is predetermined is probably unrealistic, and that firm-specificity of some other factor of production may be required. Here I show how firm-specificity in labor gives rise to a new source of real rigidity.[4]

On the other hand, recent years have seen an explosion in research on models that combine the New Keynesian and search and matching frameworks. Most of this literature however assumes that pricing decisions are made by a subset of firms that are *not* subject to search frictions, and that vacancy-posting decisions are made by firms that do *not* set prices.[5] Since both

[3] A marginal cost that increases with hours per employee has a real-world counterpart in those arrangements that make it more and more costly for firms to raise work hours above normal levels, such as overtime premia. See e.g. Hall (1980) and Bils (1987).

[4] Notice that firm-specific labor is different from the kind of *industry*-specific labor markets considered by Woodford (2003, 2005). In the latter case, the labor market in each industry is still perfectly competitive, and workers can still be costlessly and instantaneously relocated among firms in the same industry.

[5] See e.g. Walsh (2003b), Trigari (2005), Christoffel and Linzert (2005), Andres et al. (2006) and Thomas (2007). Examples of models that do not resort to this assumption are Krause and Lubik (2005) and Blanchard

decisions are forward-looking, disentangling them allows to simplify the models considerably. This assumption however is not innocuous. As I show here, if firms (realistically) make *both* pricing and vacancy-posting decisions, the resulting interaction gives rise to real rigidities. I also show how a standard New Keynesian Phillips curve can still be derived in this framework, using a method similar to the one developed by Woodford (2005) in his model of firm-specific capital.

In independent work, Kuester (2007) identifies a real rigidity mechanism which is similar to the one presented here. His framework features firm-worker pairs where nominal wages as well as prices are bargained in a staggered fashion. This creates a source of real rigidity in wage bargaining which works in the same way as the real rigidity in price bargaining. This allows him to increase the sluggishness of both inflation and real wages in response to macroeconomic shocks. Our papers also differ in focus. Kuester evaluates the ability of his model to match the impulse responses of macro variables, including unemployment and vacancies, to monetary shocks as identified by a structural VAR. Here, I focus on how search frictions can bring the average duration of price contracts implied by macro estimates of the New Keynesian Phillips curve closer to the micro data.

The remainder of the chapter is organized as follows. Section 3.2 presents the model. Section 3.3 analyzes individual price setting when firms make both pricing and hiring decisions. Section 3.4 analyzes the presence of real rigidities in the model and how this affects inflation dynamics. Section 3.5 calibrates the model and quantifies the extent to which search frictions contribute to resolving the macro-micro pricing conflict. Section 3.6 concludes.

2 The model

I now present a New Keynesian model with search and matching frictions in the labor market. The model therefore brings together two frameworks that have become the standard for analyzing the monetary transmission mechanism and the cyclical behavior of the labor market, respectively. Relative to a standard real business cycle (RBC) model with perfectly competitive labor markets, the New Keynesian elements are monopolistic competition and staggered price-setting on the part of firms, whereas search frictions in the labor market are represented by a matching function that constraints the ability of unemployed workers and vacant jobs to

and Gali (2006). The former uses quadratic costs of adjusting prices, rather than staggered price-setting. Such a model does not allow one to address the 'macro-micro pricing conflict', because all firms change prices in every period. Blanchard and Gali (2006) use staggered price setting, but do not analyze the presence of real rigidities in such a framework.

be matched to each other.

2.1 The matching function

The search frictions in the labor market are summarized by a matching function,

$$m(v_t, u_t),$$

where v_t is the total number of vacancies and u_t is the total number of unemployed workers. Normalizing the labor force to 1, u_t also represents the unemployment rate. The function m is strictly increasing and strictly concave in both arguments. I assume constant returns to scale in the matching function.[6] The matching rate for unemployed workers, or *job-finding rate*, is given by

$$\frac{m(v_t, u_t)}{u_t} = m\left(\frac{v_t}{u_t}, 1\right) \equiv p(\theta_t),$$

where $\theta_t \equiv v_t/u_t$ is an indicator of labor market *tightness*. Similarly, the matching rate for vacancies is given by

$$\frac{m(v_t, u_t)}{v_t} = m\left(1, \frac{1}{v_t/u_t}\right) \equiv q(\theta_t).$$

The functions $p(\theta_t)$ and $q(\theta_t)$ are increasing and decreasing in θ_t, respectively: in a tighter labor market, it is easier to find jobs and harder to find workers. Notice that $p(\theta_t) = \theta_t q(\theta_t)$.

2.2 Households

In the presence of unemployment risk, we may observe differences in consumption levels between employed and unemployed consumers. However, under the assumption of perfect insurance markets, consumption is equalized across consumers. This is equivalent to assuming the existence of a large representative household, as in Merz (1995). In this household, a fraction n_t of its members are employed in a measure-one continuum of firms. The remaining fraction $u_t = 1 - n_t$ search for jobs. All members pool their income so as to ensure equal consumption across members.

Household welfare is given by

$$H_t = u(c_t) - \int_0^1 n_t(i) v(h_t(i)) di + \beta H_{t+1} \tag{1}$$

[6] See Petrongolo and Pissarides (2001) for empirical evidence of constant returns to scale in the matching function for several industrialized economies.

where $u(\cdot)$ is strictly increasing and strictly concave, $v(\cdot)$ is strictly increasing and strictly convex, $n_t(i)$ and $h_t(i)$ represent the number of workers and hours per worker respectively in firm $i \in [0,1]$, and c_t is the Dixit-Stiglitz consumption basket,

$$c_t \equiv \left(\int_0^1 c_t(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}},$$

where $\gamma > 1$ measures the elasticity of substitution across differentiated goods. Cost minimization implies that the nominal cost of consumption is given by $P_t c_t$, where

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

is the corresponding price index. The household's budget constraint is given by

$$\frac{M_{t-1} + (1+i_{t-1})B_{t-1} + T_t}{P_t} + \int_0^1 n_t(i) w_t(i) di + \Pi_t \geq c_t + \frac{B_t + M_t}{P_t}, \tag{2}$$

where M_{t-1} and B_{t-1} are holdings of money and one-period nominal bonds, respectively, i_{t-1} is the nominal interest rate, T_t is a cash transfer from the government (which may be negative), $w_t(i)$ is the real wage paid by firm i and Π_t are aggregate real profits (which are reverted to households in a lump-sum manner).

Employed members separate from their jobs at the exogenous rate λ, whereas unemployed members find jobs at the rate $p(\theta_t)$. Therefore, the household's employment rate evolves according to the following law of motion,

$$n_{t+1} = (1-\lambda) n_t + p(\theta_t)(1-n_t). \tag{3}$$

It is useful at this point to find the utility that the marginal worker in firm i contributes to the household. Equations (1), (2) and (3) imply that

$$\frac{\partial H_t}{\partial n_t(i)} = u'(c_t) w_t(i) - v(h_t(i)) + \beta E_t \left[(1-\lambda) \frac{\partial H_{t+1}}{\partial n_{t+1}(i)} - p(\theta_t) \int_0^1 \frac{\partial H_{t+1}}{\partial n_{t+1}(j)} \frac{v_t(j)}{v_t} dj \right], \tag{4}$$

where $p(\theta_t)\frac{v_t(j)}{v_t}$ is the probability of being matched to firm j. The right hand side of equation (4) consists of the utility value of the real wage net of labor disutility, plus the continuation value of the job in firm i, minus the value of searching for other jobs.

I assume the existence of a standard cash-in-advance (CIA) constraint on the purchase of consumption goods. Assuming that goods markets open after the closing of financial markets, the household's nominal expenditure in consumption cannot exceed the amount of cash left after bond transactions have taken place,

$$P_t c_t \leq M_{t-1} + T_t - B_t. \tag{5}$$

Cash transfers are given by $T_t = M_t^s - M_{t-1}^s$, where M_t^s is exogenous money supply. The growth rate in money supply, $u_t \equiv \log(M_t^s / M_{t-1}^s)$, follows an autoregressive process,

$$u_t = \rho_m u_{t-1} + \varepsilon_t^m, \tag{6}$$

where ε_t^m is an iid monetary shock with standard deviation σ_m.

Assuming that the nominal interest rate (i.e. the opportunity cost of holding money) is always positive, equation (5) holds with equality. In equilibrium, money demand equals money supply, $M_t = M_t^s$, which implies $M_{t-1} + T_t = M_t$. Combining this with (5) and the fact that bonds are in zero net supply ($B_t = 0$), I obtain

$$P_t c_t = M_t. \tag{7}$$

2.3 Firms

Profits in firm $i \in [0, 1]$ are given by

$$\Pi_t(i) = \frac{P_t(i)}{P_t} y_t^d(i) - w_t(i) n_t(i) - \chi v_t(i) + E_t \beta_{t,t+1} \Pi_{t+1}(i), \tag{8}$$

where $P_t(i)$ and $y_t^d(i)$ are the firm's nominal price and sales, respectively, $v_t(i)$ are vacancies posted in period t and $\beta_{t,T} \equiv \beta^{T-t} \frac{u'(c_T)}{u'(c_t)}$ is the stochastic discount factor between periods t and T. Due to imperfect substitutability among individual goods, the firm faces the following demand curve for its product,

$$y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\gamma} y_t, \tag{9}$$

where aggregate demand is given by

$$y_t = c_t + \chi v_t. \tag{10}$$

Once the firm has chosen a price, it commits to supply whichever amount is demanded at that price, $y_t^s(i) = y_t^d(i)$. The firm's production technology is given by

$$y_t^s(i) = A_t n_t(i) h_t(i),$$

where $n_t(i)h_t(i)$ is total labor input. A_t is an exogenous aggregate technology shock, the log of which, $a_t \equiv \ln A_t$, follows an autoregressive process,

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{11}$$

where ε_t^a is an iid shock with standard deviation σ_a.

In each period, the individual firm loses a fraction λ of its workers and posts a number $v_t(i)$ of vacancies. Assuming that firms are large, each firm fills a fraction $q(\theta_t)$ of vacancies with certainty. New hires become productive in the following period. This reflects the time involved in searching for suitable workers and training them. Therefore, a firm's workforce $n_t(i)$ is given at the start of period t, and has the following law of motion,

$$n_{t+1}(i) = (1 - \lambda)n_t(i) + q(\theta_t)v_t(i). \tag{12}$$

Notice that, since $n_t(i)$ is predetermined and the firm is demand-constrained, in the short run the firm has to adjust hours per worker so as to provide the required amount of output. Using the firm's production function, hours per worker are given by

$$h_t(i) = \frac{y_t^d(i)}{A_t n_t(i)}. \tag{13}$$

I assume the firm has discretion over the labor effort that its workers must provide. However, workers must be compensated for this effort according to a wage schedule agreed by firm and workers. The derivation of this wage schedule is presented next.

2.3.1 Wage bargaining

As is standard in the search and matching literature, I assume that wages are determined by the Nash bargaining solution. This requires defining the surplus value derived by both employer and employee from their employment relationship. The worker surplus in units of consumption, which I denote by $S_t^w(i) \equiv \frac{\partial H_t/\partial n_t(i)}{u'(c_t)}$, is given by equation (4) divided by $u'(c_t)$, that is,

$$S_t^w(i) = w_t(i) - \underline{w}_t(i) + (1-\lambda)E_t\beta_{t,t+1}S_{t+1}^w(i),$$

where

$$\underline{w}_t(i) \equiv \frac{v(h_t(i))}{u'(c_t)} + p(\theta_t)E_t\beta_{t,t+1}S_{t+1}^w \tag{14}$$

and $S_{t+1}^w \equiv \int S_{t+1}^w(i)\frac{v_t(j)}{v_t}dj$ is average worker surplus in period $t+1$. The term $\underline{w}_t(i)$ represents the opportunity cost to the worker, and includes labor disutility and the value of searching for other jobs. On the firm's side, since all workers are the same and therefore contribute equally to the firm's revenue, the firm derives the following surplus from each worker,

$$S_t^f(i) = \frac{P_t(i)}{P_t}\frac{y_t^d(i)}{n_t(i)} - w_t(i) + (1-\lambda)E_t\beta_{t,t+1}S_{t+1}^f(i).$$

The firm surplus equals the worker's contribution to current profits plus what the worker is expected to contribute in the future should she remain in the firm (which happens with probability $1-\lambda$). Letting ξ denote the firm's bargaining power. Nash wage bargaining implies that the firm receives a fraction ξ of the joint match surplus, $S_t^f(i) + S_t^w(i)$, that is,

$$(1-\xi)S_t^f(i) = \xi S_t^w(i).$$

Combining the latter with the expressions for worker and firm surplus, I obtain the following real wage equation,

$$w_t(i) = (1-\xi)\frac{P_t(i)}{P_t}\frac{y_t^d(i)}{n_t(i)} + \xi\underline{w}_t(i), \tag{15}$$

Using the definition of $\underline{w}_t(i)$, equation (14), we can express the real wage as

$$w_t(i) = (1-\xi)\frac{P_t(i)}{P_t}\frac{y_t^d(i)}{n_t(i)} + \xi\left[\frac{v(h_t(i))}{u'(c_t)} + p(\theta_t)E_t\beta_{t,t+1}S_{t+1}^w\right]. \tag{16}$$

The real wage is a weighted average of the worker's contribution to revenue and the opportunity cost to the worker. Notice that the worker is (partially) compensated for the incurred labor disutility, $\frac{v(h_t(i))}{u'(c_t)}$. The latter is convex in $h_t(i)$, which implies that the firm finds it more and more expensive to increase output by increasing hours per worker. This could represent real-world arrangements, such as overtime premia, which are designed to make firms respect the value of workers' time (see e.g. Hall, 1980, and Bils, 1987). Due to the cost and time involved in hiring workers, overtime may be a reasonable way to expand production in the short run.

However, in prolonged periods of high demand the firm must eventually hire workers.

2.3.2 Vacancy posting decision

The firm chooses the number of vacancies $v_t(i)$ that maximizes profits (given by equation 8) subject to equation (12). This yields the following first order condition,

$$\chi = q(\theta_t) E_t \beta_{t,t+1} \frac{\partial \Pi_{t+1}(i)}{\partial n_{t+1}(i)}. \tag{17}$$

The contribution of the marginal worker to profits is given by

$$\frac{\partial \Pi_t(i)}{\partial n_t(i)} = \left(-\frac{\partial w_t(i)}{\partial n_t(i)} \right) n_t(i) - w_t(i) + (1-\lambda) E_t \beta_{t,t+1} \frac{\partial \Pi_{t+1}(i)}{\partial n_{t+1}(i)}. \tag{18}$$

Notice in particular that, in a context of monopolistic competition and infrequent price adjustment, once the firm has set a price its revenue is independent of $n_t(i)$. Therefore, the contribution of the marginal worker to flow profits is given, not by the marginal revenue product of the worker (as in standard RBC models), but by the reduction in the wage bill.[7] From equations (16) and (13), such reduction is given by

$$\left(-\frac{\partial w_t(i)}{\partial n_t(i)} \right) n_t(i) = (1-\xi) \frac{P_t(i)}{P_t} \frac{y_t^d(i)}{n_t(i)} + \xi \frac{v'(h_t(i))}{u'(c_t)} h_t(i). \tag{19}$$

That is, an additional worker reduces the wage bill in two ways: first, the same revenue must be shared among more workers; second, the same level of output can be produced with a smaller number of hours per worker, which in turn reduces the real wage to be paid to each worker. Using (16), (17), (18) and (19) the firm's vacancy posting decision becomes

$$\frac{\chi}{q(\theta_t)} = E_t \beta_{t,t+1} \left\{ \xi \left[\frac{v'(h_{t+1}(i)) h_{t+1}(i) - v(h_{t+1}(i))}{u'(c_{t+1})} - p(\theta_{t+1}) \beta_{t+1,t+2} S_{t+2}^w \right] + (1-\lambda) \frac{\chi}{q(\theta_{t+1})} \right\}. \tag{20}$$

According to equation (20), firms' incentives to hire are driven by fluctuations in the term $\frac{v'(h_t(i))h_t(i) - v(h_t(i))}{u'(c_t)}$, which represents the gap between marginal and average labor disutility (in terms of consumption) multiplied by hours per worker. The convexity of $v(h)$ implies that $v'(h)h - v(h)$ is positive and strictly increasing in h. Therefore, as hours increase, firms have a stronger incentive to hire more workers in order to prevent labor costs from increasing too

[7] This result is analogous to the one in Woodford's (2005) model of firm-specific capital, where the marginal contribution of capital to flow profits is given by the marginal reduction in the wage bill, rather than the marginal revenue product of capital.

much.

2.3.3 Pricing decision

As is standard in the New Keynesian literature, I use the Calvo (1983) model of staggered price setting. Each period, a randomly selected fraction δ of firms cannot change their price. Therefore, δ also represents the probability that a firm is not able to change its price in the following period. This allows me to write the part of the firm's profits that depends on its current price as

$$E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{t,T} \xi \left[\left(\frac{P_t(i)}{P_T} \right)^{1-\gamma} y_T - n_T(i) \frac{v\left(\left(\frac{P_t(i)}{P_T} \right)^{-\gamma} \frac{y_T}{A_T n_T(i)} \right)}{u'(c_T)} \right], \tag{21}$$

where I have used equation (16) to substitute for the real wage in (8), and I have used equations (9) and (13) to write demand and hours per worker in terms of the current price, $P_t(i)$. When a firm has the chance to reset its price, it chooses $P_t(i)$ so as to maximize (21). The first order condition is given by the standard pricing equation in the Calvo model,

$$E_t \sum_{T=t}^{\infty} \delta^{T-t} \beta_{t,T} P_T^{\gamma} y_T \left[\frac{P_t^*(i)}{P_T} - \frac{\gamma}{\gamma - 1} mc_{T,t}(i) \right] = 0, \tag{22}$$

where $P_t^*(i)$ is the pricing decision and

$$mc_{T|t}(i) \equiv \frac{v'\left(\left(\frac{P_t^*(i)}{P_T} \right)^{-\gamma} \frac{y_T}{A_T n_{T|t}(i)} \right)}{u'(c_T) A_T} \tag{23}$$

is the firm's real marginal cost in period T, conditional on the firm not having changed its price since period t (similarly for $n_{T|t}(i)$). Therefore, real marginal costs are given by the ratio between the marginal rate of substitution between consumption and leisure, $\frac{v'(h_T)}{u'(c_T)}$, and the marginal product of labor, A_T. Notice that the firm's pricing decision does not depend on its bargaining power, ξ. This is because, under Nash wage bargaining, firm surplus is proportional to the joint surplus of the employment relationship, which is independent of ξ.

3 Relative dynamics of the firm

In what follows, I consider a first order approximation of the equilibrium conditions around a zero-inflation steady state. For any variable $e_t(i)$, let $\hat{e}_t(i) \equiv \log(e_t(i)/e)$ denote its log-deviation from its steady state value, e. Also let $\tilde{e}_t(i) \equiv \hat{e}_t(i) - \hat{e}_t$ denote the value of $\hat{e}_t(i)$ relative to its cross-sectional average, $\hat{e}_t \equiv \int_0^1 \hat{e}_t(i)di$. I assume the following functional forms for preferences over consumption and labor, as well as the matching function,

$$u(c) = \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}},$$

$$v(h) = \frac{h^{1+\eta}}{1+\eta},$$

$$m(v,u) = v^{\epsilon}u^{1-\epsilon},$$

where $\sigma, \eta > 0$ and $\epsilon \in (0,1)$. Therefore, η represents the convexity of labor disutility.

Log-linearization of the firm's pricing decision, equation (22), yields

$$\log P_t^*(i) = (1-\delta\beta)E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \left[\widehat{mc}_{T|t}(i) + \log P_T\right]. \tag{24}$$

Equation (23) and our functional forms imply that the real marginal cost in period $T \geq t$ of a firm that has not changed its price since period t can be expressed as

$$\widehat{mc}_{T|t}(i) = \widehat{mc}_T - \eta\gamma(\log P_t^*(i) - \log P_T) - \eta\tilde{n}_{T|t}(i), \tag{25}$$

where

$$\widehat{mc}_T = \eta\hat{y}_T + \sigma^{-1}\hat{c}_T - (1+\eta)a_T - \eta\hat{n}_T \tag{26}$$

is the average real marginal cost. Notice that a firm's relative marginal cost is decreasing in its relative stock of workers, $\tilde{n}_{T|t}(i)$. Having more workers allows the firm to produce a certain amount of output with a smaller number of hours per worker, which reduces the marginal labor disutility of its workers and therefore its marginal costs. I now combine (24) and (25) to obtain

$$(1+\eta\gamma)\log P_t^*(i) = (1-\delta\beta)E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \left[\widehat{mc}_T + (1+\eta\gamma)\log P_T - \eta\tilde{n}_{T|t}(i)\right]. \tag{27}$$

This expression for a firm's pricing decision is very similar to the one produced by a standard New Keynesian model (see e.g. Walsh 2003a, chap. 3). The only difference is the presence of the $E_t\tilde{n}_{T|t}(i)$ terms, which reflects the fact that a firm's marginal cost is decreasing in its relative number of workers. These additional terms complicate the analysis in the following way. In order to determine $\log P_t^*(i)$, we need to solve for the expected path of $\tilde{n}_{T|t}(i)$. The latter however depends on the firm's current and future expected vacancy posting decisions, which in turn depend on the price chosen today. Solving for the firm's pricing decision therefore requires that one considers the effect of a firm's relative price on the evolution of its relative employment stock.

In what follows, I adapt Woodford's (2005) solution method for a firm's relative dynamics to the present context.[8] Notice that, in a log-linear approximation, the firm's pricing decision is a linear function of the state of the economy and its individual state, $\hat{n}_t(i)$. On the other hand, since price-setters are randomly chosen, their average employment stock coincides with the economy-wide average employment stock. Therefore, it is plausible to guess that a firm's pricing decision, relative to the average pricing decision, is proportional to its relative employment stock,

$$\log P_t^*(i) = \log P_t^* - \tau^* \tilde{n}_t(i). \tag{28}$$

I now log-linearize the vacancy posting decision, equation (20), and rescale the resulting expression by $\frac{y}{n}$ to obtain

$$\frac{s_v}{\lambda}(1-\epsilon)\hat{\theta}_t = \beta E_t \xi \left[\frac{1}{\mu} \left(\eta \hat{h}_{t+1}(i) + \frac{\eta \sigma^{-1}}{1+\eta} \hat{c}_{t+1} \right) - p(\theta)\beta S_y^w \left[\epsilon \hat{\theta}_{t+1} + \hat{\beta}_{t+1,t+2} + \hat{S}_{t+2}^w \right] \right] + \frac{s_v}{\lambda} E_t \hat{\beta}_{t,t+1} + \beta(1-\lambda)\frac{s_v}{\lambda}(1-\epsilon)E_t\hat{\theta}_{t+1}, \tag{29}$$

where $\mu \equiv \frac{\gamma}{\gamma-1}$ is the monopolistic mark-up, $s_v \equiv \frac{\chi v}{y}$ is the steady-state share of vacancy posting costs in GDP, and $S_y^w \equiv \frac{nS^w}{y}$.[9] Notice that $\hat{h}_{t+1}(i)$ is the only idiosyncratic term in equation (29). $\hat{h}_{t+1}(i)$ will depend on the firm's demand in $t+1$ (which in turn depends on its price in $t+1$) as well as on its stock of workers at the beginning of $t+1$. It is now possible to obtain the following result.[10]

Proposition 1 *Let relative pricing decisions be given by equation (28), up to a log-linear ap-*

[8] Woodford (2005) uses his method in a model with where capital, rather than labor, is firm-specific.

[9] In the derivation of equation (29), I have used the fact that, in the steady state, $mc = \frac{v'(h)}{u'(c)}\frac{1}{A} = \mu^{-1}$. Since $A = \frac{y}{nh}$, it follows that $\frac{v'(h)}{u'(c)}h = \frac{1}{\mu}\frac{y}{n}$. I have also used the law of motion of employment in the steady state, $q(\theta)v = \lambda n$.

[10] The proofs of all propositions are in the Appendix.

proximation. Then the relative employment stock of any firm evolves according to

$$\tilde{n}_{t+1}(i) = -\tau^n \left(\log P_t(i) - \log P_t\right), \tag{30}$$

where

$$\tau^n = \frac{\gamma\delta}{1-\gamma(1-\delta)\tau^*}. \tag{31}$$

Intuitively, firms with a higher price in the current period also expect to have a higher price in the next period, which means that they also expect lower demand. Anticipating this, these firms post a number of vacancies that leaves them with a smaller workforce than the average firm in the next period.

Proposition 1 allows me to write the expected path of a price-setter's relative employment stock in the following way,

$$\begin{aligned} E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \tilde{n}_{T|t}(i) &= \tilde{n}_t(i) + \delta\beta E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \tilde{n}_{T+1|t}(i) \\ &= \tilde{n}_t(i) - \delta\beta\tau^n E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \left(\log P_t^*(i) - \log P_T\right). \end{aligned} \tag{32}$$

Using expression (32) in equation (27), I can write the firm's pricing decision as

$$(1+\phi)\log P_t^*(i) = (1-\delta\beta)E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \left[\widehat{mc}_T + (1+\phi)\log P_T\right] - (1-\delta\beta)\eta\tilde{n}_t(i), \tag{33}$$

where $\phi \equiv \eta\gamma - \delta\beta\eta\tau^n$. Equation (33) is the solution to the firm's pricing decision, given its individual state, $\tilde{n}_t(i)$, and the state of the economy. Averaging (33) across price-setters, and using the fact that the latter are randomly chosen, I obtain

$$(1+\phi)\log P_t^* = (1-\delta\beta)E_t \sum_{T=t}^{\infty} (\delta\beta)^{T-t} \left[\widehat{mc}_T + (1+\phi)\log P_T\right]. \tag{34}$$

Substracting (34) from (33) yields

$$(1+\phi)(\log P_t^*(i) - \log P_t^*) = -(1-\delta\beta)\eta\tilde{n}_t(i).$$

This is consistent with my initial guess, equation (28), only if

$$\tau^* = \frac{(1-\delta\beta)\eta}{1+\eta\gamma-\delta\beta\eta\tau^n}. \tag{35}$$

Therefore, if relative pricing decisions and relative employment stocks are to have a solution, the latter is given by equations (28) and (30), respectively, where the parameters τ^* and τ^n must satisfy equations (31) and (35). I now analyze whether such a solution exists.

3.1 Existence of solution

Using (31) to substitute for τ^n in (35), I obtain the following equation for τ^*,

$$\tau^* = \frac{(1-\delta\beta)\eta}{1+\eta\gamma-\delta\beta\eta\left(\frac{\gamma\delta}{1-\gamma(1-\delta)\tau^*}\right)}.$$

This can be written as

$$a(\tau^*)^2 + b\tau^* + c = 0, \tag{36}$$

where

$$a \equiv (1+\eta\gamma)\gamma(1-\delta), \tag{37}$$

$$b \equiv -\left[1+\gamma(2-\delta-\delta\beta)\eta\right], \tag{38}$$

$$c \equiv (1-\delta\beta)\eta. \tag{39}$$

The quadratic equation (36) has two solutions. The latter are real numbers if and only if $b^2 - 4ac > 0$. The following result establishes that this is indeed the case.

Proposition 2 *Let the parameters a, b and c in equation (36) be given by equations (37), (38) and (39), respectively, where* $\eta > 0$, $\gamma > 1$ *and* $0 < \beta, \delta < 1$. *Then the two solutions of equation (36) are real numbers.*

Equation (36) has therefore two real solutions, given by

$$(\tau_1^*, \tau_2^*) = \left(\frac{-b-\sqrt{b^2-4ac}}{2a}, \frac{-b+\sqrt{b^2-4ac}}{2a}\right).$$

It is also possible to show that the solutions for both τ^* and τ^n have to be positive. To see this, define

$$\tau_1^n(\tau^*) \equiv \frac{1+\eta\gamma-\frac{(1-\delta\beta)\eta}{\tau^*}}{\delta\beta\eta},$$

$$\tau_2^n(\tau^*) \equiv \frac{\gamma\delta}{1-\gamma(1-\delta)\tau^*}.$$

The function $\tau_1^n(\tau^*)$ is obtained by solving for τ_1^n in equation (35). The solutions for τ^n and τ^* are given by the two points of intersection of both functions in (τ^*, τ^n) space. Both functions are increasing in τ^*. For $\tau^* < 0$, $\tau_1^n(\tau^*) > \frac{1+\eta\gamma}{\delta\beta\eta}$ and $\tau_2^n(\tau^*) < \gamma\delta$. Since $\frac{1+\eta\gamma}{\delta\beta\eta} > \gamma\delta$, there can be no solution for $\tau^* < 0$. But if $\tau^* > 0$, then $\tau_2^n(\tau^*) > \gamma\delta > 0$, which implies that τ^n must be positive too.

3.2 Convergence

Equation (36) has two real solutions, τ_1^* and τ_2^*, where $\tau_1^* < \tau_2^*$. However, only τ_1^* implies convergent dynamics. To see this, notice that a firm's relative price and employment stock evolve according to

$$\begin{bmatrix} E_t\tilde{P}_{t+1}(i) \\ \tilde{n}_{t+1}(i) \end{bmatrix} = \begin{bmatrix} \delta + (1-\delta)\tau^*\tau^n & 0 \\ -\tau^n & 0 \end{bmatrix} \begin{bmatrix} \tilde{P}_t(i) \\ \tilde{n}_t(i) \end{bmatrix}.$$

This system implies convergent dynamics only if the eigenvalues of the matrix are inside the unit circle. These eigenvalues are 0 and $\delta + (1-\delta)\tau^*\tau^n$. Since $\delta + (1-\delta)\tau^*\tau^n > 0$ (as a result of both τ^* and τ^n being positive), a non-explosive solution must satisfy $\delta + (1-\delta)\tau^*\tau^n < 1$, or simply

$$\tau^*\tau^n < 1.$$

Using equation (31), this requires in turn

$$\tau^* < \frac{1}{\gamma}. \tag{40}$$

Define

$$F(\tau^*) \equiv a(\tau^*)^2 + b\tau^* + c,$$

where a, b and c are given by equations (37), (38) and (39), respectively. Since $F(\tau^*)$ is a convex function, it follows that $F(\tau^*) < 0 \Leftrightarrow \tau^* \in (\tau_1^*, \tau_2^*)$, where τ_1^*, τ_2^* are the two roots of $F(\tau^*)$. Evaluating $F(\cdot)$ at $\frac{1}{\gamma}$, I obtain

$$\begin{aligned} F\left(\frac{1}{\gamma}\right) &= (1+\eta\gamma)\frac{1}{\gamma}(1-\delta) - \left[\frac{1}{\gamma} + (2-\delta-\delta\beta)\eta\right] + (1-\delta\beta)\eta \\ &= -\frac{\delta}{\gamma} < 0. \end{aligned}$$

It follows that $\tau_1^* < \frac{1}{\gamma} < \tau_2^*$, which means that τ_2^* violates (40) and therefore implies explosive dynamics. As emphasized by Woodford (2005), in order for a log-linear approximation around the steady-state to be an accurate approximation of the model's exact equilibrium conditions, the dynamics of firms' relative prices and employment stocks must remain forever near enough to the steady state. Since τ_2^* violates this condition, from now onwards I will set $\tau^* = \tau_1^*$.

4 Inflation dynamics and real rigidities

The average pricing decision, equation (34), can be written as

$$(1+\phi)\log P_t^* = (1-\delta\beta)\left[\widehat{mc}_t + (1+\phi)\log P_t\right] + \delta\beta E_t(1+\phi)\log P_{t+1}^*, \tag{41}$$

The Calvo model of staggered price-setting implies the following law of motion for the price level,

$$P_t^{1-\gamma} = \delta P_{t-1}^{1-\gamma} + (1-\delta)\left(P_t^*\right)^{1-\gamma}.$$

This admits the following log-linear approximation,

$$\pi_t = \frac{\delta}{1-\delta}\left(\log P_t^* - \log P_t\right), \tag{42}$$

where $\pi_t \equiv \log(P_t/\log P_{t-1})$ is the inflation rate. Combining (41) and (42), I obtain the familiar *New Keynesian Phillips curve*,

$$\pi_t = \kappa\widehat{mc}_t + \beta E_t\pi_{t+1}, \tag{43}$$

where

$$\kappa \equiv \frac{(1-\delta\beta)(1-\delta)}{\delta}\frac{1}{1+\phi}, \tag{44}$$

$$\phi \equiv \eta\gamma - \delta\beta\eta\tau^n \tag{45}$$

and average real marginal costs, $\widehat{mc}_t$, are given by equation (26).

The parameter ϕ has two components, $\eta\gamma$ and $\delta\beta\eta\tau^n$. The term $\eta\gamma$ reflects the fact that labor disutility, and hence the real wage, are convex functions of hours worked ($\eta > 0$). In other words, the *marginal* real wage is increasing in hours. Since hours are increasing in output, the firm's marginal cost curve becomes upward-sloping. In other words, the model displays *real rigidities* in the sense of Ball and Romer (1990). Real rigidities have the effect of reducing the size of individual price changes in response to the same macroeconomic fluctuations. To see this, take a price-setter that is considering a reduction in its price. For a given overall price level, a reduction in the firm's price increases its sales and therefore, for a given employment stock, the required amount of hours per worker. This increases the firm's marginal costs through the increase in workers' marginal disutility of labor. The anticipated rise in current and future expected real marginal costs leads the firm to choose a smaller price cut than the one initially considered. This mechanism is absent in the case of a frictionless labor market, where firms can hire as much labor as they want at the market hourly wage.

The term $\delta\beta\eta\tau^n$ reflects the fact that the position of a firm's marginal cost curve depends on its stock of workers, by affecting how many hours per worker are needed to produce a certain amount of output. This term has the effect of accelerating price adjustment. To see this, take the same firm considering a price cut. From Proposition 1, today's price cut leads the firm to expect a larger relative employment stock, and by equation (25) a lower marginal cost in future periods. Holding everything else constant, this would lead the firm to choose an even larger price cut than initially considered.

It is possible to show however that the first effect always dominates the second. Using the definition of τ^n, equation (31), I can write

$$\begin{aligned} \eta\gamma - \delta\beta\eta\tau^n &= \eta\gamma - \delta\beta\eta\left(\frac{\gamma\delta}{1-\gamma(1-\delta)\tau^*}\right) \\ &= \eta\gamma\left(1 - \frac{\delta^2\beta}{1-\gamma(1-\delta)\tau^*}\right). \end{aligned}$$

The latter expression is positive only if the expression in brackets is. Given that τ^* must be smaller than $\frac{1}{\gamma}$ in order for the model to have convergent dynamics, it follows that

$$1 - \frac{\delta^2\beta}{1-\gamma(1-\delta)\tau^*} > 1 - \frac{\delta^2\beta}{1-\gamma(1-\delta)\frac{1}{\gamma}} = 1 - \delta\beta > 0.$$

It follows that the parameter ϕ in expression (44) is strictly positive. Therefore, the real rigidities arising from search frictions and the firm-specificity of labor have the effect of reducing the slope of the New Keynesian Phillips curve.

4.1 Comparison to a standard New Keynesian model

As is well-known, a New Keynesian model with a perfectly competitive labor market produces the following inflation equation,[11]

$$\pi_t = \kappa_{nk} \widehat{mc}_t^{nk} + \beta E_t \pi_{t+1}, \tag{46}$$

where

$$\kappa_{nk} \equiv \frac{(1-\delta\beta)(1-\delta)}{\delta} \tag{47}$$

and

$$\widehat{mc}_t^{nk} = \left(\eta + \sigma^{-1}\right) \hat{c}_t - (1+\eta) a_t \tag{48}$$

are average real marginal costs.[12] Therefore, the inflation equation has the same form in both models. What is different is the slope of the inflation equation, which can be seen by comparing expressions (44) and (47). In the model with search frictions, the presence of real rigidities ($\phi > 0$) reduces the slope of the inflation equation. The flip side of the coin is that, given an econometric estimate of κ, the standard model implies a larger fraction of sticky prices, δ. This in turn implies a longer average duration of price contracts, $\frac{1}{1-\delta}$. As emphasized by Altig et al. (2004) and Woodford (2005), the standard New Keynesian model with perfectly competitive factor markets requires an unrealistically long duration of prices in order to match econometric estimates of κ. It is the contrast between the long price duration needed for the standard model to match the macro evidence and the short price duration found in micro data that Altig et al. (2004) have called the 'macro-micro pricing conflict'. As I have shown, the introduction of search frictions into the New Keynesian model helps resolve this conflict. The next section quantifies the importance of this mechanism.

5 Quantitative analysis

5.1 Calibration

I calibrate the model to US data. As emphasized by the recent literature on the cyclical properties of the search and matching model, a monthly model frequency is better able to

[11] See e.g. Walsh (2003a).

[12] Since there is no vacancy posting in the standard model, output equals consumption, $\hat{y}_t = \hat{c}_t$. Also, since the standard model makes no distinction between employment and hours per employee, real marginal costs do not depend on $\hat{n}_t$.

capture the dynamics of labor market flows in the US than a quarterly one.[13] I therefore assume a monthly frequency for the model.

Following most of the RBC literature, I set the discount factor to 4% per quarter, or $\beta = 0.99^{1/3}$. I also choose standard values for the intertemporal elasticity of substitution, $\sigma = 1$, and the autocorrelation of the technology shock, $\rho_a = 0.95^{1/3}$.

Calibrating the convexity of labor disutility, η, requires more attention. In a standard RBC framework, this parameter represents the inverse of the elasticity of labor supply. This interpretation does not carry over to this context, because it is the firm, not the workers, who sets hours per worker. It is however possible to derive an alternative interpretation for η. Given the wage equation (16) and the form of the labor disutility function, $v(h) = (1+\eta)^{-1}h^{1+\eta}$, the marginal wage is given by

$$\frac{\partial w_t(i)}{\partial h_t(i)} = \xi \frac{h_t^\eta(i)}{u'(c_t)}.$$

Therefore, η represents the elasticity of the marginal wage with respect to hours per worker. Using US manufacturing data, Bils (1987) constructs a measure of marginal wages by assuming an overtime premium of 50% and estimating by how much average overtime hours increase following an increase in average hours per worker. He then estimates an elasticity of marginal wages with respect to hours per worker of 1.39. Therefore, I set η to 1.4.

Regarding the New Keynesian side of the model, following Klenow and Kryvtsov (2004) I assume that firms change prices every 1.5 quarters, or 4.5 months, which implies $\delta = \frac{4.5-1}{4.5} = 0.78$. As in Woodford, I choose a monopolistic mark-up of $\mu = 1.15$, which implies $\gamma = \frac{\mu}{1-\mu} = 7.67$. Following Cooley and Quadrini (1999), the autocorrelation of the monetary shock is set to $\rho_m = 0.49^{1/3}$.

Given the values of η, β, γ and δ, equations (35) and (31) jointly imply $\tau^* = 0.072$ and $\tau^n = 6.80$. From equation (45), the parameter ϕ equals 3.35. From equation (44), the slope of the Phillips curve equals $\kappa = 0.015$. This compares to a slope of $\kappa_{nk} = 0.064$ in the standard New Keynesian model.

The parameters that describe the labor market $(\lambda, p(\theta), \epsilon)$ are calibrated as in Thomas (2007), based also on US data (see Table 1 for details). In the absence of direct evidence on the bargaining power parameter, ξ, I follow most of the literature and set it equal to ϵ, which would guarantee efficiency in the absence of price stickiness and monopolistic competition. It is however important to emphasize that ξ has no effect on the parameter ϕ, and therefore on how the fraction of price stickiness, δ, maps into the slope of the New Keynesian Phillips curve, κ. Finally, the steady-state share of vacancy posting costs in GDP, s_v, is derived from equation

[13] See e.g. Hall (2005), Shimer (2005) and Gertler and Trigari (2006).

	Value	Description		Value	Description
β	$0.99^{1/3}$	discount factor	ξ	0.6	firm's bargaining power
η	1.4	elasticity of marginal wages	ρ_a	$0.95^{1/3}$	AC of technology shock
δ	0.78	fraction of sticky prices	ρ_m	$0.49^{1/3}$	AC of monetary shock
γ	7.67	elasticity of demand curves	τ^*	0.072	elast. of relative pricing decisions
σ	1	intertemporal elast. of subst.	τ^n	6.80	elasticity of relative employment
λ	0.035	job separation rate	ϕ	3.35	(net) real rigidity
$p(\theta)$	0.30	job finding rate	κ	0.015	slope of Phillips curve (PC)
ϵ	0.6	elasticity of matching function	κ_{nk}	0.064	slope of PC, standard model
s_v	0.013	hiring costs/GDP			

Table 1: Parameter values

(20) in the steady state.

5.2 Model response to shocks

The main objective of this paper is to quantify the extent to which search frictions and the resulting real rigidities help resolve the macro-micro pricing conflict, as measured by the divergence between micro estimates of the average frequency of price adjustment and the model frequency implied by macro estimates of the New Keynesian Phillips curve. However, it is also interesting to simulate the economy's response to monetary and technology shocks, in order to assess the plausibility of these responses. I turn to this now.[14]

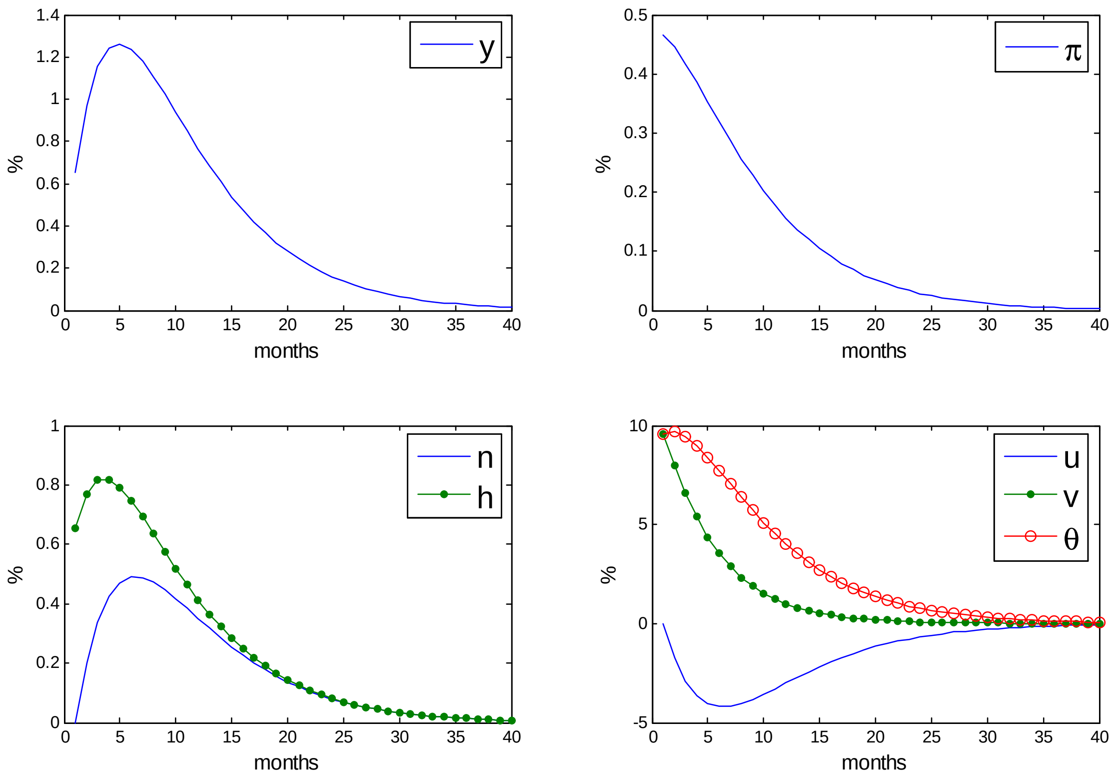
5.2.1 Monetary shocks

Figure 1 plots the economy's response to a 1% shock to money growth. The output response is hump-shaped, with a peak after almost two quarters. Inflation jumps on impact and then decays slowly. This contradicts the sluggish response of inflation to monetary shocks as identified for instance by Christiano et al. (2005). This problem is shared by the standard New Keynesian model, and in both cases the reason is the absence of inherent persistence in the inflation process.

Both labor margins increase following the shock. However, hours respond more strongly than employment. As shown by Trigari (2005), employment is more volatile than hours conditional on monetary shocks. The reason for this counterfactual behaviour may be related to the fact that wages are flexible in this framework. As emphasized by Shimer (2005), period-by-

[14] In order to simulate the response to shocks, the model is log-linearized around a zero inflation steady state. The log-linear equations are described in the Appendix.

Figure 1: Impulse responses to a monetary shock



period Nash wage bargaining tends to mute the response of unemployment to shocks. Since employment is the symmetric of unemployment, this would also explain why employment does not vary much in response to a monetary shock. Therefore, some form of wage rigidity may be needed so as to amplify the effect of monetary shocks on employment. Since hours per worker are given by $\hat{h}_t = \hat{y}_t - \hat{n}_t$ conditional on monetary shocks, a stronger employment response would also weaken the hours response given the same expansion in aggregate demand. Analyzing the role of real wage rigidities in this framework however is beyond the scope of this paper.[15]

5.2.2 Productivity shocks

Figure 2 plots the economy's response to a 1% positive technology shock. The improvement in productivity leads firms to charge lower prices, bringing inflation down. This creates a persistent surge in aggregate demand. However, the increase in aggregate demand is lower than the increase in productivity, which reduces firms' total labor input requirements. These results are consistent with a body of evidence starting with Gali (1999), although the sign of the labor input response to a technology shock remains a matter of controversy in the literature. Once again, the hours response is stronger than the employment response. In this case, the employment response is particularly weak, especially when compared to the response to a monetary shock. Again, this is probably reflecting the flexible nature of wages and the resulting dampening of employment fluctuations.

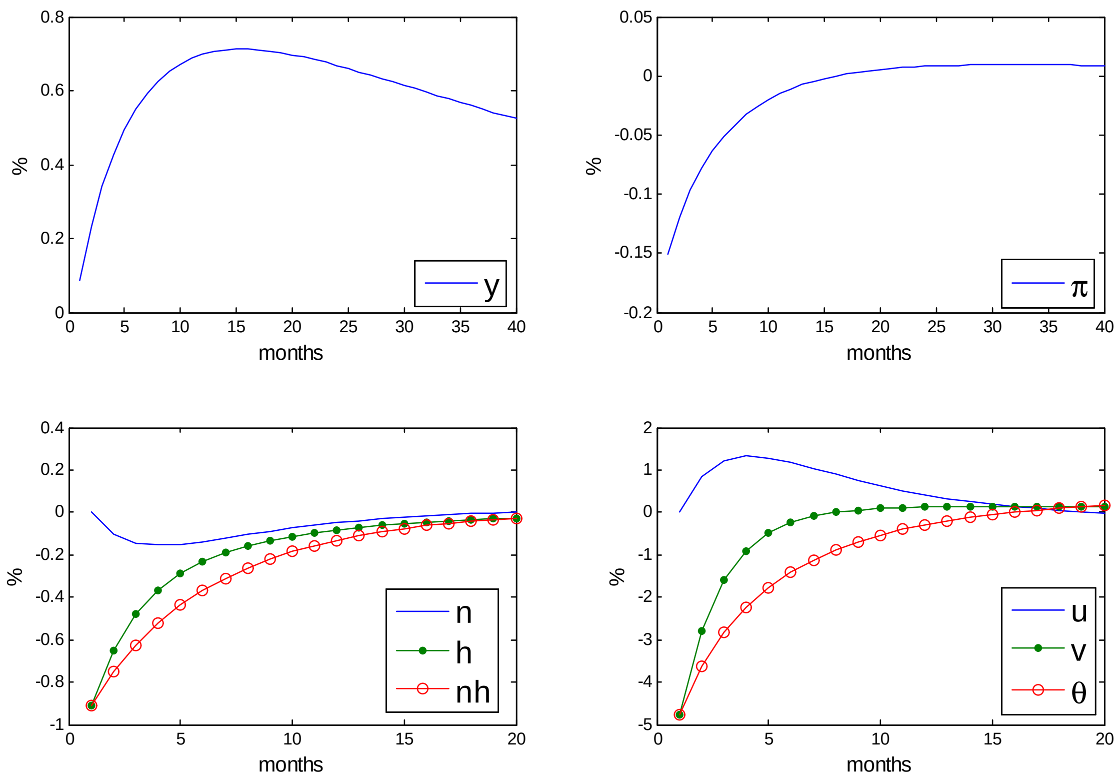
5.3 Inference about the frequency of price adjustment

I now tackle the central issue of this paper, which is how much search frictions contribute to resolving the macro-micro pricing conflict. Following Woodford (2005), I do so by comparing the respective average frequencies of price adjustment implied by the model with search frictions and the standard New Keynesian model, given hypothetical estimates of the slope of the New Keynesian Phillips curve.

Most empirical estimates of the New Keynesian Phillips curve use the (log) labor share of GDP as a proxy for average (log) real marginal costs, which are not observable, exploiting the fact that average real marginal costs in the standard New Keynesian model are indeed equal to the model's labor share of GDP. When one is comparing the average frequency of price adjustment implied by the standard and an alternative model, as long as the alternative model

[15] See Kuester (2007) for an analysis of how the interaction of staggered nominal wage bargaining and real rigidity in wage bargaining gives rise to real wage rigidity, and how this helps increase the volatility of the extensive margin of labor.

Figure 2: Impulse responses to a technology shock



produces an expression for the average real marginal cost which is equal to the labor share of GDP, one can then use the same value of the slope of the New Keynesian Phillips curve in order to infer price adjustment frequencies. This is the approach followed by Altig et al. (2004) and Woodford (2005).

This presents a complication for the comparison between the model with search frictions and the standard model, because in the former case average (log) real marginal costs are generally different from the (log) labor share of GDP. Real marginal costs are given by the marginal rate of substitution between consumption and leisure, $\frac{h_t^\eta(i)}{c_t^{-1/\sigma}}$, divided by the marginal product of labor, $\frac{y_t(i)}{n_t(i)h_t(i)}$. This can be written as

$$mc_t(i) = \frac{w_t(i)n_t(i)}{y_t(i)} \frac{h_t^\eta(i)c_t^{1/\sigma}}{w_t(i)/h_t(i)}, \tag{49}$$

where $w_t(i)$ is total (not hourly) compensation. This would imply the following log-linear approximation of average real marginal costs,

$$\widehat{mc}_t = \hat{s}_l + (1+\eta)\hat{h}_t + \sigma^{-1}\hat{c}_t - \hat{w}_t, \tag{50}$$

where $\hat{s}_l \equiv \hat{w}_t + \hat{n}_t - \hat{y}_t$ is the log deviation of the labor share of GDP from its steady state value.[16] The problem of using the right hand side of equation (50) as a proxy for real marginal costs is that, in order to identify the slope of the Phillips curve (κ), one needs to assume a value for either η (the convexity of labor disutility) or σ (the intertemporal elasticity of substitution), parameters which are not directly observable and can only be estimated at best. This would introduce an element of arbitrariness in the estimation of κ.

In order to avoid this problem, I start by noticing that the bargaining power parameter has no effect on the parameter ϕ, and therefore on the mapping between the fraction of sticky prices, δ, and the slope of the New Keynesian Phillips curve, κ. Therefore, my analysis of average frequencies of price adjustment remains completely unaffected if I assume that firms have all the bargaining power, $\xi = 1$. In this case, worker surplus in every firm drops to zero and workers are exactly compensated for their labor disutility (in terms of consumption),

$$w_t^{\xi=1}(i) = \frac{1}{1+\eta} \frac{h_t^{1+\eta}(i)}{c_t^{-1/\sigma}}.$$

[16] In the more general case of a Cobb-Douglas production function with elasticity α with respect to labor, the marginal product of labor is $\alpha\frac{y_t(i)}{n_t(i)h_t(i)}$. Real marginal costs would be proportional to the right hand side of equation (49), which would imply again equation (50).

κ	Search frictions	Standard NK
0.025	0.696	0.855
0.02	0.733	0.869
0.015	0.776	0.887

Table 2: Fraction of sticky prices implied by different estimates of the slope of the inflation equation

The ratio of the marginal rate of substitution to the average hourly wage is now given by

$$\frac{h_t^\eta(i) c_t^{1/\sigma}}{w_t^{\xi=1}(i)/h_t(i)} = 1 + \eta.$$

This, together with equation (49), implies the following expression for real marginal costs,

$$mc_t^{\xi=1}(i) = \frac{w_t(i) n_t(i)}{y_t(i)} \left(1 + \eta\right).$$

Log-linearizing and averaging, I finally obtain

$$\widehat{mc}_t^{\xi=1} = \hat{s}_l.$$

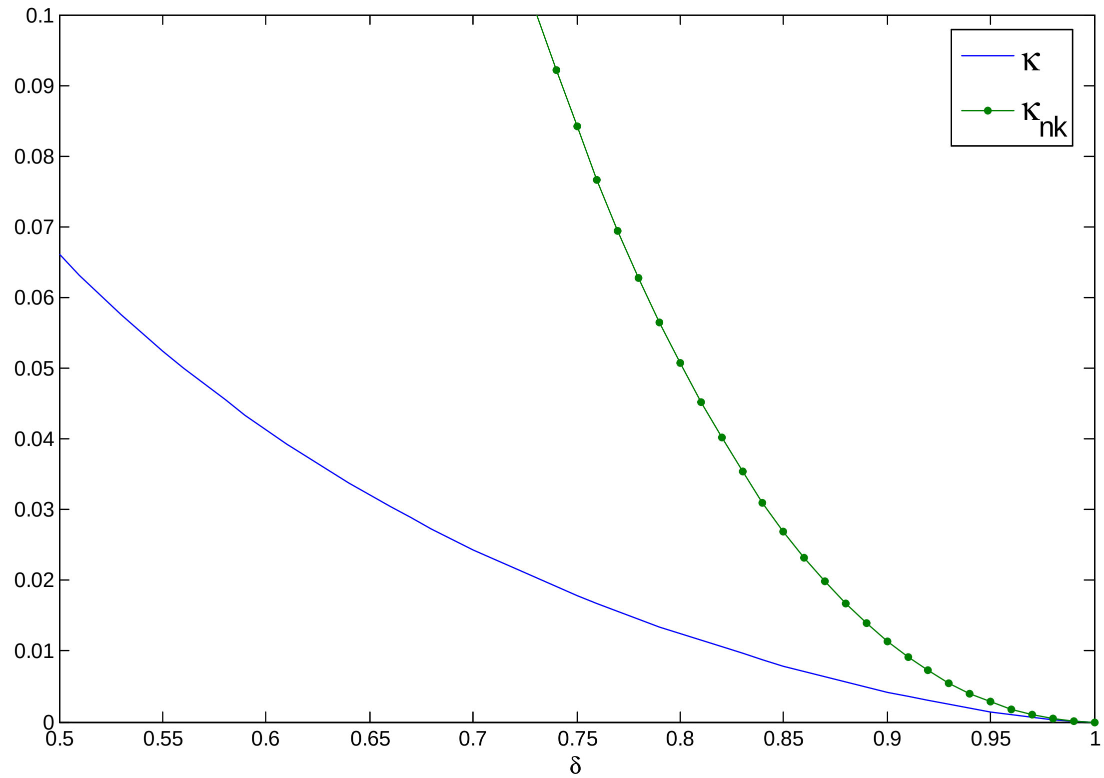
Therefore, under the assumption that firms have full bargaining power, the (log)share of GDP is a correct proxy for average (log)real marginal costs *both* for the standard New Keynesian model and the model with search frictions. As a consequence, if one were to estimate the New Keynesian Phillips curve, the estimate of κ would be the same regardless of which model one had in mind. I can then use the same value of κ in order to infer the average frequency of price adjustment implied by the model with search frictions and the standard model.

Figure 3 plots the slope of the inflation equation in each model as a function of δ, under my calibration of the other structural parameters. In the model with search frictions, the presence of real rigidities reduces the slope of the inflation equation relative to the standard New Keynesian model. The flip side of the coin is that, given any estimate of κ, the standard model implies a larger fraction of sticky prices, δ.

Table 2 displays the value of δ implied by several hypothetical estimates of κ for each model. I assume values of κ that are close to those provided by the empirical literature, and the last row uses the value of κ implied by my calibration.[17]

[17] Using US data, Gali and Gertler (1999) provide an estimate of 0.023, whereas Sbordone (2004) obtains an estimate of 0.025.

Figure 3: Slope of the New Keynesian Phillips curve as a function of the fraction of sticky prices



κ	Search frictions	Standard NK
0.025	3.3	6.9
0.02	3.7	7.6
0.015	4.5	8.8

Table 3: Average duration of price contracts, in months

Given the values of δ, I can calculate the corresponding average duration of price contracts, $\frac{1}{1-\delta}$. Table 3 shows the average duration of price contracts for each value of κ and for each model.

In all cases, the average duration of price contracts implied by the model with search frictions is roughly *half* of that implied by the standard model. For instance, under my baseline value of κ, the model with search frictions implies an average price duration of 4.5 months.[18] This is roughly the average frequency of price adjustment found in micro data (Bils and Klenow, 2004; Klenow and Kryvtsov, 2004). The standard model however would imply an average duration of price contracts of almost 9 months, which is almost one and a half quarters longer than what micro data suggests is a plausible duration. These results suggest that search frictions and the resulting real rigidities may be useful in reconciling the average frequency of price adjustment implied by macroeconometric estimates of the New Keynesian Phillips curve with the frequencies usually found in micro data.

6 Conclusion

A recent literature has pointed to the following deficiency in the standard New Keynesian model. In order to match macro-econometric estimates of the slope coefficient of the New Keynesian Phillips curve, one needs to assume an average duration of price contracts which is much longer than the average duration typically found in the micro data. Some authors have called this the 'macro-micro pricing conflict'. The same literature has blamed this conflict on the lack of *real rigidities* (in the sense of Ball and Romer, 1990) in the standard model, which stems from the assumption of perfectly competitive markets for homogenous factors of production.

This paper shows how departing from the assumption of a perfectly competitive labor market can generate real rigidities and how this helps resolve the pricing conflict. I do so by introducing search and matching frictions in the labor market into the New Keynesian model. Because of the time involved in hiring workers, in the short run firms can only adjust hours per worker so

[18] Notice that this is by construction: in my baseline calibration, I assume a pricing frequency of 4.5 months.

as to meet demand. Workers however must be compensated for their labor disutility. Under the realistic assumption that labor disutility is convex in hours worked, short-run marginal cost curves become upward-sloping. That is, the model gives rise to real rigidities. This mechanism leads firms to make smaller price changes in response to the same macroeconomic impulses, because they internalize the effect of their pricing (production) decisions on their own marginal costs. Once all pricing decisions are aggregated, real rigidities have the effect of reducing the slope of the New Keynesian Phillips curve. Therefore, for the same estimate of this slope, the model with search frictions implies an average duration of price contracts which is shorter than in the standard model and therefore closer to the micro evidence.

For future research, it would be interesting to introduce firing costs into the present context. By affecting the marginal cost of adjusting employment, firing costs will affect how firms allocate their labor-input needs between employment and hours. This in turn will affect the degree of real rigidities in the New Keynesian model and therefore the relationship between the average frequency of price adjustment and the slope of the Phillips curve. Given the size of firing costs in countries like Germany, France, Italy and Spain, this may be an important topic in the context of research on the monetary transmission mechanism in the euro area.

7 Appendix

7.1 Proof of Proposition 1

From equation (29) in the text, we can write the firm's vacancy posting decision as

$$\begin{aligned}\frac{s_v}{\lambda}(1-\epsilon)\hat{\theta}_t &= \beta E_t \xi \left[\frac{1}{\mu}\left(\eta \tilde{h}_{t+1}(i) + \eta \hat{h}_{t+1} + \frac{\eta \sigma^{-1}}{1+\eta}\hat{c}_{t+1}\right) - p(\theta)\beta S_y^w \left[\epsilon \hat{\theta}_{t+1} + \hat{\beta}_{t+1,t+2} + \hat{S}_{t+2}^w\right]\right] \\ &\quad + \frac{s_v}{\lambda} E_t \hat{\beta}_{t,t+1} + \beta(1-\lambda)\frac{s_v}{\lambda}(1-\epsilon)E_t \hat{\theta}_{t+1}, \qquad \text{(A1)}\end{aligned}$$

where $\tilde{h}_{t+1}(i) = \hat{h}_{t+1}(i) - \hat{h}_{t+1}$ is the firm's relative number of hours per worker. Hours per worker admit the following exact log-linear representation

$$\hat{h}_t(i) = \hat{y}_t^d(i) - a_t - \hat{n}_t(i).$$

Therefore, I can write $\tilde{h}_t(i) = \tilde{y}_t^d(i) - \tilde{n}_t(i)$. This becomes

$$\tilde{h}_t(i) = -\gamma \tilde{P}_t(i) - \tilde{n}_t(i)$$

once I use the fact that $\tilde{y}_t^d(i) = -\gamma \tilde{P}_t$. The firm's expected relative price is given by

$$\begin{aligned}E_t \tilde{P}_{t+1}(i) &= \delta E_t \left(\log P_t(i) - \log P_{t+1}\right) + (1-\delta)E_t\left(\log P_{t+1}^*(i) - \log P_{t+1}\right) \\ &= \delta E_t\left(\tilde{P}_t(i) - \pi_{t+1}\right) + (1-\delta)E_t\left(\log P_{t+1}^*(i) - \log P_{t+1}^* + \frac{\delta}{1-\delta}\pi_{t+1}\right) \\ &= \delta \tilde{P}_t(i) - (1-\delta)\tau^* \tilde{n}_{t+1}(i).\end{aligned}$$

In the second equality I have used the fact that, in the Calvo model, $\log P_{t+1}^* - \log P_{t+1} = \frac{\delta}{1-\delta}\pi_{t+1}$, where $\pi_t \equiv \log(P_t/P_{t+1})$ is the inflation rate. In the third equality I have used $\log P_{t+1}^*(i) - \log P_{t+1}^* = -\tau^* \tilde{n}_{t+1}(i)$. Expected relative hours are then given by

$$\begin{aligned}E_t \tilde{h}_{t+1}(i) &= -\gamma E_t \tilde{P}_{t+1}(i) - \tilde{n}_{t+1}(i) \qquad \text{(A2)} \\ &= -\gamma \delta \tilde{P}_t(i) - \left[1 - \gamma(1-\delta)\tau^*\right]\tilde{n}_{t+1}(i).\end{aligned}$$

Averaging (A1) across all firms and substracting the resulting expression from (A1) yields $E_t \tilde{h}_{t+1}(i) = 0$. Combining this with (A2), I finally obtain

$$\tilde{n}_{t+1}(i) = -\frac{\gamma \delta}{1 - \gamma(1-\delta)\tau^*}\tilde{P}_t(i).$$

7.2 Proof of Proposition 2

Using equations (37), (38) and (39), the inequality $b^2 > 4ac$ can be written as

$$[1+\gamma(2-\delta-\delta\beta)\eta]^2 > 4(1+\eta\gamma)\gamma(1-\delta)(1-\delta\beta)\eta.$$

This in turn can be written as

$$(1+\eta\gamma)^2 + [(1-\delta-\delta\beta)\eta\gamma]^2 > 2(1+\eta\gamma)\left[(1-\delta)(1-\delta\beta)+\delta^2\beta\right]\eta\gamma.$$

This can be written as

$$\begin{gathered}(1+\eta\gamma)\left\{1+\eta\gamma-\left[(1-\delta)(1-\delta\beta)+\delta^2\beta\right]\eta\gamma\right\} \\ +\eta\gamma\left\{(1-\delta-\delta\beta)^2\eta\gamma-\left[(1-\delta)(1-\delta\beta)+\delta^2\beta\right](1+\eta\gamma)\right\} > 0.\end{gathered}$$

I can write the latter as

$$\begin{gathered}(1+\eta\gamma)\left\{1+\delta\left[1-\delta\beta+\beta(1-\delta)\right]\eta\gamma\right\} \\ -\eta\gamma\left\{\delta\left[1-\delta+\beta(1-\delta\beta)\right]\eta\gamma+\left[(1-\delta)(1-\delta\beta)+\delta^2\beta\right]\right\} > 0.\end{gathered}$$

This can be expressed as

$$\begin{gathered}1+(\eta\gamma)^2\,\delta\left\{1-\delta\beta+\beta(1-\delta)-\left[1-\delta+\beta(1-\delta\beta)\right]\right\} \\ +\eta\gamma\left\{\delta\left[1-\delta\beta+\beta(1-\delta)\right]+1-\left[(1-\delta)(1-\delta\beta)+\delta^2\beta\right]\right\} > 0.\end{gathered}$$

Cancelling terms, I can finally write

$$1+(\eta\gamma)^2\,\delta^2(1-\beta)^2+2\eta\gamma\delta\left[1-\delta\beta+\beta(1-\delta)\right] > 0,$$

which holds for any $\delta, \beta \in [0,1]$.

7.3 The model in log-linear form

I now obtain the log-linear equations of the model with search frictions.

- Inflation,

$$\pi_t = \kappa\widehat{mc}_t + \beta E_t\pi_{t+1}.$$

- Average real marginal costs,

$$\widehat{mc}_t = \eta\hat{y}_t + \sigma^{-1}\hat{c}_t - (1+\eta)a_t - \eta\hat{n}_t.$$

- Aggregate demand,

$$\hat{y}_t = s_c\hat{c}_t + s_v\hat{v}_t,$$

where $s_v \equiv \frac{\chi v}{y}$ and $s_c = 1 - s_v$.

- Private consumption,

$$\hat{c}_t = \hat{c}_{t-1} + u_t - \pi_t.$$

- Vacancies,

$$\hat{v}_t = \hat{\theta}_t + \hat{u}_t.$$

- Unemployment,

$$\hat{u}_t = -\frac{n}{u}\hat{n}_t,$$

where $u = \frac{\lambda}{\lambda + p(\theta)}$ and $u = 1 - n$.

- Employment,

$$\hat{n}_{t+1} = (1 - \lambda - p(\theta))\hat{n}_t + \lambda\epsilon\hat{\theta}_t,$$

- Labor market tightness,

$$\frac{s_v}{\lambda}(1-\epsilon)\hat{\theta}_t = \frac{s_v}{\lambda}E_t\hat{\beta}_{t,t+1} + \beta\left\{\xi\left[\frac{1+\eta}{\mu}\hat{h}_{t+1} + \frac{\sigma^{-1}}{\mu}\hat{c}_{t+1} - s_{\underline{w}}\widehat{\underline{w}}_{t+1}\right] + (1-\lambda)\frac{s_v}{\lambda}(1-\epsilon)\hat{\theta}_{t+1}\right\}.$$

- Average hours per worker,

$$\hat{h}_t = \hat{y}_t - a_t - \hat{n}_t.$$

- Outside opportunities,

$$s_{\underline{w}}\widehat{\underline{w}}_t = \frac{1}{\mu}\left(\hat{h}_t + \frac{\sigma^{-1}}{1+\eta}\hat{c}_t\right) + p(\theta)\beta S_y^w\left[\epsilon\hat{\theta}_t + E_t\left(\hat{\beta}_{t,t+1} + \hat{S}_{t+1}^w\right)\right],$$

where $s_{\underline{w}} \equiv \frac{n\underline{w}}{y}$ and $S_y^w \equiv \frac{nS^w}{y}$.

- Average worker surplus,

$$S_y^w\hat{S}_t^w = s_w\hat{w}_t - s_{\underline{w}}\widehat{\underline{w}}_t + (1-\lambda)\beta S_y^w E_t(\hat{\beta}_{t,t+1} + \hat{S}_{t+1}^w),$$

where $s_w \equiv \frac{nw}{y}$.

- Average real wage,

$$s_w \hat{w}_t = (1-\xi)\left(\hat{y}_t - \hat{n}_t\right) + \xi s_{\underline{w}} \widehat{\underline{w}}_t.$$

- Stochastic discount factor,

$$\hat{\beta}_{t,t+1} = \sigma^{-1}(\hat{c}_t - \hat{c}_{t+1}).$$

- Money growth shock,

$$u_t = \rho_m u_{t-1} + \varepsilon_t^m,$$

where $\varepsilon_t^m \sim iid(0, \sigma_m^2)$.

- Technology shock,

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a,$$

where $\varepsilon_t^a \sim iid(0, \sigma_a^2)$.

References

[1] Altig, David, Lawrence J. Christiano, Martin Eichenbaum and Jesper Linde (2004). Firm-Specific Capital, Nominal Rigidities and the Business Cycle," working paper.

[2] Andres, Javier, Rafael Doménech and Javier Ferri, 2006, Price Rigidity and the Volatility of Vacancies and Unemployment, working paper.

[3] Ball, Laurence and David Romer, 1990, Real Rigidities and the Non-Neutrality of Money, *Review of Economic Studies*, 57(2), 183-203.

[4] Bils, Mark, 1987, The Cyclical Behavior of Marginal Cost and Price, *American Economic Review*, 77(5), 838-855.

[5] Bils, Mark and Pete Klenow, 2004, Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112, 947-985.

[6] Blanchard, Olivier and Jordi Gali, 2006, A New Keynesian Model with Unemployment, working paper.

[7] Calvo, Guillermo, 1983, Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics*, 12(3), 383-398.

[8] Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, 113(1), 1-45.

[9] Christoffel, Kai and Tobias Linzert, 2005, The Role of Real Wage Rigidities and Labor Market Frictions for Unemployment and Inflation Dynamics, ECB Discussion Paper 556.

[10] Cooley, Thomas F. and Vincenzo Quadrini, 1999, A neoclassical model of the Phillips curve relation, *Journal of Monetary Economics*, 44(2), 165-193.

[11] Eichenbaum, Martin and Jonas Fisher, 2004, Evaluating the Calvo Model of Sticky Prices, NBER working paper 10617.

[12] Gali, Jordi, 1999, Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?, *American Economic Review*, 89(1), 249-271.

[13] Gali, Jordi and Mark Gertler, 1999, Inflation Dynamics: A Structural Econometric Analysis, *Journal of Monetary Economics*, 44(2), 195-222.

[14] Gertler, Mark and Antonella Trigari, 2006, Unemployment Fluctuations with Staggered Nash Wage Bargaining, working paper.

[15] Hall, Robert E., 1980, Employment Fluctuations and Wage Rigidity, *Brookings Papers on Economic Activity*, 1, 91-123.

[16] Hall, Robert E., 2005, Employment Fluctuations with Equilibrium Wage Stickiness. *American Economics Review*, 95(1), 50-65.

[17] Klenow Peter J. and Oleksiy Kryvtsov, 2005, State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation? NBER Working Paper 11043.

[18] Krause, Michael and Thomas Lubik, The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions, in press, *Journal of Monetary Economics*.

[19] Kuester, Keith, 2007, Real price and wage rigidities in a model with matching frictions, ECB working paper no. 720.

[20] Merz, Monika, 1995, Search in the labor market and the real business cycle, *Journal of Monetary Economics*, 36(2), 269-300.

[21] Petrongolo, Barbara and Christopher A. Pissarides, 2001, Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature*, 39(2), 390-431.

[22] Pissarides, Christopher A., 2001. *Equilibrium Unemployment Theory*, MIT Press.

[23] Sbordone, Argia, 2004, Do Expected Future Marginal Costs Drive Inflation Dynamics?, unpublished, Rutgers University.

[24] Shimer, Robert, 2005, The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review*, 95(1), 25-49.

[25] Thomas, Carlos, 2007, Search and matching frictions and optimal monetary policy, LSE, working paper.

[26] Trigari, Antonella, 2005, Equilibrium Unemployment, Job Flows and Inflation Dynamics, mimeo.

[27] Walsh, Carl, 2003a, *Monetary Theory and Policy*, MIT Press.

[28] Walsh, Carl, 2003b, Labor Market Search and Monetary Shocks, in: *Elements of Dynamic Macroeconomic Analysis*, S. Altug, J. Chadha and C. Nolan (eds.), Cambridge University Press, 451-486.

[29] Woodford, Michael, 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

[30] Woodford, Michael, 2005, Firm-Specific Capital and the New-Keynesian Phillips Curve, *International Journal of Central Banking*, 1(2), 1-46.