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American Economic Development Since the Civil War or the Virtue of Education

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Abstract
This paper is the first empirical framework that explains the phenomenon of fast growth combined with the demographic transition occurring in the United States since 1860. I propose a structural model that unifies those events through the role of education: the key feature is that parental education determines simultaneously fertility, mortality and children's education, so that the accumulation of education from one generation to another explains both fast growth and the reduction of fertility and mortality rates. Using original data, the model is estimated and fits in a remarkable way income, the distribution of education and age pyramids. Moreover, some historical data on Blacks, assumed to constitute the bottom of the distribution of education, show that the model predicts correctly the joint distribution of fertility and education, and that of mortality and education. Comparisons with the PSID suggest that the intergenerational correlation of income is also well captured. Thus, this microfunded growth model based on human capital accumulation accounts for many traits of American economic development since 1860. In a second step, I investigate the long-run influence of income inequality on growth. Because children's human capital is a concave function of parental income, income inequality slows down the accumulation of human capital across generations and hence growth. Simulations show that this effect is large.

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1 Introduction

The goal of the paper is to assess the microfoundations of a recurrent fact in economic history of nations, namely the combination of an economic take-off and a demographic transition. Its causes have been much debated by demographers, historians or economists, and it is only recently that some theoretical models have emerged to explain both population and income dynamics on the long run. Nevertheless, those models have only received a partial assessment until now.

This paper fills the gap by providing a model of the joint dynamics of income, the distributions of fertility, mortality and schooling, and calibrates the model on original data from the United States over the period 1860-2000. What has been the mechanism driving the demographic transition, large educational investments and sustained income growth? What is the influence of income inequality on the previous dynamics?

Fundamentally, this paper is at the intersection of two strands of the literature, the empirical macroeconomic literature on education and the unified theory of long-term growth. The first strand typically considers reduced-form growth regressions where education has generally an impact on the sole income, while the second strand is pure theory and needs to be assessed. Briefly, the paper is a first attempt to make unified growth theory and empirics of growth meet.

In that respect, the framework encompasses major traits of the literature on long-run growth and its relationships with inequality. Ultimately, this literature explains how production factors accumulate over time and interact with each other at the microeconomic level: technology, population, education, physical capital, as well as their distributions. In a seminal paper, Oded Galor and David Weil (2000) provide a theoretical microfunded explanation for both the economic take-off and the decline of fertility, which is based on the interaction between technology and education. They ignore an important aspect of the demographic transition, mortality reduction, as well as the role of physical capital and the income distribution. In a following paper, Oded Galor and Omer Moav (2003) study the relationship between inequality and growth and argue that the latter has been non-monotonic in the course of Western Europe development, which has moved from a physical capital-intensive economy to a human capital-intensive one. As before, tractability purposes set some limits and the authors exclude population dynamics from their analysis. One step beyond, David de la Croix and Matthias Doepke (2003) envisage the interaction between fertility, income distribution and growth. They show a causal impact of the income distribution on growth via differential fertility between rich and poor people: this differential is likely to increase as a consequence of higher inequality, creating more low educated people on the long-term, finally reducing aggregate human capital and growth. Importantly, reduction in mortality is not taken into account. As it explains half of the demographic transition, it cannot be neglected from an empirical perspective. If they derive some qualitatively plausible results, they focus on theory and do not fit the data. Lastly, Jesus Fernandez-Villaverde (2001) is the only one paper that studies both endogenous fertility and mortality from an empirical perspective, but his analysis abstracts from the link between income distribution and growth.

To sum up, there does not exist any model assessing the joint dynamics of economic growth, population dynamics, and inequality. What misses in this literature is therefore an empirical perspective on the whole socio-economic system, which provides some realistic and credible micro-foundations to observed aggregate data. The strength of this paper is to derive plausible estimates of structural parameters regarding existing knowledge on them, which gives some support to the underlying mechanism.

What is exactely this mechanism linking growth, population and inequality? Households choose between quantity and quality of children and save for future consumption, subject to mortality risks. Those choices and the latter risks differ across households since they are conditional on parental education. Consequently, poor households invest more likely in quantity of children than in their education, which is relatively more expensive[1]. Dynamically, this heterogeneity has important repercussions since the model can exhibit polarized equilibria if the cost of education is high enough. As it is found to be low in the United States, every dynasty converges towards the maximal amount of education, insuring fast growth and a demographic transition on the long-term.

Given this unique equilibrium, what matters is the dynamics of convergence. In that respect the paper shows theoretically that income inequality has sizeable effects on the transitory dynamics because it slows down accumulation of human capital. This is because children human capital is a concave function of parental income, an assumption supported empirically, so that the larger income inequality the smaller average human capital in the economy at each period.

Empirically, the paper relies on an original databaset that picks up GDP from 1860, as well as fertility and survival probabilities at various ages for Blacks and Whites, age pyramids corrected for migrations per age, and the distribution of education[2] taken from Christian Morrisson and Fabrice Murtin (2006). The model is calibrated and fits the dynamics of the aggregates such as income or age pyramids, or the marginal distribution of schooling. In fact, it also captures the joint distribution of schooling and fertility and that of schooling and mortality, if one assumes that Blacks have been in the bottom decile of the distribution of education throughout the period. Comparison with the PSID show that intergenerational correlation of income over the last twenty years is also well

[1] see Omer Moav (2005) for a similar story.

[2] This distribution uses four quantiles: those who have not attended school, those with only Primary schooling, those with only Secondary schooling, and people with higher education.

fit. With respect to growth, the model shows that education plays a significant role both because of its level and its accumulation, and that inequality has a sizeable negative impact on growth.

Section 2 describes the historical facts relevant to this model, which is presented in Section 3 together with its major dynamical properties and the role of inequality on transitory dynamics. Section 4 depicts its estimation and proposes some counterfactual analyses in order to gauge the impact of inequality on growth. Last section concludes.

2 Trends in population dynamics and educational attainment since 1860

2.1 Education

In the middle of the nineteenth century, the United-States were the most educated country in the world. In 1870, illiteracy was about 20 % and mass primary schooling was already completed. According to the US Department of Education (1993), enrolment rate of young people aged between 5 and 17 was about 75% in 1900, a figure that reflects full enrolment in elementary schools for the 5-13 years old, and a 10% enrolment rate in secondary school for the remaining group. Then in the first half of the twentieth century, the United States experienced a dramatic development of secondary schooling. The percentage of high school graduates relatively to the 17 years old population was about 25% in 1925, 50% at the beginning of Second World War, and reached its current level of 80% in the early 1970s. Following this wave, the college graduation rate has gone from about 10% in 1960 to 30% in 2000, the bulk of this increase taking place in the first half of this period (about two thirds).

Factors that have contributed to the fast development of education are numerous. In spite of the long-lasting racial segregation of Black, it is believed that a widely shared and deeply enrooted egalitarian vision of education contributed a lot to its spread. In its "Bill for the more general diffusion of knowledge" (1779), Thomas Jefferson advocated the benefits of education for democracy in such terms: "the most effectual means of preventing [tyranny] would be, to illuminate, as far as practicable, the minds of people at large, and more especially to give them knowledge of those facts...they may be enabled to know". In an enlightning paper, Claudia Goldin and Lawrence F. Katz (2003) explain the causes of the american educational system's success story: apart from the latter cultural trait that promoted egalitarianism and in practice gender equality, decentralization and local financing of schools, which became free very early in time, haved played a key role in US educational achievement. Interestingly, it appears that the developments of elementary and secondary education were not the outcome of mandatory schooling laws, which often lagged behind phases of enrolment increase and were not binding anyway. Similarly, the contribution of the state to funding was modest at the beginning. Indeed, the share of the state in the funding of elementary and secondary schools has raised from 20% in 1890 up to 50% in 2000, and has stayed almost constant and equal to 30% for higher education. To sum up, the most plausible way of viewing education in the US over that period would be to consider it as a private good, more than as a public good.

Importantly, it is likely that the price of this private good has decreased over time. On one hand, there has been a clear increase in expenditures per pupil, following the real increases of teachers' wage and administrative costs, as well as the reduction of class size. For instance the pupil/teacher ratio in elementary and secondary schooling has gone from 35 in 1870 to 15 in 1995. This entails a clear rise of the direct cost of education. On the other hand, it is very likely that the opportunity cost of education has decreased along the period and that this effect has been the strongest as mentionned hereafter.

2.2 Fertility and Mortality

In most countries, the demographic transition is composed of a first phase where mortality is decreasing, followed by fertility in a second phase. A traditional explanation advocated by demographers is that the decline in infant mortality triggers that in fertility, building on the hypothesis that each household has a desired number of surviving offspring. In the United-States fertility rates have declined from 7 children in 1800 to 3.5 in 1900 with an acceleration around 1840s, to reach its current value of about two children per woman in 2000. In contrast, data on infant mortality start only in 1850, which is too late in time to state upon their long-run relationships.

However, this explanation is challenged by the lack of regularity in demographic patterns of Western European countries. In some countries, fertility started rising before declining, or stayed constant in some others, a picture hard to reconcile with a steadily decreasing infant mortality. In that respect, Jesus Fernandez-Vilaverde (2001) shows that in the English case fertility granger-causes infant mortality rather than the other way around, and Matthias Doepke (2004) assesses via a structural analysis that infant or child mortality are unable to explain fertility's decline.

The main trigger of fertility's decline is often advocated to be the increase of investments into children's human capital. As argued by Oded Galor and Omer Weil (1999, 2000) and Oded Galor (2004), a rising demand for education in the course of technological change has triggered parental investments in education as well as a substitution effect from children quantity to children quality. It is unfortunate that there exists no educational data for the United-States in the first half of the nineteenth century, where the development of mass primary schooling has taken place. In parallel of the rising demand for human capital, it is often mentionned in the literature that a falling opportunity cost of education is likely to have fostered fertility decline. Hazan and Berdugo (2002) and Matthias Doepke (2005) provide some quantitative evidence that the rising wage's gap between adult and children along the development process, as well as educational and child labor policies, had a strong impact on fertility rates. Similarly, Oded Galor and Omer Weil (1996) argue that the increase of women labor participation along with the rise of their real wage, has increased the cost of bearing children, making the choice of high fertility relatively less attractive than that of high educational investments.

The fall of mortality rates is mainly due to the reduction of normal causes of death rather than to the elimination of famines, as explained in Wrigley and Schofield (1981). There has been a controversy on whether this should be attributed to an increase of calories consumption or to scientific progress of medecine. Since consumption includes both of them in the model, this undeterminacy does not appear very problematic in the following.

3 The Model

3.1 Consumer's program

There are 4 periods in life of 20 years each. Work takes place between 20 and 60 (period 1 and 2), consumption and fertility choices at the beginning of period 1. Lifetime is stochastic, and s_{t+j}^{t} stands for the probability that an individual born in period t will be alive at the end of period $t+j$, with $j \in \{0,1,2\}$. For empirical motives, these survival probabilities are derived from the Cox model, and death rate are constant within each cohort of age. If λ_{t+j}^{t} is the death rate

j periods after birth at period t and z_{t+j} are some observed characteristics, then $\lambda_{t+j}^t = \lambda_j \exp(-z'_{t+j}\beta)$ where λ_j is a given constant, and one has

$$\begin{aligned} s_t^t &= \exp(-\lambda_0 \exp(-z'_t \pi_0)) \\ s_{t+j}^t &= s_{t+j-1}^t \exp(-\lambda_j \exp(-z'_{t+j}\pi_j)), \quad j = 1, 2 \end{aligned}$$

In practice one retains a single individual component z_{t+j}, individual consumption, but other factors such as education or average education in the society could be considered.

At the beginning of period 2 a representative agent born in year t with human capital h^t maximizes utility over her lifetime, while caring about both quantity n_{t+1} and human capital h^{t+1} of her children. Each agent represents a high number of comparable households such that n_{t+1} can be a continous variable (the mean number of born children among that kind of household). Raising $n_{t+1}s_{t+1}^{t+1}$ surviving children has a time cost $\phi w_{t+1}^t h^t n_{t+1} s_{t+1}^{t+1}$, where ϕ stands for the proportion of lost wage for each child. Providing e_{t+1} years of schooling to these children has a monetary cost $\tau_{t+1} w_{t+1} e_{t+1} n_{t+1} s_{t+1}^{t+1}$, which is fixed, ie does not depend on parental human capital. The parameter τ_{t+1} depends on the pupils/teachers ratio, on teachers' human capital, as well as on other institutional features that affect the opportunity cost of education such as child labor policies. It may vary across time, it is parametrized by its initial and final values in respectively 1860 and 2000, and it decreases linearly between those dates[3][4]. Raising children is subject to the time constraint[5] $\phi n_{t+1} s_{t+1}^{t+1} < 1$ (each household has one unit of time). Production of human capital is a concave

[3] An equivalent assumption is that ϕ increases over time with τ constant. What matters is in fact the variation of the ratio ϕ/τ.

[4] Other functional forms has been tested such as heterogeneity in the parameter τ with respect to grades, ie primary, secondary and higher education. In terms of fitting the data, no significant benefits have been noticed and the simplest form has been retained.

[5] as it has never been found binding in practice, this constraint is dismissed in the following analysis.

fonction of years of schooling. Two others externalities are likely to play a role: one is the social return to education, ie spillovers of the community average human capital, and the other is a direct transmission of human capital from parents to children. Following David de La Croix and Matthias Doepke (2003) we retain the following production function for human capital

$$h^{t+1} = (\theta + e_{t+1})^{\eta} (h^t)^{\rho} (\bar{h}^t)^{\kappa} \varepsilon$$

where ε is an log-normal ability shock of mean 1 and variance σ^2. Abstracting from the latter and from education spillovers, an uneducated household has therefore θ^{η} units of human capital.

An household also saves assets a_{t+i}^t during period i of her working life in order to consume during her retirement. Due to accidental death before 60, an household can receive unvolontary transfers. All these transfers $tr_{t+1,i}^e$ are pooled and mutualized so that there is no intergenerational transfers from parents to children[6]. The utility function u is assumed to be logarithmic. The program thus writes:

$$\underset{c_{t+1}^t, c_{t+2}^t, c_{t+3}^t, e_{t+1}, n_{t+1}}{Max} \mathrm{E} \left[u(c_{t+1}^t) + \beta u(c_{t+2}^t) + \beta^2 u(c_{t+3}^t) + \gamma u(s_{t+1}^{t+1} n_{t+1} h^{t+1}) \right]$$

$$\begin{aligned} s.t.\ c_{t+1}^t + a_{t+1}^t + \tau_{t+1} w_{t+1} e_{t+1} n_{t+1} s_{t+1}^{t+1} &= w_{t+1} h^t (1 - \phi n_{t+1} s_{t+1}^{t+1}) + tr_{t+1,1}^e \\ c_{t+2}^t + a_{t+2}^t &= w_{t+2}^e h^t + (1 + r_{t+1}) a_{t+1}^t + tr_{t+1,2}^e \\ c_{t+3}^t &= (1 + r_{t+2}^e) a_{t+2}^t \\ 0 \leq e_{t+1} &\leq 16 \end{aligned}$$

[6] for simplicity I assume that these transfers are mutualized within each cohort of age, otherwise this would mean that each cohort of age would rationally forecast what will be the savings of the following cohorts of age, conditionally on their own human capital investments. This would make the model untractable.

Notice that the solutions to this program depend on parental human capital: the joint distribution of $(c_{t+i}^t, a_{t+i}^t, h^{t+1}, n_{t+1})$ is conditionnal on h^t. From the same perspective recall that life expectancy also depends on h^t. Thus, the key feature of this microfunded growth model is to provide a link between generations based on human capital accumulation, as well as to describe how other variables such as fertility, mortality and savings depend on education.

For simplicity, it has been assumed that the elasticities of altruism with respect to quantity and quality of children were the same, contrary to assumptions made for instance by Robert Barro and Gary Becker (1989). It enables to derive some simple closed-form dynamics to educational attainment across generations. It also means that the altruistic term should not be viewed as dynastic utility, but rather as utility derived from total bequests in the form of education.

Let's state clearly upon three limitations of the model: the non-existence of financial bequests, the trivial calendar for fertility and educational investments, and rationality. First, one assumes that individuals do not make any financial bequest, and that after 60 individuals consume all their savings before death. Importantly, this rule out potential positive effects of savings on growth if savings are a convex function of wealth, as stated by François Bourguignon (1981). But introducing savings would have made the empirical model fairly untractable by introducing a second conditionning. Moreover, financial bequests and wealth effects in general are unable to account for the decline of fertility as argued by Oded Galor (2005).

Second, it is clear that sociological evolutions such as delays in nuptialities have played a role in fertility's decline, but this evolution could be related to the time evolution of the cost of education, since fertility and education are determined simultaneously. Moreover, educational levels are exclusively chosen by parents, so that individuals do not have the possibility to invest into their own education in the form of on-the-job training for instance; in particular, this entails that children or adult mortality does not have any impact since it affects fertility or educational bequests in the same way.

Last, the agent has to form expectations on survival probabilities, mutualized transfers, the values of capital returns, and wages at the beginning of the following periods. Assuming that the agent takes into account her gain in life expectancy by consumming more hinders the derivation of explicit solutions to the program because survival probabilities are non-linear functions of consumptions. Similarly, rational expectations would greatly complicate the empirical tractability of the program since transfers and rental prices on subsequent periods would logically depend on all consumers' decisions; therefore each agent should solve not only her program but also that of every agent in order to derive the recursive equilibria of the economy. A certain degree of myopia is believed to be a more realistic assumption. Hence I consider survival probabilities to be exogenous and rule out general equilibrium effects; as a result, all agents assume ex-ante the stationarity of survival probabilities, transfers, wages and capital returns, though the latter evolve in time but in an ex-post way. It gives

$$\begin{aligned} s_{t+j}^{t,e} &= s_t^{t-j} \\ tr_{t+1,i}^{e} &= tr_{t,i} \\ w_{t+2}^{e} &= w_{t+1} \\ r_{t+2}^{e} &= r_{t+1} \end{aligned}$$

In practice this assumption is inocuous since factors prices evolve slowly over time, so that expected prices are very close to realised ones. Some simulations have shown that extrapolative expections change neither the spirit of the model nor its results, but have the unpleasant property of making some two-states cycles appear in the long-term. Similarly, a stochastic maximization algorithm has been used to derive the program's solution with endogenous survival probabilities. Its solutions are very close to the explicit ones, but its numerical burden does not allow to achieve the final estimation in a reasonable amount of time.

3.2 Aggregates

In order to simplify the exposition, I describe below the case where the flow of migrants is null. At each date the working population is composed of two vintages of human capital distributions. Let us call F^{t+1}_{new} the cumulative distribution function of human capital for the new cohort born in $t+1$. Then

$$F^{t+1}_{new}(h) = \int_0^{+\infty} 1_{h^{t+1}(h^t)\leq h} dF^t_{new}(h^t)$$

Total human capital among the labor force is simply the mixture of two distributions

$$dF_{t+1}(h) = p_{t+1}\frac{s^t_{t+1}(h)dF^t_{new}(h)}{\int_0^{+\infty} s^t_{t+1}(h)dF^t_{new}(h)} + (1-p_{t+1})\frac{s^{t-1}_{t+1}(h)dF^{t-1}_{new}(h)}{\int_0^{+\infty} s^{t-1}_{t+1}(h)dF^{t-1}_{new}(h)}$$

where p_{t+1} is the relative weight of the cohort aged between 20 and 40 with respect to population aged between 20 and 60 at date $t+1$. The number of births is the sum of children conditionnally on the survival of parents

$$N_{t+1} = N_t \int_0^{+\infty} s^t_{t+1}(h)n_{t+1}(h)dF^t_{new}(h)$$

which provides the total population at date $t+1$

$$P_{t+1} = \sum_{i=0}^{3} N_{t+1-i}\int_0^{+\infty} s^{t+1-i}_{t+1}(h)dF^{t+1-i}_{new}(h) = \sum_{i=0}^{3} P^i_{t+1}$$

where P^i_{t+1} the total population born in $t+1-i$ that has survived until the end of period $t+1$. Consequently, the relative weight of cohort aged 20-40 in the active population is simply

$$p_{t+1} = \frac{P^1_{t+1}}{P^1_{t+1} + P^2_{t+1}}$$

Production uses a Cobb-Douglas production function

$$Y_{t+1} = K^{\alpha}_{t+1}(A_{t+1}L_{t+1})^{1-\alpha}$$

Firms maximize profits, wages and interest rates are equal to their marginal product

$$\begin{aligned} w_{t+1} &= (1-\alpha)k^{\alpha}_{t+1} \\ r_{t+1} &= \alpha k^{\alpha-1}_{t+1} \end{aligned}$$

where $k_{t+1} = K_{t+1}/A_{t+1}L_{t+1}$ is physical capital per unit of effective labor. The market-clearing conditions for labor and capital apply. Labor is equal to the total of work hours times human capital

$$L_{t+1} = N_t \int_0^{+\infty} hs^t_{t+1}(h)(1-\phi n_{t+1}(h)s^{t+1}_{t+1}(h))dF^t_{new}(h) + N_{t-1}\int_0^{+\infty} hs^{t-1}_{t+1}(h)dF^{t-1}_{new}(h)$$

while physical capital depreciates at rate δ and is augmented by households' savings during working life

$$K_{t+1} = (1-\delta)K_t + N_t \int_0^{+\infty} s^t_{t+1}(h)a^t_{t+1}(h)dF^t_{new}(h) + N_{t-1}\int_0^{+\infty} s^{t-1}_{t+1}(h)a^{t-1}_{t+1}(h)dF^{t-1}_{new}(h)$$

The rate of growth in technology A_t may depend on mean aggregate human capital in an endogenous growth context, as well as on other determinants such as lagged technology or population externality. Following the Schumpeterian growth literature, education's level augment the growth rate of technological progress. A simple specification is

$$A_{t+1} = A_t e^{\nu (H_t)^{\mu}}$$

Therefore this model encompasses both exogenous and endogenous growth models if μ is respectively equal to or different from 0.

3.3 The Poverty Trap and the Economic Take-off

With a logarithmic utility function, consumer's program can be solved explicitely as shown in appendix[7]. On a first step a "deterministic version" of the model is studied in order to connect with the previous literature. Therefore one rules out ability shocks for a while by setting $\sigma = 0$ and one simplifies the dynamics by excluding human capital externalities, ie $\rho = \kappa = 0$.

The introduction of a fixed monetary cost for education and a time cost for raising children naturally generates a trade-off between quantity and quality of children, which is conditional on parental wage: the fixed cost of education is relatively higher for poor parents, while the cost of raising a large number of children is higher for rich households because each child consumes a fixed proportion of parental wage. Explicitely one has

$$\begin{aligned}
e_{t+1} &= e^*_{t+1} \text{ if } 0 \leq e^*_{t+1} \leq 16 \\
&= 0 \text{ if } e^*_{t+1} < 0 \\
&= 16 \text{ if } e^*_{t+1} \geq 16 \\
e^*_{t+1} &= \left(\frac{-1}{1-\eta} + \frac{\eta \phi}{\tau(1-\eta)} h^t \right)
\end{aligned}$$

[7] To allow higher degree of risk aversion one has to solve consumer's program numerically with a much bigger numerical burden.

Education thus depends linearly and positively on parental human capital[8], and on three other parameters, the return to education η, the time cost of children ϕ, the opportunity cost of education τ.

What are the dynamical properties of the economy? There are three possible cases depicted on figure 1. When the cost of education is very high, the only equilibrium is illiteracy for everyone, when it is medium, a fraction gets some education on the long term and the others become illiterate, when it is low, everybody reaches higher education on the long term. The first two regimes are characterized by the existence of a poverty trap for at least a fraction of the population, the third one is called the economic take-off. Formally one has the following proposition

Proposition 1 *Given some initial distribution of education* $(e_{i,0})_i$*, there exists two tresholds* τ^m *and* τ^M *such that* $\forall i$ $e_{i,\infty} = 16$ *if* $\tau < \tau^m$ *and* $e_{i,\infty} = 0$ *if* $\tau > \tau^M$*. Otherwise, there is a polarized equilibria characterized by a threshold* $\bar{e}$ *such that* $e_{i,\infty} = 0$ *if* $e_{i,0} < \bar{e}$ *and* $e_{i,\infty} = e^{\infty} > 0$ *if* $e_{i,0} \geq \bar{e}$

Proof. There exists τ^M such that for any $\tau > \tau^M$ $e^*_{t+1} < e_t$ whatever e_0 comprised between 0 and 16, τ^M is simply the solution of $\frac{-1}{1-\eta} + \frac{\eta\phi}{\tau(1-\eta)}(1+16)^\eta = 16$. In that case the sequence $(e^*_t)_t$ converges towards $-\infty$, hence $(e_t)_t$ towards 0 whatever the initial condition e_0. Similarly there exists a threshold τ^m such that for any $\tau < \tau^m$, $e^{*,m}_{t+1} > e_t$ whatever e_0 comprised between 0 and 16, τ^m is the solution of $\frac{-1}{1-\eta} + \frac{\eta\phi}{\tau^m(1-\eta)} = 0$. Then $(e^*_t)_t$ converges towards $+\infty$, hence $(e^m_t)_t$ towards 16 whatever the initial condition e_0. In the intermediate case where $\tau^m < \tau < \tau^M$, $e^*_{t+1} = e_t$ has at least a solution for some e_t. It is well-known that the first solution, if not the only one, to this equation is a non-stable equilibrium since the function $e_t \longmapsto (1+e_t)^\eta$ is concave. So there exists a value $\bar{e}$ such that $(e^*_t)_t$ tends to $-\infty$ for any initial condition below $\bar{e}$,

[8] decreasing returns to schooling are a fundamental requirement for that.

and $(e_t^*)_t$ tends to a limit $l \in]0, +\infty]$ for any initial condition above $\bar{e}$. Hence $(e_t)_t$ converge towards 0 if $e_0 < \bar{e}$ and $(e_t)_t$ converge towards a strictly positive number if $e_0 > \bar{e}$. ■

It is interesting to notice that the dynamics of education can have different qualitative behaviours with respect to structural parameters. It is similar to stories put forward by David de la Croix and Matthias Doepke (2002) as well as Omer Moav (2005). The poverty trap mechanism channeled by educational investment echoes other stories, where the same result is driven by convex savings as described by Francois Bourguignon (1981), or by imperfections on the credit market as depicted by Oded Galor and Joseph Zeira (1993) or Thomas Piketty (1997). In particular, one should stress that in this framework poverty traps are likely to be created simply because of agents' preferences and fertility choices, and not because of non-convex technologies. Modifying the environment characteristics such as the cost of education is likely to trigger economic take-off. In that case, every household invests more likely in education than in children quantity, a process that historically fits the socio-economic dynamics of the United States. As a result, children are at least as educated as their parents.

3.4 Income Inequality and Educational Attainment

This subsection examines the impact of income inequality on dynamics of education. In the literature, income inequality is traditionnally viewed as an incentive to accumulate wealth, or on the other hand as a harmful characteristic of an economy where credit market are imperfect. Oded Galor and Omer Weil (2004) reconcile both ideas by explaining that inequality is good for growth on early stages of development when physical capital is the prime engine of growth; latter on, inequality is bad for growth because it prevents poor households who are credit-constrained to invest into education.

As suggested by simulations from David de la Croix and Matthias Doepke (2002), inequality can have a negative impact on growth independantly of any non-convexity of the technology. Classicaly a non-convexity creates polarized equilibria and initial inequality has a an impact on long-term growth rate. Hereafter, teh key idea is that inequality has an impact on the speed of convergence towards long-term equilibrium. This simply stems from diminishing returns to education: the larger the variance of ability shocks, the lower the average stock of human capital in next generation[9]. Mathematically this is because the larger support a concave function is averaged on, the smaller its average. This has an impact on all subsequent generations, creating a slowdown of convergence towards the steady-state. Therefore I show the following proposition:

Proposition 2 *In an economic take-off regime assume that σ is small enough. With decreasing returns to education, average human capital is in each period a decreasing function of income inequality, proxied by the variance of ability shocks.*

Proof. A change of variable $z_{i,t} = \frac{\tau(1-\eta)}{\eta\phi}\left(\frac{-1}{1-\eta} + e^*_{i,t}\right)$ conveys the following dynamics $z_{i,t+1} = (a+b_1 z_{i,t})^\eta \varepsilon_{i,t}$ with $a = \theta - \frac{1}{1-\eta}$ and $b_1 = \frac{\eta\phi}{\tau(1-\eta)}$.Conditionally on the initial distribution $z_{i,0}$, one first shows that E $[z_{i,2}|z_{i,0}]$ is a decreasing function of σ. One has

$$\begin{aligned} E\left[z_{i,2}|z_{i,0}\right] &= E\left[(a+b_1 z_{i,1})^\eta \varepsilon_{i,1}|z_{i,0}\right] \\ &= E\left[(a+b_1 z_{i,1})^\eta|z_{i,0}\right] \text{ by independance} \\ &= E\left[(a+b_1(a_1+b_1 z_{i,0})^\eta \varepsilon_{i,0})^\eta\left|z_{i,0}\right.\right] \\ &= E\left[(a+b_2\varepsilon_{i,0})^\eta\left|z_{i,0}\right.\right] \end{aligned}$$

with $b_2 = b_1(a+b_1 z_{i,0})^\eta$. Providing that $a + b_2 > 0$, for small enough ability

[9] even if the average stock of education remains the same.

shocks a Taylor expansion brings

$$\begin{aligned} E\left[z_{i,2}|z_{i,0}\right] &= (a+b_2)^{\eta} + \eta\,(a+b_2)^{\eta-1} b_2 E\left[\varepsilon_{i,0}-1\right] + \frac{1}{2}\eta(\eta-1)\,(a+b_2)^{\eta-2} b_2^2 E\left[(\varepsilon_{i,0}-1)^2\right] + o(V\left[\varepsilon_{i,0}^2\right]) \\ &= (a+b_2)^{\eta} - \frac{1}{2}\eta(\eta-1)\,(a+b_2)^{\eta-2} b_2^2 + \frac{1}{2}\eta(\eta-1)\,(a+b_2)^{\eta-2} b_2^2 e^{\sigma^2} + o(V\left[\varepsilon_{i,0}^2\right]) \end{aligned}$$

given that the moment of order k of a log-normal variable with parameters (μ, σ^2) is $e^{k\mu+k^2\frac{\sigma^2}{2}}$, that in the present case $\mu = -\frac{\sigma^2}{2}$ to insure $E\left[\varepsilon_{i,t}\right] = 1$, so that E $\varepsilon_{i,0}^2 = e^{\sigma^2}$. Then it follows from $\eta < 1$ that $E\left[z_{i,2}|z_{i,0}\right]$ is a decreasing function of σ^2. Now, noticing that $E\left[z_{i,t+1}|z_{i,0}\right] = E\left[(a+b_1 z_{i,t})^{\eta}|z_{i,0}\right] \simeq (a+b_1 E\left[z_{i,t}|z_{i,0}\right])^{\eta}$ following a first-order Taylor expansion, it turns out by recurrence that $\forall t\ E\left[z_{i,t}|z_{i,0}\right]$ is a decreasing function of σ^2 since $b_1 > 0$ and $\eta > 0$, and so is $e_{i,t}^*$. In order to deal with achieved education, it is necessary to assume that σ is small enough so that $\forall i,t\ e_{i,t}^* > 0$ and Taylor expansions are valid approximations, otherwise it would be possible to exhibit some case where σ has a positive impact on average human capital. Then, note $e_{i,t}^{*,0}$ the deterministic process corresponding to $\sigma = 0$ and $\Phi : x \mapsto \min(x, 16)$ the function for upper bounding. For small σ a Taylor expansion provides $E\left[e_{i,t}|e_{i,0}\right] = E\left[\Phi(e_{i,t}^*)|e_{i,0}\right] \simeq \Phi(e_{i,t}^{*,0}) + E\left[(e_{i,t}^* - e_{i,t}^{*,0})\Phi'(e_{i,t}^{*,0})|e_{i,0}\right] = \Phi(e_{i,t}^{*,0}) - p_t e_{i,t}^{*,0} + p_t E\left[e_{i,t}^*|e_{i,0}\right]$ where p_t is the proportion of dynasties strictly below the upper bound in the dynamics of $e_{i,t}^{*,0}$, since $\Phi'(x) = 0$ for $x \geq 0$ and 1 otherwise. This shows that in first approximation $E\left[e_{i,t}|e_{i,0}\right]$ is a decreasing function of σ.

■

The former proposition shows a negative impact of inequality on average educational attainment with diminishing returns to education and idiosyncratic shocks; notice that inequality could have an ambiguous or even positive effect in a poverty trap regime, more generally if the constraint $e_{i,t}^* > 0$ is binding or Taylor approximations are no longer valid because income shocks have a large variance. Next figure illustrates the former proposition by simulating the educational dynamics assuming or not some random shocks in the economic take-off regime. The boxplot represents the 25th, 50th, 75th percentiles as well as the extremes of the distribution. One clearly notices the large dynamical impact of inequality on average education, hence growth. It does not alter convergence, but slows it down.

This original effect of inequality on transitory dynamics of education relies heavily on functional forms, namely that education in next generation is a concave function of current education, or a linear function of parental income. If this function were convex, then one would easily derive the opposite result[10]. Therefore it is necessary to address this issue by displaying some empirical evidences. Table 1 provides the result of intergenerational regressions where the dependant variable is years of schooling and control variables are a quadric in father's years of schooling or father's normalized income; controls for father's experience and age of children (cohort effect) were added. I retained two cohorts of Males aged between 30 and 50 taken from the PSID (SRC and SEO samples). The results support strongly a linear or concave intergenerational transfer function with respect to father's income[11], albeit data are unconclusive with respect to father's education which turned out to be unsignificant[12].

Certainly a more careful analysis is needed, taking into account heterogeneity in returns to parental schooling, assessing the dynamic choices of education and focusing on the functional form explicitely. It would provide a better idea of the underlying process at work as well as the magnitude of ability shocks[13]. However those regressions indicate that the functional forms used in the model are plausible ones.

[10] since $\eta > 1$ in the former proof.

[11] Because of censoring in children's education, a tobit model has been applied to each cohort, but differences with OLS are often small.

[12] as well as mother's education.

[13] see Stephen V. Cameron and James J. Heckman (2001) and Pedro Carneiro and James J. Heckman (2002) in particular.

4 Assessing the Economic Development of the United States 1860-2000

4.1 The data

Data are derived from a collection of statistical sources. The educational distribution is given by Christian Morrisson and Fabrice Murtin (2006) who use total enrolments in primary, secondary and higher education as well as age pyramids to derive enrolment rates at the world level since 1870. Interestingly their statistics match those of Claudia Goldin in the Historical Statistics of the United-States (2006) at the starting point of the latter source, which is based on a survey run by the US Census. Indeed, they find that in 1940 people displaying respectively elementary schooling (grades 1 to 8), secondary and higher education represent 61.2, 27.5 and 11.3 percents of the labour force, while official figures are respectively 62.9, 26.7 and 10.4 percents. Therefore one reasonnably can think that the original series on education are accurate, even since their beginning in 1870. To strengthen this assertion, I present below the proportions of high school graduates in the 17 years-old cohort, one derived from Claudia Goldin (1999), the other from author's calculation. They turn out to be very similar in practice excepted after the Second World War, which is not problematic because after 1940 educational statistics rely on surveys and not on the summation of past enrolment rates. Thus, this new data on American distribution of education since 1860 is reliable and makes possible the estimation of the role of education over almost a century and a half.

Age pyramids are taken from Brian R. Mitchell (2003) and aggregated into 4 groups as in the model. Fertility rates and mortality, more precisely survival probabilities at the age of 20, 40 and 60, are taken from the Historical Statistics of the United-States (2006). Last, the net migration flows by age are derived from figures deduced from Jean-Claude Chesnais (1986), Imre Ferenczi and Walter F. Willcox (1931) for the period 1840-1930, and from the Historical Statistics of the United-States (2006) afterwards. They are needed in order to adjust the simulated population at different ages with migrations flows taken as exogenous variables. All figures, GDP per capita, population, education and demographic rates, are then averaged over the last 20 years; thus figures in 2000 represent in fact an average value over the period 1980-2000[14].

4.2 Estimation

In order to reduce the number of parameters to estimate, some of them are fixed prior to the estimation. Though another version of the maximization program could deal with non closed-form solutions of the consumer's program, risk aversion is set equal to one because it reduces very much the numerical burden; time preference parameter β is fixed to 0.4, which corresponds to a 4.5% annual discount rate. Capital elasticity α is set equal to the classical value of 0.3 and the capital depreciation rate is $\delta = 0.85$ or about 10% annually, which is in tune with Greenwood et al. (1997).

Another arising difficulty was the initial expectations of factor prices and the standard errors of initial differences in fertility Δn or survival probabilities Δs. Unknown expected factor prices were chosen to have reasonnable values[15] and initial differences were obtained by identifying Black people with the first decile of the human capital distribution. One has $\Delta n = -0.5$ and $\Delta s = 0.15$, which means a difference of one child in terms of fertility rate between Blacks and Whites in 1860, and 15% more mortality for Blacks at 20.

The estimation is achieved by a simulated method of moments as one cannot

[14] educational figures for the period 1840-1870 are an extrapolation based on the observed constant level of enrolment among White 5-17 years old pupils on the period 1850-1900.

[15] r^e is fixed to an annual 5.5% and w is equal to GDP per capita divided by a 0.6 participation rate.

derive explicit expressions of the moments to match: average income per capita; average fertility and fertility of Blacks; average conditionnal survival probability at 20, 40 and 60 as well as their value for Blacks; population between 0 and 20, 20 and 40, 40 and 60 years-old; the distribution of education split into four quantiles.

Given the number of parameters to estimate, prior calibration is important to guess starting values of the algorithm and insure numerical convergence. Once starting values are guessed, a minimum distance estimator is applied. If Z stands for the vector of moments to match and $\tilde{Z}$ for the simulated moments, then the parameters $\hat{\theta}$ are solutions of

$$\hat{\theta} = \arg\min_{\theta} \psi(Y,\theta)' \; \Omega \; \psi(Y,\theta)$$

where $\psi(Y,\theta) = Z - \tilde{Z}$ depends on both structural parameters θ and state variables Y. This minimization uses a stochastic search algorithm that avoids the discretization of parameters' space, which would be computationnally very hard to cope with given that there are 17 parameters. This search algorithm is relatively quick to converge if properly parametrized. Then one has to derive some standard errors. Following Christian Gourieroux and Alain Monfort (1995), the estimator is consistent and asymptotically normally distributed, namely $\sqrt{T}(\hat{\theta} - \theta) \rightsquigarrow N(0, Q)$ where the asymptotic optimal covariance matrix of the estimator is $Q = \left(\frac{\partial \psi(Y,\theta)}{\partial \theta}' \; \hat{\Omega}^* \; \frac{\partial \psi(Y,\theta)}{\partial \theta} \right)^{-1}$ and $\hat{\Omega}^* = V[\psi(Y,\hat{\theta})]^{-1}$. In the former matrix the derivative is computed numerically.

Table 2 presents the results and figures 4 to 7 the matched moments. I find that $\phi = 0.155$ which means that each child costs 15,5% of a parent's time endowment, or about 8% of household's total time. This is in tune with Robert Haveman and Barbara Wolfe (1995) who find a time cost of about 12% of household's time. The costs of education turned out to be very sensible parameters and the results suggest that they have been reduced by 50% along the period since $\tau_{1860} = 0.102$ and $\tau_{2000} = 0.049$. In 2000, this figure corresponds to a realistic pupil/teacher ratio of 20.4, were teachers paid exclusively by pupils' parents; in 1870, this cost corresponds to an implicit 47% wage gap between child and adult's earnings, given an observed pupil/teacher ratio of 30 at the same date. This wage gap is very plausible. Human capital of uneducated people is found to be equal to $\theta = 0.6$; taking a value smaller than 1 clearly augments the return of the first years of schooling, a feature in tune with high returns to primary education observed worlwide and depicted by George Psacharopoulos and Harry A. Patrinos (2002). The elasticity of the human production function is $\eta = 0.542$, which is coherent with the estimates given by Martin Browning et al. (1999). The externatlity of human capital on society is found to be low by Daron Acemoglu and Joshua D. Angrist (2000), and is calibrated to 0.1 by David de la Croix and Matthias Doepke (2003); though I find it to be a little bit higher since $\kappa = 0.17$, while $\rho = 0.02$ suggests that intergenerational transmission of human capital are mostly driven by incentives and/or preferences rather than by a direct transmission of education.

Importantly, the variance of the ability shocks matches the evolution of income inequality throughout the period. Francois Bourguignon and Christian Morrisson (2002) propose a Gini of 0.490 in 1880 that declines to 0.409 in 1980 before recovering at 0.429 in 2000. At the same dates I find that the simulated Gini is respectively equal to 0.460, 0.385 and 0.362; given that I consider disposable income in the simulations, the levels and the trend are very close, though the recent increase of inequality is not reproduced.

Moreover, I find an intergenerational correlation of earnings of 0.397 in 2000 in tune with the study of Gary Solon (1992) and a plausible R^2 of 0.26 when a Mincerian regression is run. Interestingly, simulations suggest that intergenerational correlation of earnings has increased up to 0.60 until Second World War then has decreased, perhaps because of decreasing inequality of the marginal income distributions.

Globally, the model fits almost perfectly the joint dynamics of income, fertility, mortality, population and education[16]. This gives credit to the structural model depicted above, which parameters are all plausible. Thus it is possible to explain and replicate the socio-economic development of the United States in a simple model where accumulation of education drives both income and population dynamics.

4.3 Inequality and growth

What is the impact of inequality on long-term growth? In order to answer to this question, some experiments are run where the variance of ability shocks vary from 0.2 to 0.8 while other parameters are those estimated previously. Table 3 reports the annual long-term growth rate for the period 1860-2000, as well as average educational attainment in 2000. All initial conditions in 1860 are the same across simulations. To give an idea, the R^2 of Mincerian Regression in 1860 varies from 0.82 when $\sigma = 0.2$ to 0.43 when $\sigma = 1$.

It turns out that inequality in ability, in other words income inequality, has a strong and negative impact on growth and educational attainment, which are respectively diminished by 18% and 32% when σ goes from 0.2 to 0.8. This impact is however not linear: there is a threshold effect starting at a level of Gini around 0.300, say at a level corresponding roughly to disposable income inequality in France or Germany in 2000. Over this limit, inequality has a much bigger impact and the augmentation of the Gini coefficient by 1 percentage point reduces the growth rate by 0.2 percentage point each year[17], and average

[16] on the figures, predicted variables are in plain and observed ones are in dots.

[17] This represents 25% of the cross-countries correlation found by Robert J. Barro (2000),

schooling attainment by around a year over the period. Thus, inequality introduces a noise in the economy that slowdowns convergence towards mass higher education and fast long-term growth.

It is fruitful to compare the long-run outcomes of two simulations for respectively $\sigma = 0.2$ and $\sigma = 0.8$, which are two acceptable extreme cases. The percentages of people attaining higher education in 2000 are respectively 100% and 33%, the average fertility rates are respectively 2.2 and 3.5, the survival probability at 20 respectively 0.99 and 0.76. These results mean that the important levels of inequality at each period prevent investments into education that cumulate over generations, thus maintain a sizeable proportion of agents into poverty. For instance in the most equal economy illiteracy disappears by 1900 while in the most unequal illiteracy rate is still equal to 10% in 1960. Thus income inequality has a deterrent effect on growth via the reduction of education accumulation on the long-term.

5 Conclusion

The paper provides a consistent framework for the interactions of economic growth, fertility, mortality and the education distributions, which embeds all major traits of unified growth theory. Theoretically, this model shows that the crucial lever of long-term development is the cost of education, that can lead to an homogenous distribution of education in the long run - made of either illiterate people or Doctors -, or on the contrary generate a polarized society. This model has been calibrated on data from the United States since 1860, and all state variables have been predicted satisfactorily, with plausible values for the structural parameters. I show that income inequality has a significant

though there are no reason to expect them to be equal. With different structural parameters and in particular in a poverty trap regime, some simulations show a much bigger impact of inequality on growth.

and negative impact on growth and average educational attainment, because it slowdowns accumulation of human capital within dynasties. As a whole, this framework suggests that a model of education's accumulation is able to explain most of American economic development since the Civil War.

Regarding future work, I believe that this kind of structural model can provide an interesting evaluation tool of mid and long-term consequences of public policies, especially for developping countries. However, there are many ways in which the United States differ from "average" countries. One important singularity of this country has been the role of democratic ideals shared and promoted by the founding fathers, for whom education had to play a key role in the establishment of the New Republic. At the same period, only Prussia, in a lesser extent France, had promoted education for moral purposes, or for military ones...For instance the development of mass education in England had much to do with a capitalist lobbying on the government, which aimed at raising workmen's productivity through enhanced education. Over the twentieth century, the extension of mandatory schooling reflected most often competition or contagion effects between states, or a Schumpeterian catch-up effect as shown by Fabrice Murtin and Martina Viarengo (2006). Therefore, in a more general framework the role of the State should probably be refined. A setup for endogenous taxation and public investments into education should be considered, as well as the formation of governments themselves as it is theoretically achieved by Francois Bourguignon and Thierry Verdier (2000). This more general model will be calibrated on 60 countries using the same sources as in this paper.

APPENDIX

A Consumer's program solution

Mortality risks can be easily included following the seminal paper by Menahem Yaari (1965), with survival probabilities exogenously determined previously. Assuming that agents consume their whole pension at the moment of their death mortality risk does not interfere during retirement. Then lifetime utility simply is

$$U = s_{t+1}^{t,e} u(c_{t+1}^t) + \beta s_{t+2}^{t,e}[u(c_{t+2}^t) + \beta^2 u(c_{t+3}^t)] + \gamma s_{t+1}^{t,e} u(s_{t+1}^{t+1,e} n_{t+1} h^{t+1}) \quad (1)$$

Dropping the upper t index for birth except for survival probabilities, FOCs are respectively

$$\begin{aligned}
\frac{c_{t+2}}{c_{t+1}} &= \beta(1+r_{t+1})\frac{s_{t+2}^{t,e}}{s_{t+1}^{t,e}} && (2)\\
\frac{c_{t+3}}{c_{t+2}} &= \beta(1+r_{t+2}) && (3)\\
\frac{s_{t+1}^{t+1,e} n_{t+1} h_{t+1}}{c_{t+1}} &= \frac{\gamma h^{t+1}}{\phi w_{t+1} h^t + \tau w_{t+1} e_{t+1}} \quad (\frac{\partial}{\partial n_{t+1}}) && (4)\\
\frac{s_{t+1}^{t+1,e} n_{t+1} h_{t+1}}{c_{t+1}} &= \frac{\gamma \eta h^{t+1}}{\tau w_{t+1}(\theta + e_{t+1})} \quad (\frac{\partial}{\partial e_{t+1}}) && (5)
\end{aligned}$$

from which one deduces the optimal unconstrained investment into education

$$e_{t+1}^* = \frac{-1}{1-\eta} + \frac{\eta\phi}{\tau(1-\eta)} h^t \quad (6)$$

Consider the case where the constraints $0 < e_{t+1}^* < 16$ are not binding. From the second-period budget constraint and equations (2) and (5) one gets the solutions to the consumer's problem

$$
\begin{aligned}
e_{t+1} &= e^*_{t+1} \\
c_{t+1} &= \frac{[(1+r_{t+1})w_{t+1} + w^e_{t+2}]h^t + [tr^e_{t+1,1}(1+r_{t+1})w_{t+1} + tr^e_{t+1,2}]}{(1+r_{t+1})[1+(\beta+\beta^2)\frac{s^{t,e}_{t+2}}{s^{t,e}_{t+1}} + \frac{\gamma}{s^{t+1,e}_{t+1}}]} \\
c_{t+2} &= \beta(1+r_{t+1})\frac{s^{t,e}_{t+2}}{s^{t,e}_{t+1}}c_{t+1} \\
a_{t+1} &= \frac{1}{1+r_{t+1}}\left((1+\beta)c_{t+2} - w^e_{t+2}h^t - tr^e_{t+1,2}\right) \\
a_{t+2} &= \beta c_{t+2} \\
n_{t+1} &= \frac{\gamma\eta}{\tau w_{t+1}(\theta + e_{t+1})}\frac{c_{t+1}}{s^{t+1,e}_{t+1}}
\end{aligned}
$$

In that case the education given to children and their number turn out to be respectively an increasing and decreasing functions of parental human capital.

When the constraints are bounding, consumptions and savings are not modified, only the sharing rule between fertility and education. When $e^*_{t+1} < 0$, children receive no education. That occurs for all households such that

$$h^t \leq \frac{\tau}{\phi\eta}$$

In that case, maximization with respect to the number of children gives

$$n_{t+1} = \frac{\gamma}{\phi w_{t+1}h^t}\frac{c_{t+1}}{s^{t+1,e}_{t+1}}$$

In the case where $e^*_{t+1} > 16$, then $e_{t+1} = 16$ and the number of children is given by

$$n_{t+1} = \frac{\gamma}{\phi w_{t+1}h^t + \tau w_{t+1}16}\frac{c_{t+1}}{s^{t+1,e}_{t+1}}$$

B Figures

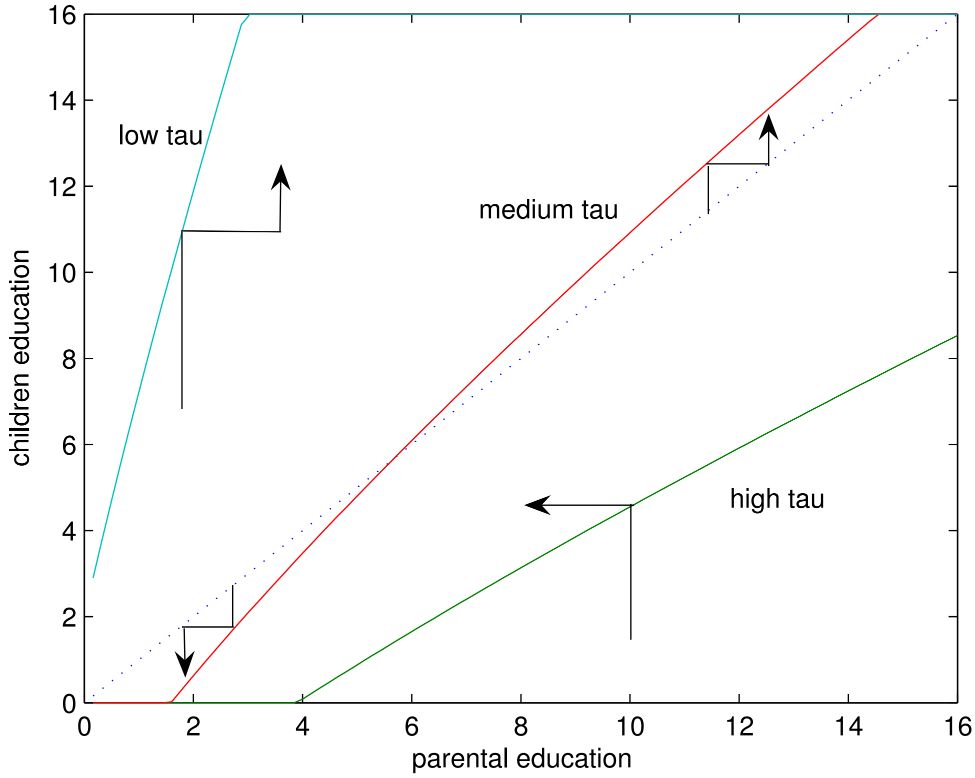

Figure 1: The Three Dynamics

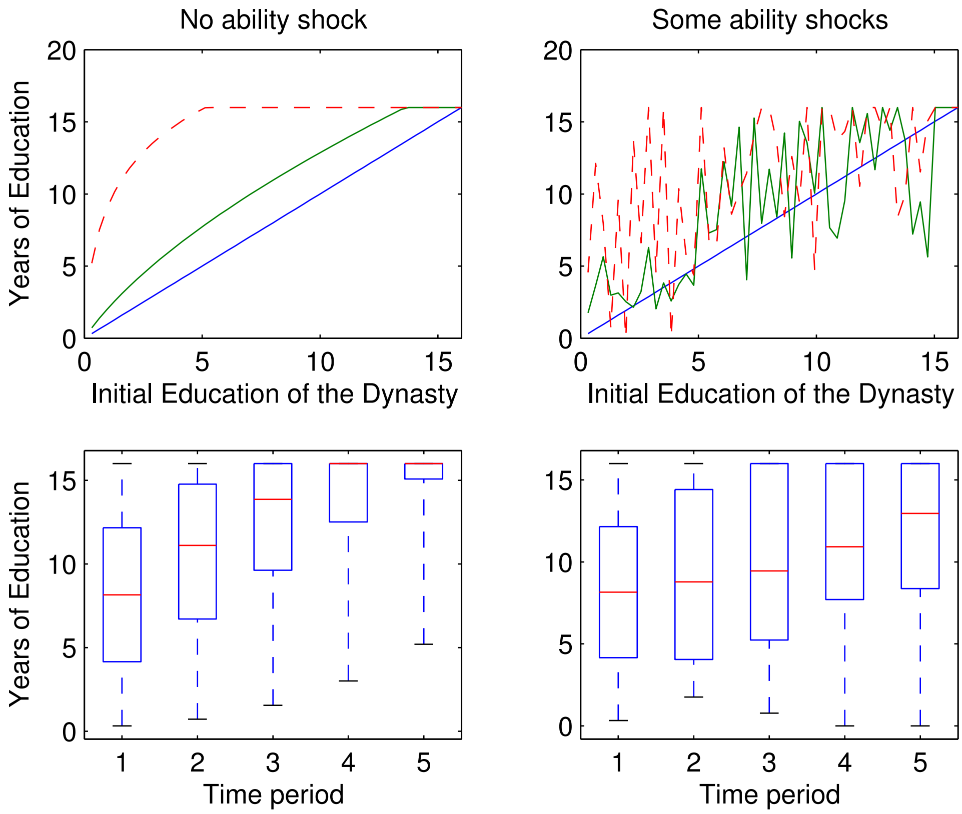

Figure 2: Income Inequality and the Distribution of Education

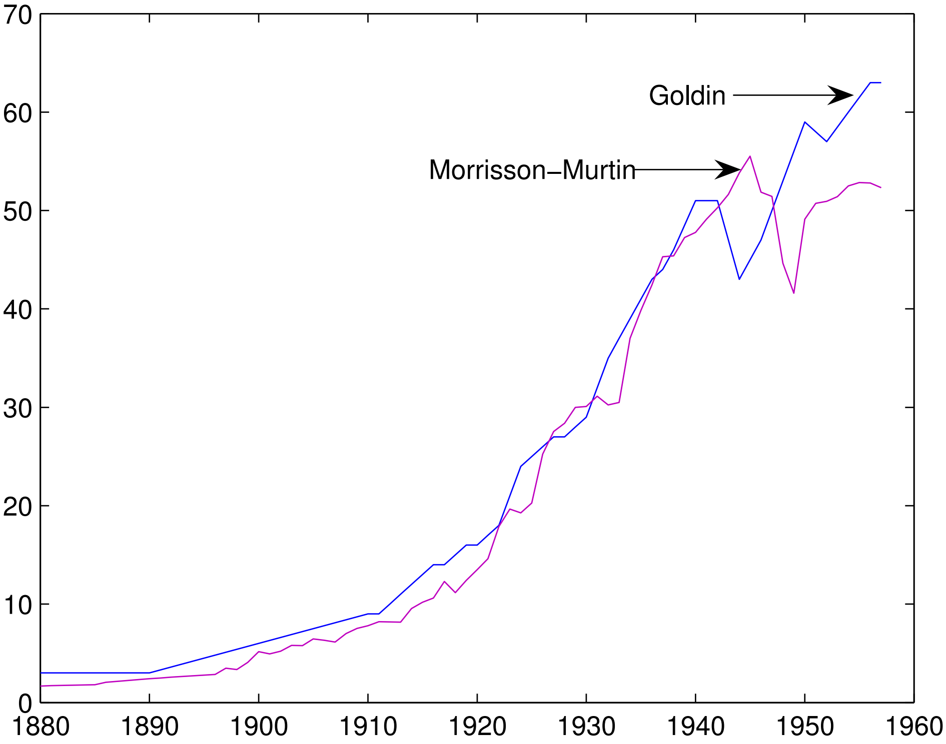

Figure 3 : Comparison of high-school graduation rates

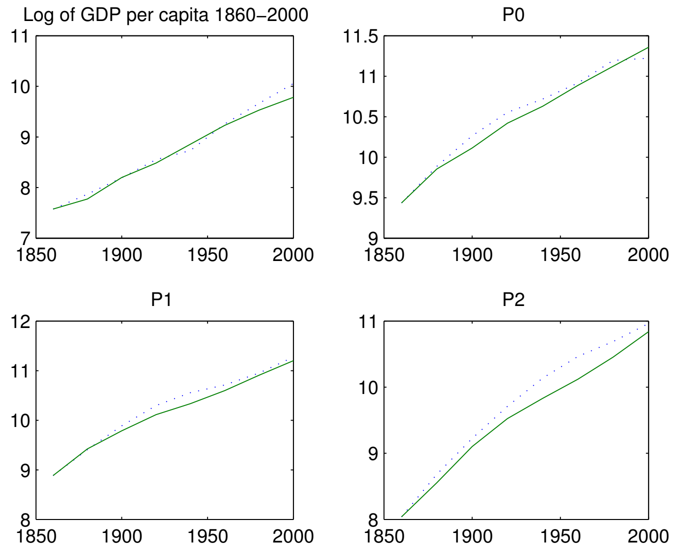

Figure 4: Observed and Simulated Income and Population by Age

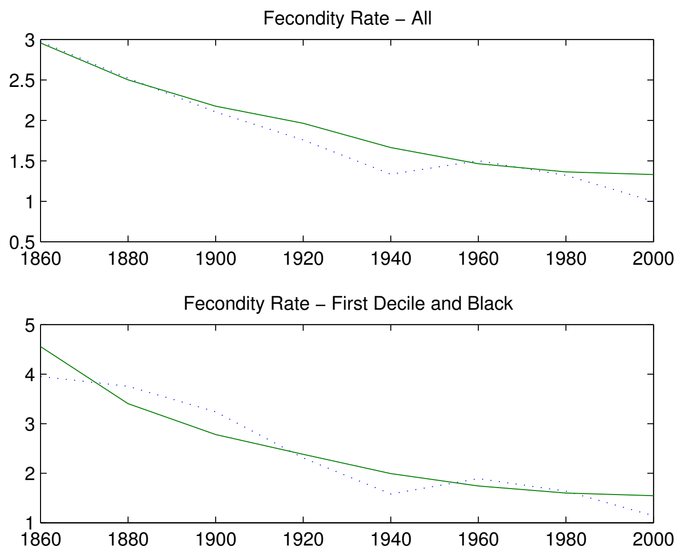

Figure 5 : Observed and Simulated Fertility Rates

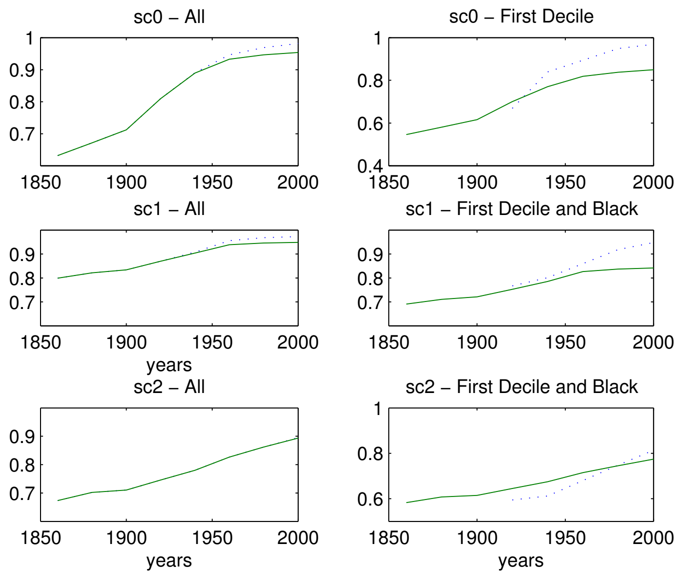

Figure 6: Observed and Simulated Conditional Survival Probabilities

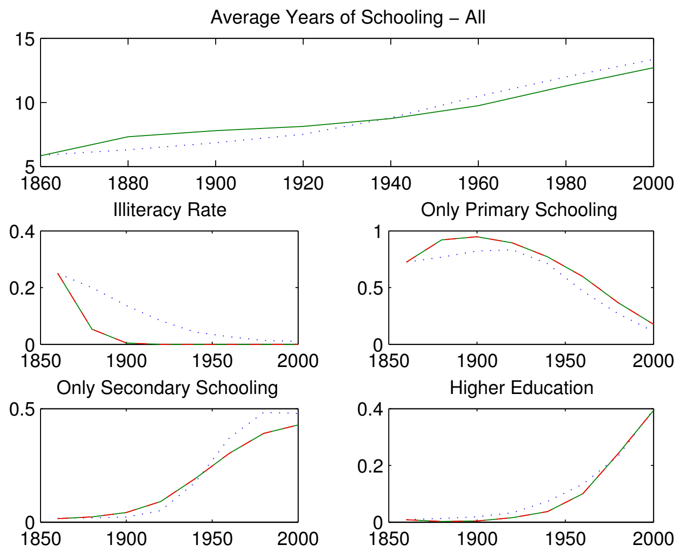

Figure 7: Observed and Simulated Distributions of Education

C Tables

Table 1 - Tobit Estimation of Intergenerational Determinants of Education - 2 Male Cohorts (SRC-SEO)

	Males aged 30−39 in 2001		Males aged 40−49 in 2001	
Father's Income	1.80^{**} (0.748)	-	1.952^{***} (0.390)	-
Father's Squared Income	-0.075 (0.264)	-	-0.138^{*} (0.080)	-
Father's Education	-	0.245 (0.280)	-	- -0.002 (0.194)
Father's Squared Education	-	0.006 (0.011)	-	- 0.011 (0.007)
Son's Age	0.081^{*} (0.042)	0.096^{**} (0.037)	0.051 (0.043)	0.086^{**} (0.040)
Father's Age	0.048 (0.033)	-	-0.098^{**} (0.049)	-
Father's Squared Age	-0.001^{*} (0.000)	-	0.002^{**} (0.001)	-
N	308		348	

*** (resp. ** and *) means significant at 1% (resp. 5% and 10%).

Table 2 - Minimum Distance Estimates of Structural Parameters

Education	τ_{1860}	τ_{2000}	θ	σ	η	κ	ρ
	0.102 (0.004)	0.049 (0.003)	0.52 (0.05)	0.61 (0.04)	0.542 (0.010)	0.17 (0.02)	0.02 (0.01)
Mortality	λ_0	λ_1	λ_2	π_0	π_1	π_2	
	1.06 (0.15)	0.32 (0.14)	0.63 (0.11)	13.2 (3.1)	1.50 (1.14)	0.78 (0.34)	
Technology	μ	ν					
	0.15 (0.02)	0.08 (0.03)					
Fertility	ϕ	γ					
	0.155 (0.005)	0.60 (0.01)					

Table 3 - Income Inequality and Growth

	Standard Error of Ability Shocks σ					
	0.2	0.4	0.5	0.6	0.7	0.8
Gini in 1860	0.313	0.367	0.409	0.436	0.472	0.510
Gini in 2000	0.114	0.237	0.300	0.362	0.423	0.481
Annual Growth Rate 1860-2000	1.92	1.90	1.86	1.76	1.65	1.57
Average Years of Schooling in 2000	15.8	14.6	13.1	12.0	11.3	10.7

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