Problem set 4

This problem set is due at 12 noon ** Late problem sets are not accepted— no exceptions. See the syllabus for details.

In this problem set, you are asked to work through a basic "principal-agent" model of a firm-worker relationship (see e.g. Milgrom and Roberts, 1992, Ch. 7; Borjas, 2000, pp. 445-446; Polachek and Siebert, 1993, Sec. 9.2). This is supposed to complement my rather verbal lectures on insurance, incentives and selection in employment relations.

Consider a farm (firm) that hires a worker to work on its land for a day. The output the worker will produce depends on the effort (e) exerted that day. The worker can either shirk ($e = 0$) or work ($e = 1$). The value y of the day's output depends on both effort and chance, but the worker can be expected to be more productive if working. To be precise, if the worker shirks ($e = 0$), (s)he produces £100 worth of output with probability 1/4 and £60 worth of output with probability 3/4. By working ($e = 1$), the worker can turn faith around and produce £100 with probability 3/4 and £60 with probability 1/4. We can summarize this as follows:

	$e = 0$	$e = 1$
y	probability	
£100	1/4	3/4
£60	3/4	1/4

At the start of the day, the firm proposes a wage contract to the worker. This contract could either specify a fixed wage w independently of output y, or a wage w that is contingent on (varies with) output y. If the worker accepts the proposed contract, the firm's profits are $y - w$. At the start of the day, the firm does not know what the value of output y will be. The actual wage w that has to be paid at the end of the day may be unknown as well if it depends on y (according to the agreed contract). Thus, profits $y - w$ are generally subject to risk. We assume that the firm is risk-neutral and chooses contracts as to maximize expected profits. These expected profits are $[y - w] = [y] - [w]$. For example, if the firm and worker agree on a fixed wage of £60 and the firm knows the worker will shirk, expected profits are

$$(1/4) \cdot £100 + (3/4) \cdot £60 - £60 = £10.$$

The worker derives utility $\sqrt{x}$ if his or her day's earnings are £x for sure. If the worker accepts the contract proposed by the firm, the day's earnings x are the wages w specified by the contract minus the monetary costs of effort. These effort costs are £0 if the worker shirks ($e = 0$) and £10 if the worker works ($e = 1$). Wages w, and therefore wages net of effort costs, may be uncertain, in which case the worker maximizes expected utility. For example, if the firm and

worker agree on a contract that pays $w = £80$ to the worker if $y = £100$ and $w = £70$ if $y = £60$, then the worker will have (ex ante) expected utility

$$(3/4) \cdot \sqrt{80 - 10} + (1/4) \cdot \sqrt{70 - 10} = (3/4) \cdot \sqrt{70} + (1/4) \cdot \sqrt{60}$$

if (s)he decides to work ($e = 1$). If the worker rejects the contract proposed by the firm, (s)he can earn a fixed (non-random) wage £70 net of effort cost elsewhere in the economy that day. Thus, the worker will not accept any contract that delivers an expected utility that is strictly less than $\sqrt{70}$. This is called the worker's "participation constraint".

1. First, ignore moral hazard problems and suppose that the firm and worker can write and enforce contracts that do not only specify a compensation (wage) scheme but also a required effort level.

   (a) First, suppose that the firm decides that it wants the worker to shirk ($e = 0$) and proposes a contract that requires (actually, allows) the worker to do so. The contract either specifies a fixed wage w independent of y or wage payments w that are contingent on y. What contract would the firm propose? What are the risk-sharing properties of this contract? Give intuition for this result.

   [Hint: The contract should maximize profits subject to the worker participation constraint. Formally, you could solve this by solving a constrained maximization problem (using the Lagrange method). In this case, however, it is easy to first reason (verbally) which type of wage contract— either a fixed-wage or output-contingent contract— the firm prefers and then find a contract of this type that maximizes profits under the worker's participation constraint.]

   (b) What contract would the firm propose in the case it wants (and therefore requires in the contract) that the worker works ($e = 1$)?

   (c) Does the firm propose the "shirking" contract of (a) or the "working" contract of (b)? [In other words: does the firm want the worker to shirk or to work?]

   (d) Is this contract chosen by the firm (Pareto) efficient? Explain.

2. In reality, there is usually asymmetric information. The worker knows whether (s)he is working, but the firm only observes output and cannot disentangle the effects of chance and worker effort on output. In this case, contracts cannot be contingent on actual effort. They can only specify wage payments w, possibly contingent on output y.

   (a) First, suppose the firm restricts attention to fixed-wage contracts, that is, contracts that promise payment of a fixed wage w independently of output y. What level of effort will the worker choose under any fixed-wage contract (s)he finds acceptable? Which fixed-wage contract will the firm choose?

(b) Alternatively, suppose that the firm proposes a contract that pays all output to the worker, that is $w = y$. If the worker would accept this contract, would (s)he work or shirk? Is this contract acceptable to the worker (that is, does it satisfy the worker's participation constraint)?

(c) How does the efficiency of the contract in (b) compare to that of the optimal fixed-wage contract in (a)?

[Hint: Simply compare the firm's expected profits and the worker's expected utility between both contracts.]

(d) What are the risk-sharing and incentive properties of both contracts? Do you think either contract will be chosen by the firm? If not, how does the contract chosen by the firm compare to these two contracts in terms of the insurance and incentives provided? [Here, you only have to give a qualitative answer.] In particular also, would the firm choose a contract that induces the worker to work? Is the firm's optimal contract efficient among the contracts that do not (cannot) specify effort levels directly (that is, subject to the asymmetric information constraint)?

(e) Shortly discuss how asymmetric information affects efficiency in this example.

3. Finally, suppose there are actually two types of workers. The first, called the "normal" worker, is just like the worker we have considered so far. The second, called the "lazy" worker, is the same as the normal worker, except that (s)he is extremely lazy: the costs of effort are so large that (s)he will always shirk, however strong the incentives (dependence of wages on output) in the contract. The firm cannot tell directly whether it is dealing with a normal or a lazy worker. However, if the worker is offered a choice of contracts, his or her choice of contract may be informative.

(a) Suppose that the firm does not offer the worker a single contract, but rather a "menu of contracts" from which the worker can choose the contract (s)he prefers. In particular, suppose that the firm offers a menu of two contracts, (i) a contract that specifies a fixed wage $w = £70$ and (ii) a contract that specifies $w = y$.

What contract will a normal worker choose? And what contract will a lazy worker choose? In both cases, check that the worker's participation constraint is satisfied.

(b) Suppose that the firm only offers the second contract, $w = y$. What happens?