Schooling, Nation Building, and Industrialization

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Abstract

We model a two-region country where value is created through bi-
ilateral production between masses and elites. Industrialization requires
the elites to finance schools and the masses to attend them. While
schools always raise productivity, only the implementation of schools in
both regions renders the masses mobile across regions (“unified school-
ing”). Alternatively, schools can be implemented in one region alone
(“regional education”) or the dominant group at the regional level can
choose to implement schooling in its own region but refuse to share
the associated costs and benefits within the wider country-level group
(“secession”). We show that if the industrialization shock generates
strong incentives for the masses of both regions to attend school, then
unified schooling is implemented whenever the dominant elite is the
same at the country and at the regional level. If instead the bour-
geoisie is dominant in one region and the nobility is dominant at the
country level, the bourgeoisie of that region may promote the secession
of the region. For smaller productivity shocks, we show that only the
masses of one region may have incentives to attend school. In that case,
the elites of that region also choose to favour secession. Empirically,
our model predicts that we should not observe countries implementing
schooling in only part of their territory, as this is dominated either by
its implementation in all the territory or by the secession of the region
supporting schooling.

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1 Introduction

Political scientists, historians and anthropologists have extensively discussed the issue of the historical genesis of Nations and Nationalism (see e.g. Smith, 2000, for a summary of the debate). While “perennialists” argue that national identities have existed for a long period of time (see e.g. Armstrong, 1982, or Hastings, 1997), “modernists” situate the birth of Nations and Nationalism during Industrialization.

In particular, Gellner (1964, 1983) has been very influential in arguing that both Nations and Nationalism result from the implementation of mass educational systems to get workers ready for industrialization. As stated by Breuilly (2006, p. xxxiv), “Gellner insisted that industrialization required or entailed cultural homogenization based on literacy in a standardized vernacular language conveyed by means of state supported mass education”. At the same time, workers become mobile through schooling because they acquire a common national identity that enables them to communicate with each other. In addition, as mass education is expensive, Gellner (1983) argues that the minimum size for a viable modern political unit is determined by the ability to finance such an educational system. More recently, Breuilly (1993) has criticized Gellner’s theory and other theories of nationalism because they failed to stress that nationalism is about power and state control, and has argued that “the central task is to relate nationalism to the objectives of obtaining and using state power” (Breuilly, 1993, p. 1). However, Breuilly (1993) chooses not to develop a theory and provides instead a typology of different historical cases.

We contribute to the literature by developing a theoretical model that relates nation building and industrialization, and aims at the same time at presenting nation-building as resulting from the interaction of social groups holding power.

To this purpose, we model a two-region economy populated by masses and by two elite groups (nobility and bourgeoisie). Regions are heterogeneous in the size of their bourgeoisie. Political power is in the hands of one of the elite groups, referred to as the “dominant group”, which is not necessarily the same at the regional and at the country level. The dominant group decides how the costs of schooling are shared within the elite. Value is created through bilateral production between the members of the elites and the members of the masses. Initially, the country is a rural society. Production takes place only within each region. There are clear rules establishing how the masses share production with the nobility, but the property rights of bourgeois are not well established, and mass members can grab the
entire surplus from the match with a bourgeois with a positive probability (“stealing”).

The economy is hit by a productivity shock representing an industrialization opportunity which can raise the productivity of the masses. In order to be more productive, mass members need however to attend school. In addition, educated mass members cannot steal from the bourgeois. The set-up of the schooling system can only be financed by the elites, but mass members decide whether to attend school or not.

The politically dominant country-level elite can choose to implement schooling in one region only (“regional education”) in which case only within-region production is possible. Alternatively, it can choose to implement schools in both regions (“unified education”), which creates a common national identity and makes it possible for the masses of one region to produce with the other region’s bourgeois. Finally, we consider the possibility that the dominant region-level elite implements schooling in its own region but refuses to share the associated costs and benefits within the wider country-level group (“secession”).

Our set-up assumes that the bourgeoisie benefits more from schooling than the nobility: while both enjoy a higher match productivity, the bourgeoisie gains stable property rights and therefore no longer fears losing its production to the masses. Moreover, under unified schooling the match pool of the bourgeoisie increases. This points to a potential conflict of interests on school implementation between the bourgeoisie and the nobility.

We show that this potential conflict of interest materializes under unified schooling, and as a result schooling is more likely to be implemented under that system when the bourgeoisie is the dominant group. Instead, if only one region gets educated, no conflict arises at equilibrium because the masses are not willing to get schooled when schooling would go against the interest of the nobility. Hence, under regional schooling whether or not schools are implemented does not depend on the identity of the dominant elite group.

Across educational systems, we show that if the industrialization shock generates incentives for the masses of both regions to attend school, then unified schooling is preferred to both regional education and to secession whenever the dominant elite is the same at the country and at the regional level. This simply results from our assumption that unified schooling is technologically superior to the other two alternative systems, in the sense that it is the only system that generates mass mobility.

Still, despite this technological advantage, unified schooling can still be dominated by secession in two different types of cases. First, we show that after a weak industrialization shock and in the presence of sufficiently het-
heterogeneous regions, the masses of one region may have stronger incentives to attend school under secession than under unified schooling. Also, if the bourgeoisie is dominated at the country level but dominant at the regional level, the secession of the region will be the preferred outcome for the regional bourgeoisie whenever the industrialization shock is not high enough to render mobility very desirable and low levels of stealing from the bourgeoisie before industrialization generate incentives for the masses to attend schools.

Empirically, as regional education is never an equilibrium outcome, our model predicts that we should not observe countries implementing schooling in only part of their territory, as this is dominated either by its implementation in all the territory or by the secession of the region supporting schooling.

Finally, we discuss other forms of heterogeneity across regions and their effects on nation building and secession. Our results are robust to different pre-industrialization property rights for the bourgeoisie, differences in sizes across the nobility and masses. However, if productivity shocks are unequally distributed across regions - a case that seems to be historically relevant - secession becomes more likely. Transfers from the more advanced region to the less advanced region are too costly to offset the savings in educational costs.

In addition to the political science, history, and anthropology literatures, this project relates to a growing literature which uses standard econometric and economic modelling tools to underline the historical importance of educational systems for nation-building (see e.g. Aspachs-Bracons et al., 2008, and Clots-Figueras and Masella, 2009) or language choices (see Ortega and Tangeras, 2008) or studies the allocation of power across groups (see e.g. Acemoglu and Robinson, 2001 and 2008).

The remainder of the paper is organized as follows. In section 2 we develop the basic model and describe when regional and unified schooling are implementable. Section 3 develops the equilibrium schooling outcome implemented by the dominant elite. Section 4 allows for the possibility of secession and shows when secession occurs. In section 5 we discuss alternative forms of heterogeneity. In section 6 we confront our results with some case studies and conclude. Most proofs are relegated to a technical appendix.
2 The Model

We study a country with two regions $i = 1, 2$. In each region, there are three social groups, namely the masses $M = M_1 + M_2$ and the elite which is split into the nobility $N = N_1 + N_2$ and the bourgeoisie $B = B_1 + B_2$. Political power is in the hands of one of the elite groups, which is referred to as the “dominant” group. The dominant group holds power for historical reasons and is not necessarily the majority elite group. Moreover, while there is one dominant group at the country level, this group is not necessarily dominant in both regions.\footnote{An interesting case from a historical point of view is when the nobility is dominant at the state level, but the bourgeoisie is dominant in one of the two regions.} Let $M > N + B$. We normalize the total size of the elite in the country to $N + B = 1$. For simplicity, we assume that in both regions both the nobility and the masses have the same size, i.e. $N_1 = N_2 = \frac{N}{2}$ and $M_1 = M_2 = \frac{M}{2}$. Without loss of generality, we assume that $B_1 > B_2$.

Value is created through bilateral production between members of the elites and members of the masses. Initially, the country is a "rural" society. Production takes place only within each region and the surplus from each match is normalized to 1. Stable sharing rules have evolved between the masses and the nobility. However, the protection of property rights of the bourgeoisie is less complete than those of the nobility. While the sharing rule stipulates that a member of the masses who is matched to a member of the elite keeps $\beta$ of the surplus generated from the match, the entire surplus might be stolen from a bourgeois (but not from a nobleman) with probability $\alpha$.

This rural society is now hit by a productivity shock of size $\sigma$ representing the industrial revolution, which can bring the productivity of matches to $1 + \sigma$. However, a match between an elite member and a member of the masses has a productivity equal to $1 + \sigma$ only if the member of the masses attends school. Otherwise, the productivity of the match remains equal to 1. We also assume that schooling creates stable property rights for the bourgeoisie, and thus that the educated members of the mass lose any chance of stealing.

The set-up of a schooling system can only be financed by the elites, and the members of the masses decide whether to attend school or not.

There are two periods in our model: in the first period the productivity shock is observed and the schooling decision is made. If schools are built, production takes place only in the second period. If schools are not built, production takes place in both periods but the match productivity stays at
the rural level. All agents have a discount factor of $\delta$.

2.1 Payoffs if schools are not implemented

Let $\Psi_j (j = B, N, M_i)$ denote the payoff for group $j$ members when schooling is not implemented. In this case, any nobleman produces an output of 1 with each of the $M/2$ members of the masses living in his region, and gets a proportion $1 - \beta$ of the output. As a result, his payoff is

$$\Psi_N = (1 - \beta)(1 + \delta)\frac{M}{2}. \quad (1)$$

For a bourgeois, the payoff is the same as for a nobleman, except that with probability $\alpha$ the output is fully appropriated by the member of the masses, i.e.

$$\Psi_B = (1 - \alpha)(1 - \beta)(1 + \delta)\frac{M}{2}. \quad (2)$$

Finally, for a member of the masses in region $i$, the pay-off is:

$$\Psi_{M_i} = (1 + \delta) \left( \frac{N}{2} \beta + B_i(\beta(1 - \alpha) + \alpha) \right) \quad (3)$$

i.e. the member of the masses receives $\beta$ from each match with one of the $N/2$ noblemen in the region, and either 1 or $\beta$ (with respective probabilities $\alpha$ and $1 - \alpha$) from each match with the $B_i$ bourgeois living in region $i$.

2.2 Schools

The dominant group chooses whether or not schooling is implemented and how to split the schooling costs among the elite. We assume that the dominant group cannot force the dominated group to pay for schooling if with this payment the dominated group would prefer a world without schooling. This implies that the maximum schooling costs that can be imposed on the dominated elite group leave this group indifferent between the implementation of schooling and the absence of schools.

We also assume that each of the elite groups acts as a single group at the country level, i.e. each group equally shares across regions the benefits from production and the costs from schooling.

Schools can be implemented either in both regions, or in one region only. We assume that the implementation of schools in both regions creates a common identity across regions, which enables the masses of each region to produce with the bourgeois from both regions. This is referred to as
a "unified" schooling system, and denoted by $U$. Instead, if schooling is implemented only in one region, no common identity is created, and thus the masses of each region can only produce with the bourgeois of the same region. This is referred to as a "regional" schooling system, and denoted by $R_i$ ($i = 1, 2$). In both cases, the masses can only produce with the nobility of their region of origin.

2.2.1 Payoffs from schooling

Let $\Pi_{j}^{k}$ denote the payoffs from schooling for group $j = B, N, M_i$ under organizational system $k = U, R_i$. Similarly, denote by $I_e^k$ the cost of setting up schooling system $k$ for a member of the elite group $e = N, B$. We can now calculate the benefits from schooling of each group under the different systems.

After attending school in a unified system, any member of the masses foregoes production in the first period and in the second period (discounted by $\delta$) appropriates a fraction $\beta$ of the amount $1 + \sigma$ produced with each of the $N/2$ noblemen in his region and each of the $B$ bourgeois in the country:

$$\Pi_{M_i}^{U} = \beta (1 + \sigma) \delta \left( \frac{N}{2} + B \right) \quad i = 1, 2. \quad (4)$$

Similarly, any bourgeois pays $I_B^U$ schooling set-up costs, and appropriates a fraction $1 - \beta$ of the amount $1 + \sigma$ produced with the $M$ members of the mass in period 2, i.e.,

$$\Pi_{B}^{U} = -I_B^U + (1 - \beta)(1 + \sigma)\delta M, \quad (5)$$

while the nobility’s payoff depends on its own investment $I_N^U$ and is associated to matches with a smaller pool of members of the mass, namely the $M/2$ living in the nobleman’s region:

$$\Pi_{N}^{U} = -I_N^U + (1 - \beta)(1 + \sigma)\delta \frac{M}{2}. \quad (6)$$

Under region-$i$ schooling, the payoff of any member of the masses in region $i$ is

$$\Pi_{M_i}^{R_i} = \beta (1 + \sigma) \delta \left( \frac{N}{2} + B_i \right) \quad i = 1, 2. \quad (7)$$

where the only difference with (4) is that now only production with the bourgeois in region $i$ is possible.
In turn, each of the $B_i$ region-$i$ bourgeois gets $(1 - \beta)(1 + \sigma)$ in the second period with each of the $M/2$ educated members of the masses in that region, while each of the $B_{-i}$ in the other region gets $(1 + \delta)(1 - \alpha)$ with the $M/2$ uneducated masses of that region. Then, given cross-subsidization across regions, the payoff of a bourgeois is given by the weighted average of these two terms plus the setting-up cost $I^B_i$, i.e.

$$\Pi^B_i = -I^B_i + (1 - \beta)(\delta(1 + \sigma)B_i + (1 + \delta)(1 - \alpha)B_{-i}) \frac{M}{2B}$$

for $i = 1, 2$. \hfill (8)

In turn, each of the $N/2$ region-$i$ noblemen gets $\delta(1 - \beta)(1 + \sigma)$ with each of the $M/2$ educated masses of that region, while each of the $N/2$ noblemen in the other region gets $(1 + \delta)(1 - \beta)(1 + \sigma)$ with each of the $M/2$ uneducated masses of that region, which leads to the payoff

$$\Pi^N_i = -I^N_i + (1 - \beta)(\delta \sigma + 1 + 2\delta) \frac{M}{4}$$

for $i = 1, 2$. \hfill (9)

### 2.3 School attendance by masses

The masses of region $i$ are willing to get educated whenever the payoffs from schooling are higher than the payoffs from no-schooling $\Pi^M_i \geq \Psi^M_i$. This leads to a minimum threshold on the size of the productivity shock for the masses to be willing to get educated. Equalizing (3) and (4), the threshold for unified schooling is:

$$\sigma^U_{M_i} = \frac{2(1 + \delta)(B_i \alpha(1 - \beta) - \beta B_{-i})}{\beta(N + 2B_i)\delta} + \frac{1}{\delta}$$

for $i = 1, 2$. \hfill (10)

Similarly, from (3) and (7), the threshold for region-$i$ schooling is:

$$\sigma^R_{M_i} = \frac{2B_i \alpha(1 - \beta)(1 + \delta)}{\delta \beta(N + 2B_i)} + \frac{1}{\delta}$$

for $i = 1, 2$. \hfill (11)

Due to the increased match pool, masses are willing to get schooled earlier under unified than under regional schooling.

**Lemma 1** $\sigma^R_{M_i} > \sigma^U_{M_i}$ for $i = 1, 2$

**Proof.** By simple algebra. \hfill ■

Lemma 2 shows that the masses of the region with the bigger size of the bourgeoisie have a higher cutoff. This also implies that the cutoff of the masses of region 1 determines when unified schooling is possible.
Lemma 2 \( \sigma^U_{M_1} > \sigma^U_{M_2} \) always and \( \sigma^{R_1}_{M_1} > \sigma^{R_2}_{M_2} \) always.

Proof. Simple algebra shows that \( \sigma^U_{M_1} > \sigma^U_{M_2} \iff \sigma^{R_1}_{M_1} > \sigma^{R_2}_{M_2} \iff B_1 > B_2 \) which is true by assumption. 

The underlying intuition is as follows. The masses lose from education because education eliminates the possibility of stealing from the bourgeoisie. This loss is bigger for the masses with the bigger size of the bourgeoisie. Under the unified education this effect is reinforced by the gains from education in terms of a higher match productivity- which is equally enjoyed by the masses of both regions - and an increased match pool which is bigger for the masses with the smaller bourgeoisie. Under regional schooling gains from education only stem from higher match productivities and hence this gain is now more important in the region with a bigger bourgeoisie, however, this gain does not off-set the loss due to the elimination of stealing.

Finally, Lemma 3 provides a full ranking of the thresholds of the masses:

Lemma 3 The cutoffs for the school attendance of the masses rank as follows:

1. \( \sigma^U_{M_2} < \sigma^U_{M_1} < \sigma^{R_2}_{M_2} < \sigma^{R_1}_{M_1} \) if \( B_2 > \frac{-1+\sqrt{1+4(1-B_1)B_1}}{2} \), or when both 
   \( B_2 < \frac{-1+\sqrt{1+4(1-B_1)B_1}}{2} \) and \( \alpha < \overline{\alpha} = \frac{B_2(1-B_2-B_1)}{(1-\beta)(1-B_1-B_2)} \) are satisfied,

2. \( \sigma^U_{M_2} < \sigma^{R_2}_{M_2} < \sigma^{U}_{M_1} < \sigma^{R_1}_{M_1} \) if both \( B_2 < \frac{-1+\sqrt{1+4(1-B_1)B_1}}{2} \) and \( \alpha > \overline{\alpha} \)

Proof. See Appendix 2.4 Education thresholds for the elites

In this subsection we study the minimum size of the productivity shock that makes the elite willing to provide schooling under the assumption that the masses get schooled when schools are built. Once these thresholds are derived, we will compare them to the thresholds of the masses. Schooling is implemented only if the productivity shock lies above the maximum threshold of the masses and the elites.

The minimum productivity shock that makes the elite indifferent between implementing unified schools or not is such that \( \Psi_e = \Pi^U_e \) with \( e = N, B \). From (2), (1), (5), and (6), the thresholds for the bourgeoisie and the bourgeoisie and the nobility are
\[ \sigma_B^U = \frac{I_B^U + (1 - \beta) \left[ (1 - \alpha)(1 + \delta) \left( \frac{M}{\delta} \right) - \delta M \right]}{M(1 - \beta)\delta} \] (12)

\[ \sigma_N^U = \frac{2I_N^U + (1 - \beta)M}{(1 - \beta)\delta M} \] (13)

Similarly, from (2), (1), (8), and (9), the thresholds under region-i schooling are

\[ \sigma_B^R_i = \frac{2B_i I_B^R + (1 - \beta)(1 - \alpha(1 + \delta))M}{\delta(1 - \beta)M} \quad \text{for } i = 1, 2 \] (14)

\[ \sigma_N^R_i = \frac{4I_N^R_i + (1 - \beta)M}{\delta(1 - \beta)M} \quad \text{for } i = 1, 2 \] (15)

All these thresholds depend on how much the elite has to pay for setting up the schools and hence depending on the size of these costs might be bigger or smaller than the threshold for schooling for the masses. However, it will be useful to understand how these thresholds compare when schooling comes for free to the elite. We will refer to these thresholds as \( \sigma_k^e \).

**Lemma 4** If \( I_B^k = 0 \), the bourgeoisie always prefers schooling to no schooling when the masses are willing to go. If \( I_N^k = 0 \), the nobility always prefers region-i schooling to no schooling when the masses are willing but might want to implement unified schools later than the masses.

**Proof.** See appendix.

Lemma 4 reveals that under unified schooling there might be a conflict of interest within the elite concerning whether or not schooling is implemented and if this happens whether or not schooling is implemented depends on whether or not the bourgeoisie or the nobility is the dominant group. Under regional schooling, the elite agrees when to build schools and the only political issue is how to split the costs of schooling.

### 2.5 Costs of schooling for the elite

The dominant elite group determines how the costs of education are split within the elite. However, the dominant group cannot oblige the dominated group to pay for education if this payment makes the dominated group worse-off than the situation with no education. In other words, the dominant group will always try to make the dominant group pay the maximum amount
possible for education. The following notation will come in handy to describe the optimal strategy of the dominant group (where for the time being we neglect the incentives of the masses. For the final schooling outcome we have to add the incentives that the masses indeed want to go to school, which tells us that schooling is impossible for all $\sigma < \sigma_{M_i}^{k}$).

**Notation 1.** We denote by

- $\tilde{I}^k_e$ the maximum acceptable payment for education by a member of the elite group $e$ under schooling system $k$. This amount leaves elite group $e$ indifferent between schooling and no schooling.
- $\tilde{I}^k_{-e}$ the amount of copayment of education by a member of the dominant elite group $e$ after imposing payment $\tilde{I}^k_e$ on each member of the dominated elite group $-e$.
- $\tilde{\sigma}^k_e$ the minimum productivity shock that makes elite group $e$ willing to pay the entire cost of schooling.
- $J$
- $\tilde{\sigma}^k_e$ the minimum productivity shock that makes elite group $e$ willing to cofinance education when group $-e$ is paying $\tilde{I}^k_e$.
- $\sigma^k_e$ the minimum productivity shock for which elite group $e$ is willing to implement education if it does not have to pay for schooling.

Let $e$ refer to the dominant elite group and $-e$ to the dominated elite group. Then educational costs are split as follows:

- For very high productivity shocks, $\sigma > \max\{\tilde{\sigma}^k_e, \tilde{\sigma}^k_{-e}\}$, $I^k_e = 0$, schooling is entirely financed by the dominated group: each group member of the dominated group with size $-E$ will have to pay $\tilde{I}^k_e$ since the masses of both regions get educated.
- If $\max\{\tilde{\sigma}^k_e, \tilde{\sigma}^k_{-e}\} = \tilde{\sigma}^k_{-e}$,
  - then for $\max\{\tilde{\sigma}^k_e, \tilde{\sigma}^k_{-e}\} < \sigma < \tilde{\sigma}^k_{-e}$, the dominant group has to cofinance education paying $\tilde{I}^k_e$ while the dominated group pays $\tilde{I}^k_{-e}$. The value of $\tilde{I}^k_e$ for the different political regimes is $\tilde{I}^k_e = \frac{M - \tilde{I}^k_{-e}(-E)}{E}$ where $E$ is the size of the dominant elite group and $-E$ is the size of the dominated elite group.
- if \( \max[\sigma_{e_k}^k, \sigma_{-e_k}^k] = \sigma_{-e_k}^k \) and \( \max[\sigma_{e_k}^k, \sigma_{-e_k}^k] = \sigma_{-e_k}^k \), then for \( \sigma < \sigma_{-e_k}^k \), the dominant group wants education, but the dominated group is made worse off with education, so the dominant group fully pays the educational costs, namely \( \frac{M}{E} \).

- In all other cases the dominant elite group has no interest in implementing schooling.

It is straightforward to calculate the values of these shocks and payments which are displayed in table 1. For the time being we ignore nonnegativity constraints on \( \hat{I}_c^k \) when calculating \( \hat{I}_c^k \) and \( \hat{\sigma}_c^k \). This approach has the advantage that \( \hat{\sigma}_c^k = \sigma_B^k = \sigma_N^k \) for all political regimes \( k \), but, as we will see below it might lead to unnatural rankings of the cutoffs, in particular to \( \sigma_B^k < \sigma_N^k \) (Lemma 5 part 1). This is of no importance, since \( \hat{\sigma}_c^k \) is irrelevant in these cases.

<table>
<thead>
<tr>
<th>( \sigma_{e_k}^k )</th>
<th>Unified education</th>
<th>Region-( i ) education</th>
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<td>( \sigma_B^k )</td>
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<td>( \sigma_N^k )</td>
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Table 1

Lemma 5 shows that two different rankings of the thresholds are possible.

**Lemma 5** For \( k = U, R_i \)

1. \( \sigma_B^k < \sigma_B^k < \sigma_N^k < \sigma_N^k \) if \( 2 > H^k \)

2. \( \sigma_B^k < \sigma_B^k < \sigma_N^k < \sigma_N^k < \sigma_N^k \) if \( 2 < H^k \)
where \( H^k \) is given by

\[
H^U = (1 - \beta)(1 + \delta)(1 + \alpha)B \tag{16}
\]

\[
H^R_i = 2(1 - \beta)(1 + \delta)\alpha B_i \tag{17}
\]

Proof. By simple algebra. ■

Remark 1 Case 2 of lemma 5 cannot occur under region-2 schooling. Indeed from \( B_2 < B_1 \), \( 2(1 - \beta)(1 + \delta)\alpha B_2 < (1 - \beta)(1 + \delta)\alpha B \). Then \( 2(1 - \beta)(1 + \delta)\alpha B_2 > 2 \) would imply that \( (1 - \beta)(1 + \delta)\alpha B > 2 \) which cannot hold since \( \delta \leq 1 \).

Within groups, the payoff from schooling for a given elite group in a given schooling system \( k \) is decreasing in the amount paid by the group, which explains why \( \sigma^k_B < \sigma^k_h, \sigma^k_B < \sigma^k_h, \sigma^k_N < \sigma^k_N \) and \( \sigma^C_N < \sigma^N_N \) always hold.\(^2\)

Across groups, for a given investment in education, the gain from schooling for the bourgeoisie is larger than for the nobility because the bourgeoisie is the only group facing a threat of stealing in the absence of education and because it is the only group that might gain production partners with schooling. This explains why \( \sigma^k_B < \sigma^k_N \) always holds.

The attractiveness of schooling for the bourgeoisie relative to the nobility is particularly high when (i) stealing is very likely, (ii) the agents discount the future to a small extent, as the future gains for schooling are higher for the bourgeoisie than for the nobility, and (iii) the size of the bourgeoisie is large, as the per capita burden from education for a bourgeois is reduced. For this reason, when \( H^k > 2 \) is satisfied (which by Remark 1 is only possible for \( k = U, R_1 \)), the thresholds of the nobility are systematically larger than the thresholds of the bourgeoisie, and, in particular, \( \sigma^C_B < \sigma^C_N \) holds, i.e. a bourgeoisie bearing the full cost of education is more willing to set up schools than a nobility that does not have to pay any cost. Instead, for \( H^k < 2 \), the attractiveness of education is more similar for both groups, and \( \sigma^C_B > \sigma^C_N \).

\(^2\)Note that \( \sigma^k_B < \sigma^C_B \) and \( \sigma^k_N < \sigma^C_N \) do not necessarily hold. This is due to the way the thresholds for copayment are derived. We made the assumption that the payments for schooling made by the dominated group are such that the dominated group is indifferent between education and no education, without imposing a nonnegativity constraint on these payments. Indeed, whenever \( \sigma^k_B > \sigma^C_B \) or \( \sigma^k_N > \sigma^C_N \) holds \( \sigma^C_B \) (resp. \( \sigma^C_N \)) will not be relevant thresholds in the sense that the nobility is not willing to pay any cost of education (resp. the bourgeoisie is willing to pay more than the full cost of education).
2.6 Provision of education by the elite

We are now in a position to represent the decision on education provision by the elite in a given organizational form \( k \).

2.6.1 Bourgeoisie dominant

Figure 1 represents the decision on education provision by the elites when the bourgeoisie is dominant and \( H^k < 2 \). For \( \sigma > \sigma_N^k \) the nobility is willing to pay the full cost of education, and thus the bourgeoisie puts the full cost on the nobility. We will see later on that this analysis also applies to secession (Section 4).
burden on the nobility. For $\tilde{\sigma}_{N} = \tilde{\sigma}_{B} < \sigma < \tilde{\sigma}_{N}$, the bourgeoisie can only impose part of the investment on the nobility, namely $\tilde{I}_{N} \geq 0$ and has to finance the rest of the payment $\tilde{I}_{B}$. Instead, for $\sigma < \tilde{\sigma}_{N} = \tilde{\sigma}_{B}$ education is not provided by the elites.

In turn, Figure 2 represents the outcome for $H^{k} > 2$ (only possible for $k = U, R_{1}$), a situation in which the payoffs from education for the bourgeoisie relative to the nobility are particularly high. In this case, the elite is willing to provide education if and only if $\sigma > \tilde{\sigma}_{B}$. The main difference with the preceding case is that for $\tilde{\sigma}_{B} < \sigma < \tilde{\sigma}_{N}$, the bourgeoisie is willing to provide education even if it has to bear the full burden. In addition, in this area, the nobility becomes actually worse-off after the implementation of education.
2.6.2 Nobility dominant

Figure 3 represents the case where the nobility is dominant and $H^k < 2$. In this case, the elite is willing to provide education if and only if $\sigma > \sigma_N^k$. This provision is fully financed by the bourgeoisie if $\sigma > \sigma_B^k$ and partially financed by each group otherwise $(\overline{I}_N^k, \overline{I}_B^k)$.

For $H^k < 2$ (only possible for $k = U$) education is provided if and only if $\sigma > \sigma_N^k$ and always fully funded by the bourgeoisie.
A simple look at the figures reveals that for $H^k < 2$ the elite agrees when to provide education (Figures 1 and 3). However, for $H^k > 2$, which by remark 1 can only apply to unified and region-1 schooling, the bourgeoisie is willing to fully finance education when the nobility does not even want education ($\tilde{\sigma}_B^k < \bar{\sigma}_B^k$). Whether or not this conflict materializes depends on the willingness of the masses to attend schools, a question to which we turn next.

2.7 Equilibrium education

Taking the incentives of the masses into account, Proposition 1 presents equilibrium unified education:
Proposition 1  Unified schooling is implemented

1. for \( \sigma > \max \left[ \widetilde{\sigma}_c^U, \sigma_{M_1}^U \right] \) if \( H^U < 2 \)

2. for \( \sigma > \max \left[ \widetilde{\sigma}_B^U, \sigma_{M_1}^U \right] \) if \( H^U > 2 \) and the bourgeoisie is dominant

3. for \( \sigma > \max \left[ \sigma_N, \sigma_{M_1}^U \right] \) if \( H^U > 2 \) and the nobility is dominant

or equivalently for \( \sigma > \max \left[ \min \left[ \widetilde{\sigma}_B^U, \sigma_{M_1}^U \right], \sigma_{M_1}^U \right] \) if the bourgeoisie is dominant and for \( \sigma > \max \left[ \widetilde{\sigma}_N^U, \sigma_{M_1}^U, \sigma_N \right] \) when the nobility is dominant

Proof. See Appendix. ■

Similarly, Proposition 2 presents equilibrium region-i education:

Proposition 2  Region-1 schooling is implemented

1. for \( \sigma > \max(\sigma_{M_1}^{R_1}, \widetilde{\sigma}_{B}^{R_1}) \) if \( H^{R_1} < 2 \).

2. for \( \sigma > \sigma_{M_1}^{R_1} \) if \( H^{R_1} > 2 \)

Region-2 schooling is implemented if and only if \( \sigma > \max(\sigma_{M_2}^{R_2}, \widetilde{\sigma}_{B}^{R_2}) \)

Proof. See Appendix. ■

Corollary 1  1. Under unified schooling, when both \( H^U > 2 \) and \( B_1 < \frac{\beta B_2}{\alpha(1 - \beta)} \) hold, education is more likely to be implemented if the bourgeoisie is dominant. In the rest of the cases, the same level of education is implemented no matter the identity of the dominant group.

2. Under region-i schooling, the level of education implemented is independent of the identity of the dominant group.

Proof. See Appendix. ■

Due to the incentives of the masses to attend school, the conflict of interest when to implement schools between the two elite groups can only materialized under unified schooling.
3 Unified vs. Region-\(i\) education

3.1 Region-1 vs. region-2 schooling

Under regional education, either region-1 or region-2 might become educated. While the masses of region 2 are willing to go for education earlier than the masses of region 1, the thresholds for the elites are weakly lower in region 1 than in region 2. Moreover, \(H_{R1} > H_{R2}\). This implies that whenever the masses are willing to get educated in region 1 and the elite is willing to provide education, the dominant elite will choose \(R_1\) over \(R_2\).

**Lemma 6** If region–1 schooling is implementable, then the dominant elite always prefers region–1 schooling to region–2 schooling.

**Proof.** See appendix. ■

Under regional education, the incentives of the masses and the elites are no longer aliened. While the masses are willing to get educated in region 2 first, the elite benefits more from education in region 1 because masses are no longer mobile and hence it is the region with the bigger bourgeoisie that gives higher payoffs from education for the elite. However, by Lemma 3 \(\sigma_{M1}^U < \sigma_{M1}^R\) always, hence whenever the masses of region 1 are willing to go for regional education, there are also willing to go for unified schooling. Moreover, Lemma 7 establishes that unified schooling is also the preferred option of the dominant elite.

**Lemma 7** If unified schooling is implementable, then the dominant elite always prefers unified schooling to region–\(i\) schooling.

**Proof.** See appendix. ■

Unified schooling leads to education in both regions versus education in one region only. This by itself leads to higher benefits for both elite groups but also to higher education costs since schools have to be set up in both regions. By Lemma 7, the extra benefits always outweigh the extra costs. The fact that unified education also induced mobility of the masses when matched with the bourgeoisie is crucial for this result. The increased match pool increases the bourgeoisie’s willingness to pay for education both when it is dominated as well as when it is dominant. Consequently, the potentially remaining educational costs for a dominant nobility are reduced, while the willingness for co-payment by a dominant bourgeoisie is higher. Education in both regions without an increased match pool would not always dominate regional education in region 1, but only for sufficiently high productivity.
costs. However, regional education in both regions is always dominated by unified schooling. The overall educational costs in both systems are the same, but unified schooling leads to higher benefits for the bourgeoisie due to the increased match pool. Hence, any elite group will prefer unified schooling to regional schooling in both regions without mobility of the masses.\(^4\)

By Lemma 7 whenever unified schooling is implementable, regional schooling will not be implemented. We will never observe region-1 schooling and can only observe region-2 schooling when it is implementable but unified schooling is not. Proposition 3 summarizes the results.

**Proposition 3** The productivity shock will lead to unified schooling whenever it is implementable. Region-1 schooling will never be observed. Region-2 schooling will be observed for high enough initial stealing \((\alpha > \frac{\beta B_2 (1+B_2-B_1)}{(1-\beta)((1-B_1)B_1-(1+B_2)B_2)})\) and a sufficiently small bourgeoisie in region 2 \((B_2<\frac{-1+\sqrt{1+4(1-B_1)B_1}}{2})\) and a not too big productivity shock such that \(\sigma_{M_2}^R < \sigma < \sigma_{M_1}^R\).

**Proof.** This follows directly from Lemma 3 and Lemma 7. \(\blacksquare\)

### 4 Seccession

So far, we have assumed the existence of inter-regional transfers within elite groups leading to a perfect equalization of payoffs across regions within elite groups. In this section, we study whether the region-\(i\) dominant elite has actually incentives to avoid such redistribution by accompanying the implementation of schooling in region \(i\) by the political secession of this region. We assume that after region-\(i\) secession, no cross-border production can take place.

Since there are no interregional matches after secession, the cutoffs for the masses to be willing to go to school under region-\(i\) secession (denoted by \(S_i\)) are the same than under regional education, i.e. \(\sigma_{M_1}^{S_1} = \sigma_{M_1}^R > \sigma_{M_2}^{S_2} = \sigma_{M_2}^R\). Instead, the payoff of region-\(i\) bourgeoisie associated to implementing schooling through secession are:

\[
\Pi_{B_i}^{S_i} = -I_{B_i}^{S_i} + \delta(1+\sigma)(1-\beta)\frac{M}{2} \tag{18}
\]

i.e., the bourgeoisie invests \(I_{B_i}^{S_i}\) in the set-up of schools in region-\(i\) and gets the proceeds from the future high-productivity matches with region-\(i\) masses. Similarly, the payoff from region-\(i\) secession for the bourgeoisie

\(^4\)A formal proof of these results is available from the authors by request.
is:

\[ \Pi_{Ni}^S = -I_{Ni}^S + \delta(1 - \beta)(1 + \sigma) \frac{M}{2}. \]  

Equalizing (18) and (19) respectively to (2) and (1), the productivity thresholds for the implementation of schooling with region-\(i\) secession are:

\[ \sigma_{Bi}^{S_i} = \frac{2I_{Bi}^S + (1 - \beta)(1 - \alpha(1 + \delta))M}{\delta(1 - \beta)M} = \frac{2I_{Bi}^S + 1 - \alpha(1 + \delta)}{\delta} \]  
\[ \sigma_{Ni}^{S_i} = \frac{2I_{Ni}^S + (1 - \beta)M}{\delta(1 - \beta)M} = \frac{2I_{Ni}^S}{\delta(1 - \beta)M} + \frac{1}{\delta} \]

Following the same steps as in section 2.5, Table 2 displays the cutoffs for free education, full payment and partial payment and the corresponding educational costs under \(S_i\).

<table>
<thead>
<tr>
<th>(S_i)</th>
<th>region-(i) Secession</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{Bi}^{S_i})</td>
<td>(\frac{1 - \alpha(1 + \delta)}{\delta})</td>
</tr>
<tr>
<td>(\sigma_{Ni}^{S_i})</td>
<td>(\frac{2I_{Ni}^S}{\delta(1 - \beta)M} + \frac{1}{\delta})</td>
</tr>
<tr>
<td>(\sigma_{ci}^{S_i})</td>
<td>(\frac{2I_{ci}^S}{\delta(1 - \beta)M} + \frac{1}{\delta})</td>
</tr>
<tr>
<td>(I_{Bi}^{S_i})</td>
<td>(\frac{(1 - \beta)(\delta \sigma - (1 - \alpha(1 + \delta)))M}{2})</td>
</tr>
<tr>
<td>(I_{Ni}^{S_i})</td>
<td>(1 - (1 - \beta)(\delta \sigma - (1 - \alpha(1 + \delta)))M )</td>
</tr>
<tr>
<td>(I_{ci}^{S_i})</td>
<td>(\frac{2I_{ci}^S}{\delta(1 - \beta)(\delta \sigma - (1 - \alpha(1 + \delta)))M} )</td>
</tr>
</tbody>
</table>

Observe that \(\sigma_{Bi}^{S_i} < \sigma_{Ni}^{S_i} < \sigma_{M_i}^{S_i}\), and hence both elite groups will agree to favour region-\(i\) secession whenever region-\(i\) masses choose to attend school. Simple algebra shows that Lemma 5 extends also to \(k = S_i\) with \(H^{S_i} = H^{R_i}\) and that in the case of region-2 secession \(2 > H^{S_2}\) always holds.

Since secession is initiated by the dominant group in region \(i\), we study three different scenarios: (i) the bourgeoisie is dominant both at region-\(i\) and country level, (ii) the nobility is dominant both at region-\(i\) and country level, and (iii) the bourgeoisie is dominant in region \(i\) and the nobility is dominant at the country level.\(^5\)

\(^5\)We do not study here the fourth possible scenario whereby the nobility is dominant in region \(i\) and the bourgeoisie at the country level as this case is historically less relevant.
4.1 Secession versus unified schooling

We consider the choice between secession and unified schooling by the dominant elite when education can be implemented under both systems, which requires that the masses are willing to get educated under unified schooling, hence \( \sigma > \sigma^U_{M_1} \). Noting that \( H^{S_1} < H^U \), Lemma 8 compares the relevant productivity cutoff parameters under the two systems:

**Lemma 8**

1. For \( 2 > H^U \) then we have \( \sigma_N < \sigma^U_{N} = \sigma^U_{B_i} = \sigma^S_{N_i} < \min \{ \sigma^S_{N_i} = \sigma^U_{N}, \sigma^U_{B_i} \} \)

2. For \( H^{S_1} < 2 < H^U \) the ranking of the thresholds is \( \sigma^U_{B_i} < \sigma^U_{N} = \sigma^S_{B_i} < \sigma^S_{N_i} = \sigma^S_{N} = \sigma^S_{B_i} \).

3. For \( 2 < H^{S_1} \) all thresholds but \( \sigma^S_{N_i} = \sigma^S_{B_i} \) are smaller than \( \sigma_N \).

**Proof.** See appendix.

Since \( \sigma^S_{M_i} > \sigma_N \), education under secession will never be implementable for \( \sigma < \sigma_N \) and we therefore do not need to consider these parameters when looking at the incentives of the elite to choose between secession and unified education.

4.1.1 Bourgeoisie always dominant

A dominant bourgeoisie prefers region-1 secession to unified education if \( \Pi_{B_i}^{S_1} > \Pi_{B_i}^U \), which can be rewritten as

\[
I^U_{B_i} - I^S_{B_i} > (1 + \sigma)(1 - \beta)\delta \frac{M}{2}.
\] (22)

Since the bourgeoisie loses the matches of the other region when initiating secession, secession can only be interesting if it leads to sufficient savings

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6 This is without loss of generality although if parameters are such that \( \sigma^S_{M_2} < \sigma < \sigma^U_{M_1} \) and the elite is willing to implement schooling for \( \sigma < \sigma^U_{M_1} \), secession leads to schooling when unitary education is not possible. However, in this parameter region there will be region-2 education, hence the relevant comparison is between secession and region-2 education to which we turn in the next subsection. Let us first consider the comparison between secession and unified schooling. As \( \sigma^S_{M_i} = \sigma^U_{M_i} \), Lemma 3 applies here, and thus whenever \( \sigma^S_{M_2} < \sigma < \sigma^U_{M_1} \), the masses are willing to attend schools under \( S_2 \) but not under \( U \). Hence, in that case, given that \( \sigma^S_{B_i} < \sigma^S_{N_i} < \sigma^S_{M_i} \), the elites will thus choose region-2 secession for some parameter values in the area.
in terms of educational costs. Notice that the right hand side of (22) is increasing in $\sigma$, hence if secession stands any chance to be preferred it has to be for low enough productivity shocks. However, Proposition 4 shows that for a dominant bourgeoisie cost saving from secession is never sufficient to offset the associated forgone productive matches:

**Proposition 4** A dominant bourgeoisie always prefers unified schooling to secession.

**Proof.** See appendix.

4.1.2 Nobility dominant always

The nobility as a dominant group prefers secession of region $i$ to unified education whenever $\Pi^S_{Ni} < \Pi^U_N$, which can be rewritten as

$$I^U_N > I^S_{Ni} \tag{23}$$

Hence, educational costs under secession have to be smaller than under unified education. However, educational costs are always weakly bigger under secession as Proposition 5 shows. This happens because the bourgeoisie is willing to pay for unified education for a lower $\sigma$ than under secession.

**Proposition 5** A dominant nobility always weakly prefers unified schooling to secession.

**Proof.** See appendix.

4.1.3 Nobility is dominant at country level, bourgeoisie is dominant in region $i$

If the nobility is dominant at the country level but the bourgeoisie is dominant in region $i$ the bourgeoisie might want region $i$ to separate. We now study when secession of region $i$ is of interest for the bourgeoisie. While secession leads to losing valuable match partners in region $-i$ (a loss that is increasing in $\sigma$), it saves on educational costs for two reasons: on the one hand, fewer people have to be educated. On the other, the bourgeoisie can shift educational costs to the nobility under secession while it will be the principal payer of educational costs under unified schooling where it is dominated by the nobility. Hence if secession stands a chance against unified schooling, it has to be for relatively low productivity shocks (but still high,
enough such that education under secession is profitable for the bourgeoisie). This is confirmed in Proposition 6 which characterizes the incentives of the bourgeoisie (assuming the masses get educated for $\sigma > \overline{\sigma}_N$).

**Proposition 6** The preferences of the bourgeoisie are as follows

1. Let $2 > H^U$. Then

   (a) for $\overline{\sigma}_N^U = \overline{\sigma}_B^U < \sigma < \overline{\sigma}_N^i = \overline{\sigma}_N^i$, the bourgeoisie prefers to be dominated under unified schooling, as it does not even want schooling under secession.

   (b) for $\overline{\sigma}_B = \overline{\sigma}_N^i < \sigma < \sigma_i$, the bourgeoisie prefers to be dominant under region-$i$ secession to being dominated under unified schooling.

   (c) for $\sigma > \sigma_i$, the bourgeoisie prefers to be dominated under unified schooling

2. For $2 < H^U$ the bourgeoisie always prefers to be dominated under unified schooling to be dominant under secession.

where $\sigma_i$ is defined by

$$\sigma_i = \begin{cases} 
\sigma_a & \text{for } 2B_i < N \\
\min [\sigma_a, \sigma_{aa1}] & \text{for } 2B_i > N 
\end{cases}$$

(24)

and

$$\sigma_a = \frac{1}{\delta} + \frac{2 - (1-\beta)(1+\delta)B}{(1-\beta)\delta B}$$

is such that $\Pi_B(\overline{J}_B^U = \overline{J}_B^S, \overline{J}_B^S = 0)$ while

$$\sigma_{aa1} = \frac{1}{\delta} + \frac{2(B_i - B_{-i}) - (1-\beta)2BB_i(\delta + 1)}{(1-\beta)B\delta (2B_i - N)}$$

is such that $\Pi_B(\overline{J}_B^U = \overline{J}_B^S, \overline{J}_B^S = \overline{J}_B^S)$.

**Remark 2** $\min [\sigma_a, \sigma_{aa1}] = \sigma_a$ is only possible for region-1 secession.

**Proof.** See Appendix.

If $2 > H^U$ figure 1 applies to region-$i$ secession and figure 3 to unified schooling. Since $\overline{\sigma}_N = \overline{\sigma}_B < \overline{\sigma}_B = \overline{\sigma}_N$, unified schooling is implemented.
earlier by a dominant nobility than education under secession by a dominant bourgeoisie, hence for $\sigma_{N_i}^V = \sigma_{B_2}^V < \sigma < \sigma_{B_1}^S = \sigma_{N_i}^S$, the bourgeoisie prefers unified schooling. Once education becomes possible under secession, it has to be cofinanced by the bourgeoisie while under unified schooling the bourgeoisie is forced to pay its maximum willingness, leaving it just indifferent between implementing unified schooling or not. After unified schooling becomes profitable enough that copayment by the nobility is no longer possible, there will be a point when the additional match benefits from unified schooling outweigh the cost savings under secession. For $2B_1 < N$, unified education outperforms secession after educational costs under secession have dropped to zero for the dominant bourgeoisie. For $2B_1 > N$, unified schooling might outperform secession even under copayment for secession. By Remark 2 this will always happen for secession of region 2 but not necessarily for secession of region 1 that is relatively more attractive for the bourgeoisie ($\sigma_{B_1}^S < \sigma_{B_2}^S$).

If $H_i^S < 2 < H_i^U$ figure 1 applies to secession of region $i$ and figure 4 to unified schooling. Unified schooling is now especially profitable for the bourgeoisie that is totally financing schooling. The potential cost savings once education under secession becomes possible never outweigh the productivity losses due to fewer match partners.

If $H_i^S > 2$ figure 2 applies to secession of region 1 and figure 4 to unified schooling. Now schooling is very profitable for the bourgeoisie also under secession but not sufficiently profitable compared to the even more profitable unified schooling.

While the bourgeoisie might prefer secession, the nobility never prefers to be dominated under secession to being dominant under unified education. This immediately follows from the fact that the nobility does not even prefer secession when it is dominant. This happens because from the point of view of the nobility the only difference between the two systems are the educational costs and these are always higher under secession. While the nobility dislikes secession, it cannot avoid it since it is the dominated group. However whether secession is indeed implementable does not only depend on the preferences of the bourgeoisie described in Proposition 6 but also on the preferences of the masses who might simply not attend the schools. We therefore need to understand how the schooling cutoffs of the masses ($\sigma_{M_1}^S > \sigma_{M_2}^S$) compare to the cutoffs $\sigma_a$ and $\sigma_{aa_1}$. For the comparison with unified schooling, it is also important to study $\sigma_{M_1}^U$.

**Lemma 9**

1. $\sigma_{M_1}^S < \sigma_a \iff \alpha < \alpha_{M_1} = \frac{\beta(\frac{\alpha}{2}+B_i)\{(1-\beta)(1+\delta)B\}}{(1-\beta)^2(1+\delta)BB_i}$
2. $\sigma_{M2}^S > \sigma_{aa1}$ always (area $2B_2 > N$)

3. If $B_2 > B_1 (1 - (1 - \beta)(1 + \delta)B)$ then $\sigma_{M1}^S > \sigma_{aa1}$ always

4. If $B_2 < B_1 (1 - (1 - \beta)(1 + \delta)B)$ then $\sigma_{M1}^S < \sigma_{aa1}$

\[ \Leftrightarrow \alpha < \frac{\beta(N+2B_1)((B_1-B_2)-(1-\beta)(1+\delta)BB_1)}{(1-\beta)^2(1+\delta)B(2B_1-N)B_1} \]

5. $\sigma_{M1}^U < \sigma_a \Leftrightarrow \alpha < \alpha_{M1}^U = \frac{\beta(2(1+\delta)B(1-B_2))(1-\beta)B)}{2B_1(1-\beta)^2(1+\delta)B}$

6. $\sigma_{M1}^U > \sigma_{aa1}$ always for secession of region 2.

7. $\alpha_{M1}^U > \alpha_{M1}$ (always for secession of region 1)

**Proof.** See Appendix □

Lemma 9 tells us that when $\sigma_{aa1}$ is the relevant cutoff in Proposition 6 then the area where secession is preferred by the bourgeoisie can never be implemented in region 2 because the masses are not willing to go to school and it is only implementable for secession of region 1 under very specific parameters and for sufficiently low $\alpha$. Lemma 9 also tells us that stealing cannot be too important, otherwise even the area for secession where $\sigma_a$ is the relevant cutoff cannot be implemented. Proposition 7 characterizes when equilibrium secession is possible for $\sigma > \sigma_{M1}^S$

**Proposition 7** Combining overall incentives we only get equilibrium secession of region $i$ if the bourgeoisie is dominant in region $i$ but dominated at the state level and stealing is not sufficiently important so that there exists a parameter area where $\sigma_{M1}^S < \sigma < \sigma_{si}$ where $\sigma_{si}$ is defined by (24) in Proposition 6.

To summarize: secession is very unlikely when (unified) nation building is possible. Secession will only result if it implies a change in the dominant group and the productivity shock is not too high and in a pre-industrial institutional setup with fairly good property rights for the bourgeoisie. We now turn to the question whether secession dominates regional education.

### 4.2 Secession versus regional education

By Proposition 3 we can observe region-2 education under very special circumstances (requiring parameters such that $\sigma_{M2}^R < \sigma < \sigma_{M1}^U$) but we will never get region-1 education. We therefore need to examine whether secession is preferred to region-2 education. Since for $\sigma_{M2}^R < \sigma < \sigma_{M1}^U$, education
under secession is only feasible in region-2, the only case we have to study is region 2 secession versus region-2 education. We start by looking at a dominant nobility.

The nobility of region 2 prefers $S_2$ to $R_2$ whenever $\Pi_{N_2}^S > \Pi_{N_2}^R$ or equivalently

$$I_{N_2}^R - I_{N_2}^S = -\frac{1}{4} M (1 - \beta) (\sigma \delta - 1)$$

(25)

Only the masses of region 2 get educated in both cases, but under secession the benefit from the increased match productivity are not shared with the nobility of the other region. Neither are the educational costs. However, for an always dominant nobility these additional costs do not outweigh the gains from not having to share the increased match productivity. To see this, notice that a dominant nobility gets education for free for $d = d_\omega$ under both systems. Also since $g_{B_2} = g_{B_2}$ the area for co-payment by the nobility is the same in both systems, and condition (25) reduces to $\sigma > \sigma_{B_2}^ R = \sigma_{B_2}^ S$.

We now turn to a dominant bourgeoisie. The bourgeoisie prefers $S_2$ to $R_2$ whenever $\Pi_{B_2}^S > \Pi_{B_2}^R$ or equivalently whenever

$$I_{B_2}^S - I_{B_2}^R = -(1 - \beta) (\delta \sigma - 1 + (1 + \delta) \alpha) \frac{MB_1}{2B}$$

(26)

Under $R_2$ the bourgeoisie of region 2 where the masses get educated has to share the increased match productivity with the bourgeoisie of region 1 which is a motive for secession where the sharing of these benefits is avoided. However, while total costs of education are the same under $R_2$ and $S_2$, secession reduces the size of the nobility that will be forced to pay up to its maximal willingness to pay for education. But, this never offsets the first effect for a bourgeoisie that is always dominant. To see this, notice that the cutoffs for total and maximum payment by the nobility are the same under both systems. We therefore only need to look at the case when education is free for the bourgeoisie and when there is copayment. Clearly, for free education secession dominates. Under co-payment, the condition that secession is preferred reduces to $\sigma > \sigma_{B_2}^ S = \sigma_{B_2}^ S$, hence secession is always preferred.

Finally, if the bourgeoisie is dominant in region 2 and dominated at the state level, then secession additionally leads to savings of educational costs and thus the following proposition holds:

7 This is also true if the nobility was dominant in region 2 and the bourgeoisie dominant
Proposition 8 The dominant elite in region 2 always prefers $S_2$ to $R_2$.

Since the willingness to get educated of the masses coincides under $S_2$ and $R_2$, secession of region 2 always destabilizes the region-2 schooling. In other words, region-2 schooling is never an equilibrium outcome. We will observe secession of region 2 instead.

5 Robustness

The above results are derived assuming one dimension of heterogeneity, namely the size of the bourgeoisie was bigger in region 1 than in region 2. In this section we briefly discuss other forms of heterogeneity. As before we will assume that the regions are identical except in one dimension. The dimensions we look at are the initial property right institutions (represented by $\alpha$), the size of the masses / nobility and the size of the productivity shock that hits the two regions representing different arrival rates of industrialization.

It is easy to see that our results are robust to heterogeneity in $\alpha$. Now the masses of the region with a lower $\alpha$ required a lower productivity shock to be willing to go to school. The interest of the elite is not aligned, since the benefits from implementing education are higher for the bourgeoisie in the region where $\alpha$ is higher. This will lead the elite to prefer schooling in the high $\alpha$ region whenever it is implementable, but in this case unified schooling will also be implementable. The masses might be willing to get regional schooling in the low $\alpha$ region before they are willing to get unified schooling, but this imposes an upper bound on the size of the bourgeoisie. This is all very similar to our previous analysis. Whether one or two regions get educated when the masses are willing to go to school again depends on the trade-off of educational costs versus benefits from industrialization. Since total education costs are fixed but benefits increase in the size of the productivity shocks, sufficiently high productivity shocks will lead to stable unified schooling, while secession will occur for not too high productivity shocks that either would have allowed for low $\alpha$ region schooling only - hence the elite saves on interregional transfers - or unified schooling was possible but secession leads to a change of the dominant group.

at the state level. A formal proof of the result when the regionally dominant elite does not coincide with the dominant elite at the state level can be obtained from the authors upon request. Moreover, it is also true if we looked at secession of region 1 versus region-1 education in which case the dominate elite in region 1 always prefers $S_1$ to $R_1$. 

If the heterogeneity stems from the size of the masses across regions, the minimum productivity shock necessary for the masses to be willing to get region-\(i\) schooling or unified schooling respectively does not vary across regions. Now the masses will be willing to attend unified schooling whenever they are willing to attend regional schooling. Under region-\(i\) education the elite of the bigger \(M_i\) region benefits more from education, but educating this region is also more costly since more individuals have to be schooled. Unified schooling will lead to less than double education costs and big benefits due to the mobility of the masses. This leads to stable nation building for sufficiently high productivity shocks and makes secession or regional education less likely than in our benchmark setting.

If the size of the nobility differs across regions, the cutoffs for regional education do not differ across regions for the elite. However, the masses of the bigger \(N\) region are willing to get educated first. Hence regional education if possible at all, could only happen in the bigger \(N\) region but again is likely to be destabilized by secession: transfers across regions are avoided and educational costs do not double due to secession. As before sufficiently high productivity shocks will lead to stable nation building. Compared to our baseline setup we expect to observe fewer incidences of secession.

A dimension of heterogeneity that makes secession more likely and hinders stable nation building is if the speed of industrialization differs across the regions, in the sense that the regions are hit by different productivity shocks. In this setup we talk about nation building if there is unified schooling leading to the mobility of the masses as before, but the effects on match productivity are different. The match productivity of matches within regions is determined by the regional productivity shock, while the match productivity across regions is determined by the lower productivity shock, because either the available production technology of the bourgeoisie was determined by the lower productivity shock or because the masses where trained for the lower productivity shock only. Now regional schooling of the high productivity region is more attractive for all groups than regional schooling of the low productivity region. Moreover, the bigger the relative difference across region, the more likely it is that regional education dominates unified schooling. Since transfers to the less efficient region can be avoided by secession, a very unequal speed of industrialization makes nation building across both regions impossible and is likely to lead to secession.
6 Case Study: Spain versus France
to be written

Appendix

Proof of Lemma 3

We already now that $\sigma^U_{M_1} < \sigma^R_{M_1}$ from Lemma 1 and that $\sigma^k_{M_1} > \sigma^k_{M_2}$ from Lemma 2. The only remaining question is how $\sigma^R_{M_2}$ compares to $\sigma^U_{M_1}$. After some calculation the condition that $\sigma^R_{M_2} < \sigma^U_{M_1}$ can be shown to be equivalent to

$$\beta B_2(1 + B_2 - B_1) < \alpha(1 - \beta)((1 - B_1)B_1 - (1 + B_2)B_2).$$

We then need to distinguish two subcases:

1. if $B_2 \in \left(\frac{-1+\sqrt{1+4(1-B_1)B_1}}{2}, B_1\right)$, then $(1 - B_1)B_1 - (1 + B_2)B_2 < 0$, and (6) is never satisfied.

   $$\alpha < \frac{\beta B_2(1 + B_2 - B_1)}{(1 - \beta)((1 - B_1)B_1 - (1 + B_2)B_2)} < 0$$

   which is never satisfied. Thus $\sigma^U_{M_1} < \sigma^R_{M_2}$, which combined with $\sigma^R_{M_2} > \sigma^R_{M_1}$ implies $\sigma^U_{M_2} < \sigma^U_{M_1} < \sigma^R_{M_2} < \sigma^R_{M_1}$.

2. if $B_2 \in \left(0, \frac{-1+\sqrt{1+4(1-B_1)B_1}}{2}\right)$, then $(1 - B_1)B_1 - (1 + B_2)B_2 > 0$, implying that:

   $$\sigma^R_{M_2} < \sigma^U_{M_1} \Leftrightarrow \alpha > \frac{\beta B_2(1 + B_2 - B_1)}{(1 - \beta)((1 - B_1)B_1 - (1 + B_2)B_2)} \equiv \overline{\alpha}.$$

   Thus $\sigma^U_{M_2} < \sigma^U_{M_1} < \sigma^R_{M_2} < \sigma^R_{M_1}$ for $\alpha < \overline{\alpha}$ and $\sigma^U_{M_2} < \sigma^U_{M_1} < \sigma^R_{M_1}$ for $\alpha > \overline{\alpha}$.

Proof of Lemma 4

Consider first unified schooling. The relevant cut-off of the masses is $\sigma^U_{M_1}$. For the bourgeoisie $\sigma^U_B < \sigma^U_{M_1} \Leftrightarrow -\alpha(1 + B) < \beta(N + 2(B_1 - B_2)) + \ldots$
$4B_1\alpha(1-\beta)$. As $B_1 > B_2$, $\sigma_{M_1}^U < \sigma_{M_1}^U$ is always verified. For the nobility, $\sigma_{M_1}^U > \sigma_{M_1}^U \Leftrightarrow B_1 < \frac{\beta B_2}{\alpha(1-\beta)}$.

Consider next region-i schooling. For the bourgeoisie $\sigma_{B}^{R_i} < \sigma_{M_1}^{R_i} \Leftrightarrow -\beta(N + 2B_1) < 2B_i(1-\beta)$ always. For the nobility it is immediate from (11) and (15) that $\sigma_{M_1}^{R_i} > \sigma_{N}^{R_i} = \frac{1}{\delta}$.

**Proof of Proposition 1**

We know that $\sigma_{M_1}^{U} > \sigma_{M_2}^{U}$, so the relevant threshold for the masses to attend school is $\sigma_{M_1}^{U}$. (i) Assume first that $H^U > 2$. In this case, from Figures 2 and 4, the thresholds when the bourgeoisie and the nobility are dominant are respectively $\tilde{\sigma}_B^{U}$ and $\tilde{\sigma}_N^{U}$, and from Lemma 5 $\sigma_B^U < \sigma_N^U < \tilde{\sigma}_N^{U}$. In addition, it is easy to show that $\sigma_B^U < \sigma_{M_1}^{U}$ is always satisfied and that $\sigma_N^U > \sigma_{M_1}^{U} \Leftrightarrow B_1 < \frac{\beta B_2}{\alpha(1-\beta)}$. Thus, if $B_1 > \frac{\beta B_2}{\alpha(1-\beta)}$, $\sigma_N^U < \sigma_{M_1}^{U}$ and thus $\tilde{\sigma}_B^{U} < \sigma_N^U < \tilde{\sigma}_B^{U}$, which means that $\sigma_{M_1}^{U}$ is the threshold independently of the identity of the dominant group. Instead, if $B_1 < \frac{\beta B_2}{\alpha(1-\beta)}$, we have that $\sigma_B^U < \sigma_{M_1}^{U}$ and $\sigma_N^U > \sigma_{M_1}^{U}$. Thus the threshold when the nobility is dominant is $\sigma_N^U$ and the threshold when the bourgeoisie is dominant is $\max(\sigma_{M_1}^{U}, \tilde{\sigma}_B^{U}) < \sigma_N^U$, which means that education is more likely to be implemented when the bourgeoisie is dominant. (ii) Assume next that $(1-\beta)B(1+\delta)(1+\alpha) < 2$. In that case, from Figures 1 and 3, the threshold for the elites to be willing to implement education is the same ($\tilde{\sigma}_B^{U}$) no matter the dominant group. Then, education is implemented for $\sigma > \max(\tilde{\sigma}_B^{U}, \sigma_{M_1}^{U})$ no matter the dominant group.

**Proof of Proposition 2**

1. (i) Assume first that $H^{R_1} > 2$. In this case, from Figures 2 and 4, the thresholds when the bourgeoisie and the nobility are dominant are respectively $\tilde{\sigma}_B^{R_1}$ and $\tilde{\sigma}_N^{R_1}$, and from Lemma 5 $\sigma_B^{R_1} < \sigma_N^{R_1} < \tilde{\sigma}_N^{R_1}$. However, by Lemma 4 $\sigma_{M_1}^{R_1} > \sigma_N^{R_1}$. Thus the relevant threshold for education is the threshold by the masses, namely $\sigma > \sigma_{M_1}^{R_1}$ (ii) Assume next that $H^{R_1} < 2$. In that case, from Figures 1 and 3, the threshold for the elites to be willing to implement education is the same ($\tilde{\sigma}_B^{R_1}$) no matter the dominant group. Then, education is implemented for $\sigma > \max(\tilde{\sigma}_B^{R_1}, \sigma_{M_1}^{R_1})$ no matter the dominant group.

2. Under region-2 schooling $H^{R_2} \leq 2$ always, hence there is no conflict of interest between the elite groups when education should be provided.
Education is provided for $\sigma > \max \left[ \sigma_{M_2}, \sigma_{e} \right]$.

Proof of Corollary 1

The proof follows directly from the proofs of Propositions 1 and 2.

Proof of Lemma 6

We start by looking at a dominant bourgeoisie. A dominant bourgeoisie prefers $R_1$ to $R_2$ whenever

$$I_{B_1}^R - I_{B_2}^R > (1 - \beta) \frac{M}{2B} (B_1 - B_2) \left( (1 + \delta)(1 - \alpha) - \delta (1 + \sigma) \right)$$

So for $\sigma > \overline{\sigma}_B$, we have $I_{B_1}^R = I_{B_2}^R = 0$ and $R_1$ is preferred if $\sigma > \frac{1}{\beta} - \frac{(1 + \delta)\alpha}{2}$, which always holds. The same condition holds for copayment since $I_{B_1}^R = I_{B_2}^R$.

Now we look at a dominant nobility. A dominant nobility prefers $R_1$ to $R_2$ whenever $I_{N_1}^R > I_{N_2}^R$. Since $H_{R_1} > H_{R_2}$, the following schooling costs are possible

- for $\sigma > \overline{\sigma}_B$, schooling is free under both systems since $\overline{\sigma}_B < \overline{\sigma}_B$,

- $I_{N_1}^R = 0$ and $I_{N_2}^R$ for $\max \left[ \overline{\sigma}_B, \sigma_{M_1} \right] < \sigma < \overline{\sigma}_B$ hence $R_1$ is preferred,

- $I_{N_1}^R$ and $I_{N_2}^R$ for $\max \left[ \overline{\sigma}_B, \sigma_{M_1} \right] < \sigma < \max \left[ \overline{\sigma}_B, \sigma_{M_1} \right] = \overline{\sigma}_B$ and $\overline{I}_{N_1}^R < I_{N_2}^R$ so $R_1$ is always preferred.

Proof of Lemma 7

Proof. This is obvious if only unified schooling is implementable, i.e. if $\sigma_{M_1} > \sigma_{M_2} > \sigma > \sigma_{M_1} > \sigma_{M_2}$. Now suppose that both unified schooling and region-i schooling are implementable, i.e. $\sigma > \max \left[ \sigma_{M_1}, \sigma_{M_2} \right]$.

We start by considering a dominant nobility. Comparing the benefits from education, a dominant nobility prefers unified schooling to region-i schooling whenever

$$(1 - \beta) \frac{M}{4} (\sigma \delta - 1) > I_{N_1}^U - I_{N_2}^R.$$  \(27\)
The LHS is always positive since the minimum productivity shock for which the nobility is willing to implement unified schooling if it does not have to pay for it is \( \sigma_N^U = \frac{1}{\gamma} \). So for free education under unified schooling (i.e. \( \sigma > \sigma_B^U \)) unified schooling is clearly preferred. Since \( \sigma_B^U < \sigma_B^R \) \( \Leftrightarrow \)
\[-(1 - \beta)BB_i(1 + \delta)(1 - \alpha) < 2B_{-i}, \quad \sigma_B^U < \sigma_B^R \] always holds, and it only remains to check whether (27) is also true under co-payment under both systems, i.e. for \( I_N^U \) and \( I_N^R \) which can only occur if \( H_i^U < 2 \).

Then (27) becomes
\[
(1 - \beta)N(\delta \sigma - 1) > 2 - 2B(1 - \beta)(2\delta \sigma - (1 - \delta) + \alpha (1 + \delta)) + 2(\sigma \delta - 1 + \alpha(1 + \delta))(1 - \beta)B_i,
\]
or equivalently
\[
\sigma > \sigma_g = \frac{1}{\delta} + \frac{2 - 2(1 - \beta)(1 + \delta)(B + \alpha B_{-i})}{(1 - \beta)(N + 4B - 2B_i)} \delta
\]
As \( \sigma_B^U < \sigma_B^R_i \) \( \Leftrightarrow \) \(- (1 - \beta)(1 + \delta) \left[ (1 - \alpha) (B_i^2 + B_i + B_iB_{-i}) + (1 + \alpha) NB_{-i} \right] < 4B_{-i} \), and the LHS of the former inequality is always negative, \( \sigma_B^U < \sigma_B^R_i \) is always satisfied. We now show that \( \sigma_g < \sigma_B^R_i \). This is equivalent to
\[
\frac{2 - 2(1 - \beta)(1 + \delta)(B + \alpha B_{-i})}{N + 4B - 2B_i} < \frac{2 - 2\alpha(1 + \delta)(1 - \beta)B_i}{2B_i + N}
\]
which is always true since the LHS has a smaller numerator and a bigger denominator than the RHS. Hence, whenever regional education with copayment is possible, the dominant nobility prefers unified schooling. Consider now a dominant bourgeoisie. A dominant bourgeoisie prefers unified schooling whenever \( \Pi_B^U > \Pi_B^R \)
\[
(1 - \beta) \frac{M}{2B} (((1 + \sigma) \delta (2B - B_i) - (1 + \delta)(1 - \alpha)B_{-i}) > I_B^U - I_B^R_i \quad \text{for } i = 1, 2.
\]

The left hand side is positive for \( \sigma > \frac{1}{\gamma} - \frac{(1 + \delta)(B + \alpha B_{-i})}{\delta(B + B_{-i})} \) which is clearly smaller than \( \sigma_N^U = \frac{1}{\gamma} \). Hence when education is free for the bourgeoisie under both systems, namely for \( \sigma > \sigma_N^U = \sigma_N^R_i \), unified schooling is preferred. It remains to check what happens under co-payment by the bourgeoisie. In this case, unified schooling is preferred for \( \Pi_B^U(I_B^U) > \Pi_B^R_i(I_B^R_i) \) or equivalently

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8Observe that \( H_i^R < H_i^U < H_i^R \). This give rise to three cases: (i) \( H_i^U < 2 \), (ii) \( H_i^R < 2 < H_i^U \) and (iii) \( H_i^R < 2 < H_i^R \).
for
\[ \sigma > \sigma_{gg} = \frac{1}{\delta} + \frac{2 - 2(1 - \beta)(1 + \delta)(B + \alpha B_{-i})}{(1 - \beta) \delta (1 + B + 2B_{-i})} \]
with \( \sigma_{gg} < \sigma_{e}^{U} \) since \(\frac{2 - 2(1 - \beta)(1 + \delta)(B + \alpha B_{-i})}{(1 - \beta) \delta (1 + B + 2B_{-i})} < \frac{2 - (1 - \beta)(1 + \delta)(1 + \alpha)B}{(1 - \beta) \delta (1 + B)}\) given that the LHS has a smaller numerator and a bigger denominator than the RHS. As \( \sigma_{e}^{U} < \sigma_{e}^{R_{i}} \), unified schooling with copayment is always preferred to regional education with copayment by a dominant bourgeoisie. By Lemma 4 \( \sigma_{N_{i}}^{R_{i}} > \frac{\delta}{2} \), so schooling will never be implemented when it harms the nobility. This concludes the proof.

**Proof of Lemma 8** The three parameter areas follow from Lemma 5 using \( H_{S_{i}}^{N} < H_{U}^{U} \). The ordering of the thresholds is based on the following auxiliary lemma.

**Lemma 10**
1. \( \sigma_{N_{i}}^{S_{i}} = \sigma_{N}^{U} \)
2. \( \sigma_{N}^{U} > \sigma_{N_{i}}^{S_{i}} \iff (1 - \beta)(1 + \delta) B(2B_{i} - (2B_{i} + (1 + \alpha)N)) > 4B_{-i} \)
3. \( \sigma_{B}^{U} < \sigma_{B_{i}}^{S_{i}} \)
4. \( \sigma_{N_{i}}^{S_{i}} > \sigma_{B_{i}}^{S_{i}} \iff (1 - \beta) \alpha (1 + \delta) NB_{i} > N - 2B_{i} \)
5. \( \sigma_{N_{i}}^{S_{i}} < \sigma_{B}^{U} \iff 2N - 4B > (1 - \beta) B(N + 1 + \alpha)(1 + \delta) \)
6. \( \sigma_{N_{i}}^{S_{i}} < \sigma_{B}^{U} \iff 2N - 4B_{-i} > (1 - \beta)(1 + \delta) B((1 + \alpha)(N + 2B_{i}) - \alpha 4B_{i}) \)
7. \( \sigma_{N_{i}}^{S_{i}} > \sigma_{N_{i}}^{S_{i}} \) and thus \( \sigma_{N_{i}}^{S_{i}} < \sigma_{B}^{U} \Rightarrow \sigma_{N_{i}}^{S_{i}} < \sigma_{B}^{U} \)
8. If \( 2 > (1 - \beta)(1 + \delta) B(1 + \alpha) \) then \( \sigma_{N}^{U} < \sigma_{N_{i}}^{S_{i}} \)

**Proof.** Points 1 to 7 follow from simple algebra. For point 8, note that \( \sigma_{N}^{U} < \sigma_{N_{i}}^{S_{i}} \iff \frac{(1 - \beta)(1 + \delta)(2B_{i} - B(2B_{i} + (1 + \alpha)N))}{2B_{-i}} < 2 \). In addition, we have that \( (1 - \beta)(1 + \delta) B(1 + \alpha) > \frac{(1 - \beta)(1 + \delta)(2B_{i} - B(2B_{i} + (1 + \alpha)N))}{2B_{-i}} \iff (B + 1)(B(1 - \alpha) + 2\alpha B_{-i}) \)
0, which always holds. Thus \( 2 > (1 - \beta)(1 + \delta) B(1 + \alpha) \)
\[ \Rightarrow 2 > \frac{(1 - \beta)(1 + \delta)(2B_{i} - B(2B_{i} + (1 + \alpha)N))}{2B_{-i}} \iff \sigma_{N}^{U} < \sigma_{N_{i}}^{S_{i}}. \]
Proof of Proposition 4

By lemma 8 a dominant bourgeoisie either gets schooling for free under both systems for $\sigma > \sigma^N_{N_i} = \sigma^U_i$, or has to co-finance education. When $I^U_B = I^S_{B_i} = 0$ condition (22) is violated. Under copayment it is easy to see that:

\[ \frac{I^U_B - I^S_{B_i}}{4BB_i} = \frac{(2 - N(1 - \beta)(\delta\sigma - 1))(B_1 - B_{-i})}{4BB_i} \]

Observe that $(2 - N(1 - \beta)(\delta\sigma - 1)) > 0$ for $\sigma < \sigma^N_{N_i} = \sigma^U_i$ which is the region we are looking at. So $I^U_B - I^S_{B_i} < 0$ for $i = 2$ since $B_1 > B_2$ and hence unified schooling is always preferred to secession of region 2. We will now show that condition (22) is also violated with co-payment for region 1. Assume for contradiction that condition (22) holds. This would require:

\[ \frac{(2 - N(1 - \beta)(\delta\sigma - 1))(B_1 - B_2)}{4BB_1} M > (1 + \sigma)(1 - \beta)\delta M \]

which can be rewritten as:

\[ \sigma < \sigma_s = \frac{2(B_1 - B_2) + (1 - \beta)(N(B_1 - B_2) - 2\delta BB_1)}{(1 - \beta)\delta (2BB_1 + N(B_1 - B_2)) + \frac{1}{\delta}} \]

but $\sigma^N_{N_1} > \sigma_s$ (since $-8B_1B_2 < (1 - \beta)(1 + \delta)2B_12BB_1(1 - \alpha) + N(B - \alpha(B_1 - B_2))$, i.e. this is incompatible with the bourgeoisie being willing to pay for the additional cost of education.

Proof of Proposition 5

We need to show that $I^U_N \leq I^S_{N_i}$. By lemma 8 a dominant nobility either gets education for free under both systems for $\sigma > \sigma^N_{N_i} = \sigma^U_i$, or has to co-pay under secession but not gets education for free under the unified system $0 = I^U_N \leq I^S_{N_i}$ or has to co-pay under both systems. It remains to prove that $I^U_N < I^S_{N_i}$ which can be rewritten as $(2B_i - B)(\delta\sigma - 1 + \alpha(1 + \delta)) < B\delta(1 + \sigma)$. It is immediate to see that this is holds for region-2 secession as $2B_2 < B$.

For region-1 secession, we need to show that $(\delta\sigma - 1 + \alpha(1 + \delta))(B_1 - B_2) < B\delta(1 + \sigma)$ which can be rewritten as $(-1 + \alpha(1 + \delta))(B_1 - B_2) < 2B_2\delta\sigma + B\delta$. Notice that all relevant $\sigma > \frac{1}{\delta} = \sigma_N$, so if the former inequality is true for...
Proof of Proposition 6

The bourgeoisie prefers secession in region \( i \) to be dominated with unified schools whenever condition (22) holds, namely

\[
I_B^U - I_{B_i}^S > (1 + \sigma)(1 - \delta)\delta \frac{M}{2}
\]

The exact value of \( I_B^U \) and \( I_{B_i}^S \) depends on the size of the shock and the underlying parameters.

The following payment constellation may occur.

1. \( \sigma > \max[\bar{\sigma}_{N_i}, \bar{\sigma}_{B_i}] \) the bourgeoisie has to finance education totally under the unified system \( I_B^U = \frac{M}{2} \) while education is free under secession \( I_{B_i}^S = 0 \) and secession is preferred if

\[
\sigma < \sigma_a = \frac{2 - (1 - \beta)\delta B}{(1 - \beta)\delta B} = \frac{1}{\delta} + \frac{2 - (1 - \beta)(1 + \delta)B}{(1 - \beta)\delta B}
\]

(29)

2. \( \min[\bar{\sigma}_{N_i}, \bar{\sigma}_{B_i}] < \sigma < \max[\bar{\sigma}_{N_i}, \bar{\sigma}_{B_i}] \) we have to distinguish two subcases:

   (a) If \( \min[\bar{\sigma}_{N_i}, \bar{\sigma}_{B_i}] = \bar{\sigma}_{N_i} \), then \( I_B^U = \frac{M}{2} \) and \( I_{B_i}^S = 0 \). In this case secession is always preferred because the condition (22) reduces to \( \sigma > \sigma_{B_i}^S = \frac{1 - \alpha(1 + \delta)}{\delta} \) which is the condition that the bourgeoisie is willing to go for free education under secession.

   (b) If \( \min[\bar{\sigma}_{N_i}, \bar{\sigma}_{B_i}] = \bar{\sigma}_{B_i}^U \), then \( I_B^U = \frac{M}{2} \) and \( I_{B_i}^S = I_{B_i}^S \). The condition that secession is preferred becomes

\[
\sigma < \sigma_{aa1} = \frac{1}{\delta} + \frac{2(B_i - B_{-i}) - (1 - \beta)2BB_i(\delta + 1)}{(1 - \beta)B\delta (2B_i - N)} \quad \text{for } 2B_i > N
\]

(30)

\[
\sigma > \sigma_{aa2} = \frac{1}{\delta} + \frac{(1 - \beta)2BB_i(\delta + 1) - 2(B_i - B_{-i})}{(1 - \beta)B\delta (N - 2B_i)} \quad \text{for } 2B_i < N
\]

(31)
3. For $\sigma_{\text{sec}}^B < \sigma < \min[\sigma_{\text{sec}}^N, \sigma_{\text{sec}}^U]$, $I_B^U = I_B^U$ and $I_B^S = I_B^S$. Secession is always preferred in this area since the condition (22) reduces to $\sigma > \sigma_{\text{sec}}^N$, which is the condition for the bourgeoisie to be willing to go for co-payment under secession.

We need to check under which conditions the cutoffs (29), (30) and (31) are relevant cutoffs. Both $\sigma_a$ and $\sigma_{aa\text{I}}$ are upper bounds. Therefore $\sigma_a$ is not relevant if $\sigma_a < \max[\sigma_{\text{sec}}^N, \sigma_{\text{sec}}^U]$. Similarly, $\sigma_{aa\text{I}}$ is not relevant for $\sigma_{aa\text{I}} < \max[\sigma_{\text{sec}}^N, \sigma_{\text{sec}}^U]$. Since $\sigma_{aa\text{II}}$ is a lower bound it is not relevant for $\sigma_{aa\text{II}} > \sigma_{\text{sec}}^N$. Lemma 11 tells us under which conditions these cutoffs are relevant and how the relate to each other and to the different payment areas.

**Lemma 11**

1. $\sigma_a > \sigma_{\text{sec}}^U$ and $\sigma_a > \sigma_{\text{sec}}^N$ always
2. $\sigma_a < \sigma_{\text{sec}}^N \iff \sigma_{aa\text{II}} > \sigma_{\text{sec}}^N$
3. $\sigma_{aa\text{I}} < \sigma_{\text{sec}}^N \iff \sigma_{aa\text{I}} < \sigma_a \iff \sigma_a < \sigma_{\text{sec}}^N$
4. $\sigma_{\text{sec}}^B > \sigma_{\text{sec}}^U \iff \sigma_{aa\text{I}} < \sigma_{\text{sec}}^B \iff \sigma_{aa\text{I}} < \sigma_{\text{sec}}^B$
5. For secession in region 2 $\sigma_a < \sigma_{\text{sec}}^N$ and $\sigma_{aa\text{I}} < \sigma_{\text{sec}}^N$ and $\sigma_{aa\text{I}} < \sigma_{\text{sec}}^N$ always
6. $\sigma_{aa\text{II}} < \sigma_{\text{sec}}^N \iff \sigma_{\text{sec}}^B < \sigma_{\text{sec}}^U \iff \sigma_{aa\text{II}} < \sigma_{\text{sec}}^B$
7. $\min[\sigma_{\text{sec}}^N, \sigma_{\text{sec}}^U] = \sigma_{\text{sec}}^N \implies \sigma_a > \sigma > \max[\sigma_{\text{sec}}^N, \sigma_{\text{sec}}^U]$
8. $\sigma_a > \sigma_{\text{sec}}^U$ always since $2 > B(1 - \alpha)(1 - \beta)(\delta + 1)$ and $\sigma_a > \sigma_{\text{sec}}^N$ always since $2 > (1 - \beta)(1 + \delta)B$

**Proof.**

1. Simple algebra reveals that $\sigma_a < \sigma_{\text{sec}}^N \iff \sigma_{aa\text{II}} > \sigma_{\text{sec}}^N \iff$

\[
NB(1 - \beta)(1 + \delta) > 2(N - B) \tag{32}
\]

2. Simple algebra reveals that $\sigma_{aa\text{I}} < \sigma_{\text{sec}}^N \iff \sigma_{aa\text{I}} < \sigma_a \iff \sigma_a < \sigma_{\text{sec}}^N \iff$ condition (32) holds.
3. Simple algebra reveals that \( \sigma_{B_i}^S > \sigma_B^U \Leftrightarrow \sigma_{aa1} < \sigma_{B_i}^S \Leftrightarrow \sigma_{aa1} < \sigma_B^U \Leftrightarrow \)

\[
B(1-\beta)(1+\delta)(2B_i(1-\alpha)+N(1+\alpha)) > 2N - 4B_{-i} \tag{33}
\]

4. The parameter restriction to have to consider \( \sigma_{aa1} \) for secession in region 2 is \( 2B_2 > N \). It is easy to see that (32) always holds in this case. Hence by point 3 it follows that \( \sigma_a < \sigma_{N_i}^S \) always. Next \( \sigma_{aa1} < \sigma_N = \frac{1}{\delta} \Leftrightarrow (B_i - B_{-i}) < BB_i(\delta + 1) \) which is always true for \( B_i < B_{-i} \) hence it is always true for secession in region 2. Similarly the condition that \( \sigma_{aa1} < \sigma_{B_i}^S \) given by (32) always holds for secession in region 2 since \( B_1 = B_{-i} > B_2 \) and hence the right hand side is always negative for secession in region 2.

5. Simple algebra reveals that \( \sigma_{aa2} < \sigma_{N_i}^S \Leftrightarrow \sigma_{N_i}^S < \sigma_B^U \Leftrightarrow \sigma_{aa2} < \sigma_B^U \Leftrightarrow \)

\[
(1-\beta)(1+\delta)B((1+\alpha)N + 2B_i(1-\alpha)) < 2N - 4B_{-i} \tag{34}
\]

6. If \( \min(\sigma_{N_i}^S, \sigma_B^U) = \sigma_{N_i}^S \) then \( \sigma_a > \sigma > \max(\sigma_{N_i}^S, \sigma_B^U) \) since by point 1 \( \sigma_a > \sigma_B^U \).

We are now set to prove the proposition. In general, the results follow by combining the parameter restriction and the resulting ranking of the cutoffs with the insights derived from Lemma 11. Here are the details.

1. We look at the parameter area where \( 2 > H^U \). Given \( \sigma_B^U < \sigma_{B_i}^S \) it follows immediately that unified education is preferred for low \( \sigma \), namely \( \sigma_N = \sigma_B^U < \sigma < \sigma_{B_i}^S = \sigma_{N_i}^S \). Since \( \min(\sigma_B^U, \sigma_{N_i}^S) > \sigma_{B_i}^S \), \( \sigma < \sigma_{B_i}^S < \sigma < \min(\sigma_B^U, \sigma_{N_i}^S) \) the bourgeoisie pays its maximum willingness in the unified system and hence is not better off in the unified system than when there is no education while it benefits from education in secession and hence secession is preferred in this entire area. Now by point 4 of lemma 11 \( \sigma_{aa1} > \sigma_B^U \) so for \( 2B_i > N \) if \( \min(\sigma_B^U, \sigma_{N_i}^S) = \sigma_B^U \) secession of region 1 is also preferred for \( \sigma_{aa1} > \sigma > \sigma_B^U \). If \( \sigma_{aa1} < \sigma_{N_i}^S \) then by point 3 of lemma 11 we also have \( \sigma_a < \sigma_{N_i}^S \), so \( \sigma_a \) is not a relevant cutoff and secession is preferred for \( \sigma_{B_i}^S < \sigma < \sigma_{aa1} \). If on the
other hand \( \sigma_{aa1} > \gamma_{S_{N_1}} \) which by point 5 of lemma 11 cannot happen for secession of region 2 but by point 3 of lemma 11 can happen for secession of region 1 in which case \( \sigma_a > \gamma_{S_{B_1}} \) also, then secession is preferred for \( \gamma_{S_{B_1}} < \sigma < \sigma_a \). Similarly, if \( \min (\gamma_{U_{B_1}}, \gamma_{S_{N_1}}) = \gamma_{S_{N_1}} \) secession is preferred in the entire area \( \min (\gamma_{U_{B_1}}, \gamma_{S_{N_1}}) < \sigma < \max (\gamma_{U_{B_1}}, \gamma_{S_{N_1}}) \) and by point 7 of lemma 11 we also have \( \gamma_{S_{N_1}} < \sigma_a \) so secession is preferred for \( \gamma_{S_{B_1}} < \sigma < \sigma_a \).

Similarly, for \( 2B_i < N \) and if \( \min (\gamma_{U_{B_i}}, \gamma_{S_{B_i}}) = \gamma_{S_{B_i}} \) then by point 7 of lemma 11 we must have that \( \gamma_{S_{N_1}} < \sigma_a \) hence secession is preferred for \( \gamma_{S_{B_i}} < \sigma < \sigma_a \).

If \( 2B_i < N \) and \( \min (\gamma_{U_{B_i}}, \gamma_{S_{B_i}}) = \gamma_{U_{B_i}} \) then by point 6 of lemma 11 we also have \( \sigma_{aa2} < \gamma_{U_{B_i}} \) since we are in the parameter region where \( \gamma_{S_{N_1}} < \gamma_{U_{B_i}} \) and by point 2 since \( \sigma_{aa2} < \gamma_{U_{B_i}} < \gamma_{S_{N_1}} \) we also have \( \gamma_{S_{N_1}} < \sigma_a \) and hence secession is preferred for \( \gamma_{S_{B_i}} < \sigma < \sigma_a \).

2. We now look at the parameter area where \( H^{S_i} < 2 < H^U \). In this parameter constellation under the unified system the bourgeoisie always fully finances education. Since \( \gamma_{U_{B_i}} < \gamma_{S_{B_i}} \) by point 4 of lemma 11 the cutoff \( \sigma_{aa1} \) is never relevant. Combining this with point 3 of lemma 11 and the fact that \( \gamma_{U_{B_i}} < \gamma_{S_{B_i}} \) we also have \( \gamma_{S_{N_1}} > \sigma_a \), hence \( \sigma_a \) is not a relevant cutoff and the unified system is always preferred for \( 2B_i > N \). Now let \( 2B_i < N \). Then by point 6 of lemma 11 \( \sigma_{aa2} > \gamma_{U_{B_i}} \). Could it be the case that \( \sigma_{aa2} < \gamma_{S_{N_1}} \) and hence by point 2 we also had \( \gamma_{S_{N_1}} < \sigma_a \) and hence secession would be preferred for \( \sigma_{aa2} < \sigma < \sigma_a \)? Notice that the cutoff (29) gives us the point of intersection of

\[
\Pi^U_B (I^U_B = \frac{M}{B}) = -\frac{M}{B} (1 - (1 - \beta) \delta B) + M (1 - \beta) \delta \sigma
\]

and

\[
\Pi^S_B (I^S_{B_i} = 0) = \delta (1 - \beta) \frac{M}{2} + (1 - \beta) \frac{M}{2} \sigma.
\]

It is easy to check that the intercept of (36) is higher and the slope half of the one of (35).
The cutoffs (30) and (31) gives the point of intersection of $\Pi_B^U(I_B^U = \frac{M}{\delta \tau})$
with
\[
\Pi_S^B \left( I_B^S = \frac{\Pi_B^S}{\Pi_B^U} \right) = -\frac{M}{4B_i} (2 - (1 - \beta)(2B_i\delta - N)) + (1 - \beta)(2B_i + N)\delta M \sigma
\]
for $2B_i > N$ and $2B_i < N$ respectively. The conditions $2B_i > N$ and $2B_i < N$ also determine the slope of (37) which is lower than the one of (35) for $2B_i > N$ and higher for $2B_i < N$. The intercept of (37) is always higher than the intercept of (35) for secession of region 2 while the direction is ambiguous for secession of region 1.

Comparing $\Pi_B^S \left( I_B^S = \frac{\Pi_B^S}{\Pi_B^U} \right)$ defined by (37) with $\Pi_B^U \left( I_B^U = \frac{M}{\delta \tau} \right)$ defined by (35) we learn that the outcome $\sigma_{aa_2} < \sigma < \sigma_a$ combined with the fact that $\sigma_B^U < \sigma_B^S$ might only possible for secession in region 1 and only for the special case where the intercept of (37) is lower than of (35) and $2B_i < N$ so that the slope of (37) is higher than of (35). But given the parameter constellations under consideration, in particular the second half of the condition to be in this case namely $2 < (1 - \beta)(1 + \delta)B(1 + \alpha)$ implies that for secession in region 1 we can never be in the area where $\sigma_{aa_2}$ is the cutoff to consider, because we are always in the area where $\sigma_{aa_1}$ is the relevant cutoff to be considered. In this area $2B_i > N \iff 2(2B_i + B) > 2$ and $(1 - \beta)(1 + \delta)B(1 + \alpha) < 2(2B_i + B)$ for $B_i = B_1$ always. Notice that the left hand side of the condition is highest for $\alpha = \delta = 1$ and the condition becomes $(1 - \beta)4B < 4B_i + 2B$ and it is definitely true for $B_i = B_1$ because $4B_1 > 2B$. This means that the only possible outcome is that unified schooling is always preferred in this area.

3. Finally we study the parameter area $H^S_i > 2$. In this case we always have $\sigma_B^U < \sigma_N^S < \sigma_{N_i}^S$, hence the bourgeoisie always fully finances education unified schooling. We also have $\sigma_{B_i}^S < \sigma_N^S$, so that the bourgeoisie is always willing to go for co-payment for $\sigma_N^S < \sigma < \sigma_{N_i}^S$ and will get education for free for $\sigma > \sigma_{N_i}^S$.

In this parameter area $\sigma_{B_i}^S > \sigma_B^U$ always. Then by point 4 of lemma 11 $\sigma_{aa_1}$ is never a relevant cutoff and neither is $\sigma_a$ by point 3, so unified schooling is always preferred. Now by point 6 of lemma 11 $\sigma_{aa_2} > \sigma_B^U$. Can it be the case that secession is preferred for $\sigma_{aa_2} < \sigma < \sigma_a$? As argued in the proof of point 2 this latter outcome is only possible for
secession in region 1 and only for the special case where the intercept of (37) is lower than of (35) and $2B_1 < N$. However, given the parameter constellations under consideration, in particular the second condition to be in this case implies that $2B_i > N \iff 2(2B_i + B) > 2$ and we can prove that $(1 - \beta)\alpha (1 + \delta) \leq 2(2B_i + B)$ always. Notice that the left hand side is biggest for $\alpha = \delta = 1$ but in this case the condition becomes $(1 - \beta)2B_i < 2B_i + B$ which is always true. Hence we never can be in the region where $\sigma_{aa2}$ is a relevant cutoff, so unified schooling is always preferred.

**Proof of Lemma 9**

1. Simple algebra gives that
   \[ \sigma_{M_i}^2 < \sigma_a \iff (1 - \beta)(1 + \delta) B (B_i \alpha (1 - \beta) + \beta (\frac{N}{2} + B_i)) < 2\beta(\frac{N}{2} + B_i). \]
   Isolating $\alpha$ gives the condition in the lemma.

2. Simple algebra that $\sigma_{M_i}^2 > \sigma_{aa1}$ \iff
   \[ (1 - \beta)(1 + \delta) B ((2B_1 - N)B_i \alpha (1 - \beta) + 2B_i \beta (\frac{N}{2} + B_i)) > 2(B_1 - B_{-1}) \beta (\frac{N}{2} + B_i) \]
   (38)
   If the condition holds at $\alpha = 0$ then it always holds. At $\alpha = 0$ the condition becomes
   \[ (1 - \beta)(1 + \delta) BB_i > (B_i - B_{-1}) \]
   (39)
   which is always true for secession of region 2. Hence for $2B_2 > N$ we always have $\sigma_{aa1} < \sigma_{M_2}^2$

3. If (39) holds for $i = 1$ then $\sigma_{aa1} < \sigma_{M_1}^2$.

4. Assume (39) is violated, then $\sigma_{aa1} > \sigma_{M_1}^2$. Condition (38) is also violated for sufficiently low $\alpha$, in particular for $\alpha < \frac{\beta(\frac{N+2B_1}{2})(1+\delta)BB_1}{(1-\beta)\beta(1+\delta)B_1}$.

5. Simple algebra gives $\sigma_{M_i}^2 < \sigma_a$
   \[ \iff \beta(1 + B) > \left( \beta \left( \frac{(1+B_1 - B_2)}{2} (1 + \delta) \right) + \alpha B_1 (1 - \beta)(1 + \delta) \right) (1 - \beta) B \]
   the result follows by isolating $\alpha$. Notice that the condition is always true for $\alpha = 0$
6. \( \sigma_{M_1}^U > \sigma_{w_1} \iff \)

\[
\begin{align*}
\beta \left( (1 + B_1 - \delta B_2) (2B_1 - N) + (BB_1 (\delta + 1)) (1 - \frac{N}{2}) \right) \\
+ B_1 \alpha (1 - \beta)(1 + \delta) (2B_1 - N) \\
> 2(B_i - B_{-i}) \beta(1 - \frac{N}{2})
\end{align*}
\]

This is always true for \( i = 2 \).

7. \( \alpha_{M_1^i} > \alpha_{M_1} \) for secession of region \( i \) follows from simply algebra.

References


