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Evaluating Urban Transport Improvements: Cost-Benefit Analysis in the Presence of Agglomeration and Income Taxation

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Abstract
There is a substantial empirical literature quantifying the positive relationship between city size and productivity. The paper draws out the implications of this productivity relationship for evaluations of urban transport improvements. A theoretical model is developed and used to derive a wider cost-benefit measure that includes productivity effects. The order of magnitude of such effects is illustrated by calculations in a simple computable equilibrium model. It is argued that productivity effects, particularly when combined with distortionary taxation, are quantitatively important, substantially increasing the gains that are created by urban transport improvements.

JEL classification: R200, R420
Keywords: Agglomeration, productivity, urban transport

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1. Introduction
There is considerable evidence of the productivity benefits of clustering economic activity together into dense spatial units. Indeed, the existence of such agglomeration economies is the main economic basis for the existence of cities. Against this benefit are the costs associated with moving around within cities, and perhaps also costs of getting goods into and out of the city. The trade-off between these sources of benefit and cost is one of the main determinants of city size. This trade-off suggests that a transport improvement may enable a city to become larger, and thereby increase the extent to which agglomeration benefits are achieved. The objective of this paper is to show how these arguments change the way in which urban transport improvements should be appraised. In addition to the effects captured in a standard cost-benefit analysis, a transport improvement increases productivity for new city workers and also for existing city workers now reaping the benefits of a larger agglomeration. These productivity effects interact with distortions created by income taxation, reinforcing the gains further. We argue that these wider benefits of transport improvements, absent from a standard analysis, may be quantitatively significant.

The argument is organised as follows. We start (section 2) with a short review of the evidence on the productivity effects of urban centres. Section 3 outlines a model of urban equilibrium and section 4 derives analytically the effects of a transport improvement in a simplified version of the model. Section 5 reports numerical simulations of the effects of a transport improvement, establishing the quantitative significance of the mechanisms analysed.

2. The productivity/ city size relationship
The productivity advantage of large cities is illustrated by the example of London. Table 1 reports data for each of the London NUTS 3 regions relative to the national average. GDP per full-time employee is 35% above the national average in Inner London West, and around 7% above in the poorer regions of Outer London East and North East, and South. The GDP figures are not an adequate measure of productivity as they include allocation of non-labour income across regions. Concentrating on labour income we see, in column 2, that London’s advantage in earnings is even greater, with Inner London West being nearly 80% above the national average. This measure overstates productivity differences as it fails to control for the occupational and skill composition of the labour force. Column 3 goes some way to correcting for this by computing an index of earnings, constructed from 3 digit occupational
groups with occupational composition held constant across all spatial units. Looking across these indices, Inner London East and West are around 41% and 34% above the national average, while the outer regions are up to 20% above. Econometric work controlling additionally for skills finds very substantial productivity premia in London relative to the national average (see Rice and Venables 2004).

Table 1: Productivity and Earnings in London, 1998-2001, UK = 100.

<table>
<thead>
<tr>
<th></th>
<th>GDP per employee hour worked</th>
<th>Hourly earnings</th>
<th>Earnings index (controlling for occupational composition)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner London West</td>
<td>135.0</td>
<td>178.8</td>
<td>141.2</td>
</tr>
<tr>
<td>Inner London East</td>
<td>120.0</td>
<td>147.9</td>
<td>134.0</td>
</tr>
<tr>
<td>Outer London E &amp; NE</td>
<td>107.3</td>
<td>107.3</td>
<td>107.6</td>
</tr>
<tr>
<td>Outer London S</td>
<td>107.1</td>
<td>116.3</td>
<td>109.5</td>
</tr>
<tr>
<td>Outer London W &amp; NW</td>
<td>126.7</td>
<td>129.6</td>
<td>119.7</td>
</tr>
</tbody>
</table>


The facts sketched out for London are just one example of a general relationship between the scale (or density) of economic activity and productivity, the micro-economic foundations of which come from several sources -- through thick labour markets, input-output linkages, or knowledge spillovers. Rosenthal and Strange (2004) review the empirical literature. A large number of studies use cross-section data on cities (typically but not exclusively in the US) to relate productivity to city size. Findings vary according to the extent to which researchers are able to control for the quality of inputs – capital stock and the skill of the labour force – and typically yield the result that doubling city size increases productivity by an amount in the range 3 - 8%, i.e. an elasticity of productivity with respect to city size of between 0.04 and 0.11. For the UK, Rice and Venables (2004) find a corresponding elasticity of 0.04. Studies by Ciccone and Hall (1996) and Ciccone (2002) relate productivity to the spatial density of economic activity, using data for the US and for Europe respectively, and find an elasticity of productivity with respect to density of around 0.06.

While most studies are done at the aggregate city level, some focus on particular
sectors (e.g., Nakumura 1985, Henderson 1986, 2003), distinguishing between urbanisation effects (the effect of city size as a whole) and localisation effects (the effect of the scale of operation of a particular sector in the city). Generally, the latter are more pronounced, and vary across sectors; for example, Henderson finds quite large effects in US high-tech sectors, and no effects in more traditional sectors.

While the debate on the magnitude of these effects is far from closed, there is enough evidence to suggest a positive city size/productivity relationship. The relationship suggests several ways in which a transport improvement might effect productivity. One is that by improving links between firms within the city the effective density of the cluster rises. The other is that, by relaxing constraints on access to the centre, overall city employment is increased. This paper concentrates on the second of these mechanisms, essentially because of the existence of robust empirical estimates of the elasticity of productivity with respect to city employment. We argue that it creates two effects that need to be valued in cost-benefit analyses of urban transport. The first is the agglomeration externality, and the second a tax distortion, arising as the benefits (extra income) and costs (extra commuting) of urban employment are split by an income tax wedge. Our analysis shows how to include these effects in appraisals of transport improvements, and points to the undervaluation of benefits if they are ignored.1

3. Urban equilibrium

How can a situation in which productivity is higher in the city than in the hinterland be consistent with equilibrium? The answer given by standard urban economics models (as developed by Alonso 1964, see Fujita and Thisse 2002 for a recent review) is that the city imposes its own costs such that, at the margin, individuals are indifferent between city and non-city locations. The mechanism is the commuting cost that urban workers incur in travelling to their jobs in the central business district (CBD). The city expands up to the point at which these are high enough that a worker is indifferent between locating at the edge of the city and commuting to the CBD, or living (and working) in a non-city location. Although transport costs secure indifference at the margin, the city as a whole generates an economic

1 For a survey of transport evaluation see Small (1999). The arguments in this paper depend on market failures, not simply the derived market benefits that are captured by standard techniques (Mohring 1993).
surplus from its productivity advantage. The surplus arises as workers who live closer to the city centre pay lower commuting costs. However, this surplus is all captured by rent/house prices so that, in equilibrium, the entire surplus accrues to owners of the immobile factor – land.2

A formal description of this is as follows. Geographical space is divided into an arbitrarily large number of units some of which are in the city and others outside. The units are indexed by \( s \). The area of unit \( s \) is \( a(s) \), and it may be occupied either by \( x(s) \) households or by firms offering \( y(s) \) jobs. These area and density parameters are exogenous.

The productivity (output per worker) of a job at location \( s \) depends on the total number of jobs in the city according to the function \( q(\phi(s)) \), where

\[
\phi(s) = \sum_{z} y(z) \psi(s,z). \tag{1}
\]

\( \psi(s,z) \) is a decreasing function of the distance between locations \( s \) and \( z \). \( \phi(s) \) is therefore the sum of all the jobs in the city, \( y(s) \), weighted by the function \( \psi(s,z) \) of their distance from location \( s \). If \( q \) is an increasing function, then productivity in unit \( s \) rises the more jobs there are in the city, and the closer they are to unit \( s \).3

If the wage at location \( s \) is \( w(s) \), other inputs are ignored,4 and firms earn no abnormal profits, the commercial rent (per unit area) is \( r_x(s) \),

\[
r_x(s) = \left[ q(\phi(s)) - w(s) \right] y(s)/a(s). \tag{2}
\]

The term in square brackets is the surplus per job, and there are \( y(s)/a(s) \) jobs per unit area.

An individual living in \( s \) and working in \( z \) has utility

\[
u(s, z) = w(z) - t(w(z)) - c(s,z) - r_x(s)a(s)/x(s). \tag{3}
\]

---

2 This assumes a perfectly elastic supply of labour to the city. If this does not hold then wages will be bid up, and the surplus divided between owners of land and labour.

3 For theory models that generate particular forms of this relationship see Fujita and Thisse (2002), Fujita, Krugman and Venables (1999).

4 For the effects of intermediate inputs see Fujita, Krugman and Venables (1999).
The right-hand side is wage income net of tax, commuting costs, and rent. The tax paid on income is a function of the wage, $t(w(z))$; the cost of commuting from $s$ to $z$ is $c(s,z)$; the residential rent per unit area is $r_x(s)$ and lot size in unit $s$ is $a(s)/x(s)$. Free movement of workers equalises real income across residential units, the mechanism being adjustment of land rents. Outside the city a worker has real income given by the constant

$$\bar{w} - t(\bar{w}) - \bar{c} - \bar{r}\bar{a}/\bar{x}$$

so, by free mobility,

$$w(z) - t(w(z)) - c(s,z) - r_x(s)a(s)/x(s) = \bar{w} - t(\bar{w}) - \bar{c} - \bar{r}\bar{a}/\bar{x}.$$ (4)

This expression can be rearranged to give residential rent

$$r_x(s) = \left[ w(z) - t(w(z)) - c(s,z) - (\bar{w} - t(\bar{w}) - \bar{c} - \bar{r}\bar{a}/\bar{x}) \right] x(s)/a(s).$$ (5)

Unit $s$ has either commercial or residential use according to which offers the higher return. Furthermore, land in the city must yield rent at least as great as land in the alternative 'outside' use so, for $s$ in the city,

$$\max[r_x(s), r_y(s)] \geq \bar{r}.$$ (6)

Finally, the supply of workers in the city equals labour demand, so

$$\sum_s x(s) = \sum_s y(s).$$ (7)

A special case of this – on which we will focus – is one in which the spatial structure of the city is such that all workers who commute travel to a common central unit, (unit 0), so commuting costs at $s$ are $c(s) = c(s, 0)$. Notice, from (4), that in this case the wage no longer depends on the location of the job, $z$, but instead takes a common value throughout the city, denoted $w$. The residential rent gradient therefore comes directly from commuting costs and the density of activity as,

\footnote{We assume that the direct utility of consuming housing is the same on all residential lots, and is not included in this equation.}
\[ r_x(s) = [w - t(w) - c(s) - (\bar{w} - t(\bar{w}) - \bar{c} - \bar{c}/\bar{x})]x(s)/a(s). \quad (5') \]

Commercial rents are

\[ r_y(s) = [g(\phi(s)) - w]y(s)/a(s). \quad (2') \]

The set of spatial units now organises into three areas. Units for which \( r_y(s) > r_x(s) \) are occupied by firms, and will be referred to as the central business district (CBD). If \( s \) is a continuous variable (such as distance from the centre), then the edge of this area, denoted \( s_y \), is given by \( r_y(s_y) = r_x(s_y) \). Units for which \( r_x(s) > r_y(s) \) and \( r_x(s) > \bar{r} \) are occupied by urban workers. There is a rent gradient in this area, holding workers indifferent between units. The city edge is denoted \( s_c \), implicitly defined by \( r_x(s_c) = \bar{r} \). Other units are outside the city, with alternative land use generating rent \( \bar{r} \).

4. Effects of a transport improvement

A transport improvement is modelled as a reduction in commuting costs, \( c(s) \), for some locations. In this section we put further structure on the model in order to derive analytical results about the effects of a transport improvement on real income, before returning to the more general model for the numerical simulations of section 5.

The further structure of this section involves assuming, first, that the CBD occupies zero area, so commercial density \( y(s)/a(s) \) is infinite. Second, we assume continuous and homogenous space, and let \( s \) denote the distance from the CBD. The population resident within the city at distance \( s \) from the centre is \( (1 + \theta)k s^\theta \). The parameter \( \theta \) measures the way in which population varies with distance from the centre, capturing both the amount of residential land at each distance and lot size. Thus, if the city occupies a one dimensional line and all residential lots are the same size, then \( \theta = 0 \); i.e. there are \( k \) workers at each unit distance from the centre. Alternatively, if the city is circular and all lot sizes are the same, then the population at distance \( s \) from the centre is proportional to \( 2\pi s \), the circumference of the circle of radius \( s \). This case corresponds to \( \theta = 1 \), \( k = \pi \). Parameter \( \theta \) is less than unity in a circular city in which lot size is larger further away from the CBD, and could be negative if lot size increased faster than the square of distance.

Given this formulation the total population within radius \( s \) of the centre is \( X = k s^\theta + 1 \)
(integrating across the city, see appendix 1). From now on we normalise \( k = 1 \), so a city of size \( s \), accommodates \( X = s^{\theta + 1} \) workers. Commuting costs are an increasing function of distance from the CBD, given by iso-elastic function \( c s^\lambda \), \( \lambda > 1 \). Edge commuting costs in a city with population \( X \) are therefore \( c s^\lambda = c X^{\lambda/(\theta + 1)} \). Workers are completely mobile, so the saving in commuting costs from being closer to the centre is exactly offset by rent payments, as in equation (5). We need expressions for total commuting costs and for rent, and these can be found by integrating across the city. Defining the parameter \( \gamma = (1 + \lambda + \theta)/(1 + \theta) \) these take the following form, derived in appendix 1.

### Table 1: Commuting costs and land rents with urban population \( X \).

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Linear city, constant lot size, linear transport costs, ( \theta = 0, \lambda = 1, \gamma = 2 )</th>
<th>Linear city, constant lot size, quadratic transport costs ( \theta = 0, \lambda = 2, \gamma = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>edge commuting cost =</td>
<td>( c s^\lambda = c X^{\gamma - 1} )</td>
<td>( c X )</td>
<td>( c X^2 )</td>
</tr>
<tr>
<td>rent + commuting cost.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total commuting costs</td>
<td>( X^\gamma (c/\gamma) )</td>
<td>( c X^2/2 )</td>
<td>( c X^3 (1/3) )</td>
</tr>
<tr>
<td>Total rent</td>
<td>( X^\gamma c(\gamma - 1)/\gamma )</td>
<td>( c X^2/2 )</td>
<td>( c X^3 (2/3) )</td>
</tr>
</tbody>
</table>

See appendix 1 for calculations underlying this table; \( \gamma > 1 \).

Thus, the parameter \( \gamma \) essentially measures the rate at which the city runs into diminishing returns because of commuting costs. The larger is \( \gamma \) the more convex is the relationship between population and total commuting costs, as is the case if transport costs are convex in distance, or if lot size is increasing as population is added to the edge of the city.

The equilibrium condition defining the edge of the city, \( r_x(s_x) = \bar{r} \), becomes, with equation (5), the simplifying assumption that \( a(s_x)/x(s_x) = \bar{a}/\bar{x} \), and our specification of commuting costs

\[
w - t(w) - cs^\lambda = \bar{w} - t(\bar{w}) - \bar{c}.
\]  

(8)

Since the CBD occupies no land it is also the case, from the production function, (1) with equation (2), that
\begin{equation}
    w = q(X) \tag{9}
\end{equation}

(where total employment equals city population \(X\), and \(\psi(0, 0) = 1\), so \(\phi(s) = X\)). Edge commuting costs in a city with \(X\) workers are \(cX^{\gamma-1}\) (table 1). Using this and equation (9) in (8), gives equilibrium city population \(X\) implicitly defined by

\[q(X) - t(q(X)) - cX^{\gamma-1} = \bar{w} - t(\bar{w}) - \bar{c}. \tag{10}\]

This is the key equilibrium condition, determining city size, \(X\); the condition ensures that workers are indifferent between living in a city with population \(X\), or living in the hinterland.

We are now in a position to see how changes in parameters – such as commuting costs – affect city population, productivity and real income. Differentiating (10), the effect of a transport improvement on city population is

\[\left[b'(1 - t') - (\gamma - 1)cX^{\gamma-2}\right]dX = X^{\gamma-1}dc. \tag{11}\]

The term in square brackets is negative (at a stable equilibrium), so \(dX/dc < 0\). As expected, a reduction in commuting costs increases city size.

The real income change associated with the transport improvement and consequent population increase is measured by the value of any extra output produced minus the change in the cost of commuting. (See appendix 2 for the statement of real income and the derivation of (12)). This is given by

\[dU = Xq'dX + (w - \bar{w})dX - (cX^{\gamma-1} - \bar{c})dX - (X^\gamma/\gamma)dc. \tag{12}\]

The first two terms are the change in production. The first comes from increasing productivity of existing city workers (if \(q' > 0\)) and the second from expanding city employment, in which productivity is higher by amount \((\bar{w} - \bar{w})\). Offsetting this, new city workers have to pay edge of city commuting costs, \(cX^{\gamma-1}\), rather than outside commuting costs of \(\bar{c}\). The final term is the direct cost saving of the transport improvement (the derivative of total commuting costs in table 1). Equilibrium condition (10) relates the productivity gap to commuting costs, and using this gives,
\[ \frac{dU}{dc} = -\frac{X^\gamma}{\gamma} + \frac{dX}{dc} \left[ Xq' + t(w) - t(\bar{w}) \right]. \]  

(13)

The first term is the direct effect of the transport improvement on total commuting costs. Remaining terms give the value of the change in the number of workers, creating benefit through two distinct mechanisms. The first, \(Xq'\), is the agglomeration externality, so increasing urban employment raises the productivity of existing workers if \(q' > 0\). The second is a tax distortion, which can be expressed as the difference between wages inside and outside the city times the marginal tax rate, \((w - \bar{w})t'(w)\). The intuition is that there is a tax wedge between the extra income earned by moving to the city, and extra commuting costs that are paid out of after tax income. Notice that this tax argument does not apply to leisure travel (where the choice is between consuming travel or consuming other goods, both generating utility and both paid for from after tax income) or commercial traffic (where costs and benefits are internal to a firm, and not separated by a tax wedge).

Equations (11) and (13) can be combined to give the real income change from the transport improvement as

\[ c \frac{dU}{dc} = -\frac{cX^\gamma}{\gamma} \left[ 1 + \left( \frac{\gamma}{c} \right) \frac{Xq' + t(w) - t(\bar{w})}{c(\gamma - 1)X^{\gamma - 1} - (1 - t')Xq'} \right]. \]  

(14)

although it is often more informative to look at (11) and (13) separately. We also note that the same expression can be derived by evaluating the change in land rents plus government revenue, rather than the change in output (see appendix 2).

This analysis is illustrated in the four panels of figure 1, the vertical axis of which measures costs and benefits per worker. The horizontal axis measures the number of workers who, for the purpose of these figures, each occupy one unit of land in a linear city with linear commuting costs, \(\theta = 0, \eta = 1, \gamma = 2\); the horizontal axis is therefore also distance from the CBD, point 0. In each figure the upward sloping rays through the origin are the costs of commuting to the CBD from each of these locations.

In figure 1a the wage gap between city workers and outsiders is assumed constant at \((w - \bar{w})\), given by the horizontal line. The size of the city is determined at point \(X\), where the wage gap is equal to the travel costs of the most distant city worker. At point \(X\) no further workers want to be employed in the city. Workers located closer to the CBD face lower commuting costs but higher rents, as given by the distance between the horizontal line
More precisely, this is the extra rent paid by a city dweller, over and above that paid by an outsider.

A transport improvement has the effect of shifting the commuting cost schedule downwards, as illustrated in figure 1b. In this figure we assume that there is no productivity effect, so \( q' = 0 \) and the wage gap remains constant at \( (w - \bar{w}) \), and no taxation, so \( \iota' = 0 \). Areas in the figure correspond to terms in equations (12) and (13) as follows:

\[
\begin{align*}
\text{Area } \alpha &= X^{\gamma}/\gamma & \text{Direct cost saving.} \\
\text{Area } \beta + \eta &= (w - \bar{w})dX & \text{Extra output from new city workers.} \\
\text{Area } \eta &= (cX^{\gamma-1} - \bar{c})dX & \text{Commuting cost of new city workers.}
\end{align*}
\]

To a first order approximation benefit \( \beta + \eta \) and cost \( \eta \) net out as, absent taxes, the benefits of moving to the city equal the marginal cost, so the only benefit is the direct cost saving. For a non-marginal change there is also the second order gain, \( \beta \), giving real income change \( \alpha + \beta \). In our framework \( \alpha + \beta \) is the value of the improvement that would be obtained from a standard cost-benefit analysis.

Figure 1c adds an endogenous productivity effect to the picture. Agglomeration benefits mean that increasing city size increases its productivity, so that instead of a constant wage gap \( (w - \bar{w}) \) there is now the ‘wage gap curve’ illustrating the relationship \( q(X) - \bar{w} \). There is now an induced productivity response which has two consequences. First, the change in city size from the transport improvement is larger. And second, the higher productivity of existing city workers raises real income by area \( \delta \).

\[
\text{Area } \delta = Xq'dX & \text{Induced productivity gain.}
\]

The real income gain is now \( \alpha + \beta + \delta \).

Figure 1d completes the illustration, adding a tax wedge. The productivity gain from the city is the curve wage gap whereas workers’ decisions are taken on the basis of the post tax wage gap curve. Consequently, the wage and output increment produced by marginal city

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More precisely, this is the extra rent paid by a city dweller, over and above that paid by an outsider.
workers now exceeds their commuting costs. Commuters only receive the net of tax share of
the extra output they produce, but pay the whole of commuting costs. The difference, area \( \epsilon \),
accrues to government as tax revenue, so,

\[
\text{Area } \epsilon = \left[ t(w) - t(w') \right] dX = t'(w)(w - w') dX. \quad \text{Tax revenue.}
\]

Notice also that the appropriate tax rate is that paid on the increment to earnings of new city
workers, so is the marginal rate \( t'(w) \), not an average rate. The real income gain is now \( \alpha + \beta 
+ \delta + \epsilon \). Comparing figures 1a and 1c we see that productivity and tax effects generate
additional benefits \( \delta + \epsilon \), and also increase the size of the increase in employment created by a
given size transport improvement.

5. A computable equilibrium model

We now move from the linear approximations of equations (11) - (14) to the results of
numerically implementing the more general model of section 3. In a city with a more complex
geography, what are the effects of a non-marginal change in commuting costs on some, but not
all, of the transport links in the city?

The economy has 20,000 spatial cells and a transport network defined by four lines
that intersect at a single point. In equilibrium, this point will form the centre of the CBD to
which workers commute. The CBD occupies an endogenously determined number of cells
around this intersection, and cells further out are residential, until the (endogenous) city edge
is reached. We initially assume that commuting costs are linear in distance, all cells are the
same area, and business occupancy (workers per cell) is greater than residential occupancy.
Production technology takes the form

\[
q(\phi(s)) = a + b \left[ \sum_{z} \frac{y(z)}{d(s,z)} \right]^\mu
\]

(15)

where \( d(s, z) \) is the distance between two points in the CBD so the term in square brackets is a
measure of the economic mass of the CBD. The parameter \( \mu \) measures the impact of this mass
on average productivity, so agglomeration effects are present if \( \mu > 0 \).

Figure 2 illustrates the initial equilibrium. The horizontal axis gives locations on one
of the four transport lines, and the vertical gives land rents. The CBD occupies cells within \( s_y \)
of the centre, and rents in this area are given by \( r_s \). Because of proximity effects in the production function, rents peak at the centre of the CBD. Residential rents are given by \( r_s \), declining linearly to the city edge \( s_x \), where they equal outside rents \( r_F \). The area of the residential district is determined by city population, and the area of the CBD determined by the number of people working there – both endogenous variables. Equilibrium values to be calculated are city population and employment, productivity, wages, and commercial and residential rents.

Although the model is not calibrated to real data, key parameters have been chosen to be of realistic orders of magnitude. Thus:

- City/ non-city productivity gap: 20%
- Commuting cost share of city income: 7%
- Residential rent share of city income: 18%
- Commercial rent share of city income: 7%
- Residential rent share of non-city income: 15%
- Commercial rent share of non-city income: 3%

The key parameter is the elasticity of productivity with respect to city employment, and we report results with this ranging in value from 0 (a benchmark case) to 0.077.

**Effect of a transport improvement**

The experiment we undertake is to reduce commuting costs by 20% on one of the four lines in the city. Figure 3 illustrates effects. Residential rent gradients on this line are flattened, as the relative advantage of cells closer to the CBD is reduced. The city edge then expands as more locations can access the CBD sufficiently cheaply. Increased CBD employment has a positive productivity effect, and this is transmitted into CBD land rents, giving the upwards shift illustrated. The increase also shifts residential rent levels upwards. While the change in slope of the residential rent gradient occurs only along the line that experiences the transport improvement, the upwards shift affects all lines. Thus, increased CBD productivity causes a further expansion in city size as the return to commuting along all lines increases.

The magnitude of the real income effects are given table 2. The first row is a benchmark case, and the second and third look at tax and productivity effects separately. Remaining rows combine tax and productivity effects, looking at successively higher elasticities of productivity with respect to CBD employment. In the benchmark case the
change in commuting costs (the direct cost saving offset by an increase in numbers commuting) is 0.35% of initial city income and there is a substantial increase in CBD employment, of nearly 6%. The overall gain is 0.44% of base city income, exceeding the direct effect because of the presence of a non-marginal ‘triangle’, like $\beta$ in figure 1b.

The presence of 33% marginal tax wedge (row 2) increases the total gain by nearly three-quarters. The productivity gap between city and non-city workers is 20%, so increasing urban employment by 5.8% gives an increase in tax revenue which amounts to 0.308% of initial city income. In the third row the elasticity of productivity with respect to CBD employment is set at 0.045, and tax rates assumed to be zero. The additional gain then comes from employment growth raising productivity, and its effect is to make the overall gain more than twice the size of the change in commuting costs alone. The remaining three columns have both the tax wedge ($t' = 0.33$) and a productivity response, set at increasingly high rates ($q' = 0.022, 0.045, 0.077$). The higher the elasticity of productivity with respect to employment the greater the increase in city employment and the greater the overall economic gain. When the elasticity reaches 0.077 the overall gain amounts to 1.88% of initial city income, approximately 5 times the value of the change in commuting costs, and 4 times larger than the base case welfare gain (0.44% of income, column 1).
Table 2: Real income gains from transport improvement. \((\gamma = 2)\).

<table>
<thead>
<tr>
<th>(q' = 0, t' = 0)</th>
<th>1: Commuting cost reduction (%) base income</th>
<th>2: Urban Employment change (%)</th>
<th>3: Value of productivity increase (%) base income</th>
<th>4: Increase in tax revenue (%) base income</th>
<th>5: Full effect (%) base income</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.351</td>
<td>5.91</td>
<td>0</td>
<td>0</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>(q' = 0, t' = 0.33)</td>
<td>0.332</td>
<td>5.8</td>
<td>0</td>
<td>0.308</td>
<td>0.738</td>
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<tr>
<td>(q' = 0.045, t' = 0.00)</td>
<td>0.347</td>
<td>7.9</td>
<td>0.354</td>
<td>0</td>
<td>0.909</td>
</tr>
<tr>
<td>(q' = 0.022, t' = 0.33)</td>
<td>0.329</td>
<td>6.65</td>
<td>0.145</td>
<td>0.389</td>
<td>0.987</td>
</tr>
<tr>
<td>(q' = 0.045, t' = 0.33)</td>
<td>0.327</td>
<td>7.45</td>
<td>0.337</td>
<td>0.481</td>
<td>1.277</td>
</tr>
<tr>
<td>(q' = 0.077, t' = 0.33)</td>
<td>0.324</td>
<td>9.15</td>
<td>0.709</td>
<td>0.662</td>
<td>1.881</td>
</tr>
</tbody>
</table>

Residential density, commuting cost gradients and the change in city size.

The simulations reported in table 2 were conducted with commuting costs linear in distance, and residential units spread at constant density along each of the travel lines – essentially like the linear city case \((\gamma = 2)\) of section 4. The effect of these assumptions is to give quite large increases in commuting and in CBD employment following the transport improvement. This quantity response is smaller if the parameter \(\gamma\) is larger. As we discussed in section 4, \(\gamma\) captures several underlying parameters. Thus, \(\gamma\) is higher if commuting costs are a convex function of distance, \(\lambda > 1\); a proportional reduction in commuting costs then brings a smaller increase in city size and employment, because city expansion always moves the marginal resident to a steeper part of the commuting cost schedule. Alternatively, \(\gamma\) is larger if residential population density falls off sufficiently sharply with distance from the centre – so the parameter \(\theta\) is negative. Table 3 reports results where \(\gamma = 3\), supported by either commuting costs increasing with the square of distance \(\lambda = 2\), or equivalently by population density.
density falling off with $\theta = -1/2$.

This change more than halves the change in city employment, bringing it down to around 3%. In the central case in which the tax rate is 0.33 and the elasticity of productivity is 0.045 the overall gain amounts to 0.53% of base income, as compared to 1.28% in table 2. However, the change also reduces the value of the direct change in commuting costs – more commuters are close to the centre so receive relatively little gain from the commuting cost reduction. Thus the ratio of the full effect (column 5) with tax and productivity effects present ($t' = 0.33, q' = 0.045$) to that with them absent is 0.53/0.214 = 2.5. The analogous number in table 2 is 1.277/0.44 = 2.9. Thus, while dampening the employment effect reduces the overall gain from the project, the ‘multiplier’ effect due to tax and productivity change remains comparable, and very large.

Table 3: Real income gains from transport improvement: robustness. ($\gamma = 3$).

<table>
<thead>
<tr>
<th></th>
<th>1: Commuting cost reduction</th>
<th>2: Urban Employment change</th>
<th>3: Value of productivity increase</th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>% base income</td>
<td>%</td>
<td>% base income</td>
<td>% base income</td>
<td>% base income</td>
</tr>
<tr>
<td>$q' = 0$,</td>
<td>0.18</td>
<td>2.91</td>
<td>0</td>
<td>0</td>
<td>0.214</td>
</tr>
<tr>
<td>$t' = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q' = 0.0$</td>
<td>0.172</td>
<td>2.91</td>
<td>0</td>
<td>0.122</td>
<td>0.324</td>
</tr>
<tr>
<td>$t' = 0.33$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q' = 0.045$</td>
<td>0.177</td>
<td>3.33</td>
<td>0.148</td>
<td>0</td>
<td>0.395</td>
</tr>
<tr>
<td>$t' = 0.00$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q' = 0.045$</td>
<td>0.17</td>
<td>3.33</td>
<td>0.148</td>
<td>0.172</td>
<td>0.53</td>
</tr>
<tr>
<td>$t' = 0.33$</td>
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</table>

6. Conclusions and qualifications

The results of this paper suggest that there are significant gains from urban transport improvements, over and above those that would be contained in a standard cost-benefit appraisal. Productivity in the city may be increased by additional urban employment; this externality raises the productivity of existing as well as incoming workers, creating real income gain. In addition, the combination of income taxation and an urban/ non-urban
productivity differential is a source of gain, even if productivity levels are constant. Moving workers to higher productivity jobs raises real income and raises tax revenue; extra commuting costs are incurred, but the presence of a tax wedge in the equilibrium condition between the wage gap and the marginal commuting cost means that the increase in output exceeds the increase in commuting costs. The calculations undertaken in the paper, using econometric estimates of the relationship between city size and productivity, suggest that these effects are large – typically yielding total gains several times larger than those that would be derived from a standard cost-benefit analysis.

These large effects must however be qualified by several remarks. The first is that, in the model, journeys are made only for commuting. If journeys are also made for other reasons then this increases the benefits calculated in a standard cost benefit, while holding the other effects constant. The multiplier associated with the tax and productivity effects is therefore correspondingly smaller.

Second, our modelling assumes that productivity outside the city remains unchanged. Thus, while expanding city employment raises productivity, reducing employment outside the city does not reduce outside productivity. Obviously, if activities outside the city experience a reduction in productivity following a decline in employment levels, then the overall productivity effect would be reduced. The model would then suggest that transport investments have additional benefit in cities where the employment productivity relationship is strongest. The reality here is likely to be complex. The mechanisms work not only by drawing more people into the city as a whole, but also by enabling more of the city’s initial inhabitants to work in the CBD. Transport improvements increase the effective density of activity – in London terms, making docklands closer to the City. The productivity relationship varies across sectors, and is generally strongest in those sectors that are clustered in large cities. All this means that, as usual, implementation of policy based on these arguments requires careful identification of where the market failures – tax wedges and agglomeration externalities – are largest.

Despite these qualifications, the message of this paper is clear. The same forces that cause cities to exist – agglomeration benefits – provide additional effects that should be included in urban transport appraisal. Estimating their exact size remains the subject of future work, but to ignore them is surely to miss one of the benefits of urban transport improvements.
Appendix 1:

The population in a city of radius $s$ is:

$$X = \int_0^s (1 + \theta) c z^\theta \, dz = s^{1 + \theta} \quad \text{so} \quad X^{1/(1 + \theta)} = s.$$ 

The cost of commuting distance $z$ is $cz^\lambda$; constant commuting costs per unit distance are $\lambda = 1$, and $\lambda > 1$ implies convex commuting costs. Total commuting costs, $C$, are:

$$C = \int_0^s (1 + \theta) c z^\lambda z^\theta \, dz = \frac{c(1 + \theta)}{1 + \lambda + \theta} s^{1 + \lambda + \theta} = \frac{c X^\gamma}{\gamma}, \quad \gamma = (1 + \lambda + \theta)/(1 + \theta).$$

Edge commuting costs are $cz^\lambda = c X^\gamma - 1$. At each point rent plus commuting costs equal edge commuting costs. Total rent, $R$, is therefore $c X^\gamma$ minus total commuting costs, i.e.

$$R = X^\gamma c(\gamma - 1)/\gamma.$$ 

Appendix 2:

Real income in the economy is given by:

$$U = Xq(X) - X^\gamma c/\gamma + (\bar{w} - \bar{c})(\bar{X} - X)$$

$$= X^\gamma c(\gamma - 1)/\gamma + Xt(q(X)) + (\bar{X} - X)t(\bar{w}) + \bar{X}[\bar{w} - \bar{c} - t(\bar{w})]$$

The first expression is output in the economy (in which $\bar{X}$ is total employment) minus commuting costs. The second is the sum of urban land rents (table 1), tax revenue, and the disposable income that would be earned if all workers, $\bar{X}$, were in the outside region. The two expressions are equal, as can be seen by using equation (10). Equation (12) comes from differentiating the first expression. Equivalently, differentiation of the second gives

$$\frac{dU}{dc} = X^\gamma(\gamma - 1)/\gamma + \frac{dX}{dc}[c(\gamma - 1)X^{\gamma - 1} + t(q(X)) - t(\bar{w}) + \bar{X}'q]$$

Using $dX/dc$ from equation (11) and rearranging gives equation (14) of the text.

Appendix 3:

Simulation: Gauss code available on request from the author.

In the production function, $a = 1$, $b = 0.1$, $\mu = 0.2, 0.3, 0.4$, elasticity of output per worker with
respect to CBD employment computed numerically in neighbourhood of initial equilibrium. Residential density 1/5 of CBD density. Other parameters as described in text.

References:
Figure 1a: Urban equilibrium

Figure 1b: Net gains from transport improvement
Figure 1c: Net gains from transport improvement with endogenous productivity

Figure 1d: Net gains from transport improvement with endogenous productivity and tax wedge \((t)\)
Figure 2: Rent gradients

Figure 3: Rent gradients following a transport improvement
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